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Estimation for stochastic volatility model: Quasi-likelihood and asymptotic quasi-likelihood approaches

Raed Alzghool *

Department of Basic Sciences and Humanities, College of Engineering, University of Dammam, Saudi Arabia
Department of Mathematics, Faculty of Science, Al-Balqa' Applied University, Jordan

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Abstract For estimation of the stochastic volatility model (SVM), this paper suggests the quasi-likelihood (QL) and asymptotic quasi-likelihood (AQL) methods. The QL approach is quite simple and does not require full knowledge of the likelihood functions of the SVM. The AQL technique is based on the QL method and is used when the covariance matrix Σ is unknown. The AQL approach replaces the true variance–covariance matrix Σ by nonparametric kernel estimator of Σ in QL.

1. Introduction

Consider the stochastic volatility process y_t which satisfies the stochastic volatility model

$$y_t = e^{\frac{h_t}{2}}\eta_t, \quad t = 1, 2, 3, \ldots, T \tag{1.1}$$

and

$$h_t = \gamma + \phi h_{t-1} + \delta_t, \quad t = 1, 2, 3, \ldots, T. \tag{1.2}$$

Furthermore, η_t are independent and identically distributed (i.i.d) with $E(\eta_t) = 0$ and $V(\eta_t) = \sigma_\eta^2$, and δ_t are i.i.d with $E(\delta_t) = 0$ and $V(\delta_t) = \sigma_\delta^2$. For estimation and application of the stochastic volatility model (SVM) (see Jacquire et al., 1994; Breidt and Carriquiry, 1996; Sandmann and Koopman, 1998; Pitt and Shepard, 1999; Papanastastiou and Ioannides, 2004; Alzghool and Lin, 2008; Chan and Grant, 2015; Pinho et al., 2016) Sandmann and Koopman (1998) introduced the Monte Carlo maximum-likelihood procedure. Davis and Rodriguez-Yam (2005) proposed another estimation technique that relies on the likelihood function.

This paper applies the quasi-likelihood (QL) and asymptotic quasi-likelihood (AQL) approaches to SVM. The QL approach relaxes the distributional assumptions but has a restriction that assumes that the conditional variance process is known. To overcome this limitation, we suggest a substitute technique, the AQL methodology, merging the kernel technique used for parameter estimation of the SVM. This AQL

* Address: Department of Basic Sciences and Humanities, College of Engineering, University of Dammam, Saudi Arabia.
E-mail addresses: raalzghool@uod.edu.sa, raedalzghool@bau.edu.jo
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Table 1 The QL and AQL estimates; the RMSE of each estimate is given below that estimate.

	γ	ϕ	μ	σ_δ	σ_ϵ	γ	ϕ	μ	σ_δ	σ_ϵ
True	−0.821	0.90	−1.271	0.675	2.22	-0.411	0.95	−1.271	0.484	2.22
QL	−0.809	0.901	−1.366	0.344	2.15	-0.417	0.950	−1.144	0.382	2.05
	0.108	0.013	0.157	0.331	0.123	0.080	0.010	0.147	0.104	0.205
AQL	−0.821	0.896	−1.257	0.330	2.34	-0.429	0.943	−1.360	0.342	2.25
	0.108	0.015	0.088	0.158	0.347	0.085	0.014	0.120	0.111	0.148
True	−0.736	0.90	−1.271	0.363	2.22	-0.368	0.95	−1.271	0.260	2.22
QL	−0.889	0.881	−1.199	0.321	2.02	-0.511	0.931	−1.185	0.318	2.01
	0.176	0.022	0.099	0.046	0.23	0.159	0.021	0.098	0.061	0.23
AQL	−0.850	0.876	−1.279	0.293	2.16	−0.496	0.927	−1.284	0.309	2.16
	0.231	0.038	0.051	0.089	0.124	0.181	0.030	0.049	0.063	0.129
True	−0.706	0.90	−1.271	0.135	2.22	-0.353	0.95	−1.271	0.096	2.22
QL	−0.695	0.905	−1.043	0.040	2.21	-0.364	0.946	−1.660	0.070	2.17
	0.017	0.006	0.247	0.095	0.12	0.019	0.006	0.404	0.026	0.13
AQL	−0.889	0.872	−1.111	0.28	2.09	−0.504	0.927	−1.125	0.295	2.10
	0.329	0.049	0.164	0.153	0.164	0.224	0.034	0.150	0.167	0.202
True	−0.147	0.98	−1.271	0.166	2.22	-0.141	0.98	−1.271	0.061	2.22
QL	−0.169	0.977	−1.327	0.072	2.23	-0.140	0.979	−1.705	0.018	2.22
	0.027	0.004	0.155	0.094	0.12	0.003	0.001	0.450	0.043	0.12
AQL	−0.225	0.965	−1.342	0.316	2.13	−0.238	0.961	−1.336	0.310	2.11
	0.109	0.019	0.083	0.130	0.15	0.125	0.023	0.074	0.156	0.251

methodology enables a substitute technique for parameter estimation when the conditional variance process is unknown.

This paper is structured as follows. The QL and AQL approaches are introduced in Section 2. The SVM estimation using the QL and AQL methods, reports of simulation outcomes, and numerical cases are presented in Section 3. The QL and AQL techniques are applied to a real data set in Section 4. Section 5 summarizes and concludes the paper.

2. The QL and AQL methods

In this section, we introduce the QL and AQL methods.

2.1. The QL Method

Let the observation equation be given by

$$\mathbf{y}_t = \mathbf{f}_t(\boldsymbol{\theta}) + \zeta_t, \quad t = 1, 2, 3 \ldots, T. \tag{2.1.1}$$

ζ_t is a sequence of martingale difference with respect to $\mathcal{F}_t, \mathcal{F}_t$ denotes the σ-field generated by $\mathbf{y}_t, \mathbf{y}_{t-1}, \ldots, \mathbf{y}_1$ for $t \geqslant 1$; that is, $E(\zeta_t|\mathcal{F}_{t-1}) = E_{t-1}(\zeta_t) = 0$; where $\mathbf{f}_t(\boldsymbol{\theta})$ is an $\mathcal{F}_{t-1}$ measurable; and $\boldsymbol{\theta}$ is a parameter vector, which belongs to an open subset $\Theta \in R^d$. Note that $\boldsymbol{\theta}$ is a parameter of interest. We assume that $E_{t-1}(\zeta_t\zeta_t') = \boldsymbol{\Sigma}_t$ is known. Now, the liner class $\mathcal{G}_T$ of the estimating function (EF) can be defined by

$$\mathcal{G}_T = \left\{ \sum_{t=1}^{T} \mathbf{W}_t(\mathbf{y}_t - \mathbf{f}_t(\boldsymbol{\theta})) \right\}$$

and the quasi-likelihood estimation function (QLEF) can be defined by

$$\mathbf{G}_T^*(\boldsymbol{\theta}) = \sum_{t=1}^{T} \dot{\mathbf{f}}_t(\boldsymbol{\theta}) \boldsymbol{\Sigma}_t^{-1}(\mathbf{y}_t - \mathbf{f}_t(\boldsymbol{\theta})), \tag{2.1.2}$$

where $\mathbf{W}_t$ is $\mathcal{F}_{t-1}$-measurable and $\dot{\mathbf{f}}_t(\boldsymbol{\theta}) = \partial \mathbf{f}_t(\boldsymbol{\theta})/\partial \boldsymbol{\theta}$. Then, the estimation of $\boldsymbol{\theta}$ by the QL method is the solation of the QL equation $\mathbf{G}_T^*(\boldsymbol{\theta}) = 0$ (see Hedye, 1997).

If the sub-estimating function spaces of $\mathcal{G}_T$ are considered as follows,

$$\mathcal{G}_t = \{\mathbf{W}_t(\mathbf{y}_t - \mathbf{f}_t(\boldsymbol{\theta})), \ t = 1, 2, 3 \ldots, T\},$$

then the QLEF in the space $\mathcal{G}_t$ can be defined by

$$\mathbf{G}_{(t)}^*(\boldsymbol{\theta}) = \dot{\mathbf{f}}_t(\boldsymbol{\theta}) \boldsymbol{\Sigma}_t^{-1}(\mathbf{y}_t - \mathbf{f}_t(\boldsymbol{\theta})) \tag{2.1.3}$$

and the estimation of $\boldsymbol{\theta}$ by the QL method is the solation of the QL equation $\mathbf{G}_{(t)}^*(\boldsymbol{\theta}) = 0$.

A limitation of the QL method is that the nature of $\boldsymbol{\Sigma}_t$ may not be obtainable. A misidentified $\boldsymbol{\Sigma}_t$ could result in a deceptive inference about parameter $\boldsymbol{\theta}$. In the next subsection, we introduce the AQL method, which is basically the QL estimation assuming that the covariance matrix $\boldsymbol{\Sigma}_t$ is unknown.

2.2. The AQL method

The QLEF (see (2.1.2) and (2.1.3)) relies on the information of Σ_t. Such information is not always accessible. To find the QL when $E_{t-1}(\zeta_t\zeta_t')$ is not accessible, Lin (2000) proposed the AQL method.

Definition 2.2.1. Let $\mathbf{G}_{T,n}^*$ be a sequence of the EF in $\mathcal{G}$. For all $\mathbf{G}_T \in \mathcal{G}$, if

$$(E\dot{\mathbf{G}}_T)^{-1}(E\mathbf{G}_T\mathbf{G}_T)'(E\dot{\mathbf{G}}_T')^{-1} - (E\dot{\mathbf{G}}_{T,n}^*)^{-1}(E\mathbf{G}_{T,n}^*\mathbf{G}_T^{*\prime})(E\dot{\mathbf{G}}_{T,n}^{*\prime})^{-1}$$

is asymptotically non-negative definite, $\mathbf{G}_{T,n}^*$ can be denoted as the asymptotic quasi-likelihood estimation function (AQLEF) sequence in $\mathcal{G}$, and the AQL sequence estimates $\boldsymbol{\theta}_{T,n}$ by the AQL method is the solution of the AQL equation $\mathbf{G}_{T,n}^* = 0$.

Suppose, in probability, $\boldsymbol{\Sigma}_{t,n}$ is converging to $E_{t-1}(\zeta_t\zeta_t')$. Then,

$$\mathbf{G}_{T,n}^*(\boldsymbol{\theta}) = \sum_{t=1}^{T}\dot{\mathbf{f}}_t(\boldsymbol{\theta})\boldsymbol{\Sigma}_{t,n}^{-1}(\mathbf{y}_t - \mathbf{f}_t(\boldsymbol{\theta})) \tag{2.2.1}$$

expresses an AQLEF sequence. The solution of $\mathbf{G}_{T,n}^*(\boldsymbol{\theta}) = 0$ expresses the AQL sequence estimate $\{\boldsymbol{\theta}_{T,n}^*\}$, which converges to $\boldsymbol{\theta}$ under certain regular conditions.

In this paper, the kernel smoothing estimator of $\boldsymbol{\Sigma}_t$ is suggested to find $\boldsymbol{\Sigma}_{t,n}$ in the AQLEF ((2.2.1)). A wide-ranging appraisal of the Nadaraya–Watson (NW) estimator-type kernel estimator is available in Härdle (1990) and Wand and Jones (1996). By using these kernel estimators, the AQL equation becomes

$$\mathbf{G}_{T,n}^*(\boldsymbol{\theta}) = \sum_{t=1}^{T}\dot{\mathbf{f}}_t(\boldsymbol{\theta})\hat{\boldsymbol{\Sigma}}_{t,n}^{-1}(\hat{\boldsymbol{\theta}}^{(0)})(\mathbf{y}_t - \mathbf{f}_t(\boldsymbol{\theta})) = 0. \tag{2.2.2}$$

The estimation of $\boldsymbol{\theta}$ by the AQL method is the solution to (2.2.2). Iterative techniques are suggested to solve the AQL equation (2.2.2). Such techniques start with the ordinary least squares (OLS) estimator $\hat{\boldsymbol{\theta}}^{(0)}$ and use $\hat{\boldsymbol{\Sigma}}_{t,n}(\hat{\boldsymbol{\theta}}^{(0)})$ in the AQL equation (2.2.2) to obtain the AQL estimator $\hat{\boldsymbol{\theta}}^{(1)}$. Then, update $\hat{\boldsymbol{\theta}}^{(0)}$ by $\hat{\boldsymbol{\theta}}^{(1)}$. Repeat this a few times until it converges.

The next section presents the parameter estimation of SVM using the QL and AQL methods.

3. Parameter estimation of SVM

In the following, we present the parameter estimation of SVM, which include non-linear and non-Gaussian models. We propose the QL and AQL approaches for SVM estimation. The estimations of states and unknown parameters are considered without any distribution assumptions about processes, and the estimation is based on different scenarios in which the conditional covariance of the error terms are assumed to be known or unknown.

3.1. Parameter estimation of SVM using the QL method

The stochastic volatility model is given by

$$y_t = e^{\frac{h_t}{2}}\eta_t, \quad t = 1,2,3\ldots,T \tag{3.1.1}$$

and

$$h_t = \gamma + \phi h_{t-1} + \delta_t, \quad t = 1,2,3\ldots,T \tag{3.1.2}$$

δ_t are i.i.d with $E(\delta_t) = 0$ and $V(\delta_t) = \sigma_\delta^2$.

The SVM in (3.1.1) can be transformed into a linear model as follows:

$$\ln(y_t^2) = h_t + \ln\eta_t^2, \quad t = 1,2,3\ldots,T. \tag{3.1.3}$$

Abramovitz and Stegun (1970) showed that if $\eta_t \sim N(0,1)$, then $E(\ln\eta_t^2) = -1.2704$ and $Var(\ln\eta_t^2) = \pi^2/2$. Now, assume that $\varepsilon_t = \ln\eta_t^2 + 1.2704$. Thus, $E(\varepsilon_t) = 0$. However, if η_t has an unknown distribution, then $E(\ln\eta_t^2) = \mu$ and $Var(\ln\eta_t^2) = \sigma_\epsilon^2$. Therefore, let $\epsilon_t = \ln\eta_t^2 - \mu$. For this scenario, the martingale difference is

$$\begin{pmatrix}\epsilon_t\\ \delta_t\end{pmatrix} = \begin{pmatrix}\ln(y_t^2) - h_t - \mu\\ h_t - \gamma - \phi h_{t-1}\end{pmatrix}.$$

First, to estimate h_t, the QLEF is given by

$$\begin{aligned}G_{(t)}(h_t) &= (-1,1)\begin{pmatrix}\sigma_\epsilon^2 & 0\\ 0 & \sigma_\delta^2\end{pmatrix}^{-1}\left(\ln(y_t^2) - h_t - \mu h_t - \gamma - \phi h_{t-1}\right)\\ &= \sigma_\epsilon^{-2}(\ln(y_t^2) - h_t - \mu) + \sigma_\delta^{-2}(h_t - \gamma - \phi h_{t-1}).\end{aligned} \tag{3.1.4}$$

Given that $\hat{h}_0 = 0$, the initial values $\psi_0 = (\gamma_0, \phi_0, \sigma_{\delta_0}^2, \mu_0, \sigma_{\epsilon_0}^2)$, and $\hat{h}_{t-1}$ is the QL estimation of h_{t-1}, the QL estimation of h_t is the solation of $G_{(t)}(h_t) = 0$,

$$\hat{h}_t = \frac{\sigma_{\delta_0}^2(\ln(y_t^2) - \mu_0) + \sigma_{\epsilon_0}^2(\phi_0\hat{h}_{t-1} + \gamma_0)}{\sigma_{\delta_0}^2 + \sigma_{\epsilon_0}^2},$$
$$t = 1,2,3\ldots,T. \tag{3.1.5}$$

Second, using $\{\hat{h}_t\}$ and $\{y_t\}$, and considering γ, μ, and ϕ as unknown parameters, the QLEF can be given by

$$G_T(\mu,\gamma,\phi) = \sum_{t=1}^{T}\begin{pmatrix}-1 & 0\\ 0 & -1\\ 0 & -h_{t-1}\end{pmatrix}\begin{pmatrix}\sigma_{\epsilon_0}^2 & 0\\ 0 & \sigma_{\delta_0}^2\end{pmatrix}^{-1}\begin{pmatrix}\ln(y_t^2) - h_t - \mu\\ h_t - \gamma - \phi h_{t-1}\end{pmatrix}.$$

The QL estimation of μ, γ, and ϕ is the solation of $G_T(\mu,\gamma,\phi) = 0$. Therefore,

$$\hat{\mu} = \frac{\sum_{t=1}^{T}\ln(y_t^2) - \sum_{t=1}^{T}\hat{h}_t}{T}, \quad t = 1,2,3\ldots,T. \tag{3.1.6}$$

$$\hat{\phi} = \frac{\sum_{t=1}^{T}\hat{h}_t\sum_{t=1}^{T}\hat{h}_{t-1} - T\sum_{t=1}^{T}\hat{h}_{t-1}\hat{h}_t}{1^2 - T\sum_{t=1}^{T}\hat{h}_{t-1}^2}, \quad t = 1,2,3\ldots,T. \tag{3.1.7}$$

$$\hat{\gamma} = \frac{\sum_{t=1}^{T}\hat{h}_t - \hat{\phi}\sum_{t=1}^{T}\hat{h}_{t-1}}{T}, \quad t = 1,2,3\ldots,T. \tag{3.1.8}$$

Further,

$$\hat{\sigma}_\delta^2 = \frac{\sum_{t=1}^{T}(\hat{\delta}_t - \bar{\hat{\delta}})^2}{T-1}, \tag{3.1.9}$$

$$\hat{\sigma}_\epsilon^2 = \frac{\sum_{t=1}^{T}(\hat{\epsilon}_t - \bar{\hat{\epsilon}})^2}{T-1}, \tag{3.1.10}$$

where $\hat{\epsilon}_t = \ln(y_t^2) - \hat{h}_t - \hat{\mu}$, and $\hat{\delta}_t = \hat{h}_t - \hat{\gamma} - \hat{\phi}\hat{h}_{t-1}$, $t = 1,2,3,\ldots,T$.

$\hat{\psi} = (\hat{\mu}, \hat{\gamma}, \hat{\phi}, \hat{\sigma}_\delta^2, \hat{\sigma}_\epsilon^2)$ is an updated initial value in the iterative procedure. The initial values h_0 and $\boldsymbol{\psi}_0$ might be affected by the estimation results of SVM. For an extensive discussion on assigning initial values in the QL estimation procedures, see Alzghool and Lin (2011).

Table 2 The QL and AQL estimates; the RMSE of each estimate is given below that estimate.

		γ	ϕ	μ	σ_δ	σ_ϵ
	True	−0.141	0.98	−1.271	0.061	2.220
$T=20$	QL	−0.147	0.976	−1.273	0.067	2.127
		0.032	0.024	0.405	0.019	0.573
	AQL	−0.294	0.828	−1.056	0.399	1.952
		0.268	0.206	0.260	0.547	0.426
$T=40$	QL	−0.145	0.978	−1.286	0.069	2.143
		0.022	0.010	0.264	0.015	0.415
	AQL	−0.249	0.917	−1.052	0.397	1.991
		0.174	0.086	0.248	0.421	0.379
$T=60$	QL	−0.147	0.976	−1.273	0.067	2.127
		0.018	0.006	0.201	0.013	0.334
	AQL	−0.225	0.943	−1.074	0.387	2.012
		0.125	0.048	0.219	0.356	0.354
$T=80$	QL	−0.144	0.979	−1.290	0.070	2.162
		0.016	0.004	0.171	0.013	0.283
	AQL	−0.214	0.954	−1.088	0.382	2.037
		0.099	0.032	0.203	0.308	0.342
$T=100$	QL	−0.144	0.979	−1.285	0.070	2.163
		0.015	0.004	0.156	0.012	0.253
	AQL	−0.211	0.958	−1.110	0.368	2.050
		0.094	0.027	0.180	0.279	0.322

Table 3 Estimation of $\psi=(\mu,\gamma,\phi,\sigma_\delta^2,\sigma_\epsilon^2)$ for pound/dollar exchange rate data.

	γ	ϕ	σ_δ	μ	σ_ϵ
QL	−0.0250	0.974	0.0210	−1.27	2.140
AQL	−0.078	0.977	0.224	−1.042	2.12
AL	−0.0227	0.957	0.0267		
MCL	−0.0227	0.975	0.0273		

3.2. Parameter estimation of SVM using the AQL method

Consider the SVM given by ((3.1.1)) and ((3.1.2)) and the same argument listed under ((3.1.2)). First, to estimate h_t, the AQLEF sequence is given by

$$G_{(t)}(h_t)=(-1,1)\boldsymbol{\Sigma}_{t,n}^{-1}\begin{pmatrix}\ln(y_t^2)-h_t-\mu\\ h_t-\gamma-\phi h_{t-1}\end{pmatrix}$$

Given $\hat{h}_0=0,\boldsymbol{\theta}^{(0)}=(\gamma_0,\phi_0,\mu_0)$, $\boldsymbol{\Sigma}_{t,n}^{(0)}=\mathbf{I}_2$, and $\hat{h}_{t-1}$ is the AQL estimation of h_{t-1}, the AQL estimation of h_t is the solation of $G_{(t)}(h_t)=0$; that is,

$$\hat{h}_t=\frac{\ln(y_t^2)-\mu_0+\phi_0\hat{h}_{t-1}+\gamma_0}{2},\quad t=1,2,3\ldots,T. \tag{3.2.3}$$

Second, using the kernel estimation method, we find

$$\hat{\boldsymbol{\Sigma}}_{t,n}(\boldsymbol{\theta}^{(0)})=\begin{pmatrix}\hat{\sigma}_n(y_t) & \hat{\sigma}_n(y_t,h_t)\\ \hat{\sigma}_n(h_t,y_t) & \hat{\sigma}_n(h_t)\end{pmatrix}.$$

Third, to estimate the parameters $\boldsymbol{\theta}=(\gamma,\phi,\mu)$, we use $\{\hat{h}_t\}$ and $\{y_t\}$ and the AQLEF sequence

$$G_T(\gamma,\phi,\mu)=\sum_{t=1}^{T}\begin{pmatrix}-1 & 0\\ 0 & -1\\ 0 & -\hat{h}_{t-1}\end{pmatrix}\hat{\boldsymbol{\Sigma}}_{t,n}^{-1}\begin{pmatrix}\ln(y_t^2)-\hat{h}_t-\mu\\ \hat{h}_t-\gamma-\phi\hat{h}_{t-1}\end{pmatrix}.$$

The AQL estimation of γ,ϕ, and μ is the solation of $G_T(\gamma,\phi,\mu)=0$. Then $\boldsymbol{\theta}^{(0)}=(\gamma_0,\phi_0,\mu_0)$ is updated and replaced by the $\hat{\boldsymbol{\theta}}=(\hat{\gamma},\hat{\phi},\hat{\mu})$, the estimate of $\boldsymbol{\theta}=(\gamma,\phi,\mu)$. The estimation procedure will be iteratively repeated until it converges.

In the following, the setup for this simulation study is similar to the design used by Rodriguez-Yam (2003). Samples of size $T=500$ are taken, and the mean and root mean squared errors (RMSE) for $\hat{\phi},\hat{\gamma},\ \hat{\mu},\sigma_\delta$, and σ_ϵ are calculated, where $N=1000$ independent samples.

In Table 1, QL represents the QL estimate and AQL represents the AQL estimate.

The results in Table 1 confirm that QL and AQL have succeeded in SVM parameter estimation.

The effect of sample size on parameter estimation is considered. Samples of sizes $T = 20, 40, 60, 80$, and 100 were generated.

The results in Table 2 show that the RMSE decreases when the sample size increases.

4. Application to SVM

The QL and AQL methods developed in the previous section are applied to real-life data, where the data are modeled by SVM (1.1) and (1.2). The data are the pound/dollar exchange rates $x_t, t = 1, \ldots, 945$ from 1/10/1981 to 28/6/1985 (see Davis and Rodriguez-Yam, 2005; Rodriguez-Yam, 2003; Durbin and Koopman, 2001).

In the literature, SVM (1.1) and (1.2) are used to model $y_t = log(x_t) - log(x_{t-1})$, where $\eta_t \sim N(0, 1)$ and $\psi = (\mu, \gamma, \phi, \sigma_\delta^2, \sigma_\epsilon^2)$ is the parameter.

Table 3 gives the estimates of ψ obtained using different methods. QL represents the estimate obtained using the QL method, AQL represents the AQL estimate, AL is the estimate found by maximizing the approximate likelihood as suggested by Davis and Rodriguez-Yam (2005), and the MCL estimate is found by maximizing the likelihood estimate as suggested by Durbin and Koopman (2001). Note that the AL and MCL outcomes are given in Rodriguez-Yam (2003).

The estimates of γ, ϕ, and σ_δ by the QL, AL and MCL methods are conceded. These three estimates are based on the same assumption where both η_t and δ_t are independent. However, the AQL estimates are a little dissimilar from the QL, AL, and MCL estimates.

The QL and AQL estimates are carried out in diverse model sceneries. The first scenario assumes that $cov(\eta_t, \delta_t) = 0$ and the second scenario assumes that $cov(\eta_t, \delta_t) = \sigma(h_t, y_t)$. To know which model scenario is suitable, we need to examine whether we can adapt $cov(\eta_t, \delta_t) = 0$. We compute the $\hat{\epsilon}_t$ and $\hat{\delta}_t$ given by QL approach and find that $\hat{\epsilon}_t$ and $\hat{\delta}_t$ are not independent with a significant correlation coefficient $r = 0.89$ at the 0.01 level. Thus, assuming that ϵ_t and δ_t are independent is not effective and using the QL technique for these data is not suitable. Therefore, the estimation using the AQL technique is accepted more than the QL estimates.

5. Summary

In this paper, we presented the estimation of parameters in SVMs using two alternative approaches. The study has shown that the QL and AQL estimating procedures are easy to apply, especially when the SVM's probability structure cannot be fully identified. Results from the simulation study show that the AQL technique is a competent estimation procedure. The technique can escape the threat of possible misspecification of $\mathbf{\Sigma}$ by using the kernel estimator of covariance matrixes to substitute the true $\mathbf{\Sigma}$ in the QL and thus make the parameter estimation more efficient in SVMs.

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