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ORIGINAL ARTICLE

Numerical comparison for the solutions of anharmonic vibration of fractionally damped nano-sized oscillator

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Abstract In this paper, we consider the nonlinear vibrations of nano-sized cantilever. The elastic force is considered anharmonic, deriving from a Morse potential and the nonlinearity is attributed to the Casimir force. The solution is established for viscous and fractional damping by making use of He's polynomials which are calculated from homotopy perturbation method (HPM). The solution procedure explicitly reveal the complete reliability and simplicity of the proposed algorithm. Moreover, comparison with variational iteration method (VIM) shows that both the techniques are in full agreement with each other.

1. Introduction

The mechanical properties of the micro- and nano-devices can be described in terms of classical or quantum mechanics (Cleland, 2003; Draganescu and Capalnasan, 2003; Draganescu, 2006; Drăgănescu et al., 2010; Ke and Espinosa, 2004; Ghorbani and Nadjfi, 2007; He, 2008a). It is an established fact (Draganescu and Capalnasan, 2003; Draganescu, 2006; Drăgănescu et al., 2010; Ke and Espinosa, 2004) that mechanical motion of the elements of micro- and nano-devices is examined inconnection with nonlinear forces of quantum nature similar to the Casimir force. Moreover, the anelastic properties of materials are nonlinear in nature (see Draganescu, 2006; Drăgănescu et al., 2010; Ke and Espinosa, 2004; He, 2008a, and the references therein). The Casimir effect consists in the electrical polarization of two perfectly conducting bodies, the Casimir force taking significant values when the separation between these bodies is reduced to less than 100 nm. On the other hand (Cleland, 2003; Draganescu and Capalnasan, 2003; Draganescu, 2006; Drăgănescu et al., 2010; Ke and Espinosa, 2004; Ghorbani and Nadjfi, 2007; He, 2008a), it was found that in materials like plastics and nano-wires, the most adequate kind of damping is the fractional damping. Recently, Drăgănescu et al. (2010) used Adomian's decomposition method for solving the governing problem. It is worthmentiong that Adomian's scheme is coupled with number of complexities including evaluation of the so-called Adomian's polynomials. He (2008a,b, 2006, 2005, 2004a,b, 2000) developed the homotopy perturbation method (HPM)

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by merging the standard homotopy and perturbation. The HPM (Ghorbani and Nadjfi, 2007; He, 2008a,b, 2006, 2005, 2004a,b, 2000; Mohyud-Din, 2009; Mohyud-Din et al., 2009; Mohyud-Din and Noor, 2007; Mohyud-Din and Noor, 2009; Mohyud-Din et al., 2009; Yıldırım, 2009; Yıldırım, 2008; Yıldırım, 2008; Xu, 2007; Abbasbandy, 2007; Abbasbandy, 2007; Abdou and Soliman, 2005; Abdou and Soliman, 2005) has been successfully applied to a wide class of nonlinear problems. In a subsequent work, Ghorbani and Nadjfi (2007) established He's polynomials which are calculated from homotopy perturbation method (HPM), are compatible with Adomian's polynomials but are much more user friendly (see Ghorbani and Nadjfi, 2007; Mohyud-Din et al., 2009; Mohyud-Din and Noor, 2007; Mohyud-Din and Noor, 2009; Mohyud-Din et al., 2009 and the references therein). It is to be highlighted that Noor and Mohyud-Din (2008), Mohyud-Din et al. (2010) made the elegant coupling of He's polynomials and correction functional of variatioanl iteration method (VIM) and introduced one of the most reliable modified version of VIM. It is also explained (Noor and Mohyud-Din, 2008; Mohyud-Din et al., 2010) that this modified version is easier to implement and is very effective to tackle the nonlinear terms. The basic motivation of this paper is the extension of He's polynomials for solving anharmonic vibration equation of a nano-sized oscillator with fractional damping. The solution procedure explicitly reveal the complete reliablity of the proposed algorithm. Moreover, we have also applied variational iteration method (VIM) on the same problem and it is observed that results obtained by both the techniques are in good agreement with each other.

2. Homotopy perturbation method (HPM) and He's polynomials

To explain the He's homotopy perturbation method, we consider a general equation of the type,

$$L(u) = 0, \tag{1}$$

where L is any integral or differential operator. We define a convex homotopy $H(u,p)$ by

$$H(u,p) = (1-p)F(u) + pL(u), \tag{2}$$

where $F(u)$ is a functional operator with known solutions v_0, which can be obtained easily. It is clear that, for

$$H(u,p) = 0, \tag{3}$$

we have

$$H(u,0) = F(u), \tag{4}$$

$$H(u,1) = L(u). \tag{5}$$

This shows that $H(u,p)$ continuously traces an implicitly defined curve from a starting point $H(v_0, 0)$ to a solution function $H(f,1)$. The embedding parameter monotonically increases from zero to unit as the trivial problem $F(u) = 0$, is continuously deforms the original problem $L(u) = 0$. The embedding parameter $p \in (0,1]$ can be considered as an expanding parameter (Ghorbani and Nadjfi, 2007, 2008a,b, 2006, 2005, 2004a,b, 2000; Mohyud-Din, 2009; Mohyud-Din et al., 2009; Mohyud-Din and Noor, 2007; Mohyud-Din and Noor, 2009; Mohyud-Din et al., 2009; Yıldırım, 2009; Yıldırım, 2008; Yıldırım, 2008; Xu, 2007; Noor and Mohyud-Din, 2008; Mohyud-Din et al., 2010). The homotopy perturbation method uses the homotopy parameter p as an expanding parameter (He, 2008a,b, 2006, 2005, 2004a,b, 2000) to obtain

$$u = \sum_{i=0}^{\infty} p^i u_i = u_0 + pu_1 + \cdots, \tag{6}$$

if $p \to 1$, then (6) corresponds to (2) and becomes the approximate solution of the form,

$$f = \lim_{p\to 1} u = \sum_{i=0}^{\infty} u_i. \tag{7}$$

It is well known that series (7) is convergent for most of the cases and also the rate of convergence is dependent on $L\ (u)$ (see He, 2008a,b, 2006, 2005, 2004a,b, 2000). We assume that (7) has a unique solution. The comparisons of like powers of p give solutions of various orders. In sum, according to Ghorbani and Nadjfi (2007), He's HPM considers the nonlinear term $N(u)$ as

$$N(u) = \sum_{i=0}^{\infty} p^i H_i = H_0 + pH_1 + p^2 H_2 + \cdots, \tag{8}$$

where H_n's are the so-called He's polynomials (Ghorbani and Nadjfi, 2007), which can be calculated by using the formula

$$H_n(u_0, \ldots, u_n) = \frac{1}{n!}\frac{\partial^n}{\partial p^n}\left(N\left(\sum_{i=0}^{n} p^i u_i\right)\right)_{p=0}, \quad n = 0, 1, 2, \ldots \tag{9}$$

3. The nonlinear model

Consider the elastic force acting in case of oscillations of the nano-devices originates in the Morse potential (Draganescu and Capalnasan, 2003; Draganescu, 2006; Drăgănescu et al., 2010):

$$V(x) = D(\exp(-2\alpha x) - 2\exp(-\alpha x)), \tag{10}$$

where D is the dissociation energy, α is the anharmonicity constant and x is the displacement from the equilibrium position. It is established that the Casimir force (Cleland, 2003; Draganescu and Capalnasan, 2003; Draganescu, 2006; Drăgănescu et al., 2010; Ke and Espinosa, 2004; Ghorbani and Nadjfi, 2007; He, 2008a) between two perfectly conducting plates without roughness, is an attractive force given by:

$$F = \frac{\pi^2 hcA}{240z^4}, \tag{11}$$

when acts between two perfectly conducting plates without roughness, and by:

$$F = \frac{\pi^3 hcR}{360z^3}, \tag{12}$$

when acting between a sphere and a plate, both perfectly conducting and having smooth surfaces. Here $\hbar = h/2\pi$ is the Planck constant, c is the velocity of the light, A is the area of the plates, R radius of the sphere; z is the distance between the two plates, and between the sphere and the plate respectively. Our model is composed from a nano-sized one-dimensional oscillator consisting of a conducting sphere of mass m and radius R, situated on a horizontal conducting surface. The sphere is suspended by means of a vertical elastic wire which produces a force originating in a Morse potential (1); a Casimir force is present between the sphere and the conducting. The distance between the center of the sphere and the surface is $d \gg R$. Consider that a perturbing force F acts on the sphere. Consider that a fractional damping force of damping constant λ (Cleland, 2003; Draganescu and Capalnasan, 2003; Draganescu, 2006; Drăgănescu et al., 2010; Ke and Espinosa, 2004; Ghorbani and Nadjfi, 2007; He, 2008a) is also present. We will denote by x the instantaneous vertical displacement of the sphere from the equilibrium position to the conducting surface. The one-dimensional motion of the sphere can be expressed by the differential equation:

$$m\frac{d^2}{dt^2}x+\lambda\frac{d^\mu}{dt^\mu}x+2\alpha D(\exp(-2\alpha x)-\exp(-\alpha x))-\frac{c}{(d-x)^3}=F(t) \tag{13}$$

where the second term is the fractional damping force, the third is the anharmonic elastic force, the forth is the Casimir force and $F(t)$ is an external excitation force. The fractional damping from (12) is expressed by means of a fractional derivative operator (d^μ/dt^μ), μ being a fractional number. As a typical value of $\mu=0.5$ can be taken; for $\mu=1$, we obtain the classical viscous damping force. The constant a is a parameter which will be discussed in the next section. The motion is possible for $x<d-R$. We consider the case of a harmonic excitation with angular frequency ω and we will denote by ω_n the pseudo-natural angular frequency of the system in absence of the Casimir force and damping:

$$F(t)=F_0\sin(\omega t) \tag{14}$$

where F_0 is the amplitude of the force. Accordingly, (12) becomes:

$$m\frac{d^2}{dt^2}x+\lambda\frac{d^\mu}{dt^\mu}x+2\alpha D(\exp(-2\alpha x)-\exp(-\alpha x))-\frac{c}{(d-x)^3}=F_0\sin(\omega t). \tag{15}$$

In case of the small oscillations, around the equilibrium position, the nonlinear term can be expanded into series, which leads to the fact (Drăgănescu et al., 2010) that (14) can be written as:

$$\frac{d^2}{dt^2}x+\frac{\lambda}{m}\frac{d^\mu}{dt^\mu}x-2\frac{\alpha|D|}{m}\left[-\alpha x+\frac{3}{2}(\alpha x)^2-\frac{7}{6}(\alpha x)^3+\frac{16}{24}(\alpha x)^4\right]-\frac{c}{md^3}\left(3\frac{x}{d}+5\frac{x^2}{d^2}+10\frac{x^3}{d^3}+15\frac{x^4}{d^4}\right)=F_0\sin(\omega t). \tag{16}$$

The pseudo-natural angular frequency of the system is:

$$\omega_n=\sqrt{\frac{2\alpha^2|D|}{m}-\frac{c}{md^3}}. \tag{17}$$

4. Fractional derivatives

The fractional derivative was first introduced by Leibniz, and then was studied by mathematicians like Liouville and Riemann. With the aid of Riemann–Liouville definition ([Cleland, 2003; Draganescu and Capalnasan, 2003; Draganescu, 2006; Drăgănescu et al., 2010; Ke and Espinosa, 2004; Ghorbani and Nadjfi, 2007; He, 2008a), the fractional derivative operator $D^\mu_{t,a}$, which is a linear operator, may be written as:

$$D^\mu_{t,a}x(t)=\begin{cases}\frac{1}{\Gamma(n-\mu)}\frac{d^n}{dt^n}\int_a^t(t-y)^{n-\mu-1}f(y)dy, & n-1<\mu<n,\\ \frac{d^n}{dx^n}f(t) & \mu=n,\end{cases} \tag{18}$$

where n is an integer. This definition of the fractional derivative is in fact more general, since it gives the fractional differentiation operator $D^\mu_{t,a}$ for positive μ, and the fractional integration operator for negative values of μ. The fractional differentiation operator exhibits the following properties:

$$D^\mu_{t,a}D^v_{t,a}=D^{\mu+n}_{t,a} \tag{19}$$

$$D^0_{t,a}x(t)=x(t) \tag{20}$$

$$D^\mu_{t,a}f(t)g(t)=\sum_{k=0}^{\infty}\binom{k}{\mu}D^k_{t,a}f(t)D^{\mu-k}_{t,a}g(t) \tag{21}$$

It is worth to mentioning that different values of a greatly affects (Drăgănescu et al., 2010) differentiation operators results, with different differentiation rules. The value $a=-\infty$ gives the $D^\mu_{t,-\infty}$ operator, to which the following differentiation rule corresponds:

$$D^\mu_{t,-\infty}\exp(ut)=u^\mu\exp(ut) \tag{22}$$

where u is a complex quantity, $u\in C$. The following differentiation rules result:

$$\begin{aligned}&D^\mu_{t,-\infty}\exp(iat)=a^\mu\exp\left(i\left(at+\frac{\pi}{2}\mu\right)\right),\\&D^\mu_{t,-\infty}\sin(\omega t)=\omega^\mu\sin\left(\omega t+\frac{\pi}{2}\mu\right),\\&D^\mu_{t,-\infty}\cos(\omega t)=\omega^\mu\cos\left(\omega t+\frac{\pi}{2}\mu\right).\end{aligned} \tag{23}$$

For $a=0$ the differentiation operator

$$D^\mu_t=D^\mu_{t,0}, \tag{24}$$

results, which will be used in our calculation since the process investigated by us corresponds to positive time values. For this kind of fractional derivative one finds

$$D^\mu_t t^k=\frac{n!}{\Gamma(k-\mu+1)}t^{k-\mu}, \tag{25}$$

The differentiation rule for the constant function $f(t)=1$ is

$$D^\mu_t 1=\frac{n!}{\Gamma(1-\mu)}t^{-\mu}, \tag{26}$$

and this is valid for $\mu\geqslant 0$ and $t>0$; here Γ is the Euler's Gamma function (Drăgănescu et al., 2010). In this paper the differentiation operator $D^\mu_{t,-\infty}=d^\mu/dt^\mu$ which corresponds to $a=-\infty$ will be used;

5. Implementation of the method

We will solve Eq. (16) by using He's polynomials with the initial conditions

$x(0)=A$ and $\dot{x}(0)=0$.

$$\frac{d^2}{dt^2}x-\frac{d^2}{dt^2}x_0=p\left\{\begin{array}{l}-\frac{\lambda}{m}\frac{d^\mu}{dt^\mu}x+2\frac{\alpha|D|}{m}\left[\alpha x-\frac{3}{2}(\alpha x)^2+\frac{7}{6}(\alpha x)^3-\frac{16}{24}(\alpha x)^4\right]\\+\frac{c}{md^3}\left(3\frac{x}{d}-5\frac{x^2}{d^2}-10\frac{x^3}{d^3}-15\frac{x^4}{d^4}\right)+F_0\sin(\omega t)-\frac{d^2}{dt^2}x_0\end{array}\right\} \tag{27}$$

$$x(t)=x_0(t)+px_1(t)+p^2x_2(t)+p^3x_3(t)+\cdots \tag{28}$$

We can substitute (28) into (27) and then we can find terms of the power of p gives.

$$p^0:\frac{d^2}{dt^2}x_0-\frac{d^2}{dt^2}x_0=0 \tag{29}$$

$$p^1:\frac{d^2}{dt^2}x_1=-\frac{\lambda}{m}\frac{d^\mu}{dt^\mu}x_0+2\frac{\alpha|D|}{m}\left[\alpha x_0-\frac{3}{2}(\alpha x_0)^2+\frac{7}{6}(\alpha x_0)^3-\frac{16}{24}(\alpha x_0)^4\right]+\frac{c}{md^3}\left(3\frac{x_0}{d}-5\frac{x_0^2}{d^2}-10\frac{x_0^3}{d^3}-15\frac{x_0^4}{d^4}\right)+F_0\sin(\omega t)-\frac{d^2}{dt^2}x_0 \tag{30}$$

$$p^2 : \frac{d^2}{dt^2}x_2 = -\frac{\lambda}{m}\frac{d^\mu}{dt^\mu}x_1 + 2\frac{\alpha|D|}{m}\left[\alpha x_1 - \frac{3}{2}\alpha^2(2x_0x_1) + \frac{7}{6}\alpha^3(3x_0^2x_1) - \frac{16}{24}\alpha^4(4x_0^3x_1)\right] + \frac{c}{md^3}\left(3\frac{x_1}{d} - 5\frac{(2x_0x_1)}{d^2} - 10\frac{(3x_0^2x_1)}{d^3} - 15\frac{(4x_0^3x_1)}{d^4}\right) \tag{31}$$

$\vdots,$

where p^i's are He's polynomials. The solution of (27) can be obtained by setting $p = 1$ in (28):

$$x(t) = x_0(t) + x_1(t) + x_2(t) + x_3(t) + \dots \tag{32}$$

We now successively obtain

$$x_0(t) = A - \frac{F_0}{m}\frac{1}{\omega^2}\sin(\omega t), \tag{33}$$

$$x_1(t) = \frac{F_0}{m\omega^4}\omega^\mu \sin\left(\omega t + \frac{\pi}{2}\mu\right) - \frac{2\alpha|D|}{m}T_1 - \frac{c}{md^3}T_2, \tag{34}$$

where T_1 and T_2 are

$$\begin{aligned} T_1 = &-\frac{1}{2}\alpha A t^2 + \frac{\alpha F\sin(\omega t)}{m\omega^4} + \frac{15}{128}\frac{\alpha^4F^4t^2}{m^4\omega^8} + \frac{3}{4}\alpha^2A^2t^2 \\ &- 3\frac{\alpha^2AF\sin(\omega t)}{m\omega^4} + \frac{3}{8}\frac{\alpha^2F^2(\cos(\omega t))^2}{\omega^6m^2} \\ &+ \frac{3}{8}\frac{\alpha^2F^2t^2}{m^2\omega^4} - \frac{7}{12}\alpha^3A^3t^2 + \frac{7}{2}\frac{\alpha^3A^2F\sin(\omega t)}{m\omega^4} \\ &- \frac{7}{8}\frac{\alpha^3AF^2(\cos(\omega t))^2}{\omega^6m^2} - \frac{7}{8}\frac{\alpha^3AF^2t^2}{m^2\omega^4} + \frac{7}{54}\frac{\alpha^3F^3(\sin(\omega t))^3}{\omega^8m^3} \\ &+ \frac{7}{9}\frac{\alpha^3F^3\sin(\omega t)}{\omega^8m^3} + \frac{5}{16}\alpha^4A^4t^2 - \frac{5}{2}\frac{\alpha^4A^3F\sin(\omega t)}{m\omega^4} \\ &+ \frac{15}{16}\frac{\alpha^4A^2F^2(\cos(\omega t))^2}{\omega^6m^2} + \frac{15}{16}\frac{\alpha^4A^2F^2t^2}{m^2\omega^4} \\ &- \frac{5}{18}\frac{\alpha^4AF^3(\sin(\omega t))^3}{\omega^8m^3} - \frac{5}{3}\frac{\alpha^4AF^3\sin(\omega t)}{\omega^8m^3} \\ &- \frac{5}{128}\frac{\alpha^4F^4(\sin(\omega t))^4}{\omega^{10}m^4} + \frac{5}{128}\frac{\alpha^4F^4(\cos(\omega t))^4}{\omega^{10}m^4} \end{aligned} \tag{35}$$

and

$$\begin{aligned} T_2 = &-30\frac{A^2F\sin(\omega t)}{d^3\omega^4m} + \frac{3}{2}\frac{F^2(\cos(\omega t))^2}{d^2\omega^6m^2} + \frac{3}{2}\frac{F^2t^2}{d^2\omega^4m^2} \\ &- 12\frac{AF\sin(\omega t)}{d^2\omega^4m} - 3\frac{F\sin(\omega t)}{dm\omega^4} + \frac{3}{2}\frac{At^2}{d} + \frac{21}{2}\frac{A^5t^2}{d^5} \\ &+ \frac{15}{2}\frac{A^4t^2}{d^4} + 5\frac{A^3t^2}{d^3} + 3\frac{A^2t^2}{d^2} + \frac{45}{16}\frac{F^4t^2}{d^4\omega^8m^4} \\ &- \frac{15}{16}\frac{F^4(\sin(\omega t))^4}{d^4\omega^{10}m^4} + \frac{45}{16}\frac{F^4(\cos(\omega t))^2}{d^4\omega^{10}m^4} - 40\frac{AF^3\sin(\omega t)}{d^4\omega^8m^3} \\ &- \frac{20}{3}\frac{AF^3(\sin(\omega t))^3}{d^4\omega^8m^3} + \frac{45}{2}\frac{A^2F^2t^2}{d^4\omega^4m^2} + \frac{45}{2}\frac{A^2F^2(\cos(\omega t))^2}{d^4\omega^6m^2} \\ &- 60\frac{A^3F\sin(\omega t)}{d^4\omega^4m} - \frac{10}{9}\frac{F^3(\sin(\omega t))^3}{d^3\omega^8m^3} - \frac{20}{3}\frac{F^3\sin(\omega t)}{d^3\omega^8m^3} \\ &+ \frac{15}{2}\frac{AF^2t^2}{d^3\omega^4m^2} + \frac{15}{2}\frac{AF^2(\cos(\omega t))^2}{d^3\omega^6m^2} \\ &- \frac{56}{5}\frac{F^5\sin(\omega t)}{d^5\omega^{12}m^5}\,\frac{21}{25}\frac{F^5(\sin(\omega t))^5}{d^5\omega^{12}m^5} - \frac{28}{15}\frac{F^5(\sin(\omega t))^3}{d^5\omega^{12}m^5} \\ &+ \frac{315}{16}\frac{AF^4t^2}{d^5\omega^8m^4} + \frac{315}{16}\frac{AF^4(\cos(\omega t))^2}{d^5\omega^{10}m^4} \\ &- \frac{105}{16}\frac{AF^4(\sin(\omega t))^4}{d^5\omega^{10}m^4} - 140\frac{A^2F^3\sin(\omega t)}{d^5\omega^8m^3} \\ &- \frac{70}{3}\frac{A^2F^3(\sin(\omega t))^3}{d^5\omega^8m^3} + \frac{105}{2}\frac{A^3F^2(\cos(\omega t))^2}{d^5\omega^6m^2} \\ &- 105\frac{A^4F\sin(\omega t)}{d^5\omega^4m}. \end{aligned} \tag{36}$$

The last calculations were carried out using Maple 12 Software Package. In principle, it is possible to calculate other terms.

$$x_0(t) = A - \frac{F_0}{m}\frac{1}{\omega^2}\sin(\omega t), \tag{37}$$

$$x_1(t) = \frac{F_0}{m\omega^2}\cos(\omega t) - \frac{2\alpha|D|}{m}T_1 - \frac{c}{md^3}T_2, \tag{38}$$

where T_1 and T_2 are (35) and (36). Our solution agrees with Drăgănescu et al. (2010) where Adomian's decomposition method coupled with its complexities was used.

Now, we shall apply variational iteration method (VIM) on Eq. (16) using the conditions $x(0) = A$ and $\dot{x}(0) = 0$. The correction functional (with Lagrange multiplier $s - t$) is given by

$$\begin{aligned} x_{n+1}(t) = A + &\int_0^t (s-t)\left(\frac{d^2x_n}{dt^2} + \frac{\lambda}{m}\frac{d^\mu x_n}{dt^\mu} - 2\frac{\alpha|D|}{m}\right. \\ &\left.\times\left(-\alpha x + \frac{3}{2}(\alpha x)^2 + \frac{16}{24}(\alpha x)^4\right)\right)ds \\ &+ \int_0^t (s-t)\left(\frac{c}{md^3}\left(3\frac{x}{d} + 5\frac{x^2}{d^2} + 10\frac{x^3}{d^3} + 15\frac{x^4}{d^4}\right)\right. \\ &\left. - F_0Sin(\omega t)\right)ds. \end{aligned}$$

Consequently, we get the same results as (33)–(38). Hence, it is observed that results obtained by He's polynomials and variational iteration method are in good agreement with each other.

6. Conclusion

In this study, we studied a nonlinear oscillator model with fractional damping and a nonlinearity due to Casimir force and anharmonic elastic force deriving from a Morse potential. The solution of the model was obtained with the aid of He' polynomials which aare calculated from homotopy perturbation method (HPM). Unlike classical techniques, the homotopy perturbation method leads to an analytical approximate and exact solutions of the nonlinear equations easily and elegantly without transforming the equation or linearizing the problem and with high accuracy, minimal calculation and avoidance of physically unrealistic assumptions. As a numerical tool, the method provide us with numerical solution without discretization of the given equation, and therefore, it is not effected by computation round-off errors and one is not faced with necessity of large computer memory and time. Moreover, the proposed algorithm is independent of the complexities arising in calculating Adomian's polynomials.

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