Let $J=[a,b]\subseteq\mathbb{R}$ be an interval and $0<q<1$, q-derivative of a continuous function $f:J\rightarrow\mathbb{R}$ at a point $x\in J$ is given in the following definition.

Definition 1. *(Sudsutad et al., 2015) Let $f:J\rightarrow\mathbb{R}$ be a continuous function and let $x\in J$. Then q-derivative of f at x is defined by the expression*

$$ {}_aD_qf\left(x\right)=\frac{f\left(x\right)-f\left(qx+\left(1-q\right)a\right)}{\left(1-q\right)\left(x-a\right)},x\neq a. \tag{0.1}$$

Since $f:J\rightarrow\mathbb{R}$ is a continuous function, thus we have ${}_aD_qf\left(a\right)=\lim\limits_{x\rightarrow a}{}_aD_qf\left(x\right)$. The function f is said to be q-differentiable on J if ${}_aD_qf\left(x\right)$ exists for all $x\in J$. If $a=0$ in (0.1), then ${}_0D_qf\left(x\right)=D_qf\left(x\right)$, where $D_qf\left(x\right)$ is the well-known q-derivative of f defined by the expression

$$D_qf\left(x\right)=\frac{f\left(qx\right)-f\left(x\right)}{\left(1-q\right)x},x\neq 0. \tag{0.2}$$

For more details on q-derivative given above by (0.2), we refer the reader to (Kac and Cheung, 2002).

Definition 2. *(Sudsutad et al., 2015) Let $f:J\rightarrow\mathbb{R}$ be a continuous function. A second-order q-derivative on J denoted by ${}_aD_q^2f$, provided ${}_aD_qf$ is q-differentiable on J, is defined as ${}_aD_q^2f={}_aD_q\left({}_aD_qf\right):J\rightarrow\mathbb{R}$. Similarly higher order q-derivatives on J is defined by ${}_aD_q^nf={}_aD_q\left({}_aD_q^{n-1}f\right):J\rightarrow\mathbb{R}$.*

The following result is very important to evaluate q-derivative of monomials.

Lemma 1. *(Sudsutad et al., 2015) Let $\alpha\in\mathbb{R}$ and $0<q<1$, we have*

$${}_aD_q\left(x-a\right)^{\alpha}=\left(\frac{1-q^{\alpha}}{1-q}\right)\left(x-a\right)^{\alpha-1}.$$

One can find further properties of q-derivatives in (Tariboon and Ntouyas, 2013).

Definition 3. *(Sudsutad et al., 2015) Suppose that $f:J\rightarrow\mathbb{R}$ is a continuous function. Then the definite q-integral on J is defined by*

$$\int_a^x f\left(x\right)_a d_qx=\left(x-a\right)\left(1-q\right)\sum_{n=0}^{\infty}q^nf\left(q^nx+\left(1-q^n\right)a\right) \tag{0.3}$$

for $x\in J$. If $c\in(a,x)$, then the definite q-integral on J is defined as

$$\begin{aligned}\int_c^x f\left(x\right)_a d_qx&=\int_a^x f\left(x\right)_a d_qx-\int_a^c f\left(x\right)_a d_qx\\&=\left(x-a\right)\left(1-q\right)\sum_{n=0}^{\infty}q^nf\left(q^nx+\left(1-q^n\right)a\right)\\&-\left(c-a\right)\left(1-q\right)\sum_{n=0}^{\infty}q^nf\left(q^nc+\left(1-q^n\right)a\right).\end{aligned}$$

If $a=0$ in (0.3), then we get the classical q-definite integral defined by see (Ernst, 2012).

$$\int_0^x f\left(x\right)\ {}_0d_qx=\left(1-q\right)x\sum_{n=0}^{\infty}q^nf\left(q^nx\right),x\in[0,\infty).$$

The following results hold about definite q-integrals.

Theorem 1. *(Tariboon and Ntouyas, 2013) Let* $f : J \to \mathbb{R}$ *be a continuous function. Then*

(1) ${}_aD_q \int_a^x f(t)\, {}_a d_q t = f(x)$
(2) $\int_c^x {}_aD_q f(t)\, {}_a d_q t = f(x) - f(c),\ c \in (a, x)$.

Theorem 2. *(Tariboon and Ntouyas, 2013) Suppose that* $f, g : J \to \mathbb{R}$ *are continuous functions,* $\alpha \in \mathbb{R}$*. Then, for* $x \in J$,

(1) $\int_a^x [f(t) + g(t)]\, {}_a d_q t = \int_a^x f(t)\, {}_a d_q t + \int_a^x g(t)\, {}_a d_q t$;
(2) $\int_a^x \alpha f(t)\, {}_a d_q t = \alpha \int_a^x f(t)\, {}_a d_q t$;
(3) $\int_c^x f(t)\, {}_a D_q g(t)\, {}_a d_q t = f(t) g(t)|_c^x - \int_c^x g(qt + (1-q)a)\, {}_a D_q f(t)\, {}_a d_q t,\ c \in (a, x)$.

The following is a valuable result to evaluate definite q-integrals of monomials.

Lemma 2. *(Sudsutad et al., 2015) For* $\alpha \in \mathbb{R}\backslash \{-1\}$ *and* $0 < q < 1$*, the following formula holds:*

$$\int_a^x (x-a)^\alpha \ {}_a d_q x = \left(\frac{1-q}{1-q^{\alpha+1}}\right)(x-a)^{\alpha+1}.$$

We recall the following definitions before proving our main results.

Definition 4. *(Dragomir, 2001) A mapping* $f : [a, b] \times [c, d] \to \mathbb{R}$ *is said to be convex on* $[a, b] \times [c, d]$ *if the inequality*

$$f(\lambda x + (1-\lambda)z, \lambda y + (1-\lambda)w) \leq \lambda f(x, y) + (1-\lambda) f(z, w)$$

holds for all $(x, y), (z, w) \in [a, b] \times [c, d]$ *and* $\lambda \in [0, 1]$.

Definition 5. *(Dragomir, 2001) A function* $f : [a, b] \times [c, d] \to \mathbb{R}$ *is said to be convex on co-ordinates on* $[a, b] \times [c, d]$ *if the partial mappings* $f_y : [a, b] \to \mathbb{R}, f_y(u) = f(u, y)$ *and* $f_x : [c, d] \to \mathbb{R}, f_x(v) = f(x, v)$ *are convex where defined for all* $x \in [a, b], y \in [c, d]$.

A different approach of stating convexity of f on co-ordinates on $[a, b] \times [c, d]$ is given in the definition below.

Definition 6. *(Latif and Alomari, 2009) A function* $f : [a, b] \times [c, d] \to \mathbb{R}$ *is said to be convex on co-ordinates on* $[a, b] \times [c, d]$ *if the inequality*

$$\begin{aligned} &f(tx + (1-t)y, sz + (1-s)w) \\ &\leq tsf(x, z) + t(1-s)f(x, w) + s(1-t)f(y, z) + (1-t)(1-s)f(y, w) \end{aligned}$$

holds for all $(t, s) \in [0, 1] \times [0, 1]$ *and* $(x, z), (y, w) \in [a, b] \times [c, d]$.

Definition 7. *(Özdemir et al., 2012) A function* $f : [a, b] \times [c, d] \subset \mathbb{R}^2 \to \mathbb{R}$ *is said to be quasi-convex on* $[a, b] \times [c, d]$ *if the inequality*

$$f(\lambda x + (1-\lambda)z, \lambda y + (1-\lambda)w) \leq \max\{f(x, y), f(z, w)\}$$

holds for all $(x, y), (z, w) \in [a, b] \times [c, d]$ *and* $\lambda \in [0, 1]$.

Definition 8. *(Özdemir et al., 2012) A function* $f : [a, b] \times [c, d] \to \mathbb{R}$ *is said to be quasi-convex on the co-ordinates on* $[a, b] \times [c, d]$ *if the partial mappings* $f_y : [a, b] \to \mathbb{R}, f_y(u) = f(u, y)$ *and* $f_x : [c, d] \to \mathbb{R}, f_x(v) = f(x, v)$ *are quasi-convex where defined for all* $x \in [a, b], y \in [c, d]$.

Another way of expressing the concept of co-ordinated quasi-convex functions is stated in the definition below.

Definition 9. *(Özdemir et al., 2012) A function* $f : [a,b] \times [c,d] \subset \mathbb{R}^2 \to \mathbb{R}$ *is said to be quasi-convex on co-ordinates on* $[a,b] \times [c,d]$ *if*

$$f(tx + (1-t)z, sy + (1-s)w) \leq \max\{f(x,y), f(x,w), f(z,y), f(z,w)\}$$

holds for all $(x,y), (z,w) \in [a,b] \times [c,d]$ *and* $(s,t) \in [0,1] \times [0,1]$.