Journal of King Saud University – Science 31 (2019) 653–658

Contents lists available at ScienceDirect

Journal of King Saud University – Science

journal homepage: www.sciencedirect.com

Original article

Dispersive solitary wave soliton solutions of (2 + 1)-dimensional Boussineq dynamical equation via extended simple equation method

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ARTICLE INFO

Article history:
Received 4 December 2017
Accepted 28 December 2017
Available online 2 January 2018

Keywords:
Boussineq equation
Extended simple equation method
Exact solitary wave solutions

ABSTRACT

In this article, the extended simple equation method is employed to construct solitary wave solutions of Boussineq equation, which describes gravity waves propagation on the surface of water and also explains the collision of oblique waves transformation. The extended simple equation method is a new technique which helpful to other sorts of nonlinear evolution equations in current areas of research in mathematics and physics.

1. Introduction

Nonlinear evolution equations (NLEEs) are mostly used as models to represent physical phenomena in several fields of sciences, especially in biology, solid state physics, plasma physics, plasma waves, and fluid mechanics. However, finding exact solutions of NLEEs is an tedious exercise and only in certain distinctive cases one can explicitly write down their solutions. In the last few years great improvement have been made in the progress of methods for finding the exact solutions of nonlinear equations but the advancement achieved is insufficient. However, in the last few decades important progress has been made and many powerful and efficient methods for obtaining exact solutions of NLEEs have been suggested in the literature.

It is detained that all of these methods are problem dependent, viz. some approaches work well with influenced problems but not for the others problems. Different authors used different methods to find the solitary waves solution of nonlinear evoulation equations some of these methods are, the Darboux transformation method (Gu et al., 2005), the inverse scattering transform method (Ablowitz and Clarkson, 1991), Hirota bilinear method (Hirota, 2004), Jacobi elliptic function expansion method (Liu et al., 2001), the sine–cosine method (Seadawy, 2015), the homogeneous balance method (Fan and Zhang, 1998; Seadawy, 2017), modified simple equation method (Seadawy, 2014; Lu et al., 2017; Seadawy, 2012a; Seadawy, 2016a; Ali et al., 2017), modified extended direct algebraic method (Arshad et al., 2017), the soliton ansatz method (Yuanfen, 2012; Seadawy, 2016b; Tang and Shukla, 2007; Biswas, 2010; Zhou et al., 2013), Auxiliary equation method (Helal and Seadawy, 2012; Tariq and Seadawy, 2017). The learning about solutions, structures and further properties of solitons and solitary wave solutions gained much concentration (Seadawy and El-Rashidy, 2014; Seadawy, 2012b; Saha and Sarma, 2013; Chen et al., 2003; Imed and Abderrahmen, 2012; Tian, 2017; Tian, 2016; Wang et al., 2017a,b; Jian-Min et al., 2016; Wang et al., 2016; Yang et al., 2016; Yang et al., 2017a,b; Gao et al., 2017).

In this article, we consider one such NLEE, namely, the (2 + 1)-dimensional Boussinesq equation given by

$$v_{tt} - v_{xx} - \alpha(v^2)_{xx} - v_{yy} - v_{xxxx} = 0, \quad \alpha \neq 0. \tag{1}$$

The Eq. (1) is used to study the propagation of gravity waves on the surface of water and head-on collision of an oblique waves. The (2 + 1)-dimensional Boussinesq Eq. (1) combines the mutual propagation of the standard Boussinesq equation with the dependence on a second spatial variable, as that arises in the two-dimensional

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Peer review under responsibility of King Saud University.

Production and hosting by Elsevier

https://doi.org/10.1016/j.jksus.2017.12.015
1018-3647/

Kadomstev-Petviashvili (KP) equation. The unknown function illustrated the elevation of the free surface of the fluid. Here in Eq. (1) the terms u_{xx}, u_{yy} and u_{xxxx} represents the dispersion phenomenon. In previous different authors used different methods on Eq. (1) for finding the exact solitary wave solutions such as, generalized transformation homogeneous balance method (Saha and Sarma, 2013) used to find solitary waves solution, homotopy perturbation method was used (Chen et al., 2003) to find numerical solution, extended ansatz method was employed in Liu and Dai (2010) to find exact periodic waves solutions, the Hirota bilinear method was used in Wazwaz (2010) to attain two soliton solutions, simple equation method used (Moleleki et al., 2013) to find the solitons solution of Eq. (1).

In our current work, we have employed extended simple equation method (Ali et al., 2017) to find the solitary waves solution of the Eq. (1), the obtained solutions are helpful in exploring nonlinear wave phenomena in physical sciences.

The structure of article the as follows: In Section 2, the main steps of the illustrated method are given. In Section 3, we apply the present extended simple equation method on the boussineq equation. Discussion of the results are given in Section 4. The summary of the work is given in Section 5.

2. Description of the method

In this section, we illustrated extended simple equation method to obtain the solitary wave solutions of (2 + 1) dimensional boussineq equation. Consider the nonlinear PDE in the form as:

$$F(v, v_t, v_x, v_y, v_{tt}, v_{xx}, v_{yy}, \ldots) = 0, \tag{2}$$

where F is called a polynomial function of $v(x, y, t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. The basic key steps of the described method are as follows:

Step 1: Consider traveling wave transformation

$$v(x, y, z, t) = V(\xi), \quad \xi = x + y + \omega t, \tag{3}$$

by utilizing the above transformation, the Eq. (2) is reduces into ODE as:

$$G(V, V', V'', V''', \ldots) = 0, \tag{4}$$

where G is a polynomial in $V(\xi)$ its derivatives with respect to ξ.

Step 2: Let us assume that the solution of Eq. (4) has the form as:

$$V(\xi) = \sum^{m} i = -m b_i \Psi^i(\xi) \tag{5}$$

where b_i (i = -n, -n + 1,...,-1, 0, 1,...,n) are arbitrary constants and m is a positive integer, which can be calculated by applying the homogeneous balance principle on Eq. (4).

Let Ψ satisfies the following equation.

$$\Psi'(\xi) = c_0 + c_1\Psi + c_2\Psi^2 + c_3\Psi^3 \tag{6}$$

where c_0, c_1, c_2, c_3, are arbitrary constants.

The general solutions of new simple ansatz Eq. (6) are as following:

$$\Psi(\xi) = -\frac{c_1 - \sqrt{4c_0c_2 - c_1^2}\tan\left(\frac{\sqrt{4c_0c_2-c_1^2}}{2}(\xi + \xi_0)\right)}{2c_2},$$

$$4c_0c_2 > c_1^2,\ c_3 = 0, \tag{7}$$

If $c_0 = 0, c_3 = 0$, then simple ansatz Eq. (6) reduces to Bernoulli equation, which has the following solutions:

$$\Psi(\xi) = \frac{c_1 e^{c_1(\xi+\xi_0)}}{1 - c_2 e^{c_1(\xi+\xi_0)}}, \quad c_1 > 0, \tag{8}$$

$$\Psi(\xi) = \frac{-c_1 e^{c_1(\xi+\xi_0)}}{1 + c_2 e^{c_1(\xi+\xi_0)}}, \quad c_1 < 0. \tag{9}$$

If $c_1 = 0, c_3 = 0$, then the ansatz (6) reduces to Riccati equation, which has the following solutions:

$$\Psi(\xi) = \frac{\sqrt{c_0c_2}}{c_2}\tan(\sqrt{c_0c_2}(\xi + \xi_0)), \quad c_0c_2 > 0, \tag{10}$$

$$\Psi(\xi) = -\frac{\sqrt{-c_0c_2}}{c_2}\tanh(\sqrt{-c_0c_2}(\xi + \xi_0)), \quad c_0c_2 < 0. \tag{11}$$

Step 3: Substituting Eq. (5) along with Eq. (6) into Eq. (4), and collecting the coefficients of $(\Psi)^j$, then setting coefficients equal to zero, we obtained a system of algebraic equations in parameters $b_0, b_1, b_2, b_3, \omega$ and c_i. The system of algebraic equations are solved with the help of Mathematica and we get the values of these parameters.

Step 4: By substituting of all these values of parameters and Ψ into Eq. (5). We obtained the required the solutions of Eq. (2).

3. Applications of the method

(2 + 1)-D Boussineq equation

Consider the traveling waves transformations

$$v(x, y, t) = V(\xi), \quad \xi = x + y + \omega t, \tag{12}$$

By using the above transformations in Eq. (1) and, we have the following ordinary differential equation, we have form as:

$$2\alpha V'^2 + 2\alpha VV'' + (2 - \omega^2)V'' + V^{zqprime} = 0, \tag{13}$$

Here in above we applying the homogeneous balance principle in Eq. (13), we have $m = 2$. We suppose the solution of Eq. (13) has the form as:

$$V(\xi) = b_{-2}\Psi^{-2} + b_{-1}\Psi^{-1} + b_0 + b_1\Psi + b_2\Psi^2 \tag{14}$$

Substituting Eq. (14) along Eq. (6) into Eq. (13), we obtained a system of algebraic equations in parameters, $b_0, b_1, b_2, b_{-1}, b_{-2}, \omega, \alpha, c_0, c_1, c_2, c_3, c_2$. The system of algebraic equations can be solved for these parameters, we have following solutions cases (see Figs. 1–5).

Case 1: $c_3 = 0$,

Family-I

$$\omega = \pm\sqrt{2\alpha b_0 + c_1^2 + 8c_0c_2 + 2},\ \ b_2 = 0,\ \ b_1 = 0,\ \ b_{-1} = -\frac{6c_0c_1}{\alpha},\ \ b_{-2} = -\frac{6c_0^2}{\alpha}. \tag{15}$$

Substituting Eq. (15) into Eq. (14) along with solution of Eq. (6), then the solution of Eq. (1) becomes as:

$$V_1(x,y,t) = b_0 + \frac{12c_2c_0c_1}{\alpha\left(c_1 - \sqrt{4c_0c_2 - c_1^2}\tan\left(\frac{\sqrt{4c_0c_2-c_1^2}}{2}(\xi+\xi_0)\right)\right)} - \frac{24c_2^2c_0^2}{\alpha\left(c_1 - \sqrt{4c_0c_2 - c_1^2}\tan\left(\frac{\sqrt{4c_0c_2-c_1^2}}{2}(\xi+\xi_0)\right)\right)^2},\ \ 4c_0c_2 > c_1^2 \tag{16}$$

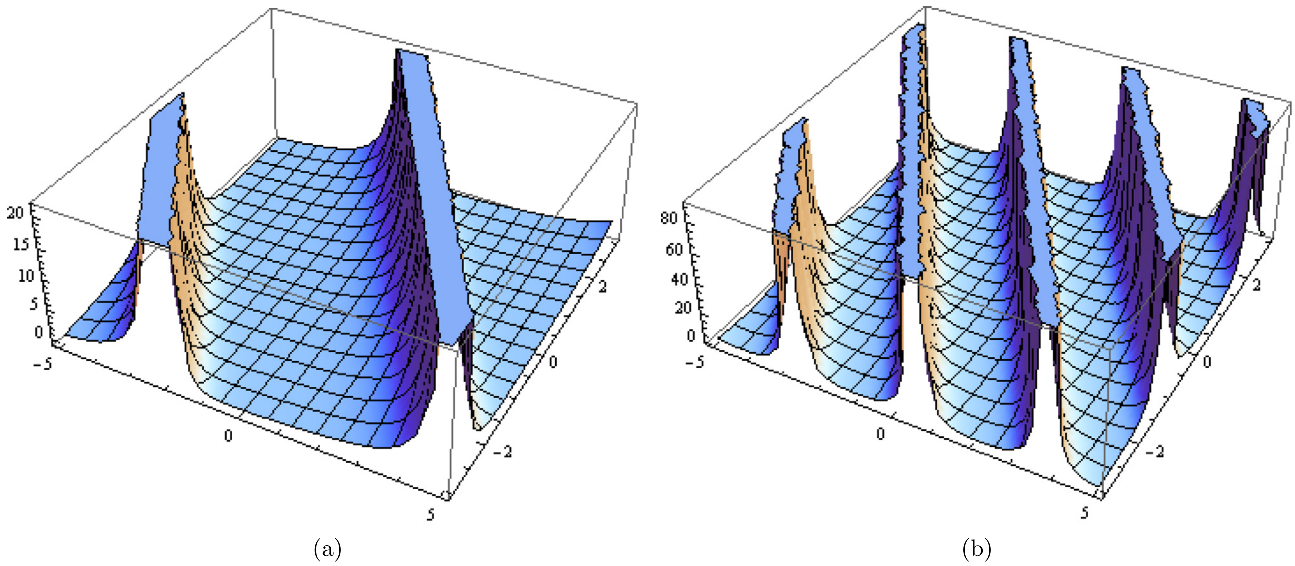


Fig. 1. Exact solitary wave solution of Eq. (16) is plotted at (a) and Eq. (18) at (b) by using the these values of parameters: $\xi_0 = 0.5, b_0 = -0.5, c_0 = 0.5, c_1 = 0.5, c_2 = 0.5, \alpha = 2$ and $\xi_0 = 0.5, b_0 = 0.5, c_0 = 1, c_0.5 = 0.5, c_2 = 1, \alpha = -2$ respectively.

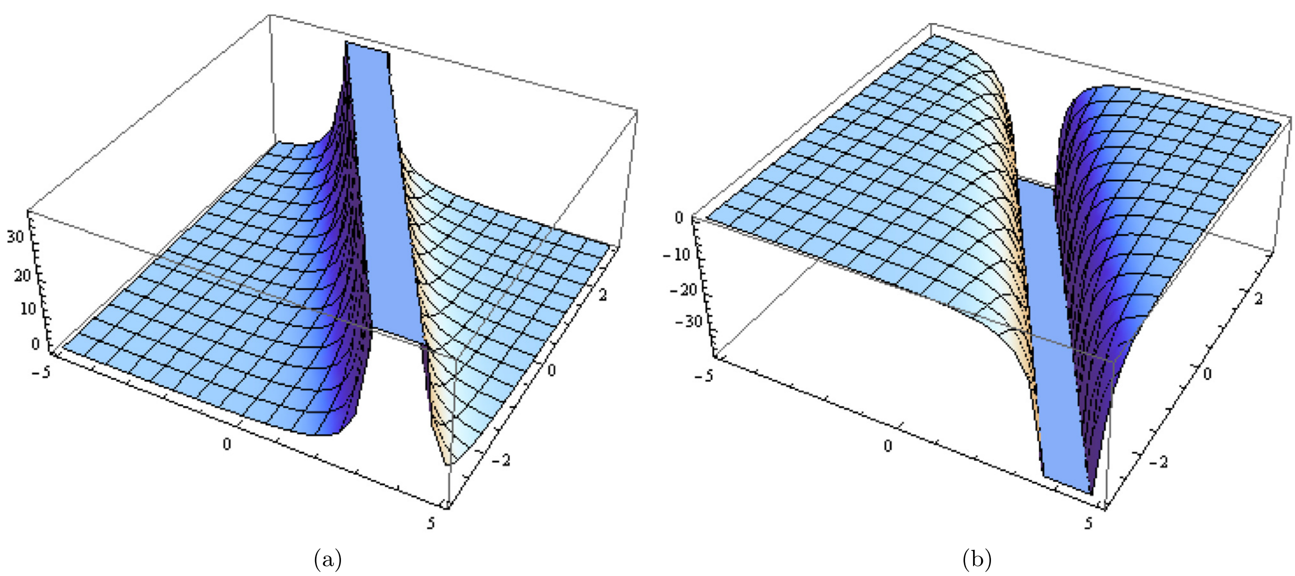


Fig. 2. Exact solitary wave solution of Eq. (20) is plotted at (a) and Eq. (21) at (b) by using the these values of parameters: $\xi_0 = 0.5, b_0 = 0.5, c_1 = 0.5, c_2 = 1, \alpha = -0.5$ and $\xi_0 = 0.5, b_0 = 0.5, c_1 = -0.5, c_2 = 1, \alpha = 0.5$ respectively.

Family-II

$$\omega = \pm\sqrt{2\alpha b_0 + c_1^2 + 8c_0c_2 + 2},\ \ b_2 = -\frac{6c_2^2}{\alpha},$$
$$b_1 = -\frac{6c_1c_2}{\alpha},\ \ b_{-1} = 0,\ \ b_{-2} = 0. \tag{17}$$

Substituting Eq. (17) into Eq. (14) along with Eq. (6) then the solution of Eq. (1) becomes as:

$$V_2(x,y,t) = b_0 + \frac{3c_1\left(c_1 - \sqrt{4c_0c_2 - c_1^2}\tan\left(\frac{\sqrt{4c_0c_2-c_1^2}}{2}(\xi+\xi_0)\right)\right)}{\alpha} - \frac{3\left(c_1 - \sqrt{4c_0c_2 - c_1^2}\tan\left(\frac{\sqrt{4c_0c_2-c_1^2}}{2}(\xi+\xi_0)\right)\right)^2}{2\alpha},\ \ 4c_0c_2 > c_1^2 \tag{18}$$

Case 2: $c_0 = c_3 = 0$,

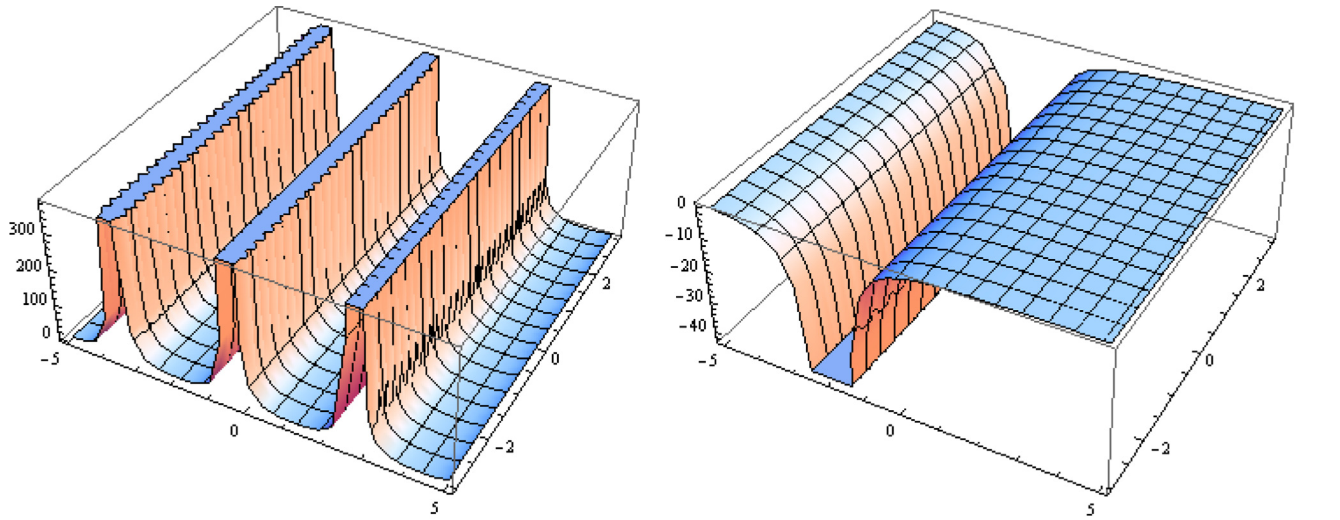
$$\omega = \pm\sqrt{2\alpha b_0 + c_1^2 + 2},\ \ b_2 = -\frac{6c_2^2}{\alpha},\ \ b_1 = -\frac{6c_1c_2}{\alpha},$$
$$b_{-1} = 0,\ \ b_{-2} = 0. \tag{19}$$

Substituting Eq. (19) into Eq. (14) along with Eq. (6) then the solution of Eq. (1) becomes as:

$$V_3(x,y,t) = b_0 - \frac{6c_2c_1^2e^{c_1(\xi+\xi_0)}}{\alpha(1-c_2e^{c_1(\xi+\xi_0)})} - \frac{6c_2^2c_1^2e^{2c_1(\xi+\xi_0)}}{\alpha(1-c_2e^{c_1(\xi+\xi_0)})^2},\ \ c_1 > 0. \tag{20}$$

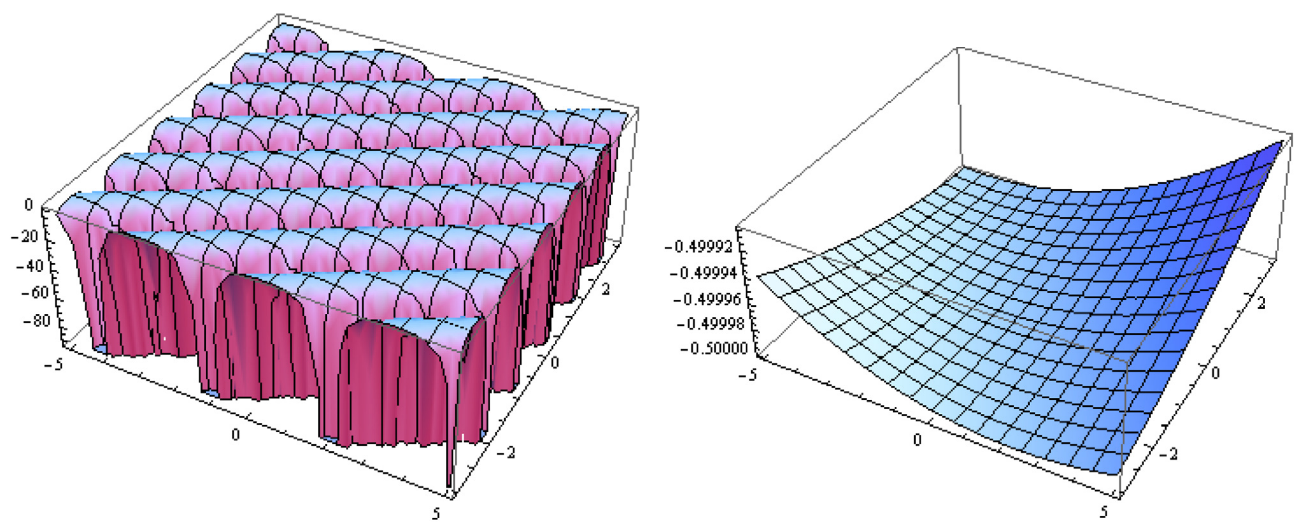
$$V_4(x,y,t) = b_0 + \frac{6c_2c_1^2e^{c_1(\xi+\xi_0)}}{\alpha(1+c_2e^{c_1(\xi+\xi_0)})} - \frac{6c_2^2c_1^2e^{2c_1(\xi+\xi_0)}}{\alpha(1+c_2e^{c_1(\xi+\xi_0)})^2},\ \ c_1 < 0. \tag{21}$$

Case 3: $c_1 = c_3 = 0$,
Family-I



(a) (b)

Fig. 3. Exact solitary wave solution of Eq. (23) is plotted at (a) and Eq. (24) at (b) by using the these values of parameters: $\xi_0 = -0.5, b_0 = 0.5, c_0 = -1, c_1 = -0.5, c_2 = 1$ $\alpha = -0.5$ and $\xi_0 = -0.5, b_0 = 0.5, c_0 = -0.01, c_1 = -0.5, c_2 = 2, \alpha = 0.5$ respectively.



(a) (b)

Fig. 4. Exact solitary wave solution of Eq. (26) is plotted at (a) and Eq. (27) at (b) by using the these values of parameters: $\xi_0 = -0.5, b_0 = -0.5, c_0 = -1, c_1 = -0.5, c_2 = 1, \alpha = 2$ and $\xi_0 = -0.5, b_0 = -0.5, c_0 = 0.01, c_1 = -0.5, c_2 = -0.01, \alpha = -0.05$ respectively.

$$\omega = \pm\sqrt{2(\alpha b_0 + 4c_0c_2 + 1)},\;\; b_2 = 0,\;\; b_1 = 0,\;\; b_{-1} = 0,\;\; b_{-2} = -\frac{6c_0^2}{\alpha}. \tag{22}$$

Substituting Eq. (22) into Eq. (14) along with Eq. (6) then the solution of Eq. (1) becomes as:

$$V_5(x,y,t) = b_0 - \frac{6c_0c_2}{\alpha\left(\tan^2(\sqrt{c_0c_2}(\xi+\xi_0))\right)},\quad c_0c_2 > 0. \tag{23}$$

$$V_6(x,y,t) = b_0 + \frac{6c_0c_2}{\alpha\left(\tanh^2(\sqrt{-c_0c_2}(\xi+\xi_0))\right)},\quad c_0c_2 < 0. \tag{24}$$

Family-II

$$\omega = \pm\sqrt{2(\alpha b_0 + 4c_0c_2 + 1)},\;\; b_2 = -\frac{6c_2^2}{\alpha},\;\; b_1 = 0,\;\; b_{-1} = 0,\;\; b_{-2} = 0. \tag{25}$$

Substituting Eq. (25) into Eq. (14) along with Eq. (6) then the solution of Eq. (1) becomes as:

$$V_7(x,y,t) = b_0 - \frac{6c_0c_2\tan^2(\sqrt{c_0c_2}(\xi+\xi_0))}{\alpha},\quad c_0c_2 > 0. \tag{26}$$

$$V_8(x,y,t) = b_0 + \frac{6c_0c_2\tanh^2(\sqrt{-c_0c_2}(\xi+\xi_0))}{\alpha},\quad c_0c_2 < 0. \tag{27}$$

Family-III

$$\omega = \pm\sqrt{2(\alpha b_0 + 4c_0c_2 + 1)},\;\; b_2 = -\frac{6c_2^2}{\alpha},\;\; b_1 = 0,\;\; b_{-1} = 0,\;\; b_{-2} = -\frac{6c_0^2}{\alpha}. \tag{28}$$

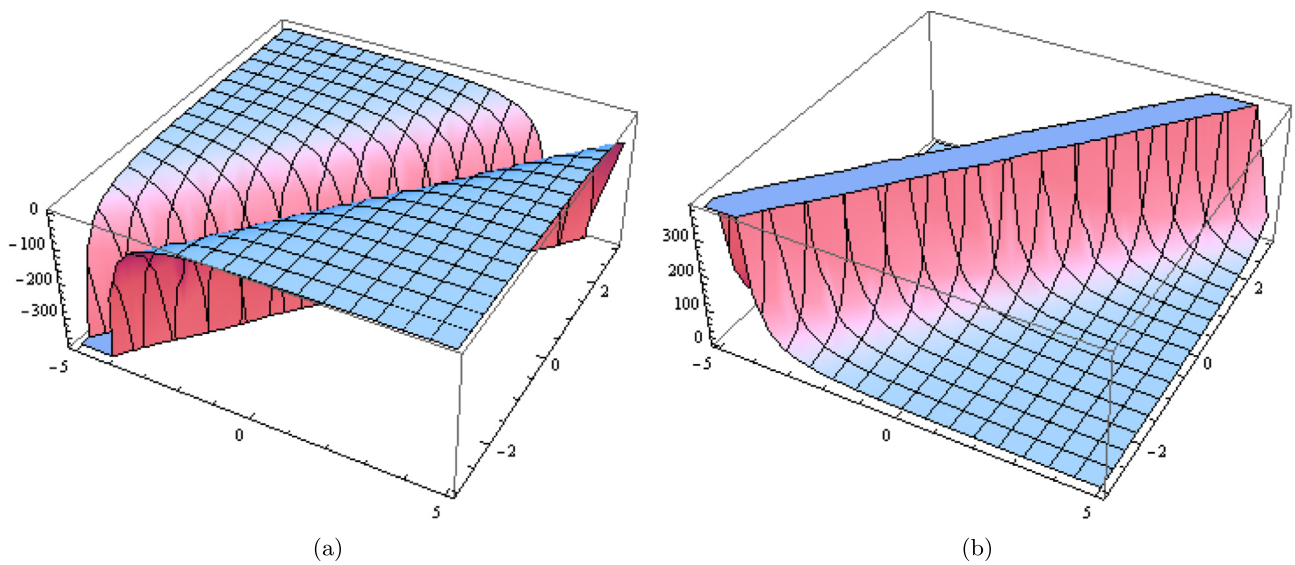


Fig. 5. Exact solitary wave solution of Eq. (29) is plotted at (a) and Eq. (30) at (b) by using the these values of parameters: $\xi_0 = -0.5, b_0 = -0.5, c_0 = 0.01, c_1 = -0.5, c_2 = 0.01$ $\alpha = 0.5$ and $\xi_0 = -0.5, b_0 = -0.5, c_0 = 0.01, c_1 = 2, c_2 = -0.5, \alpha = -0.05$, respectively.

Substituting Eq. (28) into Eq. (14) along with Eq. (6) then the solution of Eq. (1) becomes as:

$$V_9(x,y,t) = b_0 - \frac{6c_2c_0}{\mu_2(\tan^2(\sqrt{c_0c_2}(\xi+\xi_0)))} - \frac{6c_0c_2\tan^2(\sqrt{c_0c_2}(\xi+\xi_0))}{\alpha}, \quad c_0c_2 > 0. \tag{29}$$

$$V_{10}(x,y,t) = b_0 + \frac{6c_0c_2}{\alpha\left(\tanh^2(\sqrt{-c_0c_2}(\xi+\xi_0))\right)} + \frac{6c_0c_2\tanh^2(\sqrt{-c_0c_2}(\xi+\xi_0))}{\alpha}, \quad c_0c_2 < 0. \tag{30}$$

4. Discussion of the results

In this section we comparison our obtained results of $(2+1)$ boussinesq equation with article in Wang et al. (2017a). We attained that our results in Eq. (24), Eq. (27) and Eq. (30) are likely similar to the results that obtained in Eq. (19) and Eq. (23) in Wang et al. (2017a) and left behind of all our solutions are more general and new from that soutions obtained in Wang et al. (2017a).

It is exposed that our method provides an efficient and a more influential mathematical instrument for solving nonlinear evolution equations in different areas of research. It reliable and recommend a variety of exact solutions NPDEs.

5. Conclusion

In this paper, we have employed extended simple equation method to obtain exact soliton solutions of the $(2+1)$-dimensional Boussinesq equation. It is apparent from the analysis we conducted that the $(2+1)$-dimensional equation gives arise to a variety of solitary wave solutions. The achieved solitary wave solutions clarify the complex physical phenomena. We may also conclude from the section of discussion of the results that extended simple equation method is very simple, straightforward. The simplicity and power of the recent method shows that it fruitful to solve different problems in mathematics and physics.

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