**Catalogue of First order NB designs in Bloc*k*s of Size Three and Four**

* **Catalogue of First order NBDs for 3 ≤ *v* ≤ 50 and *k* = 3**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| ***v*** | **λ1** | **Set(s) of Shifts** | **b** | **Efficiency** | **Upper Bound (UB)** | **% of Upper Bound** |
| 4 | 2 | [1]t+[2]t | 6 | 0.7959 | 0.8889 | 89.5376 |
| 1 | [1]t | 3 | 0.7261 | 0.8889 | 81.6852 |
| 6 | 2 | [1,1]+[2]t+[3]t | 15 | 0.7647 | 0.8000 | 95.5875 |
| 8 | 2 | [1,1]+[2,2]+[3]t+[4]t | 28 | 0.7514 | 0.7619 | 98.6219 |
| 1 | [1,5]+[3]t | 14 | 0.6845 | 0.7619 | 89.8412 |
| 10 | 2 | [1,1]+[2,2]+[3,3]+[4]t+[5]t | 45 | 0.7375 | 0.7407 | 99.5680 |
| 12 | 2 | [1,1]+[2,2]+[3,3]+[4,4]+[5]t+[6]t | 66 | 0.7272 | 0.7273 | 99.9863 |
| 1 | [1,9]+[3,7]+[5]t | 33 | 0.6592 | 0.7273 | 90.6366 |
| 14 | 2 | [1,1]+[2,2]+[3,3]+[4,4]+[5,5]+[6]t+[7]t | 91 | 0.7193 | 0.7193 | 100.00 |
| 16 | 2 | [1,1]+[2,2]+[3,3]+[4,4]+[5,5]+[6,6]+[7]t+ [8]t | 120 | 0.7135 | 0.7135 | 100.00 |
| 1 | [1,13]+[3,11]+[5,9]+[7]t | 60 | 0.6448 | 0.7111 | 90.6764 |
| 18 | 2 | [1,1]+[2,2]+[3,3]+[4,4]+[5,5]+[6,6]+[7,7]+[8]t+ [9]t | 153 | 0.7095 | 0.7095 | 100.00 |
| 20 | 2 | [1,1]+[2,2]+[3,3]+[4,4]+[5,5]+[6,6]+[7,7]+[8,8]+ [9]t+[10]t | 190 | 0.7047 | 0.7047 | 100.00 |
| 1 | [1,17]+[3,15]+[5,13]+[7,11]+[9]t | 95 | 0.6365 | 0.7018 | 90.6954 |
| 22 | 2 | [1,1]+[2,2]+[3,3]+[4,4]+[5,5]+[6,6]+[7,7]+[8,8]+ [9,9]+[10]t+[11]t | 231 | 0.7015 | 0.7015 | 100.00 |
| 24 | 2 | [1,1]+[2,2]+[3,3]+[4,4]+[5,5]+[6,6]+[7,7]+[8,8]+ [9,9]+[10,10]+ [11]t+[12]t | 276 | 0.6997 | 0.6997 | 100.00 |
| 1 | [1,21]+[3,19]+[5,17]+[7,15]+[9,13]+[11]t | 138 | 0.6311 | 0.6957 | 90.7144 |
| 26 | 2 | [1,1]+[2,2]+[3,3]+[4,4]+[5,5]+[6,6]+[7,7]+[8,8]+ [9,9]+[10,10]+[11,11]+[12]t+[13]t | 325 | 0.6968 | 0.6968 | 100.00 |
| 28 | 2 | [1,1]+[2,2]+[3,3]+[4,4]+[5,5]+[6,6]+[7,7]+[8,8]+ [9,9]+[10,10]+[11,11]+[12,12]+ [13]t+[14]t | 378 | 0.6942 | 0.6942 | 100.00 |
| 1 | [1,25]+[3,23]+[5,21]+[7,19]+[9,17]+[11,15]+ [13]t | 189 | 0.6278 | 0.6914 | 90.8013 |
| 30 | 2 | [1,1]+[2,2]+[3,3]+[4,4]+[5,5]+[6,6]+[7,7]+[8,8]+ [9,9]+[10,10]+[11,11]+[12,12]+[13,13]+[14]t+ [15]t | 435 | 0.6923 | 0.6923 | 100.00 |
| 32 | 2 | [1,1]+[2,2]+[3,3]+[4,4]+[5,5]+[6,6]+[7,7]+[8,8]+ [9,9]+[10,10]+[11,11]+[12,12]+[13,13]+[14,14]+ [15]t+[16]t | 496 | 0.6921 | 0.6921 | 100.00 |
| 1 | [1,29]+[3,27]+[5,25]+[7,23]+[9,21]+[11,19]+ [13,17]+[15]t | 248 | 0.6250 | 0.6882 | 90.8166 |
| 34 | 2 | [1,1]+[2,2]+[3,3]+[4,4]+[5,5]+[6,6]+[7,7]+[8,8]+ [9,9]+[10,10]+[11,11]+[12,12]+[13,13]+[14,14]+ [15,15]+[16]t+[17]t | 561 | 0.6904 | 0.6904 | 100.00 |
| 36 | 2 | [1,1]+[2,2]+[3,3]+[4,4]+[5,5]+[6,6]+[7,7]+[8,8]+ [9,9]+[10,10]+[11,11]+[12,12]+[13,13]+[14,14]+ [15,15]+[16,16]+[17]t+ [18]t | 630 | 0.6884 | 0.6884 | 100.00 |
| 1 | [1,33]+[3,31]+[5,29]+[7,27]+[9,25]+[11,23]+ [13,21]+[15,19]+[17]t | 315 | 0.6233 | 0.6857 | 90.8998 |
| 38 | 2 | [1,1]+[2,2]+[3,3]+[4,4]+[5,5]+[6,6]+[7,7]+[8,8]+ [9,9]+[10,10]+[11,11]+[12,12]+[13,13]+[14,14]+ [15,15]+[16,16]+[17,17]+[18]t+[19]t | 703 | 0.6880 | 0.6880 | 100.00 |
| 40 | 2 | [1,1]+[2,2]+[3,3]+[4,4]+[5,5]+[6,6]+[7,7]+[8,8]+ [9,9]+[10,10]+[11,11]+[12,12]+[13,13]+[14,14]+ [15,15]+[16,16]+[17,17]+[18,18]+[19]t+[20]t | 780 | 0.6859 | 0.6859 | 100.00 |
| 1 | [1,37]+[3,35]+[5,33]+[7,31]+[9,29]+[11,27]+ [13,25]+[15,23]+[17,21]+[19]t | 390 | 0.6227 | 0.6838 | 91.0646 |
| 42 | 2 | [1,1]+[2,2]+[3,3]+[4,4]+[5,5]+[6,6]+[7,7]+[8,8]+ [9,9]+[10,10]+[11,11]+[12,12]+[13,13]+[14,14]+ [15,15]+[16,16]+[17,17]+[18,18]+[19,19]+[20]t+ [21]t | 861 | 0.6871 | 0.6871 | 100.00 |
| 44 | 2 | [1,1]+[2,2]+[3,3]+[4,4]+[5,5]+[6,6]+[7,7]+[8,8]+ [9,9]+[10,10]+[11,11]+[12,12]+[13,13]+[14,14]+ [15,15]+[16,16]+[17,17]+[18,18]+[19,19]+ [20,20]+[21]t+[22]t | 946 | 0.6856 | 0.6856 | 100.00 |
| 1 | [1,41]+[3,39]+[5,37]+[7,35]+[9,33]+[11,31]+ [13,29]+[15,27]+[17,25]+[19,23]+[21]t | 473 | 0.6218 | 0.6822 | 91.1463 |
| 46 | 2 | [1,1]+[2,2]+[3,3]+[4,4]+[5,5]+[6,6]+[7,7]+[8,8]+ [9,9]+[10,10]+[11,11]+[12,12]+[13,13]+[14,14]+ [15,15]+[16,16]+[17,17]+[18,18]+[19,19]+ [20,20]+[21,21]+[22]t+ [23]t | 1035 | 0.6836 | 0.6836 | 100.00 |
| 48 | 2 | [1,1]+[2,2]+[3,3]+[4,4]+[5,5]+[6,6]+[7,7]+[8,8]+ [9,9]+[10,10]+[11,11]+[12,12]+[13,13]+[14,14]+ [15,15]+[16,16]+[17,17]+[18,18]+[19,19]+ [20,20]+[21,21]+[22,22]+[23]t+[24]t | 1128 | 0.6842 | 0.6842 | 100.00 |
| 1 | [1,45]+[3,43]+[5,41]+[7,39]+[9,37]+[11,35]+ [13,33]+[15,31]+[17,29]+[19,27]+[21,25]+[23]t | 564 | 0.6213 | 0.6809 | 91.2469 |
| 50 | 2 | [1,1]+[2,2]+[3,3]+[4,4]+[5,5]+[6,6]+[7,7]+[8,8]+ [9,9]+[10,10]+[11,11]+[12,12]+[13,13]+[14,14]+ [15,15]+[16,16]+[17,17]+[18,18]+[19,19]+ [20,20]+[21,21]+[22,22]+[23,23]+[24]t+[25]t | 1225 | 0.6838 | 0.6838 | 100.00 |

**6.2 Catalogue of First order NB designs for 4 ≤ *v* ≤ 50 and *k* = 4**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| ***v*** | **λ1** | **Set(s) of Shifts** | **B** | **Efficiency** | **Upper Bound** | **% of Upper Bound** |
| 4 | 2 | [1,2,1] |  | 0.8823 | 1.0000 | 88.2300 |
| 5 | 3 | [1,1,1]+[2,2,2] | 10 | 0.9377 | 0.9377 | 100.00 |
| 6 | 1 | [1,2]t | 5 | 0.8197 | 0.9000 | 91.0778 |
| 7 | 1 | [1,2,3] | 7 | 0.875 | 0.875 | 100.000 |
| 8 | 3 | [1,1,1]+[2,2]t+[2,3]t+[3,3]t | 28 | 0.8669 | 0.8669 | 100.00 |
| 9 | 1 | [1,7,3]+[4,4,4](1/3) | 12 | 0.7053 | 0.8438 | 83.5862 |
| 10 | 2 | [1,2,7]+[4,5,4]+[1,2,7] | 30 | 0.7972 | 0.8333 | 95.6678 |
| 1 | [1,8,3]+[4,5,6](1/2) | 15 | 0.8028 | 0.8333 | 96.3399 |
| 11 | 3 | [1,1,1]+[2,2,2]+[3,3,3]+[4,4,4]+[5,5,5] | 55 | 0.8251 | 0.8251 | 100.00 |
| 12 | 1 | [1,2,3]+[4,5]t | 22 | 0.7875 | 0.8182 | 96.2479 |
| 13 | 1 | [1,2,3]+[4,5,6] | 26 | 0.8070 | 0.8125 | 99.3231 |
| 14 | 3 | [1,1,1]+[2,2,2]+[3,3,3]+[4,4,4]+ [5,5]t +[5,6]t+[6,6]t | 91 | 0.8176 | 0.8176 | 100.00 |
| 15 | 1 | [1,13,3]+[4,10,6]+[7,7,7](1/3) | 35 | 0.6993 | 0.8036 | 87.0209 |
| 16 | 2 | [1,2,13]+[4,5,10]+[7,8,7]+[1,2,13]+ [4,5,10] | 80 | 0.7788 | 0.8000 | 97.3500 |
| 1 | [1,14,3]+[4,11,6]+[7,8,9] (1/2) | 40 | 0.7645 | 0.8000 | 95.5625 |
| 17 | 3 | [1,1,1]+[2,2,2]+[3,3,3]+[4,4,4]+ [5,5,5]+[6,6,6]+[7,7,7]+[8,8,8] | 136 | 0.7972 | 0.7972 | 100.00 |
| 18 | 1 | [1,2,3]+[4,5,6]+[7,8]t | 51 | 0.7723 | 0.7941 | 97.2548 |
| 19 | 1 | [1,2,3]+[4,5,6]+[7,8,9] | 57 | 0.7869 | 0.7917 | 99.3937 |
| 20 | 3 | [1,1,1]+[2,2,2]+[3,3,3]+[4,4,4]+[5,5,5]+[6,6,6]+[7,7,7]+[8,8]t+[8,9]t+[9,9]t | 190 | 0.7945 | 0.7945 | 100.00 |
| 21 | 1 | [1,19,3]+[4,16,6]+[7,13,9]+ [10,10,10](1/3) | 70 | 0.7100 | 0.7875 | 90.1587 |
| 22 | 2 | [1,2,19]+[4,5,16]+[7,8,13]+[10,11,10] +[1,2,19]+[4,5,16]+[7,8,13] | 154 | 0.7697 | 0.7857 | 97.9636 |
| 1 | [1,20,3]+[4,17,6]+[7,14,9]+ [10,11,12](1/2) | 77 | 0.7479 | 0.7857 | 95.1890 |
| 23 | 3 | [1,1,1]+[2,2,2]+[3,3,3]+[4,4,4]+ [5,5,5]+[6,6,6]+[7,7,7]+[8,8,8]+ [9,9,9]+[10,10,10]+[11,11,11] | 253 | 0.7837 | 0.7841 | 99.9490 |
| 24 | 1 | [1,2,3]+[4,5,6]+[7,8,9]+[10,11]t | 92 | 0.7771 | 0.7826 | 99.2972 |
| 25 | 1 | [1,2,3]+[4,5,6]+[7,8,9]+[10,11,12] | 100 | 0.7783 | 0.7812 | 99.6288 |
| 26 | 3 | [1,1,1]+[2,2,2]+[3,3,3]+[4,4,4]+ [5,5,5]+[6,6,6]+[7,7,7]+[8,8,8]+ [9,9,9]+[10,10,10]+[11,11]t + [11,12]t+[12,12] t | 325 | 0.7829 | 0.7829 | 100.00 |
| 27 | 1 | [1,25,3]+[4,22,6]+[7,19,9]+[10,16,12] +[13,13,13](1/3) | 90 | 0.7077 | 0.7788 | 90.8706 |
| 28 | 2 | [1,2,25]+[4,5,22]+[ 7,8,19]+[10,11,16] +[13,14,13]+[1,2,25]+[4,5,22]+  [7,8,19]+[10,11,16] | 252 | 0.7645 | 0.7778 | 98.2900 |
| 1 | [1,26,3]+[4,23,6]+[7,20,9]+[10,17,12] +[13,14,15](1/2) | 126 | 0.7388 | 0.7778 | 94.9859 |
| 29 | 3 | [1,1,1]+[2,2,2]+[3,3,3]+[4,4,4]+[5,5,5]+[6,6,6]+[7,7,7]+[8,8,8]+[9,9,9]+ [10,10,10]+[11,11,11]+[12,12,12]+ [13,13,13]+[14,14,14] | 406 | 0.7764 | 0.7768 | 99.9485 |
| 30 | 1 | [1,2,3]+[4,5,6]+[7,8,9]+[10,11,12]+ [13,14]t | 145 | 0.7693 | 0.7759 | 99.1494 |
| 31 | 1 | [1,2,3]+[4,5,6]+[7,8,9]+[10,11,12]+ [13,14,15] | 155 | 0.7728 | 0.7750 | 99.7161 |
| 32 | 3 | [1,1,1]+[2,2,2]+[3,3,3]+[4,4,4]+[5,5,5] +[6,6,6]+[7,7,7]+[8,8,8]+[9,9,9]+ [10,10,10]+[11,11,11]+[12,12,12]+ [13,13,13]+[14,14]t+[14,15]t+[15,15]t | 496 | 0.7770 | 0.7770 | 100.00 |
| 33 | 1 | [1,31,3]+[4,28,6]+[7,25,9]+ [10,28,12]+[13,25,15]+[16,16,16](1/3) | 176 | 0.7273 | 0.7734 | 94.0393 |
| 34 | 2 | [1,2,31]+[4,5,28]+[7,8,25]+ [10,11,22]+[13,14,19]+[16,17,16]+ [1,2,31]+[4,5,28]+[7,8,25]+ [10,11,22]+[13,14,19] | 374 | 0.7603 | 0.7727 | 98.3952 |
| 1 | [1,32,3]+[4,29,6]+[7,26,9]+ [10,23,12]+[13,20,15]+[16,17,18](1/2) | 187 | 0.7331 | 0.7727 | 94.8751 |
| 35 | 3 | [1,1,1]+[2,2,2]+[3,3,3]+[4,4,4]+ [5,5,5]+[6,6,6]+[7,7,7]+[8,8,8]+ [9,9,9]+[10,10,10]+[11,11,11]+ [12,12,12]+[13,13,13]+[14,14,14]+ [15,15,15]+[16,16,16]+[17,17,17] | 595 | 0.7740 | 0.7740 | 100.00 |
| 36 | 1 | [1,2,3]+[4,5,6]+[7,8,9]+[10,11,12]+ [13,14,15]+[16,17] t | 210 | 0.7619 | 0.7714 | 98.7685 |
| 37 | 1 | [1,2,3]+[4,5,6]+[7,8,9]+[10,11,12]+ [13,14,15] + [16,17,18] | 222 | 0.7688 | 0.7708 | 99.7405 |
| 38 | 3 | [1,1,1]+[2,2,2]+[3,3,3]+[4,4,4]+[5,5,5]+[6,6,6]+[7,7,7]+[8,8,8]+[9,9,9]+ [10,10,10]+[11,11,11]+[12,12,12]+ [13,13,13]+[14,14,14]+[15,15,15]+ [16,16,16]+[17,17] t [17,18]t+[18,18]t | 703 | 0.7730 | 0.7730 | 100.00 |
| 39 | 1 | [1,37,3]+[4,34.6]+[7,31,9]+ [10,28,12]+[13,25,15]+[16,22,18]+ [19,19,19] (1/3) | 247 | 0.7096 | 0.7697 | 92.1918 |
| 40 | 2 | [1,2,37]+[4,5,34]+[7,8,31]+[10,11,28] +[13,14,25]+[16,17,22]+[19,20,19]+ [1,2,37]+[4,5,34]+[7,8,31]+[10,11,28] +[13,14,25]+[16,17,22] | 520 | 0.7567 | 0.7692 | 98.3749 |
| 1 | [1,38,3]+[4,35,6]+[7,32,9]+ [10,29,12]+[13,26,15]+[16,23,18]+ [19,20,21](1/2) | 260 | 0.7284 | 0.7692 | 94.6958 |
| 41 | 3 | [1,1,1]+[2,2,2]+[3,3,3]+[4,4,4]+[5,5,5]+[6,6,6]+[7,7,7]+[8,8,8]+[9,9,9]+ [10,10,10]+[11,11,11]+[12,12,12]+ [13,13,13]+[14,14,14]+[15,15,15]+ [16,16,16]+[17,17,17]+[18,18,18]+ [19,19,19]+[20,20,20] | 820 | 0.7669 | 0.7688 | 99.7529 |
| 42 | 1 | [1,2,3]+[4,5,6]+[7,8,9]+[10,11,12]+ [13,14,15]+[16,17,18]+[19,20]t | 287 | 0.7646 | 0.7683 | 99.5184 |
| 43 | 1 | [1,2,3]+[4,5,6]+[7,8,9]+[10,11,12]+ [13,14,15]+[16,17,18]+[19,20,21] | 301 | 0.7664 | 0.7679 | 99.8047 |
| 44 | 3 | [1,1,1]+[2,2,2]+[3,3,3]+[4,4,4]+[5,5,5]+[6,6,6]+[7,7,7]+[8,8,8]+[9,9,9]+ [10,10,10]+[11,11,11]+[12,12,12]+ [13,13,13]+[14,14,14]+[15,15,15]+ [16,16,16]+[17,17,17]+[18,18,18]+ [19,19,19]+[20,20]t+[20,21]t+[21,21]t | 946 | 0.7696 | 0.7696 | 100.00 |
| 45 | 1 | [1,43,3]+[4,40,6]+[7,37,9]+[10,34,12]+[13,31,15]+[16,28,18]+[19,25,21]+ [22,22,22](1/3) | 323 | 0.7092 | 0.7670 | 92.4641 |
| 46 | 2 | [1,2,43]+[4,5,40]+[7,8,37]+[10,11,34] +[13,14,31]+[16,17,28]+[19,20,25]+ [22,23,22]+[1,2,43]+[4,5,40]+[7,8,37]+[10,11,34]+[13,14,31]+[16,17,28]+ [19,20,25] | 690 | 0.7548 | 0.7667 | 98.4479 |
| 1 | [1,44,3]+[4,41,6]+[7,38,9]+[10,35,12]+[13,32,15]+[16,29,18]+[19,26,21]+ [22,23,24](1/2) | 345 | 0.7262 | 0.7667 | 94.7176 |
| 47 | 3 | [1,1,1]+[2,2,2]+[3,3,3]+[4,4,4]+ [5,5,5]+[6,6,6]+[7,7,7]+[8,8,8]+ [9,9,9]+[10,10,10]+[11,11,11]+ [12,12,12]+[13,13,13]+[14,14,14]+ [15,15,15]+[16,16,16]+[17,17,17]+ [18,18,18]+[19,19,19]+[20,20,20]+ [21,21,21]+[22,22,22]+[23,23,23] | 1081 | 0.7655 | 0.7663 | 99.8956 |
| 48 | 1 | [1,2,3]+[4,5,6]+[7,8,9]+[10,11,12]+[13,14,15]+[16,17,18]+[19,20,21]+ [22,23]t | 329 | 0.7614 | 0.7660 | 99.3995 |
| 49 | 1 | [1,2,3]+[4,5,6]+[7,8,9]+[10,11,12]+[13,14,15]+ [16,17,18] + [19,20,21] +[22,23,24] | 343 | 0.7641 | 0.7656 | 99.8041 |
| 50 | 3 | [1,1,1]+[2,2,2]+[3,3,3]+[4,4,4]+[5,5,5]+[6,6,6]+[7,7,7]+[8,8,8]+[9,9,9]+[10,10,10]+[11,11,11]+[12,12,12]+[13,13,13]+[14,14,14]+[15,15,15]+[16,16,16]+[17,17,17]+[18,18,18]+[19,19,19]+ [20,20,20]+[21,21,21]+[22,22,22]+ [23,23]t+[23,24]t+[24,24]t | 1225 | 0.7675 | 0.7675 | 100.00 |