# Paper MCNSBNDs-R2

*by* Khadija Noreen

**Submission date:** 12-May-2023 10:56AM (UTC+0500)
**Submission ID:** 2091096714
**File name:** Paper_MCNSBNDs_R2.docx (55.43K)
**Word count:** 3941
**Character count:** 16945

# Some New Constructions of Minimal Efficient Circular Nearly Strongly Balanced Neighbor Designs


## Abstract

Neighbor designs are popular to control neighbor effects. Among neighbor designs, strongly balanced neighbor designs are important to estimate treatment effects and neighbor effects independently. Minimal circular strongly balanced neighbor designs (MCSBNDs) can be obtained only for odd *v* (number of treatments). For *v* even, minimal circular nearly strongly balanced neighbor designs are used which satisfied all conditions of MCSBNDs except that the treatment labeled as (*v*-1) does not appear as its own neighbor. These designs can be converted directly in some other useful classes of neighbor designs. These designs are efficient to minimize the bias due to the neighbor effects.



***Key Words*:** Rule I; Rule II; Neighbor effects; CNSBNDs; CSBNDs; CBNDs.

**Mathematics Subject Classification (2010): 05B05; 62K10; 62K05.**


## 1. Introduction

If response of a treatment (treatment effect) is affected by the treatment(s) applied in neighboring units then such neighbor effects become major source of bias, in estimating the treatment effects. This bias can be minimized with the use of neighbor balanced designs, see Azais (1987), Azais *et al*. (1993), Kunert (2000) and Tomer *et al*. (2005).

- A circular design in which every treatment appears once as neighbors with all others (excluding it) is called a minimal circular balanced neighbor design (MCBND). If it also appears as its own neighbor then it is called MCSBND. MCBNDs and MCSBNDs can only be obtained for *v* odd.
- A circular design is called minimal circular nearly SBND (MCNSBND) if each treatment appears once as neighbor with other (*v*-2) treatments exactly once and (i) appear twice with only one treatment, labeled as (*v*-1), (ii) appear once as neighbor with itself except the treatment labeled as (*v*-1) which does not appear as its own neighbor. For *v* even, MCNSBNDs should be used as the best alternate of the MCSBNDs.

Rees (1967) introduced MCBNDs in serology for *v* odd. Azais *et al*. (1993) constructed some CBNDs using border plots. Jaggi *et al*. (2006) constructed some partially BNDs. Nutan (2007), Kedia & Misra (2008), Ahmed *et al*. (2009) constructed generalized neighbor designs (GNDs). Iqbal *et al*. (2009) constructed some classes of CBNDs using cyclic shifts. Akhtar *et al*. (2010) constructed CBNDs for k = 5. Meitei (2010) constructed new series of (i) CBNDs and (ii) one-sided CBNDs. Ahmed and Akhtar (2011) constructed CBNDs for k = 6. Shehzad *et al*. (2011) constructed some CBNDs. Jaggi *et al*. (2018) described some methods to construct CBNDs and circular partially BNDs. Singh (2019) developed new series of universally optimal one-sided CBNDs. Meitei (2020) presented a new series of universally optimal one-sided CBND for k = 5. Salam *et al*. (2022) introduced MCNSBNDs for (i) $v = 8i+4$, k = 4, (ii) $v = 10i+6$, k = 5, (iii) $v =$

$12i+8$, $k = 6$, (iv) $v = 2ik_1+2$, $k_1 = 4j$, $k_2 = 3$, (v) $v = 2ik_1+4$, $k_1 = 4j$, $k_2 = 4$, (vi) $v = 2ik_1+2$, $k_1 > 3$ and $k_2 = 3$, (vii) $v = 2ik_1+4$, $k_1 > 4$ and $k_2 = 4$, and (viii) $v = 2ik_1+6$, $k_1 > 5$ and $k_2 = 5$.

In this article, (i) a generator is developed which produces the MCNSBNDs in equal as well as in unequal block sizes, with smallest of size at least three, (ii) some generators are developed which produce the MCNSBNDs which can directly be converted into MCSBNDs and MCBNDs, in blocks of equal as well as in unequal sizes, where smallest block size should be at least six.

## 2. Method of Construction

Iqbal (1991) introduced method of cyclic shifts (Rule I & II) to construct experimental designs of several types. Its construction procedures are described here for MCNSBNDs, MCSBNDs and MCBNDs.

### 2.1 Rule II to obtain MCNSBNDs

Let $S_j = [q_{j1}, q_{j2}, \ldots, q_{j(k-1)}]$ and $S_i = [q_{i1}, q_{i2}, \ldots, q_{i(k-2)}]t$ be the sets, where $0 \le q_{ji} \le v-2$. If each of $0, 1, 2, \ldots, v-2$ appears once in $S^*$, where $S^* = [q_{j1}, q_{j2}, \ldots, q_{j(k-1)}, (q_{j1}+q_{j2}+ \ldots +q_{j(k-1)}) \bmod (v-1), (v-1)-q_{j1}, (v-1)-q_{j2}, \ldots, (v-1)-q_{j(k-1)}, (v-1)-[(q_{j1}+q_{j2}+ \ldots +q_{j(k-1)}) \bmod (v-1)], q_{i1}, q_{i2}, \ldots, q_{i(k-2)}, (v-1)-q_{i1}, (v-1)-q_{i2}, \ldots, (v-1)-q_{i(k-2)}]$ then it is MCNSBND. In Rule II, at least one set will contain k-2 elements which will be expressed as $[q_1, q_2, \ldots, q_{(k-2)}]t$. Here 't' is just to specify the set containing k-2 elements.

**Example 2.1:** Following MCNSBND is constructed from $S_1 = [4,5,6,7,9,10,11]$, $S_2 = [0,1,3,8,13]t$ for $v = 26$, $k_1 = 8$ & $k_2 = 7$.

Take $(v-1)$ blocks for every set of shifts to get the complete design through Rule II. Consider $0, 1, \ldots, v-2$ as 1st unit of each block. Obtain 2nd unit elements by adding 4 (mod $(v-1)$) to 1st unit elements, where 4 is the 1st element of $S_1$. Obtain 3rd unit elements by adding 5 (mod 25) to 2nd unit elements, where 5 is the 2nd element of $S_1$. Similarly add 6, 7, 9, 10 and 11, see Table 1.

**Table 1:** Blocks obtained from $S_1 = [4,5,6,7,9,10,11]$

| **Blocks** | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 0 | 1 | 2 |
| 22 | 23 | 24 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 0 | 1 | 2 | 3 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |

| **Blocks** | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **14** | **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** | **24** | **25** |
| 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 0 | 1 | 2 | 3 |
| 22 | 23 | 24 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| 19 | 20 | 21 | 22 | 23 | 24 | 0 | 1 | 2 | 3 | 4 | 5 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 0 | 1 |

For $S_2$, take ($v$-1) more blocks. Obtain the blocks as are taken from $S_1$ except one extra row containing ($v$-1) in its each cell, see Table 2.

**Table 2:** Blocks obtained from $S_2$ = [0,1,3,8,13]t

| **Blocks** | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 |

| **Blocks** | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **14** | **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** | **24** | **25** |
| 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 0 |
| 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 0 | 1 | 2 | 3 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 |

Table 1 & 2 jointly present MCNSBND for $v = 26$, $k_1 = 8$ and $k_2 = 7$, using 50 blocks.

### 2.2 Rule I to obtain MCSBNDs and MCBNDs

Let $S_j = [q_{j1}, q_{j2},\ldots, q_{j(k-1)}]$ be *i* sets, where *j* = 1, 2,…, *i* and *u* = 1, 2,…, k-1. If S* contains each of:

- 1, 2, …, *v*-1 once and $1 \le q_{ju} \le v-1$ then design will be MCBND.
- 0, 1, 2, …, *v*-1 once and $0 \le q_{ju} \le v-1$ then design will be MCSBND.

Here S* contains:

(i) All elements of $S_j$.

(ii) Sum of all elements (mod *v*) in each of $S_j$.

(iii) Complements of all elements in (i) and (ii). In Rule I, the complement of '*a*' is '*v-a*'.

**Example 2.2.** Following MCBND is constructed from $S_1$ = [4,5,6,7,9,10,11] and $S_2$ = [1,3,8] for *v* = 25, $k_1 = 8$ & $k_2 = 4$ using Rule I.

Take *v* blocks for every set of shifts to get the complete design through Rule I. Consider 0, 1, …, *v*-1 as 1st unit of each block. Obtain 2nd unit elements by adding 4 (mod 25) to 1st unit elements. Similarly add 5, 6, 7, 9, 10 and 11, see Table 3.

**Table 3:** Blocks obtained from $S_1$ = [4,5,6,7,9,10,11]

| Blocks | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 0 | 1 | 2 |
| 22 | 23 | 24 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 0 | 1 | 2 | 3 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |

| Blocks | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **14** | **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** | **24** | **25** |
| 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 0 | 1 | 2 | 3 |
| 22 | 23 | 24 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| 19 | 20 | 21 | 22 | 23 | 24 | 0 | 1 | 2 | 3 | 4 | 5 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 0 | 1 |

Take more 25 blocks for $S_2$ and obtain blocks as taken from $S_1$, see Table 4.

**Table 4:** Blocks obtained from $S_2 = [1,3,8]$

| Blocks | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** | **21** | **22** | **23** | **24** | **25** |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 0 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 0 | 1 | 2 | 3 |
| 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |

Table 3 and Table 4 jointly present MCBND for $v = 25$, $k_1 = 8$ & $k_2 = 4$.

### 2.3 Efficiency of Separability

Divecha and Gondaliya (2014) derived following expression for the efficiency of Separability (Es) which is also applicable for MCNSBNDs.

$$ES = \left[\frac{v\sqrt{v-1}-1}{v\sqrt{v-1}}\right] \times 100\%$$ , where $v$ is the number of treatments.

MCNSBND possessing Es at least 70% is considered efficient to reduce bias due to neighbor effects.

## 3. Construction of MCNSBNDs and Their Conversion into MCSBNDs and MCBNDs

Here, the procedure to obtain the sets of shifts from generators developed in Section 4 is described. Non-zero elements of generator 'A' are divided into the required number of groups such that sum of elements in each group is divisible by ($v$-1). Sets to generate MCNSBNDs are obtained by deleting one value (any) from each group containing non-zero values. The group containing '0' will remain unchanged.

MCNSBNDs which can directly be converted into MCSBNDs and MCBNDs are constructed for following cases. Here $i$ (integer) > 0 and A will be selected from Section 4.

- **For equal block sizes**

  (i) $v = 2(i+1)k-4$, $k > 5$. Divide the non-zero values of selected A into $i$ groups each of k elements. Last will contain the remaining k-2 values.

- **For two different block sizes**

(i) $v = 2ik_1+2k_2-4$, $k_1 > k_2 > 5$. Divide the non-zero values of selected A into $i$ groups each of $k_1$ elements. Last will contain the remaining $k_2$-2 values.

(ii) $v = 2ik_1+4k_2-4$, $k_1 > k_2 > 5$. Divide the non-zero values of selected A into $i$ groups each of $k_1$ elements and one group of $k_2$ elements. Last will contain the remaining $k_2$-2 values.

- **For three different block sizes**

(i) $v = 2ik_1+2k_2+2k_3-4$, $k_1 > k_2 > k_3 > 5$. Divide the non-zero values of selected A into $i$ groups each of $k_1$ elements and one group of $k_2$ elements. Last will contain the remaining $k_3$-2 values.

(ii) $v = 2ik_1+4k_2+2k_3-4$, $k_1 > k_2 > k_3 > 5$. Divide the non-zero values of selected A into $i$ groups each of $k_1$ elements and two groups of $k_2$ elements. Last will contain the remaining $k_3$-2 values.

(iii) $v = 2ik_1+2k_2+4k_3-4$, $k_1 > k_2 > k_3 > 5$. Divide the non-zero values of selected A into $i$ groups each of $k_1$ elements, one group of $k_2$ elements and one of $k_3$ elements. Last will contain the remaining $k_3$-2 values.

(iv) $v = 2ik_1+4k_2+4k_3-4$, $k_1 > k_2 > k_3 > 5$. Divide the non-zero values of selected A into $i$ groups each of $k_1$ elements, two groups of $k_2$ elements and one of $k_3$ elements. Last will contain the remaining $k_3$-2 values.

## 4. Generator to generate MCNSBNDs which cannot be converted directly into MCSBNDs and MCBNDs

**Generator 4.1:** A = [0, 1, 2, …, *m*] produces sets of shifts to obtain MCNSBNDs for every block sizes with smallest of size at least three, where $m = (v-2)/2$. The designs obtained from generator 4.1 cannot be converted directly into MCSBNDs and MCBNDs.

**Example 4.1.1.** $S_1$ = [3,5,6,7,8] and $S_2$ = [0,1,2,4]t produce MCNSBND for $v$ = 20 & k = 6 with Es = 0.7415.

**Example 4.1.2.** $S_1$ = [2,3,5,6,7,9] and $S_2$ = [0,1,4,8]t produce MCNSBND for $v$ = 22, $k_1$ = 7 & $k_2$ = 6 with Es = 0.7837.

## 5. Generators to generate MCNSBNDs which can directly be converted into MCSBNDs and MCBNDs

According to the value of *m*, generators 'A' are developed here using the logic behind Rule II, where $m = (v-2)/2$. These generators produce the sets of shifts to obtain MCNSBNDs which can directly be converted into MCSBNDs and MCBNDs.

**Generator 5.1:** A = [0, 1, 2, …, $j$-1, $j$+1, $j$+2, …, $m$, $v$-$j$] produces sets of shifts to obtain MCNSBNDs for $m \equiv 0 (\text{mod } 8), j = m/8, j \geq 1$.

**Example 5.1.** $S_1$ = [5,6,7,8,10,18,23,24], $S_2$ = [4,9,11,12,13,14,16,17], $S_3$ = [0,1,15,19,20,21,22]t obtained from A = [0,1,2,**46**,4,…,24] produce MCNSBND for $v$ = 50 & k = 9 with Es = 0.8574.

**Generator 5.2:** A = [0, 1, 2, …, 3$j$, 3$j$+2, 3$j$+3, …, $m$-1, $m$+1, $v$-(3$j$+1)] produces sets of shifts to obtain MCNSBNDs for $m \equiv 1 (\text{mod } 8), j = (m-1)/8, j \geq 1$.

**Example 5.2.** $S_1$ = [1,2,3,4,5,6,7,8,11,14], $S_2$ = [9,12,13,15,17,18,19,24], $S_3$ = [0,16,20,21,22, 23]t obtained from A = [0,1,2,…,9,**41**,11,12,…,24,**26** ] produce MCNSBND for $v$ = 52, $k_1$ = 11, $k_2$ = 9 & $k_3$ = 8 with Es = 0.8439.

**Generator 5.3:** A = [0, 1, 2, …, 5$j$+1, 5$j$+3, 5$j$+4, …, $m$-1, $m$+1, $v$-(5$j$+2)] produces sets of shifts to obtain MCNSBNDs for $m \equiv 2 (\text{mod } 8), j = (m-2)/8, j \geq 1$.

**Example 5.3.** $S_1$ = [2,3,4,5,6,7], $S_2$ = [1,8,9,11,13,15] and $S_3$ = [0,14,16,19,25]t obtained from A = [0,1,2,…,11,**25**,13,14,15,16,17,**19**] produce MCNSBND for $v$ = 38 & k = 7 with Es = 0.8513.

**Generator 5.4:** A = [0, 1, 2, …, $m$-1-$j$, $m$+1-$j$, $m$+2-$j$, …, $m$, $v$-($m$-$j$)] produces sets of shifts to obtain MCNSBNDs for $m \equiv 3 (\text{mod } 8), j = (m-3)/8, j \geq 0$.

**Example 5.4.** $S_1$ = [1,3,4,5,6,7,9], $S_2$ = [0,2,8,13]t obtained from A = [0,1,2,…,9,**13**,11] produce MCNSBND for $v$ = 24, $k_1$ = 8 & $k_2$ = 6 with Es = 0.7680.

**Generator 5.5:** A = [0, 1, 2, …, $j$, $j$+2, $j$+3, …, $m$-1, $m$+1, $v$-($j$+1)] produces sets of shifts to obtain MCNSBNDs for $m \equiv 4 (\text{mod } 8), j = (m-4)/8, j \geq 0$.

**Example 5.5.** $S_1$ = [1,3,4,5,6,7,11], $S_2$ = [0,8,9,10,23]t obtained from A = [0,1,**23**,3,4,5,6,7,8,9, 10,11,**13**] produce MCNSBND for $v$ = 26, $k_1$ = 8 & $k_2$ = 7 with Es = 0.7581.

**Generator 5.6:** A = [0, 1, 2, …, 3$j$+1, 3$j$+3, 3$j$+4, …, $m$, $v$-(3$j$+2)] produces sets of shifts to obtain MCNSBNDs for $m \equiv 5 (\text{mod } 8), j = (m-5)/8, j \geq 0$.

**Example 5.6.** $S_1$ = [3,4,6,10,11,12,13], $S_2$ = [0,1,2,7,8,9]t obtained from A = [0,1,2,3,4,**22**,6,7, …,13] produce MCNSBND for $v$ = 28 & k = 8 with Es = 0.8318.

**Generator 5.7:** A = [0, 1, 2, …, 5$j$+3, 5$j$+5, 5$j$+6, $m$, $v$-(5$j$+4)] produces sets of shifts to obtain MCNSBNDs for $m \equiv 6 (\text{mod } 8), j = (m-6)/8, j \geq 0$.

**Example 5.7.** $S_1$ = [1,2,3,4,6,7,8,13], $S_2$ = [0,5,10,11,12,20]t obtained from A = [0,1,2,…,8,**20**, 10,11,…,14] produce MCNSBND for $v$ = 30, $k_1$ = 9 & $k_2$ = 8 with Es = 0.7963.

**Generator 5.8:** A = [0, 1, 2, …, $m$-1-$j$, $m$+1-$j$, $m$+2-$j$, …, $m$-1, $m$+1, $v$-($m$-$j$)] produces sets of shifts to obtain MCNSBNDs for $m \equiv 7 (\text{mod } 8), j = (m-7)/8, j \geq 1$.

**Example 5.8.1.** $S_1 = [1,2,3,4,5]$, $S_2 = [7,8,9,10,11]$, $S_3 = [0,6,12,13]$ obtained from A = [0,1,2,…, 13,**17,16**] produce MCNSBND for $v = 32$ & k = 6 with Es = 0.8404.

Catalogues are also presented in Appendix A, B, and C.

## 6. Conversion of proposed MCNSBNDs into MCSBNDs and MCBNDs

**Conversion 6.1:** Considering the Rule II as Rule I, MCNSBNDs constructed in Section 5 for $v = 2ik-4$, $i > 1$, $k > 5$ can be converted into:

(i) MCSBNDs for $v = 2ik-5$, $k_1 = k$, $k_2 = k-2$. For it, delete '0' from the set of shifts.

(ii) MCBNDs for $v = 2ik-5$, $k_1 = k$, $k_2 = k-3$. For it, delete '0' and one more value (any) from the set containing '0'.

**Example 6.1.** MCNSBND constructed in example 5.2.1 for $v = 20$ and k = 6 through $S_1 = [1,2,3,7,10]$, $S_2 = [0,5,6,8]$t will be converted into:

(a) MCSBND for $v = 19$, $k_1 = 6$ & $k_2 = 4$, with $S_1 = [1,2,3,7,10]$, $S_2 = [5,6,8]$.

(b) MCSBND for $v = 19$, $k_1 = 6$ & $k_2 = 3$, with $S_1 = [1,2,3,7,10]$, $S_2 = [5,6]$.

## 7. Remarks

Salam *et al*. (2022) introduced MCNSBNDs for some specific cases of $3 \le k_2 \le 5$. In this article, generator is developed for MCNSBNDs in equal as well as in unequal block sizes, with smallest block size at least three. Some generators are developed MCNSBNDs for $v$ even with smallest block size at least six and these designs can directly be converted into MCSBNDs and MCBNDs for $v$ odd.

MCSBNDs require at least $v(v-1)$ experimental units for $v$ even while our proposed MCNSBNDs require $v(v-1)/2$ units. Our proposed designs lose $\frac{100}{v(v)}$% neighbor balance and save at least 50 % experimental material. Our designs possess Es at least 70% therefore, these are efficient to minimize bias due to neighbor effects.

**Acknowledgement:** Authors are thankful to anonymous Reviewer for the valuable corrections.

## References

Ahmed, R., Akhtar, M. and Tahir, M. H. (2009). Economical generalized neighbor designs of use in Serology. *Computational Statistics and Data Analysis*, 53, 4584-4589.

Ahmed, R. and Akhtar, M. (2011). Designs balanced for neighbor effects in circular blocks of size six. *Journal of Statistical Planning and Inference*, 141, 687-691.

Akhtar, M., Ahmed, R. and Yasmin, F. (2010). A catalogue of nearest neighbor balanced- designs in circular blocks of size five. *Pakistan Journal of Statistics*, 26(2), 397- 405.

Azais, J. M. (1987). Design of experiments for studying intergenotypic competition. *Journal of the Royal Statistical Society: Series B*, 49, 334-345.

Azais, J. M., Bailey, R. A. and Monod, H. (1993). A catalogue of efficient neighbor designs with border plots. *Biometrics*, 49(4), 1252- 61.

Divecha, J. and Gondaliya, J. (2014). Construction of minimal balanced crossover designs having good efficiency of separability. *Electronic Journal of Statistics*, 8, 2923-2936.

Iqbal, I. (1991) Construction of experimental design using cyclic shifts, Ph.D. Thesis, University of Kent at Canterbury, U.K.

Iqbal, I., Tahir, M. H. and Ghazali, S. S. A. (2009). Circular neighbor-balanced designs using cyclic shifts. *Science in China Series A: Mathematics*, 52 (10), 2243-2256.

Jaggi, S., Gupta V. K. and Ashraf J. (2006). On block designs partially balanced for neighbouring competition effects. *Journal of Indian Statistical Association*, 44(1), 27-41.

Jaggi, S., Pateria, D. K., Varghese, C., Varghese, Eldho. and Bhowmik, Arpan. (2018). A note on circular neighbour balanced designs. *Communications in Statistics-Simulation and Computation*, 47, 2896-2905.

Kedia, R. G. and Misra, B. L. (2008). On construction of generalized neighbor design of use in serology. *Statistics and Probability Letters*, 18, 254-256.

Kunert J (2000). Randomization of neighbour balanced designs, *Biometrical Journal*, 42(1), 111-118.

Meitei, K. K. S. (2010). Circular neighbour-balanced designs and one-sided circular neighbour-balanced designs. *Sankhyā: The Indian Journal of Statistics, Series B*, 72, 175-188.

Meitei, K. K. S. (2020). Universally Optimal One-sided Circular Neighbour-balanced Designs with k = 5. *Calcutta Statistical Association Bulletin*, 72(2), 84-88.

Nutan, S. M. (2007). Families of proper generalized neighbor designs. *Journal of Statistical Planning and Inference*, 137, 1681–1686.

Rees, D.H. (1967). Some designs of use in serology. *Biometrics*, 23, 779-791.

Salam, A., Ahmed, R., Daniyal, M. and Berihan, R. A. (2022). Minimal circular nearly strongly-balanced neighbor designs when left and right neighbor effects are equal. *Journal of Statistics Applications and Probability*, 11(3), 883-891.

Shehzad, F., Zafaryab, M. and Ahmed, R. (2011). Minimal neighbor designs in circular blocks of unequal sizes. *Journal of Statistical Planning and Inference*, 141, 3681-3685.

Singh, K. P. (2019). A neighbor balanced design and its optimal certainty contribution. *Bulletin of Pure and Applied Sciences Section - E - Mathematics & Statistics*, 38E(2), 636-640.

Tomar, J. S., Jaggi, S. and Varghese, C. (2005). On totally balanced block designs for competition effects. *Journal of Applied Statistics*, 32(1), 87-97.

# Appendix A

**Catalogue of MCNSBNDs for $6 \leq k \leq 8$ and $v \leq 60$**

| *v* | k | Sets of Shifts | Es |
|---|---|---|---|
| 20 | 6 | [5,3,15,7,3]+[1,8,10,0]t | 0.7415 |
| 32 | 6 | [1,2,3,4,5]+[7,8,9,10,11]+[13,12,6,0]t | 0.8404 |
| 44 | 6 | [21,2,3,4,7]+[5,9,10,35,11]+[1,15,19,13,20]+[14,17,12,0]t | 0.8363 |
| 56 | 6 | [12,4,25,1,10]+[14,26,21,16,11]+[17,20,15,27,8]+[9,7,19,13,5]+[6,31,18,0]t | 0.8653 |
| 24 | 7 | [6,2,3,4,13,11]+[8,9,5,1,0]t | 0.8215 |
| 38 | 7 | [1,2,3,4,5,6]+[19,17,10,25,11,14]+[7,13,9,8,0]t | 0.8513 |
| 52 | 7 | [24,2,3,4,5,6]+[26,9,23,11,12,13]+[22,16,17,19,18,20]+[15,21,1,14,0]t | 0.8510 |
| 28 | 8 | [1,2,3,4,22,7,6]+[8,10,11,12,13,0]t | 0.8318 |
| 44 | 8 | [22,2,3,4,5,6,10]+[15,7,11,1,13,14,9]+[17,18,19,20,12,0]t | 0.8474 |
| 60 | 8 | [1,25,3,4,5,6,7]+[9,29,48,27,13,15,20]+[24,26,21,14,19,22,23]+ [17,18,12,10,2,0]t | 0.8699 |

# Appendix B

**Catalogue of MCNSBNDs in two different block sizes**

| *v* | $k_1$ | $k_2$ | Sets of Shifts | Es |
|---|---|---|---|---|
| 22 | 7 | 6 | [1,9,3,6,5,4]+[8,2,11,0]t | 0.7837 |
| 36 | 7 | 6 | [18,2,3,4,5,10]+[1,9,6,15,12,13]+[11,16,8,0]t | 0.8027 |
| 50 | 7 | 6 | [24,2,46,4,5,6]+[23,9,20,7,12,13]+[1,16,15,10,17,18]+[22,8,19,0]t | 0.8140 |
| 24 | 8 | 6 | [11,2,3,4,5,6,7]+[9,13,1,0]t | 0.7680 |
| 40 | 8 | 6 | [1,2,3,4,5,9,7]+[19.18,10,12,13,15,14]+[22,11,6,0]t | 0.8996 |
| 56 | 8 | 6 | [20,2,3,4,5,7,6]+[9,10,11,12,13,14,25]+[17,26,19,1,21,27,23]+ [15,18,22,0]t | 0.8418 |
| 26 | 8 | 7 | [13,23,4,3,11,6,7]+[9,10,5,1,0]t | 0.7581 |
| 42 | 8 | 7 | [1,2,38,5,4,6,7]+[15,9,16,21,13,18,14]+[12,10,11,8,0]t | 0.8320 |
| 58 | 8 | 7 | [29,2,3,5,6,53,7]+[8,10,26,12,13,14,15]+ [17,18,19,27,21,22,23]+[25,11,20,1,0]t | 0.8650 |

# Appendix C

**Catalogue of MCNSBNDs in three different block sizes**

| *v* | $k_1$ | $k_2$ | $k_3$ | Sets of shifts | Es |
|---|---|---|---|---|---|
| 38 | 8 | 7 | 6 | [1,2,3,4,5,6,7]+[17,10,25,11,19,14]+[8,16,13,0]t | 0.8514 |
| 54 | 8 | 7 | 6 | [18,2,3,4,5,6,7]+[9,10,11,12,13,20,15]+ [36,24,19,14,22,21]+[1,25,27,0]t | 0.8722 |

| | | | | | |
|---|---|---|---|---|---|
| 40 | 9 | 7 | 6 | [19,2,18,4,5,6,8,7]+[10,11,12,13,14,15]+[22,16,1,0]t | 0.8545 |
| 58 | 9 | 7 | 6 | [1,22,3,53,6,5,7,8]+[1,29,27,25,14,15,16,17]+ [`9,12,21,2,24,23]+[26,20,11,0]t | 0.8453 |
| 42 | 9 | 8 | 6 | [1,2,38,5,4,6,7,8]+[18,9,21,13,14,16,15]+[10,19,12,0]t | 0.8575 |
| 60 | 9 | 8 | 6 | [25,16,3,22,5,23,7,8]+[10,11,12,13,14,21,2,17]+ [28,20,15,29,27,24,1]+[6,19,4,30]t | 0.8779 |
| 44 | 9 | 8 | 7 | [7,2,3,17,4,6,1,5]+[19,10,11,21,14,16,20]+[3,12,13,15,0]t | 0.8877 |

Paper MCNSBNDs-R2

ORIGINALITY REPORT

| 18% | 12% | 14% | 1% |
|---|---|---|---|
| SIMILARITY INDEX | INTERNET SOURCES | PUBLICATIONS | STUDENT PAPERS |

PRIMARY SOURCES

| | | |
|---|---|---|
| 1 | Jamshaidul Hassan, Khadija Noreen, H. M. Kashif Rasheed, Mahmood ul Hassan, Rashid Ahmed. "Construction of circular quasi rees neighbor designs which can be converted into minimal circular balanced and strongly balanced neighbor designs", Communications in Statistics - Theory and Methods, 2021<br>Publication | 3% |
| 2 | www.naturalspublishing.com<br>Internet Source | 2% |
| 3 | www.ijicc.net<br>Internet Source | 2% |
| 4 | journalskuwait.org<br>Internet Source | 1% |
| 5 | Rida Jabeen, Abid Khan, H. M. Kashif Rasheed, Mahmood Ul Hassan, Rashid Ahmed. "General construction of efficient circular partially strongly-balanced repeated measurements designs", Communications in Statistics - Simulation and Computation, 2022<br>Publication | 1% |

| | | |
|---|---|---|
| 6 | Khadija Noreen, Muhammad Sajid Rashid, Farrukh Shehzad, Mahmood Ul Hassan, Zahra Noreen, Talha Omer, Rashid Ahmed. "Algorithms to obtain generalized neighbor designs in minimal circular blocks", Communications in Statistics - Simulation and Computation, 2022<br>Publication | 1% |
| 7 | ijicc.net<br>Internet Source | 1% |
| 8 | Rida Jabeen, Zahid Bashir, H. M. Kashif Rasheed, Jigneshkumar Gondaliya, Rashid Ahmed. "Minimal circular generalized strongly balanced and their conversion into circular balanced and strongly balanced repeated measurements designs", Communications in Statistics - Theory and Methods, 2022<br>Publication | 1% |
| 9 | www.researchgate.net<br>Internet Source | 1% |
| 10 | Muhammad Abdullah, Rashid Ahmed, M. H. Tahir, Muhammad Daniyal. "Minimal non-circular balanced and strongly balanced repeated measurements designs", Communications in Statistics - Simulation and Computation, 2019<br>Publication | <1% |

| | | |
|---|---|---|
| 11 | www.tandfonline.com<br>Internet Source | <1 % |
| 12 | www.hindawi.com<br>Internet Source | <1 % |
| 13 | Abid Khan, Rashid Ahmed, Farrukh Shehzad, M. H. Tahir, Syed Shakir Ali Ghazali. "Construction of circular partially balanced-Repeated measurement designs using cyclic shifts", Communications in Statistics - Simulation and Computation, 2017<br>Publication | <1 % |
| 14 | Abid Khan, Zahid Bashir, H. M. Kashif Rasheed, Mahmood Ulhassan, Rashid Ahmed. "Construction of minimal circular nearly strongly balanced repeated measurements designs and their conversion", Communications in Statistics - Simulation and Computation, 2021<br>Publication | <1 % |
| 15 | www.econstor.eu<br>Internet Source | <1 % |
| 16 | coek.info<br>Internet Source | <1 % |
| 17 | Khadija Noreen, Muhammad Rasheed, H. M. Kashif Rasheed, Farrukh Jamal, Rashid Ahmed, Ronald Onyango. "Some New Dimensions to | <1 % |

Construct Economical Circular Weakly Balanced Neighbor Robust Designs", Mathematical Problems in Engineering, 2022
Publication

| | | |
|---|---|---|
| 18 | thenucleuspak.org.pk<br>Internet Source | <1 % |
| 19 | Zahid Bashir, Rashid Ahmed, Jigneshkumar Gondaliya, Kashif Rasheed. "Some classes of circular balanced RMDs and their conversion into circular strongly and nearly strongly balanced RMDs", Communications in Statistics - Theory and Methods, 2021<br>Publication | <1 % |
| 20 | Arpan Bhowmik, Seema Jaggi, Cini Varghese, Eldho Varghese. "Universally optimal second order neighbour designs", Model Assisted Statistics and Applications, 2013<br>Publication | <1 % |
| 21 | pjsor.com<br>Internet Source | <1 % |
| 22 | Muhammad Abdullah, H. M. Kashif Rasheed, Rashid Ahmed. "General construction of minimal balanced repeated measurements designs in non-circular periods of k different sizes", Communications in Statistics - Simulation and Computation, 2020<br>Publication | <1 % |

| | | |
|---|---|---|
| 23 | Muhammad Rasheed, Khadija Noreen, Rashid Ahmed, M. H. Tahir, Farrukh Jamal. "Some useful classes of minimal weakly balanced neighbor designs in circular blocks of two different sizes", Communications in Statistics - Theory and Methods, 2021<br>Publication | <1 % |
| 24 | www.sciencegate.app<br>Internet Source | <1 % |
| 25 | K.K. Singh Meitei. "Universally Optimal One-sided Circular Neighbour-balanced Designs with k = 5", Calcutta Statistical Association Bulletin, 2020<br>Publication | <1 % |
| 26 | Rashid Ahmed, Munir Akhtar. "Designs balanced for neighbor effects in circular blocks of size six", Journal of Statistical Planning and Inference, 2011<br>Publication | <1 % |

| | | | |
|---|---|---|---|
| Exclude quotes | Off | Exclude matches | Off |
| Exclude bibliography | On | | |