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ORIGINAL ARTICLE

The $\left(\frac{G'}{G}\right)$-expansion method for (2 + 1)-dimensional Kadomtsev–Petviashvili equation

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Abstract In this work, we apply a new method to construct the travelling wave solutions involving parameters of the (2 + 1)-dimensional Kadomtsev–Petviashvili equation. When the parameters are taken special values, the solitary waves are derived from the travelling waves. The travelling wave solutions are expressed by the hyperbolic functions, the trigonometric functions and the rational functions.

1. Introduction

The main idea of this method is that the travelling wave solutions of nonlinear equations can be expressed by a polynomial in $\left(\frac{G'}{G}\right)$ where $G = G(\xi)$ satisfies the second order linear ordinary differential equation $G'' + \lambda\, G' + \mu G = 0$ where $\xi = sx + ly - vt$. In recent years, many powerful methods to construct exact solutions of nonlinear evolution equations have been established and developed such as the Jacobi elliptic function expansion, the tanh-method, the truncated Painleve expansion and the $\left(\frac{G'}{G}\right)$-expansion method (Fan, 2000; Inc and Evans, 2004; Liu et al., 2001; Yan, 2003; Yan and Zhang, 1999; Zayed et al., 2005; Zhang et al., 2008; Abdou, 2007; Malfliet, 1992; Parkes and Duffy, 1996; Wang and Li, 2005; Chow, 1995). The rest of the Letter is organized as follows. In Section 2, we describe briefly the $\left(\frac{G'}{G}\right)$-expansion method is briefly described. In Section 3, we apply the method to the (2 + 1)-dimensional Kadomtsev–Petviashvili equation is applied. In Section 4, some conclusions are given.

2. The $\left(\frac{G'}{G}\right)$-expansion method

Now we describe the $\left(\frac{G'}{G}\right)$ expansion method for finding travelling, say in three independent variables x, y and t, and is given by

$$P(u, u_x, u_y, u_t, u_{tt}, u_{xt}, u_{xx}, \ldots) = 0 \tag{1}$$

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In the following, we give the main steps of the $\left(\frac{G'}{G}\right)$-expansion method.

step 1:
Combining the independent variables x and t into one variable $\xi = x - vt$, we suppose that

$$u(x,t) = u(\xi), \quad \xi = sx + ly - vt \tag{2}$$

The travelling wave variable (2) permits us to reduce Eq. (1) to an ODE for $G = G(\xi)$, namely

$$P(u, su', lu', -vu', v^2u'', -vu'', u'', \ldots) = 0 \tag{3}$$

step 2:
Suppose that the solution of ODE (3) can be expressed by a polynomial in $\left(\frac{G'}{G}\right)$ as follows

$$u(\xi) = \sum_{i=1}^{n} \alpha_i \left(\frac{G'}{G}\right)^i \tag{4}$$

where $G = G(\xi)$ satisfies the second order LODE in the form

$$G'' + \lambda G' + \mu G = 0 \tag{5}$$

$\alpha_n, \ldots, \lambda$ and μ are constants to be determined later $\alpha_n \neq 0$. The positive integer "n" can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in (3)

step 3:
By substituting (4) into Eq. (3) and using the second order linear ODE (5), collecting all terms with the same order $\left(\frac{G'}{G}\right)$ together, the left-hand side of Eq. (3) is converted into another polynomial in $\left(\frac{G'}{G}\right)$. Equating each coefficient of this polynomial to zero yields a set of algebraic equations for $\alpha_n, \ldots, \lambda$ and μ. By solving the algebraic equations above we obtain $\alpha_n, \ldots, v$.

3. (2 + 1)-Dimensional Kadomtsev–Petviashvili equation

We consider the (2 + 1)-dimensional Kadomtsev–Petviashvili equation in the form

$$\frac{\partial}{\partial x}(\partial_t u_+ u\partial_x u + \varepsilon^2 \partial_{xxx} u) + \delta \partial_{yy} u = 0 \tag{6}$$

The travelling wave variable given below

$$u(x,t) = u(\xi), \quad \xi = sx + ly - vt \tag{7}$$

permits us to convert Eq. (7) into an ODE for $u = u(\xi)$ and integrating twice, we have

$$c + \left(\frac{l^2\delta}{s} - v\right)u + \frac{1}{2}su^2 + \varepsilon^2 s^3 u'' = 0 \tag{8}$$

where C is the integration constant, and the first integrating constant is taken to zero. Suppose that the solution of ODE (8) can be expressed by a polynomial in $\left(\frac{G'}{G}\right)$ as follow

$$u(\xi) = \alpha_n \left(\frac{G'}{G}\right) + \ldots, \tag{9}$$

where $G = G(\xi)$ satisfies the second order LODE in the form

$$G'' + \lambda G' + \mu G = 0 \tag{10}$$

α_1, α_0, v and μ are to be determined later.

By using (9) and (10) and considering the homogeneous balance between u'' and u^2 in Eq. (8) we required that $2n = n + 2$ then $n = 2$. So we can write (9) as

$$u(\xi) = \alpha_2 \left(\frac{G'}{G}\right)^2 + \alpha_1 \left(\frac{G'}{G}\right) + \alpha_0 \tag{11}$$

By using (10) and (11) it is derived that

$$\begin{aligned} u'' = {} & 6\alpha_2 \left(\frac{G'}{G}\right)^4 + (2\alpha_1 + 10\alpha_2\lambda)\left(\frac{G'}{G}\right)^3 \\ & + \left(8\alpha_2\mu + 3\alpha_1\lambda + 4\alpha_2\lambda^2\right)\left(\frac{G'}{G}\right)^2 \\ & + \left(6\alpha_2\lambda\mu + 2\alpha_1\mu + \alpha_1\lambda^2\right)\left(\frac{G'}{G}\right) + 2\alpha_2\mu^2 + \alpha_1\lambda\mu \end{aligned} \tag{12}$$

By substituting (11) and (12) into Eq. (8) and collecting all terms with the same power of $\left(\frac{G'}{G}\right)$ together, the left-hand side of Eq. (8) is converted into another polynomial in $\left(\frac{G'}{G}\right)$. Equating each coefficient of this polynomial to zero yields a set of simultaneous algebraic equations for $\alpha_2, \alpha_1, \alpha_0, v, \lambda, \mu$ and c as follows:

$$\left(\frac{l^2\delta}{s} - v\right)\alpha_0 + \frac{1}{2}s\alpha_0^2 + \varepsilon^2 s^3(2\alpha_2\mu^2 + \alpha_1\lambda\mu) - c = 0$$
$$\left(\frac{l^2\delta}{s} - v\right)\alpha_1 + s\alpha_1\alpha_0 + \varepsilon^2 s^3(6\alpha_2\lambda\mu + 2\alpha_1\mu + \alpha_1\lambda^2) = 0$$
$$\left(\frac{l^2\delta}{s} - v\right)\alpha_2 + \frac{1}{2}s(\alpha_1^2 + 2\alpha_2\alpha_0) + \varepsilon^2 s^3(8\alpha_2\mu + 3\alpha_1\lambda + 4\alpha_2\lambda^2) = 0$$
$$s\alpha_2\alpha_1 + \varepsilon^2 s^3(2\alpha_1 + 10\alpha_2\lambda) = 0$$
$$\frac{1}{2}s\alpha_2^2 + 6\varepsilon^2 s^3\alpha_2 = 0$$

By solving the algebraic equations above yields

$$\begin{aligned} & \alpha_2 = -12\varepsilon^2 s^2, \quad \alpha_1 = -12\varepsilon^2 s^2\lambda \\ & v = \frac{1}{s}(8\varepsilon^2 s^2 + \varepsilon^2 s^4\lambda^2 + s^2\alpha_0 + l^2\delta) \\ & c = -\frac{1}{2}s(48\varepsilon^2 s^4\mu^2 + 24\varepsilon^2 s^4\mu\lambda^2 + \alpha_0^2) \\ & \quad -\frac{1}{2}s(16\varepsilon^2 s^2\alpha_0 + 2\alpha_0 + \varepsilon^2 s^2\lambda^2) \end{aligned} \tag{13}$$

λ, μ and α_0 are arbitrary constants.

By using (13), expression (11) can be written as

$$u(\xi) = -12\varepsilon^2 s^2 \left(\frac{G'}{G}\right)^2 - 12\varepsilon^2 s^2\lambda\left(\frac{G'}{G}\right) + \alpha_0 \tag{14}$$

And

$$\xi = x - \frac{1}{s}(8\varepsilon^2 s^2 + \varepsilon^2 s^4\lambda^2 + s^2\alpha_0 + l^2\delta)t.$$

Eq. (14) is the formula of a solution of Eq. (8). On solving the Eq. (10), we deduce after some reduction that

$$u(\xi) = \sqrt{\lambda^2 - 4\mu} \times \left(\frac{C_1 \sinh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \cosh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}{C_1 \cosh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \sinh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}\right) - \frac{\lambda}{2}$$

where C_1 and C_2 are arbitrary constants. Substituting the general solutions of Eq. (10) into (14) we have three types of travelling wave solutions of the (2 + 1)-dimensional Kadomtsev–Petviashvili Eq. (6) as follows:

Case 1:
When $\lambda^2 - 4\mu\ 0$

$$u(\xi) = -12\varepsilon^2 s^2(\lambda^2 - 4\mu) \times \left(\frac{C_1 \sinh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \cosh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}{C_1 \cosh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 \sinh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi} \right) - 12\varepsilon^2 s^2 \lambda - \frac{\lambda}{2} + \alpha_0$$

where $\xi = x - \frac{1}{s}(8\varepsilon^2 s^2 + \varepsilon^2 s^4 \lambda^2 + s^2\alpha_0 + l^2\delta t$. C_1, and C_2, are arbitrary constants.If C_1 and C_2 are taken as special values, the various known results in the literature can be rediscovered, for instance, if $C_1 \succ 0, C_1^2 \succ C_2^2$, then $u = u(\xi)$ can be written as

$$u(\xi) = -12\varepsilon^2 s^2(\lambda^2 - 4\mu) \times \sec h^2\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + \xi_0\right) - 12\varepsilon^2 s^2 \lambda - \frac{\lambda}{2} + \alpha_0$$

Case 2:
When $\lambda^2 - 4\mu \prec 0$

$$u(\xi) = -12\varepsilon^2 s^2(\lambda^2 - 4\mu) \times \left(\frac{-C_1 \sin\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \cos\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}{C_1 \cos\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 \sin\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi} \right) - 12\varepsilon^2 s^2 \lambda - \frac{\lambda}{2}$$

Case 3:
When $\lambda^2 - 4\mu = 0$

$$u(\xi) = \frac{-12\varepsilon^2 s^2 C_2}{(C_1 + C_2\xi)^2} - 12\varepsilon^2 s^2 \lambda - \frac{\lambda}{2} + \alpha_0$$

where C_1 and C_2 are arbitrary constants.

4. Conclusions

The solutions of these nonlinear evolution equations have many potential applications in physics. In this paper, we have seen that three types of travelling solutions of (2 + 1)-dimensional Kadomtsev–Petviashvili equation are successfully found out by using the $\left(\frac{G'}{G}\right)$-expansion method. The performance of this method is reliable, simple and gives many new exact solutions.

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