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Exponential Ratio Estimator of the Median: An Alternative to the Regression estimator of the Median under Stratified Sampling

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Abstract: This article develops statistical inference about the population median under the stratified sampling method. An exponential class of ratio estimators of the median was suggested using the combination of scalars and known supplementary information on the population median. Mean square error and bias expressions were derived theoretically and also the AOE (asymptotic optimum estimator) conditions were obtained with its mean square error and bias expressions. From both the empirical evidence and analytical approach evaluations of the AOE with other obtainable members of the suggested class of estimators show that the AOE performs better than its competitors in the literature and is also the alternative to the regression estimator of the median under a stratified random sampling scheme.

Keywords: Population Median, AOE, Bias, MSE, Efficiency, Stratified Sampling

AMS Subject Classification: 62D05

1. Introduction

Auxiliary information is commonly used in survey sampling in order to increase precision while estimating the population parameters. So, whenever this information is available and every researcher wants to utilize it in order to get more precise results. However various authors have put their sincere efforts to do the same for details see Kadilar and Cingi (2003) who amended the estimators in Upadhyaya and Singh (1999) to the sampling design stratified random sampling. Also, Singh and Vishwakarma (2008), Sharma and Singh (2015), Verma et al. (2015) suggested a new family of estimators in Stratified random sampling and Subzar *et al* (2018) have anticipated different estimators using Non-Conventional measures of dispersion for estimating finite population mean in stratified random sampling. Recently Lone et al (2021) have also proposed a general class of ratio estimator for estimating the finite population variance. So this type of sampling procedure is used when the population under study is heterogeneous, it is then usually used to make substrata, which are homogeneous within and heterogeneous between, with a view to producing smaller bound on the error of estimation for a fixed cost of the survey. Having been motivated by the above research works, the present study focuses on how we will get precise results from a heterogonous population and even that is skewed. However, we will get by using the linear regression estimator, but one thing is clear while using OLS (ordinary least square) our results are not precise because it is sensitive to extreme values. But our present study focuses on obtaining reliable results from the data having skewed distribution.

Consider a finite population with auxiliary variate (U_t) and study variable (V_t) which is divided into (M) strata containing (D_t) units in each t^{th} stratum such that $\sum_{t=1}^{M} D_t = D$. Let u_{ti} and v_{ti} represent the sample medians corresponding to the population medians Q_{ut} and with Q_{vt} correlation coefficient between $\hat{Q}_{ut}$ and $\hat{Q}_{vt}$ as ρ_{ct}. Let $f_{ut}(u_t)$ and $f_{vt}(v_t)$ be the marginal densities; $f_{ut}(Q_{ut})$ and $f_{vt}(Q_{vt})$ the probability density functions of the variables. Let $\hat{Q}_{ust} = \sum_{t=1}^{M} W_t \hat{Q}_{ut}$ and $\hat{Q}_{vst} = \sum_{t=1}^{M} W_t \hat{Q}_{vt}$ be the respective weighted sample medians corresponding to the population medians $Q_u = Q_{ust} = \sum_{t=1}^{M} W_t Q_{ut}$ and $Q_v = Q_{vst} = \sum_{t=1}^{M} W_t Q_{vt}$ where $W_t = D_t / D$, $f_t = d_t / d$ where d_t is the sample size from stratum $t = 1,2,\ldots,M$ and d is the total sample size. Also, let

$$\varphi_{0t} = \left(\hat{Q}_{vt} - Q_{vt}\right) / Q_{vt}\ ,\ \varphi_{1t} = \left(\hat{Q}_{ut} - Q_{ut}\right) / Q_{ut} \tag{1.1}$$

Such that

$$E(\varphi_{0t}) = E(\varphi_{1t}) = 0\ ,\ E\left(\varphi_{0t}^2\right) = \zeta_t C_{Qvt}^2\ ,\ E\left(\varphi_{1t}^2\right) = \zeta_t C_{Qut}^2\ , E(\varphi_{0t}\varphi_{1t}) = \zeta_t \rho_{ct} C_{Qvt} C_{Qut} \tag{1.2}$$

Where $C_{Qvt} = [Q_{vt} f_{vt}(Q_{vt})]^{-1}$, $C_{Qut} = [Q_{ut} f_{ut}(Q_{ut})]^{-1}$, $\zeta_t = (1 - f_t)/d_t$, $f_t = d_t / D_t$,

$$K_t = \rho_{ct} C_{Qvt} / C_{Qut} \tag{1.3}$$

2. Review of some existing estimators for population median in stratified sampling scheme

For the above described population different authors have proposed different estimators for population median in stratified sampling in different years whose literature is mentioned in this section, given as under

(a) The usual unbiased sample median estimator (Gross; 1980) given by $\hat{Q}_{vst}$ with variance

$$Var\left(\hat{Q}_{vst}\right) = \sum_{t=1}^{M} W_t^2 \zeta_t Q_{vt}^2 C_{Qvt}^2 \tag{2.1}$$

(b) The classical ratio median estimator (Kuk and Mak; 1989) given by

$\hat{Q}_{RS} = \sum_{t=1}^{M} W_t \hat{Q}_{vt} \left(Q_{ut} / \hat{Q}_{ut}\right)$ With bias $B\left(\hat{Q}_{RS}\right) = \sum_{t=1}^{M} W_t \zeta_t Q_{vt} C_{Qut}^2 (1 - K_t)$ and Mean square error

$$MSE\left(\hat{Q}_{RS}\right) = \sum_{t=1}^{M} W_t^2 \zeta_t Q_{vt}^2 \left[C_{Qvt}^2 + C_{Qut}^2 (1 - 2K_t)\right] \tag{2.2}$$

(c) The product median estimator [(Robson;1957), (Murthy;1964)] is defined as

$\hat{Q}_{PS} = \sum_{t=1}^{M} W_t \hat{Q}_{vt} \left(\hat{Q}_{ut} / Q_{ut}\right)$ With bias $B\left(\hat{Q}_{PS}\right) = \sum_{t=1}^{M} W_t \zeta_t Q_{vt} C_{Qut}^2 K_t$ and Mean square error

$$MSE\left(\hat{Q}_{PS}\right) = \sum_{t=1}^{M} W_t^2 \zeta_t Q_{vt}^2 \left[C_{Qvt}^2 + C_{Qut}^2 (1 + 2K_t)\right] \tag{2.3}$$

(d) The exponential ratio type median estimator (Bahl and Tuteja; 1991) defined as

$$\hat{Q}_{\exp RS} = \sum_{t=1}^{M} W_t \hat{Q}_{vt} \exp\left[\left(Q_{ut} - \hat{Q}_{ut}\right) / \left(Q_{ut} + \hat{Q}_{ut}\right)\right]$$

With bias $B\left(\hat{Q}_{\exp RS}\right) = \sum_{t=1}^{M} \left[\left(W_t \zeta_t Q_{vt} C_{Qut}^2 (3 - 4K_t)\right)/8\right]$ and Mean square error

$$MSE\left(\hat{Q}_{\exp RS}\right)=\sum_{t=1}^{M}W_t^2\zeta_t Q_{vt}^2\left[C_{Qvt}^2+\left(C_{Qut}^2/4\right)\left(1-4K_t\right)\right] \tag{2.4}$$

(e) The exponential product type median estimator (Bahl and Tuteja; 1991) defined as

$$\hat{Q}_{\exp PS}=\sum_{t=1}^{M}W_t\hat{Q}_{vt}\exp\left[\left(\hat{Q}_{ut}-Q_{ut}\right)/\left(Q_{ut}+\hat{Q}_{ut}\right)\right]$$

With bias $B\left(\hat{Q}_{\exp PS}\right)=\sum_{t=1}^{M}\left[\left(W_t\zeta_t Q_{vt}C_{Qut}^2\left(4K_t-1\right)\right)/8\right]$ and Mean square error

$$MSE\left(\hat{Q}_{\exp PS}\right)=\sum_{t=1}^{M}W_t^2\zeta_t Q_{vt}^2\left[C_{Qvt}^2+\left(C_{Qut}^2/4\right)\left(1+4K_t\right)\right] \tag{2.5}$$

(f) The chain ratio type median estimator (Kadilar and Cingi; 2003) defined as

$\hat{Q}_{CRS}=\sum_{t=1}^{M}W_t\hat{Q}_{vt}\left(Q_{ut}/\hat{Q}_{ut}\right)^2$ With bias $B\left(\hat{Q}_{CRS}\right)=\sum_{t=1}^{M}W_t\zeta_t Q_{vt}C_{Qut}^2\left(1+2K_t\right)$ and Mean square error $MSE\left(\hat{Q}_{CRS}\right)=\sum_{t=1}^{M}W_t^2\zeta_t Q_{vt}^2\left[C_{Qvt}^2+4C_{Qut}^2\left(1+K_t\right)\right]$ (2.6)

3. The proposed Median estimator in Stratified Random Sampling Scheme

While taking the motivation from the existing estimators and suggesting the new class of Exponential Ratio estimator of the median using ancillary information and the combination of scalars. The obtainable estimators are the members of the suggested class of estimator and is given as

$$\psi\left(\hbar_{tt},\theta_{tt},\vartheta_{tt},\tau_{tt}\right)=\sum_{i=1}^{M}W_t\left[\hat{Q}_{vt}\left\{\hbar_{tt}-\theta_{tt}\left(\frac{\hat{Q}_{ut}}{Q_{ut}}\right)^{\vartheta_{tt}}\exp\left[\frac{\tau_{tt}\left(\hat{Q}_{ut}-Q_{ut}\right)}{\left(\hat{Q}_{ut}+Q_{ut}\right)}\right]\right\}\right] \tag{3.1}$$

Where $\left(\hbar_{tt},\theta_{tt},\vartheta_{tt},\tau_{tt}\right)$ are the scalars chosen suitably, such that $\hbar_{tt}$ and θ_{tt} fulfil the condition

$$\hbar_{tt}=1+\theta_{tt};\qquad -\infty<\lambda_{tt}<\infty \tag{3.2}$$

In order to derive the estimated expressions of bias and mean square error for the suggested class of estimator we have expressed (3.1) in terms of (1.1), then we obtain

$$\psi\left(\hbar_{tt},\theta_{tt},\vartheta_{tt},\tau_{tt}\right)=\sum_{i=1}^{M}W_t\left[Q_{vt}\left(1+\varphi_{0t}\right)\left\{\hbar_{tt}-\theta_{tt}\left(\frac{Q_{ut}\left(1+\varphi_{1t}\right)}{Q_{ut}}\right)^{\vartheta_{tt}}\exp\left[\frac{\tau_{tt}\left[Q_{ut}\left(1+\varphi_{1t}\right)\right]-Q_{ut}}{2Q_{ut}\left(1+\varphi_{1t}/2\right)}\right]\right\}\right]$$

$$=\sum_{i=1}^{M}W_t\left[Q_{vt}\left(1+\varphi_{0t}\right)\left\{\hbar_{tt}-\theta_{tt}\left(1+\varphi_{1t}\right)^{\vartheta_{tt}}\exp\left[\frac{\tau_{tt}Q_{ut}\varphi_{1t}}{2Q_{ut}\left(1+\varphi_{1t}/2\right)}\right]\right\}\right]$$

$$=\sum_{i=1}^{M}W_t\left[Q_{vt}\left(1+\varphi_{0t}\right)\left\{\hbar_{tt}-\theta_{tt}\left(1+\varphi_{1t}\right)^{\vartheta_{tt}}\exp\left[\frac{\tau_{tt}\varphi_{1t}}{2}\left(1+\varphi_{1t}/2\right)^{-1}\right]\right\}\right]$$

Assuming that $\left|\varphi_{1t}\right|<1$ and expanding $\left(1+\varphi_{1t}\right)^{\vartheta_{tt}}$, $\left[\frac{\tau_{tt}\varphi_{1t}}{2}\left(1+\varphi_{1t}/2\right)^{-1}\right]$ and $\left(1+\varphi_{1t}/2\right)^{-1}$, we have

$$\psi(\hbar_{tt},\theta_{tt},\vartheta_{tt},\partial_{tt}) = \sum_{t=1}^{M} W_t \left[Q_{vt}(1+\varphi_{0t}) \left\{ \begin{array}{l} \hbar_{tt} - \theta_{tt}\left[1+\vartheta_{tt}\varphi_{1t} + \dfrac{\vartheta_{tt}(\vartheta_{tt}-1)}{2}\varphi_{1t}^2 + \dots\right] \times \\ \left[1 + \dfrac{\tau_{tt}\varphi_{1t}}{2}(1+\varphi_{1t}/2)^{-1} + \dfrac{\tau_{tt}^2\varphi_{1t}^2}{8}(1+\varphi_{1t}/2)^{-2}\right] \end{array} \right\} \right]$$

$$\psi(\hbar_{tt},\theta_{tt},\vartheta_{tt},\tau_{tt}) = \sum_{t=1}^{M} W_t \left[Q_{vt}(1+\varphi_{0t}) \left\{ \begin{array}{l} \hbar_{tt} - \theta_{tt}\left[1+\vartheta_{tt}\varphi_{1t} + \dfrac{\vartheta_{tt}(\vartheta_{tt}-1)}{2}\varphi_{1t}^2 + \dots\right] \times \\ \left[1 + \left[\dfrac{\tau_{tt}\varphi_{1t}}{2}\left(1 - \dfrac{\varphi_{1t}}{2} + \dfrac{\varphi_{1t}^2}{4} - \dots\right)\right] + \dfrac{\tau_{tt}^2\varphi_{1t}^2}{8}(1+\varphi_{1t})\right] \end{array} \right\} \right]$$

$$\psi(\hbar_{tt},\theta_{tt},\vartheta_{tt},\tau_{tt}) = \sum_{t=1}^{M} W_t \left[Q_{vt}(1+\varphi_{0t}) \left\{ \begin{array}{l} \hbar_{tt} - \theta_t\left[1+\vartheta_{tt}\varphi_{1t} + \dfrac{\vartheta_{tt}(\vartheta_{tt}-1)}{2}\varphi_{1t}^2 + \dots\right] \times \\ \left[1 + \dfrac{\tau_{tt}\varphi_{1t}}{2} - \dfrac{\tau_{tt}\varphi_{1t}^2}{4} + \dfrac{\tau_{tt}^2\varphi_{1t}^2}{8}\right] \end{array} \right\} \right]$$

$$\cong \sum_{t=1}^{M} W_t \left[Q_{vt}(1+\varphi_{0t}) \left\{ (\hbar_{tt} - \theta_{tt}) - \frac{\lambda_{tt}\varphi_{1t}}{2}(2\vartheta_{tt}+\tau_{tt}) - \frac{\lambda_{tt}\varphi_{1t}^2}{8}(4\vartheta_{tt}^2 - 4\vartheta_{tt} - 2\tau_{tt} + \tau_{tt}^2 + 4\vartheta_{tt}\tau_{tt}) \right\} \right]$$

$$\cong \sum_{t=1}^{M} W_t \left[Q_{vt}(1+\varphi_{0t}) \left\{ (\hbar_{tt} - \theta_{tt}) - \frac{\theta_{tt}(2\vartheta_{tt}+\tau_{tt})\varphi_{1t}}{2} - \frac{\theta_{tt}(2\vartheta_{tt}+\tau_{tt})(2\vartheta_{tt}+\tau_{tt}-2)\varphi_{1t}^2}{8} \right\} \right]$$

Neglecting terms of φ_{it} $(i = 0 \ or \ 1)$ having power greater than two, we have

$$\cong \sum_{t=1}^{M} W_t \left[Q_{vt} \left\{ \begin{array}{l} (\hbar_{tt} - \theta_{tt}) - \dfrac{\theta_{tt}(2\vartheta_{tt}+\tau_{tt})\varphi_{1t}}{2} - \dfrac{\theta_{tt}(2\vartheta_{tt}+\tau_{tt})(2\vartheta_{tt}+\tau_{tt}-2)\varphi_{1t}^2}{8} \\ +(\hbar_{tt} - \theta_{tt})\varphi_{0t} \dfrac{\theta_{tt}(2\vartheta_{tt}+\tau_{tt})\varphi_{0t}\varphi_{1t}}{2} \end{array} \right\} \right]$$

$$\cong \sum_{t=1}^{M} W_t \left[Q_{vt} \left\{ (\hbar_{tt} - \theta_{tt}) - \frac{\theta_{tt}(2\vartheta_{tt}+\tau_{tt})}{2}\left[\varphi_{1t} + \frac{(2\vartheta_{tt}+\tau_{tt}-2)\varphi_{1t}^2}{4} + \varphi_{0t}\varphi_{1t}\right] + (\hbar_{tt} - \theta_{tt})\varphi_{0t} \right\} \right] \qquad (3.3)$$

Therefore,

$$\psi(\hbar_{tt},\theta_{tt},\vartheta_{tt},\tau_{tt}) - Q_{vt} \cong \sum_{t=1}^{M} W_t \left[Q_{vt} \left\{ \begin{array}{l} (\hbar_{tt} - \theta_{tt} - 1) - \dfrac{\theta_t(2\vartheta_{tt}+\tau_{tt})}{2} \\ \left[\varphi_{1t} + \dfrac{(2\vartheta_{tt}+\tau_{tt}-2)\varphi_{1t}^2}{4} + \varphi_{0t}\varphi_{1t}\right] + (\hbar_{tt} - \theta_{tt})\varphi_{0t} \end{array} \right\} \right] \qquad (3.4)$$

Then, taking expectation of (3.4) and using (1.1), we obtain the bias for the proposed class of ratio median estimators $\psi(\hbar_{tt},\theta_{tt},\vartheta_{tt},\tau_{tt})$ to the first degree of approximation as

$$B[\psi(\hbar_{tt},\theta_{tt},\vartheta_{tt},\tau_{tt})] = E[\psi(\hbar_{tt},\theta_{tt},\vartheta_{tt},\tau_{tt}) - Q_{vt}]$$

$$\cong \sum_{t=1}^{M} W_t \left[\left(\hbar_{tt} - \theta_{tt} - 1\right) Q_{vt} + \frac{\left(1 - f_t\right)}{d_t} Q_{vt} \left\{ -\frac{\theta_{tt}\left(2\vartheta_{tt} + \tau_{tt}\right)}{2} \left[\frac{\left(2\vartheta_{tt} + \tau_{tt} - 2\right)}{4} C_{Qut}^2 + \rho_{ct} C_{Qv} C_{Qu} \right] \right\} \right]$$

$$\cong \sum_{t=1}^{M} W_t \left[\left(\hbar_{tt} - \theta_{tt} - 1\right) Q_{vt} + \frac{\left(1 - f_t\right)}{d_t} Q_{vt} C_{Qut}^2 \left\{ -\frac{\theta_{tt}\left(2\vartheta_{tt} + \tau_{tt}\right)}{2} \left[\frac{\left(2\vartheta_{tt} + \tau_{tt} - 2\right)}{4} + K_t \right] \right\} \right] \tag{3.5}$$

While squaring equation (3.4) on both sides, ignoring terms of φ_{it} $\left(i = 0 \ or \ 1\right)$ having power greater than two, taking expectation, and using (1.1), we obtain the mean square error for the suggested class of ratio median estimator to the first degree of approximation as

$$MSE\left[\psi\left(\hbar_{tt}, \theta_{tt}, \vartheta_{tt}, \tau_{tt}\right)\right] = E\left[\psi\left(\hbar_{tt}, \theta_{tt}, \vartheta_{tt}, \tau_{tt}\right) - Q_{vt}\right]^2$$

$$\cong E\left\{ \sum_{t=1}^{M} W_t \left[Q_{vt} \left\{ \left(\hbar_{tt} - \theta_{tt} - 1\right) \right\} - \frac{\theta_{tt}\left(2\vartheta_{tt} + \tau_{tt}\right)}{2} + \left(\hbar_{tt} - \theta_{tt}\right) \varphi_{1t} \right] \right\}^2$$

$$\cong \sum_{t=1}^{M} W_t^2 E \left[Q_{vt}^2 \left\{ \begin{array}{l} \left(\hbar_{tt} - \theta_{tt} - 1\right)^2 + \dfrac{\theta_{tt}^2 \varphi_{1t}^2 \left(2\vartheta_{tt} + \tau_{tt}\right)^2}{4} + \left(\hbar_{tt} - \theta_{tt}\right)^2 \varphi_{0t}^2 + 2\left(\hbar_{tt} - \theta_{tt} - 1\right)\left(\hbar_{tt} - \theta_{tt}\right)\varphi_{0t} \\ -\left(\hbar_{tt} - \theta_{tt} - 1\right)\left(2\vartheta_{tt} + \tau_{tt}\right)\theta_{tt}\varphi_{1t} - \theta_{tt}\left(2\vartheta_{tt} + \tau_{tt}\right)\left(\hbar_{tt} - \theta_{tt}\right)\varphi_{0t}\varphi_{1t} \end{array} \right\} \right]$$

$$\cong \sum_{t=1}^{M} W_t^2 \left[Q_{vt}^2 \left(\hbar_{tt} - \theta_{tt} - 1\right)^2 + \frac{\left(1 - f_t\right)}{d_t} Q_{vt}^2 \left[\begin{array}{l} \left(\hbar_{tt} - \theta_{tt}\right)^2 C_{Mvt}^2 + \dfrac{\theta_{tt}^2 \left(2\vartheta_{tt} + \tau_{tt}\right)^2}{4} C_{Mut}^2 \\ -\theta_{tt}\left(2\vartheta_{tt} + \tau_{tt}\right)\left(\hbar_{tt} - \theta_{tt}\right)\rho_{ct} C_{Mvt} C_{Mut} \end{array} \right] \right]$$

$$\cong \sum_{t=1}^{M} W_t^2 \left[Q_{vt}^2 \left(\hbar_{tt} - \theta_{tt} - 1\right)^2 + \frac{\left(1 - f_t\right)}{d_t} Q_{vt}^2 \left[\begin{array}{l} \left(\hbar_{tt} - \theta_{tt}\right)^2 C_{Mvt}^2 + \\ \dfrac{\left(2\vartheta_{tt} + \partial_{tt}\right)}{4} C_{Mut}^2 \left\{ \theta_{tt}^2 \left(2\vartheta_{tt} + \tau_{tt}\right) - 4K_t \theta_{tt} \left(\hbar_{tt} - \theta_{tt}\right) \right\} \end{array} \right] \right] \tag{3.6}$$

Now to examine the optimality condition for the suggested class of ratio median estimator, let

$$\frac{\partial MSE\left[\psi\left(\hbar_{tt}, \theta_{tt}, \vartheta_{tt}, \tau_{tt}\right)\right]}{\partial \theta_{tt}} = 0$$

So that

$$\theta_{tt}^2 C_{Qut}^2 \left(2\vartheta_{tt} + \tau_{tt}\right) = 2K_{tt}\theta_{tt}\left(\hbar_{tt} - \theta_{tt}\right) C_{Qut}^2$$

$$\Rightarrow \left(2\vartheta_{tt} + \tau_{tt}\right) = \frac{2K_t \theta_{tt} \left(\hbar_{tt} - \theta_{tt}\right)}{\theta_{tt}}$$

$$\Rightarrow \left(2\vartheta_{tt} + \tau_{tt}\right) = \frac{2K_t}{\theta_{tt}} \quad \text{Since} \quad \hbar_{tt} = 1 + \theta_{tt} \tag{3.7}$$

Putting (3.7) in (3.5), we have

$$\frac{2K_t}{\theta_{tt}} = \left(2\vartheta_{tt} + \tau_{tt}\right) \text{ and } \hbar_{t} = \left(1 + \theta_{tt}\right)$$

Then

$$B\left(\psi\left(\hbar_{tt}, \theta_{tt}, \vartheta_{tt}, \tau_{tt}\right)\right) = E\left[\left(\psi\left(\hbar_{tt}, \theta_{tt}, \vartheta_{tt}, \tau_{tt}\right)\right) - Q_{vt}\right]$$

$$\cong \sum_{t=1}^{M} W_t \left[(1+\theta_{tt}-\theta_{tt}-1)Q_{vt} + \frac{1-f_t}{d_t} Q_{vt} C_{Qut}^2 \left\{ \frac{-\theta_{tt}\dfrac{2K_t}{\theta_{tt}}\left[\dfrac{\frac{2K_t}{\theta_{tt}}-2}{4}+K_t\right]}{2} \right\} \right]$$

$$\cong \sum_{t=1}^{M} W_t \frac{1-f_t}{d_t} Q_{vt} C_{Qut}^2 \left\{ \frac{-2K_t}{2}\left[\frac{2K_t-2\theta_{tt}}{4\theta_{tt}}+K_t\right]\right\}$$

$$\cong \sum_{t=1}^{M} W_t \frac{1-f_t}{d_t} Q_{vt} C_{Qut}^2 \left\{ \frac{-K_t}{4\theta_{tt}}\left[2K_t-2\theta_{tt}+4K_t\theta_{tt}\right]\right\} \tag{3.8}$$

Hence substituting (3.7) in (3.6), we obtain

$$MSE[\psi(\hbar_{tt},\theta_{tt},\vartheta_{tt},\tau_{tt})] \cong \sum_{t=1}^{M} W_t^2 \left[\begin{array}{l} Q_{vt}^2(\hbar_{tt}-\theta_{tt}-1)^2 + \\ \dfrac{(1-f_t)}{d_t} Q_{vt}^2 \left[\begin{array}{l} (\hbar_{tt}-\theta_{tt})^2 C_{Mvt}^2 + \dfrac{2K_t(\hbar_{tt}-\theta_{tt})}{4\lambda_{tt}} C_{Mut}^2 \times \\ \left\{ \dfrac{2K_t\theta_{tt}^2(\hbar_{tt}-\theta_{tt})}{4\theta_{tt}} - 4K_t\theta_{tt}(\hbar_{tt}-\theta_{tt}) \right\} \end{array} \right] \end{array} \right]$$

$$\cong \sum_{t=1}^{M} W_t^2 \left[Q_{vt}^2(\hbar_{tt}-\theta_{tt}-1)^2 + \frac{(1-f_t)}{d_t} Q_{vt}^2 \left[(\hbar_{tt}-\theta_{tt})^2 C_{Mvt}^2 + \frac{K_t(\hbar_{tt}-\theta_{tt})}{2\theta_{tt}} C_{Mut}^2 \{-2K_t\theta_{tt}(\hbar_{tt}-\theta_{tt})\} \right] \right]$$

$$\cong \sum_{t=1}^{M} W_t^2 \left[Q_{vt}^2(\hbar_{tt}-\theta_{tt}-1)^2 + \frac{(1-f_t)}{d_t} Q_{vt}^2 \left[(\hbar_{tt}-\theta_{tt})^2 C_{Mvt}^2 - (\hbar_{tt}-\theta_{tt})^2 K_t^2 C_{Mut}^2 \right] \right]$$

$$\cong \sum_{t=1}^{M} W_t^2 \left[Q_{vt}^2(\hbar_{tt}-\theta_{tt}-1)^2 + \frac{(1-f_t)}{d_t} Q_{vt}^2 (\hbar_{tt}-\theta_{tt})^2 \left[C_{Mvt}^2 - K_t^2 C_{Mut}^2 \right] \right]$$

$$\cong \sum_{t=1}^{M} W_t^2 \left[Q_{vt}^2(\hbar_{tt}-\theta_{tt}-1)^2 + \frac{(1-f_t)}{d_t} Q_{vt}^2 (\hbar_{tt}-\theta_{tt})^2 C_{Mvt}^2 \left[1-\rho_{ct}^2 \right] \right] \tag{3.9}$$

Thus substituting (3.2) in (3.9) we obtain the asymptotic optimum MSE for the Proposed class of ratio median estimator $\psi(\hbar_{tt},\theta_{tt},\vartheta_{tt},\tau_{tt})$ as

$$MSE_{opt}[\psi(\hbar_{tt},\theta_{tt},\vartheta_{tt},\tau_{tt})] \cong \sum_{t=1}^{M} W_t^2 \left[\frac{(1-f_t)}{d_t} Q_{vt}^2 C_{Mvt}^2 (1-\rho_{ct}^2) \right]$$

$$\cong \sum_{t=1}^{M} W_t^2 \zeta_t Q_{vt}^2 C_{Mvt}^2 (1-\rho_{ct}^2) \tag{3.10}$$

Thus the mean square error of the suggested class of exponential ratio estimator of the median in stratified random sampling scheme at optimal condition has the same proficiency as the unbiased linear regression estimator as the mean square error expression at the optimal condition of the suggested estimator is the same as the expression of unbiased linear regression estimator.

4. Some existing members of the proposed class of estimators

How the existing estimators which are mentioned in this study fit into the suggested class of exponential ratio estimator of the median are presented in this section (see for details Table 1).

Note. As $(\hbar_{tt}, \theta_{tt}, \vartheta_{tt}, \tau_{tt})$ take on definite distinctive values, the suggested class of exponential ratio estimator of the median giving the same expressions of bias and mean square error which the estimators give mentioned in section 2. Thus, the study provides unified treatment towards the properties of the existing members of the suggested class of estimators point out in this study.

Table 1. Some Existing members of the suggested class of estimators

$\hbar_{tt}$	θ_{tt}	ϑ_{tt}	τ_{tt}	Estimators
1	0	ϑ_{tt}	τ_{tt}	$\hat{Q}_{vst}$ Usual Median Unbiased estimator
0	-1	-1	0	$\hat{Q}_{RS} = \sum_{t=1}^{M} W_t \hat{Q}_{vt} \left(Q_{ut} / \hat{Q}_{ut}\right)$ Classical ratio median estimator
0	-1	1	0	$\hat{Q}_{PS} = \sum_{t=1}^{M} W_t \hat{Q}_{vt} \left(\hat{Q}_{ut} / Q_{ut}\right)$ Product type median estimator
0	-1	0	-1	$\hat{Q}_{\exp RS} = \sum_{t=1}^{M} W_t \hat{Q}_{vt} \exp\left[\left(Q_{ut} - \hat{Q}_{ut}\right) / \left(Q_{ut} + \hat{Q}_{ut}\right)\right]$ Bahl and Tuteja exponential ratio median estimator
0	-1	0	1	$\hat{Q}_{\exp PS} = \sum_{t=1}^{M} W_t \hat{Q}_{vt} \exp\left[\left(\hat{Q}_{ut} - Q_{ut}\right) / \left(Q_{ut} + \hat{Q}_{ut}\right)\right]$ Bahl and Tuteja exponential product median estimator
0	-1	2	0	$\hat{Q}_{CRS} = \sum_{t=1}^{M} W_t \hat{Q}_{vt} \left(Q_{ut} / \hat{Q}_{ut}\right)^2$ Chain ratio median estimator

5. Efficiency comparisons among members of the proposed class of estimators

Here, the mean square error of the suggested exponential ratio estimator of the median at optimum condition is equated with the mean square error of some existing median estimators.

(a) Comparison of $MSE_{opt}\left[\psi(\hbar_{tt}, \theta_{tt}, \vartheta_{tt}, \tau_{tt})\right]$ **with sample median estimator** $\hat{Q}_{vst}$

Comparing (2.1) and (3.9), $(optimum)\left[\psi(\hbar_{tt}, \theta_{tt}, \vartheta_{tt}, \tau_{tt})\right]$ will be more efficient than $\hat{Q}_{vst}$

$$\sum_{t=1}^{M} W_t^2 \zeta_t Q_{vt}^2 C_{Mvt}^2 - \sum_{t=1}^{M} W_t^2 \zeta_t Q_{vt}^2 C_{Mvt}^2 \left(1 - \rho_{ct}^2\right) > 0$$

$\Rightarrow \sum_{t=1}^{M} W_t^2 \theta_t Q_{vt}^2 C_{Mvt}^2 \rho_{ct}^2 > 0$, Which is always true

(b) Comparison of $MSE_{opt}\left[\psi(\hbar_{tt}, \theta_{tt}, \vartheta_{tt}, \tau_{tt})\right]$ **with classical ratio median estimator** $\hat{Q}_{RS}$

Comparing (2.2) and (3.9), $(optimum)\left[\psi(\hbar_{tt}, \theta_{tt}, \vartheta_{tt}, \tau_{tt})\right]$ will be more efficient than $\hat{Q}_{RS}$

$$\sum_{t=1}^{M} W_t^2 \zeta_t Q_{vt}^2 \left[C_{Qvt}^2 + C_{Qut}^2 (1 - 2K_t)\right] - \sum_{t=1}^{M} W_t^2 \zeta_t Q_{vt}^2 C_{Mvt}^2 \left(1 - \rho_{ct}^2\right) > 0$$

$\Rightarrow (1 - K_t)^2 > 0$, Which is always true

(c) Comparison of $MSE_{opt}\left[\psi(\hbar_{tt}, \theta_{tt}, \vartheta_{tt}, \tau_{tt})\right]$ **with classical product median estimator** $\hat{Q}_{PS}$

Comparing (2.3) and (3.9), $(optimum)\left[\psi(\hbar_{tt}, \theta_{tt}, \vartheta_{tt}, \tau_{tt})\right]$ will be more efficient than $\hat{Q}_{PS}$

$$\sum_{t=1}^{M} W_t^2 \zeta_t Q_{vt}^2 \left[C_{Qvt}^2 + C_{Qut}^2 (1 + 2K_t)\right] - \sum_{t=1}^{M} W_t^2 \zeta_t Q_{vt}^2 C_{Mvt}^2 \left(1 - \rho_{ct}^2\right) > 0$$

$\Rightarrow (1+K_t)^2 > 0$, Which is always true

(d) Comparison of $MSE_{opt}[\psi(\hbar_{tt},\theta_{tt},\vartheta_{tt},\tau_{tt})]$ with classical exponential ratio median estimator $\hat{Q}_{\exp RS}$

Comparing (2.4) and (3.9), $(optimum)[\psi(\hbar_{tt},\theta_{tt},\vartheta_{tt},\tau_{tt})]$will be more efficient than $\hat{Q}_{\exp RS}$

$$\sum_{t=1}^{M} W_t^2 \zeta_t Q_{vt}^2 \left[C_{Qvt}^2 + \frac{C_{Qut}^2}{4}(1-4K_t) \right] - \sum_{t=1}^{M} W_t^2 \zeta_t Q_{vt}^2 C_{Mvt}^2 (1-\rho_{ct}^2) > 0$$

$\Rightarrow (1-2K_t)^2 > 0$, Which is always true

(e) Comparison of $MSE_{opt}[\psi(\hbar_{tt},\theta_{tt},\vartheta_{tt},\tau_{tt})]$ with classical exponential product median estimator $\hat{Q}_{\exp PS}$

Comparing (2.5) and (3.9), $(optimum)[\psi(\hbar_{tt},\theta_{tt},\vartheta_{tt},\tau_{tt})]$will be more efficient than $\hat{Q}_{\exp PS}$

$$\sum_{t=1}^{M} W_t^2 \zeta_t Q_{vt}^2 \left[C_{Qvt}^2 + \frac{C_{Qut}^2}{4}(1+4K_t) \right] - \sum_{t=1}^{M} W_t^2 \zeta_t Q_{vt}^2 C_{Mvt}^2 (1-\rho_{ct}^2) > 0$$

$\Rightarrow (1+2K_t)^2 > 0$, Which is always true

(f) Comparison of $MSE_{opt}[\psi(\hbar_{tt},\theta_{tt},\vartheta_{tt},\tau_{tt})]$ with classical chain ratio median estimator $\hat{Q}_{CRS}$

Comparing (2.6) and (3.9), $(optimum)[\psi(\hbar_{tt},\theta_{tt},\vartheta_{tt},\tau_{tt})]$will be more efficient than $\hat{Q}_{CRS}$

$$\sum_{t=1}^{M} W_t^2 \zeta_t Q_{vt}^2 \left[C_{Qvt}^2 + 4C_{Qut}^2 (1+K_t) \right] - \sum_{t=1}^{M} W_t^2 \zeta_t Q_{vt}^2 C_{Mvt}^2 (1-\rho_{ct}^2) > 0$$

$\Rightarrow (K_t^2 + 4K_t + 4) > 0$, Which is always true

6. Empirical Study

In this present study, for verifying the general results and also check the optimality performance i.e., (asymptotic optimum estimator) for the suggested class of exponential ratio estimator of the median $\psi(\hbar_{tt},\theta_{tt},\vartheta_{tt},\tau_{tt})$ over the existing members of the suggested class of estimators cited in this study, we the data of the population **[Source: PDS(2012)]** and the summary of the population is given in Table 2.ll

Table 2: Summary Statistics of the population

$D=144$				$d=20$			
D_1	36	D_2	36	D_3	36	D_4	36
d_1	5	d_2	5	d_3	5	d_4	5
W_1	0.25	W_1	0.25	W_1	0.25	W_1	0.25
Q_{u1}	1480	Q_{u2}	127289	Q_{u3}	54559	Q_{u4}	71615
Q_{v1}	38	Q_{v2}	3058	Q_{v3}	2033	Q_{v4}	2382
ρ_{c1}	0.7776	ρ_{c2}	0.8888	ρ_{c3}	0.8888	ρ_{c4}	0.8888

$f_{v1}(Q_{v1})$	0.007056	$f_{v2}(Q_{v2})$	0.00032023	$f_{v3}(Q_{v3})$	0.0004219	$f_{v4}(Q_{v4})$	0.0003012
$f_{u1}(Q_{u1})$	0.0001641	$f_{u2}(Q_{u2})$	0.000007827	$f_{u3}(Q_{u3})$	0.0000141	$f_{u4}(Q_{u4})$	0.00001026
C_{Mv1}	3.729562	C_{Mv2}	1.021175775	C_{Mv3}	1.16587797	C_{Mv4}	1.393809035
C_{Mu1}	4.117462984	C_{Mu2}	1.003722805	C_{Mu3}	1.299913595	C_{Mu4}	1.360970285
ζ_1	0.043056	ζ_2	0.043056	ζ_3	0.043056	ζ_4	0.043056
K_1	0.70434	K_2	0.90425	K_3	0.79715	K_4	0.91025

Considering the above population, setting $\theta_{t1} = 2$ satisfies the condition in (3.2), implies $\hbar_1 = 3$. Then set $\vartheta_{t1} = (177387/1100000)$ and $\tau_{t1} = (21/55)$ so that $(2\vartheta_{t1} + \tau_{t1}) = 2K_1/\theta_{t1}$, $K_1 = (0.70434)$ fulfils the condition in (3.7). Hereafter setting $(\hbar_{t1}, \theta_{t1}, \vartheta_{t1}, \tau_{t1}) = (3, 2, 177387/1100000, 21/55)$ in (3.1) we find an asymptotic optimum estimator (AOE) for the suggested class of estimators for the population median for strata 1 as

$$\psi(\hbar_{t1}, \theta_{t1}, \vartheta_{t1}, \tau_{t1}) = \left(3, 2, \frac{177387}{1100000}, \frac{21}{55}\right) = \hat{Q}_{v1}\left[\left\{3 - 2\left(\frac{\hat{Q}_{u1}}{Q_{u1}}\right)^{\frac{177387}{1100000}} \exp\left[\frac{21\left(\hat{Q}_{u1} - Q_{u1}\right)}{55\left(\hat{Q}_{u1} + Q_{u1}\right)}\right]\right\}\right] \tag{6.1}$$

and the first degree approximation of bias and mean square error given respectively in Table 3,

$$B[\psi(3,2,177387/1100000,21/55)] \cong \frac{1-f_1}{d_1} W_1 Q_{v1} C_{Mu1}^2 \left\{\frac{-K_1}{4\theta_{t1}}\left[2K_1 - 2\theta_{t1} + 4K_1\theta_{t1}\right]\right\} \tag{6.2}$$

$$MSE\left[\psi\left(3, 2, \frac{177387}{1100000}, \frac{21}{55}\right)\right] = \frac{(1-f_1)Q_{v1}}{d_1}\left(C_{Mv1}^2 + \frac{35216590909}{200000000000} C_{Mu1}^2 \left\{2\frac{10216590909}{12500000000} - 8K_1\right\}\right) \tag{6.3}$$

For Strata 2 we have Set $\theta_{t2} = 3$ satisfies the condition in (3.2), implies $\hbar_{t2} = 4$. Then set $\vartheta_{t2} = (20283/2000000)$ and $\tau_{t2} = (12/30)$ so that $(2\vartheta_{t2} + \tau_{t2}) = 2K_2/\theta_{t2}$, $K_2 = (0.90425)$ fulfils the condition in (3.7). Hereafter setting $(\hbar_{t2}, \theta_{t2}, \vartheta_{t2}, \tau_{t2}) = (4, 3, 20283/200000, 12/30)$ in (3.1) find an asymptotic optimum estimator (AOE) for the suggested class of estimators for the population median for strata 2 as

$$\psi(\hbar_{t2}, \theta_{t2}, \vartheta_{t2}, \tau_{t2}) = \left(4, 3, \frac{20283}{200000}, \frac{12}{30}\right) = \hat{Q}_{v2}\left[\left\{4 - 3\left(\frac{\hat{Q}_{u2}}{Q_{u2}}\right)^{\frac{20283}{200000}} \exp\left[\frac{12\left(\hat{Q}_{u2} - Q_{u2}\right)}{30\left(\hat{Q}_{u2} + Q_{u2}\right)}\right]\right\}\right] \tag{6.4}$$

and the first degree approximation of bias and mean square error given respectively in Table 3,

$$B[\psi(4,3,20283/200000,12/30)] \cong \frac{1-f_2}{d_2} W_2 Q_{v2} C_{Mu2}^2 \left\{\frac{-K_2}{4\theta_{t2}}\left[2K_2 - 2\theta_{t2} + 4K_2\theta_{t2}\right]\right\}, \tag{6.5}$$

$$MSE\left[\psi\left(4, 3, \frac{20283}{200000}, \frac{12}{30}\right)\right] \cong \frac{(1-f_2)Q_{v2}}{d_2}\left(C_{Mv2}^2 + \frac{60283}{400000} C_{Mu2}^2 \left\{5\frac{42547}{100000} - 12K_2\right\}\right) \tag{6.6}$$

For Strata 3, we have set $\theta_{t3}=4$ fulfils the condition in (3.2), implies $\hbar_{t3}=5$. Then set $\vartheta_{t3}=(3943/80000)$ and $\tau_{t3}=(15/50)$ so that $(2\vartheta_{t3}+\tau_{t3})=2K_3/\theta_{t3}$, $K_3=(0.79715)$ fulfils the condition in (3.7). Hereafter setting $(\hbar_{t3},\theta_{t3},\vartheta_{t3},\tau_{t3})=(5,4,3943/80000,15/50)$ in (3.1) we find an asymptotic optimum estimator (AOE) for the suggested class of estimators for the population median for strata 3 as

$$\psi(\hbar_{t3},\theta_{t3},\vartheta_{t3},\tau_{t3})=\left(5,4,\frac{3943}{80000},\frac{15}{50}\right)=\hat{Q}_{v3}\left[\left[\left\{5-4\left(\frac{\hat{Q}_{u3}}{Q_{u3}}\right)^{\frac{3943}{80000}}\exp\left[\frac{15(\hat{Q}_{u3}-Q_{u3})}{50(\hat{Q}_{u3}+Q_{u3})}\right]\right\}\right]\right] \tag{6.7}$$

and the first degree approximation of bias and mean square error given respectively in Table 3,

$$B[\psi(5,4,3943/80000,15/50)]\cong\frac{1-f_3}{d_3}W_3Q_{v3}C^2_{Mu3}\left\{\frac{-K_3}{4\theta_{t3}}[2K_3-2\theta_{t3}+4K_3\theta_{t3}]\right\} \tag{6.8}$$

$$MSE\left[\psi\left(5,4,\frac{3943}{80000},\frac{15}{50}\right)\right]=\frac{(1-f_3)Q_{v3}}{d_3}\left(C^2_{Mv3}+\frac{15943}{160000}C^2_{Mu3}\left\{6\frac{943}{2500}-16K_3\right\}\right) \tag{6.9}$$

and for strata 4, we have set $\theta_{t4}=4$ fulfils the condition in (3.2), implies $\hbar_{t4}=5$. Then set $\vartheta_{t4}=(13179/304000)$ and $\tau_{t4}=(35/95)$ so that $(2\vartheta_{t4}+\tau_{t4})=2K_4/\theta_{t4}$, $K_4=(0.91025)$ fulfils the condition in (3.7). Hereafter setting $(\hbar_{t4},\theta_{t4},\vartheta_{t4},\tau_{t4})=(5,4,13179/304000,35/95)$ in (3.1) we find an asymptotic optimum estimator (AOE) for the suggested class of estimators for the population median for strata 4 as

$$\psi(\hbar_{t4},\theta_{t4},\vartheta_{t4},\tau_{t4})=\left(5,4,\frac{13179}{304000},\frac{35}{95}\right)=\hat{Q}_{v4}\left[\left[\left\{5-4\left(\frac{\hat{Q}_{u4}}{Q_{u4}}\right)^{\frac{13179}{304000}}\exp\left[\frac{35(\hat{Q}_{u4}-Q_{u4})}{95(\hat{Q}_{u4}+Q_{u4})}\right]\right\}\right]\right] \tag{6.10}$$

and the first degree approximation of bias and mean square error given respectively in Table 3,

$$B[\psi(5,4,13179/304000,35/95)]\cong\frac{1-f_4}{d_4}W_4Q_{v4}C^2_{Mu4}\left\{\frac{-K_4}{4\theta_{t4}}[2K_4-2\theta_{t4}+4K_4\theta_{t4}]\right\} \tag{6.11}$$

$$MSE\left[\psi\left(5,4,\frac{13179}{304000},\frac{35}{95}\right)\right]=\frac{(1-f_4)Q_{v4}}{d_4}\left(C^2_{Mv4}+\frac{11274}{100000}C^2_{Mu4}\left\{\begin{matrix}7\frac{67372368417}{312500000000}\\-16K_4\end{matrix}\right\}\right) \tag{6.12}$$

Then, when combining all the strata, from the (6.1) to (6.12) we came to conclude that setting $\theta_{tt}=(\theta_{t1}+\theta_{t2}+\theta_{t3}+\theta_{t4})=(2+3+4+4)=13$, implies $\hbar_{tt}=(3+4+5+5=17)$. Then set

$$\vartheta_{tt}=\left(\frac{177387}{1100000}+\frac{20283}{200000}+\frac{3945}{80000}+\frac{13179}{304000}\right)=\left(\frac{1490967}{2000000}\right)$$

and $\tau_{tt}=\left(\frac{21}{55}+\frac{12}{30}+\frac{15}{50}+\frac{35}{95}\right)=\left(1\frac{4502311}{10000000}\right)$ so that $(2\vartheta_{tt}+\tau_{tt})=2K_t/\theta_{tt}$, $K_t=(3.39599)$ fulfils the condition in (3.7). Hereafter setting $(\hbar_{tt},\theta_{tt},\vartheta_{tt},\tau_{tt})=\left(17,13,\frac{1490967}{2000000},1\frac{4502311}{10000000}\right)$ in (3.1) we find an asymptotic optimum estimator (AOE) for the suggested class of estimators for the population median as

$$\psi\left(\hbar_{ut},\theta_{ut},\vartheta_{ut},\tau_{ut}\right)=\left(17,13,\frac{1490967}{2000000},1\frac{4502311}{10000000}\right)$$

$$=\sum_{t=1}^{M}W_t^{2}\hat{Q}_{vt}\left[\left[\left\{17-13\left(\frac{\hat{Q}_{ut}}{Q_{ut}}\right)^{\frac{1490967}{2000000}}\exp\left[1\frac{4502311\left(\hat{Q}_{ut}-Q_{ut}\right)}{10000000\left(\hat{Q}_{ut}+Q_{ut}\right)}\right]\right\}\right]\right] \tag{6.13}$$

and the first degree approximation of bias and mean square error given respectively in Table 3

,

$$B\left[\psi\left(15,13,\frac{1490967}{2000000},1\frac{4502311}{100000000}\right)\right]\cong\frac{1-f_t}{d_t}W_tQ_{vt}C_{Mut}^{2}\left\{\frac{-K_t}{4\theta_{ut}}\left[2K_t-2\theta_{ut}+4K_t\theta_{ut}\right]\right\} \tag{6.14}$$

$$MSE\left[\psi\left(15,13,\frac{1490967}{2000000},1\frac{4502311}{10000000}\right)\right]=\sum_{t=1}^{M}W_t^{2}\frac{\left(1-f_t\right)Q_{vt}}{d_t}\left(C_{Mvt}^{2}+\frac{294199}{4000000}C_{Mut}^{2}\left\{\frac{4971}{10}-52K_t\right\}\right) \tag{6.15}$$

Table 3. Bias and Mean Square error (MSE) values of members of the suggested class of estimators.

Estimators	Bias	MSE	%RE
$\hat{Q}_{vst}$	0.00	284303.40	100.00
$\hat{Q}_{RS}$	16.99	64711.48	439.34
$\hat{Q}_{PS}$	107.58	1083839.42	26.23
$\hat{Q}_{\exp RS}$	-7.08	102014.42	278.69
$\hat{Q}_{\exp PS}$	38.22	611578.41	46.49
$\hat{Q}_{CRS}$	339.72	2463319.41	11.54
$\psi\left(\hbar_{t1},\theta_{t1},\vartheta_{t1},\tau_{t1}\right)=\left(3,2,177387/1100000,21/55\right)$	-6.47	341.89	83156.59
$\psi\left(\hbar_{t2},\theta_{t2},\vartheta_{t2},\tau_{t2}\right)=\left(4,3,20283/200000,12/30\right)$	-57.98	88186.16	322.39
$\psi\left(\hbar_{t3},\theta_{t3},\vartheta_{t3},\tau_{t3}\right)=\left(5,4,3943/80000,15/50\right)$	-40.69	50804.88	559.60
$\psi\left(\hbar_{t4},\theta_{t4},\vartheta_{t4},\tau_{t4}\right)=\left(5,4,13179/304000,35/95\right)$	-78.95	99681.49	285.21
$\psi\left(\hbar_{ut},\theta_{ut},\vartheta_{ut},\tau_{ut}\right)=\left(17,13,\frac{1490967}{2000000},1\frac{4502311}{10000000}\right)$	-184.08	59753.60	475.79

7. Conclusion

In this study, we have suggested an exponential ratio estimator of the median in stratified random sampling, and its asymptotic conditions were also derived which shows our suggested estimator has the same expression as the unbiased linear regression estimator for mean square error. Thus our suggested exponential ratio estimator of the median using ancillary information and the combination of scalars is the general class of estimator and the existing estimators are actually the members of a proposed class of estimator and performs better at optimal conditions which we have revealed from Table 3. Thus our suggested estimator is preferred over existing estimators and is an alternative to regression estimator for practical applications in case of skewed data.

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