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An efficient approach for fractional nonlinear chaotic model with Mittag-Leffler law

P. Veeresha [a], D.G. Prakasha [b], Abdel-Haleem Abdel-Aty [c,d,*], Harendra Singh [e], Emad E. Mahmoud [f,g], Sunil Kumar [h,*]

[a] Department of Mathematics, CHRIST University, Bengaluru-560029, India
[b] Department of Mathematics, Faculty of Science, Davangere University, Shivagangothri, Davangere 577007, Karnataka, India
[c] Department of Physics, College of Sciences, University of Bisha, P.O. Box 344, Bisha 61922, Saudi Arabia
[d] Physics Department, Faculty of Science, Al-Azhar University, Assiut 71524, Egypt
[e] Department of Mathematics, Post-Graduate College Ghazipur, Uttar Pradesh 233001, India
[f] Department of Mathematics and Statistics, College of Science, Taif University, P.O. Box 11099, Taif 21944, Saudi Arabia
[g] Department of Mathematics, Faculty of Science, Sohag University, Sohag 82524, Egypt
[h] Department of Mathematics, National Institute of Technology, Jamshedpur 831014, Jharkhand, India

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ABSTRACT

In this work, we exemplify the behaviour of the nonlinear model of arbitrary order differential equations by adopting q-homotopy analysis transform method (q-HATM). In the present study, the illustrated scheme is a graceful amalgamation of Laplace transform with q-homotopy analysis algorithm and we considered arbitrary order derivative using Atangana-Baleanu (AB) operator. The suggested nonlinear system exhibits chaotic behaviour in nature with respect to considered initial conditions. Fixed point hypothesis heard present the existence and uniqueness for the attained solution. We exemplified suggested arbitrary order system with to illustrate and confirm the efficiency of the projected solution procedure. Further, the numerical simulation is illustrated and also the chaotic behaviour of the obtained result captured with respect to arbitrary order in terms of plots. The obtained results confirm the projected scheme is highly methodical, easy to implement and very powerful to exemplify the nature of the dynamical system of arbitrary order.

1. Introduction

The theory and applications of arbitrary order derivative recently magnetise the attention of many young researchers even though it was originated in Newton's time. Fractional calculus is the most influential apparatus from the last few decades to examine and exemplify the nonlinear complex phenomena, due to the auspicious assets namely, memory effect, nonlocality, analyticity and hereditary. Within the frame of FC, the most simulating leaps in science, technology and their associated areas have been arises. The fractional differential operators accomplished to exemplify the necessary development of the nonlinear phenomena having a diffusion mechanism. Many scholars begin to investigate on the FC with fundamentals and its applications due to the progress of a mathematical algorithm. The diverse definitions are anticipated by numerous pioneering for fractional calculus, and which prearranged the groundwork for FC (Liouville, 1832; Riemann, 1896; Caputo, 1969; Miller and Ross, 1993; Podlubny, 1999; Kilbas et al., 2006). The fundamental theory and applications of FC are widely illustrated in various aspects with emerging phenomena like nanotechnology (Baleanu et al., 2010), chaos theory (Esen et al., 2018), human diseases (Veeresha et al., 2019), optics (Baleanu et al., 2017), and other fields (Veeresha et al., 2019a, 2020b, 2019c; Gao, 2020). Particularly, mathematical models exemplifying diverse phenomena are analysed by the aid of notions and fundamentals of FC, for instance authors in (Singh and Srivastava, 2020) studied Liénard and D uffing equation arising

* Corresponding authors at: Department of Physics, College of Sciences, University of Bisha, P.O. Box 344, Bisha 61922, Saudi Arabia (A.–H. Abdel-Aty).

E-mail addresses: pundikala.veeresha@christuniversity.in (P. Veeresha), amabdelaty@ub.edu.sa (A.-H. Abdel-Aty), harendra059@gmail.com (H. Singh), e.mahmoud@tu.edu.sa (E.E. Mahmoud), skiitbhu28@gmail.com (S. Kumar).

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in oscillating circuit theory and presented the numerical stimulation within the frame of arbitrary order calculus, the epidemiological model of the Ebola virus is analysed by Srivastava et al. in (Srivastava et al., 2020) with the help of spectral collocation technique, the behaviour of the solution achieved for arbitrary-order Drinfeld-Sokolov-Wilson system is captured by authors in (Srivastava and Saad, 2020), Singh and his co-authors derived numerical solution for arbitrary-order Bloch equation by adopting the Jacobi polynomials (Singh and Srivastava, 2020), the overview and recent developments are effectively illustrated by senior and emeritus scholar in (Srivastava, 2020), new mathematical models of the human immune against IAV infection is nurtured within the frame of arbitrary order calculus by authors in (Srivastava et al., 2020), and researchers in (Izadi and Srivastava, 2020) presented few stimulating results with respect to nonlinear arbitrary order logistic equation by using an discretization approach.

Nowadays, there is a tremendous fascination towards the study of nonlinear dynamics, particularly chaos and fractals. Dynamics is the subject that deals with changes and systems that evolve in time. This subject originated in the mid of 1600s when Newton invented DEs. Particularly, the theory of chaos magnetized the attention of many scientists and engineers due to the discovery of the Lorenz attractor (Lorenz, 1963) and the innovation of high-speed computers. In has been proven that, chaotic signal play a vibrant role in the chaos-based information systems, and is used for the control processing and secure communication. In addition to this, authors in (Grigorenko and Grigorenko, 2003; Hammouch and Mekkaoui, 2014; Baskonus et al., 2015) demonstrated the essence of fractional order with chaos system in order to study the various physical phenomena and capture the behaviour of nature in an effective and systematic manner.

The chaotic nature of the systems is the emerging and interesting topic in the recent era. It has been studied and analysed by many senior scholars in order to illustrate the many interesting and emerging consequences arisen in diverse areas. In connection with this, authors in (Owolabi and Atangana, 2018) capture the chaotic behaviour of some fractional-order system, authors in (Bhalekar et al., 2012) illustrated the chaotic nature of Bloch equations which address the key issue in NMR relaxation problem, authors in Daftardar-Gejji et al. (2012) presented the chaotic dynamics of Chen system having fractional order.

Many nonlinear interesting phenomena arisen in related fields of science and engineering are systematically and effectively demonstrated using fractional calculus. Many pioneers defined the diverse notions for arbitrary order differential and integral. However, each definition has its own limitation. The Riemann–Liouville derivative fails to define the importance of the ICs, the Liouville-Caputo fractional operator is not related to describe singular kernel. Later Caputo and Fabrizio in 2015 overcome these limitations (Caputo and Fabrizio, 2015), and soon after many researchers applied to analyse and illustrate some stimulating nature for complex models. Recently, some authors raised some issues associated with important properties exemplifying the behaviour of nonlinear problems like non-singular kernel and non-local. In 2016, Atangana and Baleanu proposed definition with the assist of Mittag–Leffler functions, namely Atangana-Baleanu (AB) derivative (Atangana and Baleanu, 2016) and which get huge attention of the research community.

We assume the model of the equation which described the chaotic nature studied by Hammouch and Mekkaoui (Hammouch and Mekkaoui, 2018):

$$\dot{x}(t) = -2x(t) - y^2(t), \dot{y}(t) = -4x(t)z(t) + 3y(t) - z^2(t), \dot{z}(t) = 4x(t)y(t) - 7z(t) + y(t)z(t) \tag{1}$$

Authors in (Hammouch and Mekkaoui, 2018) suggested the simulation for the arbitrary order system with Caputo derivative which poses the interesting chaotic behaviour by the aid of Adams–Bashforth–Moulton scheme and also they demonstrated the circuit design. In this work, we consider with AB derivative to include non-singular kernel and non-local, and which as follows

$$\begin{aligned} &aABCD_t^{\alpha}x(t) = -2x(t) - y^2(t), \\ &aABCD_t^{\alpha}y(t) = -4x(t)z(t) + 3y(t) - z^2(t), \quad 0 < \alpha \leqslant 1, \\ &aABCD_t^{\alpha}z(t) = 4x(t)y(t) - 7z(t) + y(t)z(t), \end{aligned} \tag{2}$$

where$\alpha$is arbitrary order of the system.

In the last few decades, many advanced methods are proposed by mathematicians and physicist to examine the differential and integral equations. In this connection, Liao Shijun defined the homotopy analysis technique (Liao, 1997, 1998). It has advantageously and efficiently considered evaluating the solution for nonlinear problems. But, there is an essence of the amalgamation of this algorithm and classical transform methods. Since this method requires more computer memory and huge time for calculation work.

In this present investigation, we exemplified and evaluate the solution for the arbitrary order system describes the interesting chaotic behaviour by the help of q-HATM. The projected scheme is suggested by Singh et al. (Singh et al., 2016) with the aid of Laplace transform associated to q-HAM. Here, q is an embedded parameter defined by $q \in \left[0, \frac{1}{n}\right](n \geq 1)$, and as q increases from 0 to $\frac{1}{n}$, then the obtained results vary from the primary guess to the solution. As q gradually increases continuously toward $1/n$, the system goes through a sequence of deformations, and the solution at each stage is close to that at the previous stage of the deformation. Moreover, authors in (El-Tawil and Huseen, 2012) illustrated that the convergence region of series solutions achieved by q-HAM is increasing as q is decreased and which provides on improvisation in the classical scheme.

The suggested algorithm will reduce vast mathematical computations. The projected solution procedure is recently many researchers considered in to exemplify the behaviour of many classes of nonlinear and complex systems (Srivastava et al., 2017; Veeresha et al., 2020, 2019; Gao, 2020; Veeresha and Prakasha, 2019, 2020; Kumar et al., 2018; Prakasha and Veeresha, 2020; Kiran, 2020). Moreover, it cogently encompasses the consequences of various classical techniques such as HPM, RDTM, Adomian decomposition method and q-HAM, these shows its prodigious generality. The projected method can decrease the computation of the time and work as weigh compared the other classical scheme while conserving the decent accuracy.

2. Preliminaries

We present the essential definitions of Laplace transform (LT) and FC (Singh et al., 2018; Veeresha et al., 2020; Atangana and Alkahtani, 2015, 2016; Prakasha et al., 2020).

Definition 1. For a function $f \in H^1(a,b)$ the arbitrary order ABC derivative is described as follows:

$$aABCD_t^{\alpha}(f(t)) = \frac{\mathscr{B}[\alpha]}{1-\alpha}\int_a^t f'(\vartheta)E_{\alpha}\left[\alpha\frac{(t-\vartheta)^{\alpha}}{\alpha-1}\right]d\vartheta, \quad b > a. \tag{3}$$

Definition 2. For a $f \in H^1(a,b)$ the AB arbitrary order derivative in Riemann-Liouville sense is presented as

$$aABRD_t^{\alpha}(f(t)) = \frac{\mathscr{B}[\alpha]}{1-\alpha}\frac{d}{dt}\int_a^t f(\vartheta)E_{\alpha}\left[\alpha\frac{(t-\vartheta)^{\alpha}}{\alpha-1}\right]d\vartheta, \quad b > a. \tag{4}$$

Definition 3. The arbitrary order AB integral presented as

$$aABRI_t^{\alpha}(f(t)) = \frac{1-\alpha}{B[\alpha]}f(t) + \frac{\alpha}{B[\alpha]\Gamma(\alpha)}\int_a^t f(\vartheta)(t-\vartheta)^{\alpha-1}d\vartheta. \tag{5}$$

Definition 4. *The Laplace transform (LT) with AB operator is described as*

$$L\left[\, aABRD_t^{\alpha}(f(t))\right] = \frac{B[\alpha]}{1-\alpha}\frac{s^{\alpha}L[f(t)] - s^{\alpha-1}f(0)}{s^{\alpha} + (\alpha/(1-\alpha))}. \tag{6}$$

Theorem 1. For the Riemann-Liouville and AB derivatives, the subsequent Lipschitz conditions respectively satisfy (Grigorenko and Grigorenko, 2003)

$$\|\ aABRD_t^{\alpha}f_1(t) - \ aABRD_t^{\alpha}f_2(t)\ \| < K_1\|\ f_1(x) - f_2(x)\ \|, \tag{7}$$

and

$$\|\ aABRD_t^{\alpha}f_1(t) - \ aABRD_t^{\alpha}f_2(t)\ \| < K_2\|\ f_1(x) - f_2(x)\ \|. \tag{8}$$

Theorem 2. The arbitrary order DEs $aABRD_t^{\alpha}f_1(t) = s(t)$ has a unique solution is described by (Grigorenko and Grigorenko, 2003)

$$f(t) = 1 - \frac{\alpha}{\mathscr{B}[\alpha]}s(t) + \frac{\mu}{\mathscr{B}[\alpha]\Gamma(\alpha)}\int_a^t s(\varsigma)(t-\varsigma)^{\alpha-1}d\varsigma. \tag{9}$$

3. Solution for considered system with suggested method

In this segment, we illustrate the efficiency of the suggested algorithm to find the solution for a considered arbitrary-order non-linear chaotic system. Moreover, we capture the behaviour of the achieved results. Now, we have by the aid of Eq. (2), we have

$$\begin{aligned} &aABRD_t^{\alpha}x(t) + 2x(t) + y^2(t) = 0, \\ &aABRD_t^{\alpha}y(t) + 4x(t)z(t) - 3y(t) + z^2(t) = 0, \quad ,0 < \alpha \leqslant 1, \\ &aABRD_t^{\alpha}z(t) - 4x(t)y(t) + 7z(t) - y(t)z(t) = 0 \end{aligned} \tag{10}$$

associated to

$$x(0) = x_0(t),\ \ y(0) = y_0(t),\ \ z(0) = z_0(t). \tag{11}$$

Using the Eq. (11) after applying LT on Eq. (10), one can have

$$L[x(t)] = \frac{1}{s}(x_0(t)) + \frac{1}{\mathscr{B}[\alpha]}\left(1-\alpha+\frac{\alpha}{s^{\alpha}}\right)L\{2x(t) + y^2(t)\}$$

$$L[y(t)] = \frac{1}{s}(y_0(t)) + \frac{1}{\mathscr{B}[\alpha]}\left(1-\alpha+\frac{\alpha}{s^{\alpha}}\right)L\{4x(t)z(t) - 3y(t) + z^2(t)\}$$

$$\begin{aligned} L[z(t)] = &\frac{1}{s}(z_0(t)) \\ &-\frac{1}{\mathscr{B}[\alpha]}\left(1-\alpha+\frac{\alpha}{s^{\alpha}}\right)L\{4x(t)y(t) + 7z(t) - y(t)z(t)\}. \end{aligned} \tag{12}$$

Now, $N$is defined as

$$N^1[\varphi_1(t;q), \varphi_2(t;q), \varphi_3(t;q)] = L[\varphi_1(t;q)] - \frac{1}{s}(x_0(t))$$

$$+\frac{1}{\mathscr{B}[\alpha]}\left(1-\alpha+\frac{\alpha}{s^{\alpha}}\right)L\{2\varphi_1(t;q) + \varphi_2^2(t;q)\},$$

$$N^2[\varphi_1(t;q), \varphi_2(t;q), \varphi_3(t;q)] = L[\varphi_2(t;q)] - \frac{1}{s}(y_0(t))$$

$$+\frac{1}{\mathscr{B}[\alpha]}\left(1-\alpha+\frac{\alpha}{s^{\alpha}}\right)L\{4\varphi_1(t;q)\varphi_3(t;q)$$

$$-3\varphi_2(t;q) + \varphi_3^2(t;q)\},$$

$$N^3[\varphi_1(t;q), \varphi_2(t;q), \varphi_3(t;q)] = L[\varphi_3(t;q)] - \frac{1}{s}(\varphi_2(t;q))$$

$$-\frac{1}{\mathscr{B}[\alpha]}\left(1-\alpha+\frac{\alpha}{s^{\alpha}}\right)L\{4\varphi_1(t;q)\varphi_2(t;q)$$

$$-7\varphi_3(t;q) + \varphi_2(t;q)\varphi_3(t;q)\}. \tag{13}$$

At $\mathscr{H}(x,t) = 1$, the *m-th* order deformation equation is suggested as is given as follows

$$\begin{aligned} L[x_m(t) - \mathrm{k}_m x_{m-1}(t)] &= \hbar\Re_{1,m}\left[\vec{x}_{m-1}, \vec{y}_{m-1}, \vec{z}_{m-1}\right], \\ L[y_m(t) - \mathrm{k}_m y_{m-1}(t)] &= \hbar\Re_{2,m}\left[\vec{x}_{m-1}, \vec{y}_{m-1}, \vec{z}_{m-1}\right], \\ L[z_m(t) - \mathrm{k}_m z_{m-1}(t)] &= \hbar\Re_{3,m}\left[\vec{x}_{m-1}, \vec{y}_{m-1}, \vec{z}_{m-1}\right], \end{aligned} \tag{14}$$

where

$$\Re_{1,m}\left[\vec{x}_{m-1}, \vec{y}_{m-1}, \vec{z}_{m-1}\right] = L[x_{m-1}(t)] - \left(1 - \frac{\mathrm{k}_m}{n}\right)\left\{\frac{1}{s}(x_0(t))\right\}$$

$$+\frac{1}{\mathscr{B}[\alpha]}\left(1-\alpha+\frac{\alpha}{s^{\alpha}}\right)L\left\{2x_{m-1} + \sum_{i=0}^{m-1}y_i y_{m-1-i}\right\},$$

$$\Re_{2,m}\left[\vec{x}_{m-1}, \vec{y}_{m-1}, \vec{z}_{m-1}\right] = L[y_{m-1}(t)] + \left(1 - \frac{\mathrm{k}_m}{n}\right)\left\{\frac{1}{s}(y_0(t))\right\}$$

$$+\frac{1}{\mathscr{B}[\alpha]}\left(1-\alpha+\frac{\alpha}{s^{\alpha}}\right)L\{4\sum_{i=0}^{m-1}x_i z_{m-1-i}$$

$$-3y_{m-1} + \sum_{i=0}^{m-1}z_i z_{m-1-i}\}$$

$$\Re_{3,m}\left[\vec{x}_{m-1}, \vec{y}_{m-1}, \vec{z}_{m-1}\right] = L[z_{m-1}(t)] + \left(1 - \frac{\mathrm{k}_m}{n}\right)\left\{\frac{1}{s}(z_0(t))\right\}$$

$$-\frac{1}{\mathscr{B}[\alpha]}\left(1-\alpha+\frac{\alpha}{s^{\alpha}}\right)L\{4\sum_{i=0}^{m-1}x_i y_{m-1-i}$$

$$-7z_{m-1} + \sum_{i=0}^{m-1}y_i z_{m-1-i}\} \tag{15}$$

The Eq. (14) simplifies after employing inverse LT, as follows

$$\begin{aligned} x_m(t) &= \mathrm{k}_m x_{m-1}(t) + \hbar L^{-1}\left\{\Re_{1,m}\left[\vec{x}_{m-1}, \vec{y}_{m-1}, \vec{z}_{m-1}\right]\right\}, y_m(t) \\ &= \mathrm{k}_m y_{m-1}(t) + \hbar L^{-1}\left\{\Re_{2,m}\left[\vec{x}_{m-1}, \vec{y}_{m-1}, \vec{z}_{m-1}\right]\right\}, z_m(t) \\ &= \mathrm{k}_m z_{m-1}(t) + \hbar L^{-1}\left\{\Re_{3,m}\left[\vec{x}_{m-1}, \vec{y}_{m-1}, \vec{z}_{m-1}\right]\right\}. \end{aligned} \tag{16}$$

On clarifying the Eq. (16) with $x_0(t) = 0.7, y_0 = 0.1$ and $z_0(t) = 0$we can find the terms of

$$x(t) = x_0(t) + \sum_{m=1}^{\infty} x_m(t)\left(\frac{1}{n}\right)^m,$$
$$y(t) = y_0(t) + \sum_{m=1}^{\infty} y_m(t)\left(\frac{1}{n}\right)^m, z(t) = z_0(t) + \sum_{m=1}^{\infty} z_m(t)\left(\frac{1}{n}\right)^m. \tag{17}$$

4. Existence of solutions

Now, we consider the system (10) to illustrate the existence of the solution as follows:

$$\begin{cases} 0ABCD_t^{\alpha}[x(t)] = G_1(t,\ x), \\ 0ABCD_t^{\alpha}[y(t)] = G_2(t, y), \\ 0ABCD_t^{\alpha}[z(t)] = G_3(t, z). \end{cases} \tag{18}$$

By the help of Theorem 2, the above model transformed to the Volterra integral equation and we have

$$\begin{cases} x(t) - x(0) = \frac{(1-\alpha)}{\mathscr{B}(\alpha)}\mathscr{G}_1(t,x) + \frac{\alpha}{\mathscr{B}(\alpha)\Gamma(\alpha)}\int_0^t \mathscr{G}_1(\zeta,x)(t-\zeta)^{\alpha-1}d\zeta, \\ y(t) - y(0) = \frac{(1-\alpha)}{\mathscr{B}(\alpha)}\mathscr{G}_2(t,y) + \frac{\alpha}{\mathscr{B}(\alpha)\Gamma(\alpha)}\int_0^t \mathscr{G}_2(\zeta,y)(t-\zeta)^{\alpha-1}d\zeta, \\ z(t) - z(0) = \frac{(1-\alpha)}{\mathscr{B}(\alpha)}\mathscr{G}_3(t,z) + \frac{\alpha}{\mathscr{B}(\alpha)\Gamma(\alpha)}\int_0^t \mathscr{G}_3(\zeta,z)(t-\zeta)^{\alpha-1}d\zeta. \end{cases} \tag{19}$$

Theorem 3. The kernel $\mathscr{G}_1$ admits the Lipschitz condition and contraction if $0 \le \left(2 + \lambda_2^2\right) < 1$ gratifies.

Proof. Now, we consider $u$and u_1 to illustrate the essential result, then

$$\begin{aligned} \|\mathscr{G}_1(t,x) - \mathscr{G}_1(t,x_1)\| &= \|\left(2[x(t) - x(t_1)] + y^2(t)\right)\| \\ &\le \|2 + \lambda_2^2\|\|x(t) - x(t_1)\| \\ &\le \left(2 + \lambda_2^2\right)\|x(t) - x(t_1)\| \end{aligned} \tag{20}$$

where $\|y(t)\| \le \lambda_2$ be the bounded function. Putting $\eta_1 = 2 + \lambda_2^2$ in Eq. (20), then

$$\|\mathscr{G}_1(t,x) - \mathscr{G}_1(t,x_1)\| \le \eta_1\|x(t) - x(t_1)\| \tag{21}$$

Therefore, the Lipschitz condition is attained for $\mathscr{G}_1$. Moreover, if $0 \le (2 + \lambda_2^2) < 1$, then it leads to contraction. Further, we get

$$\begin{cases} \|\mathscr{G}_2(t,y) - \mathscr{G}_2(t,y_1)\| \le \eta_2\|y(t) - y(t_1)\|, \\ \|\mathscr{G}_3(t,z) - \mathscr{G}_3(t,z_1)\| \le \eta_3\|z(t) - z(t_1)\|. \end{cases} \tag{22}$$

The recursive form of Eq. (19) described as follows

$$\begin{cases} x_n(t) = \frac{(1-\alpha)}{\mathscr{B}(\alpha)}\mathscr{G}_1(t,x_{n-1}) + \frac{\alpha}{\mathscr{B}(\alpha)\Gamma(\alpha)}\int_0^t \mathscr{G}_1(\zeta,x_{n-1})(t-\zeta)^{\alpha-1}d\zeta, \\ y_n(t) = \frac{(1-\alpha)}{\mathscr{B}(\alpha)}\mathscr{G}_2(t,y_{n-1}) + \frac{\alpha}{\mathscr{B}(\alpha)\Gamma(\alpha)}\int_0^t \mathscr{G}_2(\zeta,y_{n-1})(t-\zeta)^{\alpha-1}d\zeta, \\ z_n(t) = \frac{(1-\alpha)}{\mathscr{B}(\alpha)}\mathscr{G}_3(t,z_{n-1}) + \frac{\alpha}{\mathscr{B}(\alpha)\Gamma(\alpha)}\int_0^t \mathscr{G}_3(\zeta,z_{n-1})(t-\zeta)^{\alpha-1}d\zeta. \end{cases} \tag{23}$$

The associated initial conditions are

$$x(0) = x_0(t), y(0) = y_0(t) and z(0) = z_0(t) \tag{24}$$

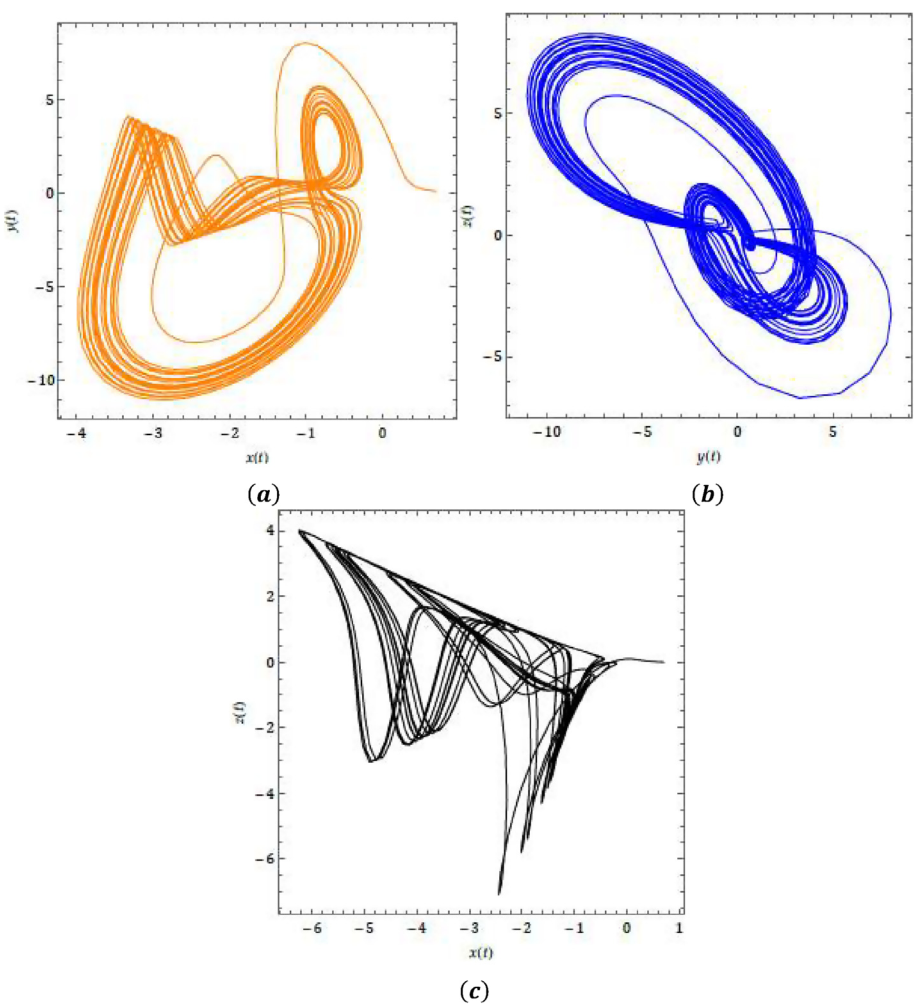

Fig. 1. Chaotic nature of the suggested model (***a***) $x - y$, (***b***) $y - z$ and (c) $x - z$ at $\hbar = -1, n = 1$ and $\alpha = 0.9$.

The successive difference between the terms is described as

$$\begin{cases} \phi_{1n}(t) = x_n(t) - x_{n-1}(t) \\ = \frac{(1-\alpha)}{\mathscr{B}(\alpha)}(\mathscr{G}_1(t,x_{n-1}) - \mathscr{G}_1(t,x_{n-2})) + \frac{\alpha}{\mathscr{B}(\alpha)\Gamma(\alpha)}\int_0^t \mathscr{G}_1(\zeta,x_{n-1})(t-\zeta)^{\alpha-1}d\zeta, \\ \phi_{2n}(t) = y_n(t) - y_{n-1}(t) \\ = \frac{(1-\alpha)}{\mathscr{B}(\alpha)}(\mathscr{G}_2(t,y_{n-1}) - \mathscr{G}_2(t,y_{n-2})) + \frac{\alpha}{\mathscr{B}(\alpha)\Gamma(\alpha)}\int_0^t \mathscr{G}_2(\zeta,y_{n-1})(t-\zeta)^{\alpha-1}d\zeta, \\ \phi_{3n}(t) = z_n(t) - z_{n-1}(t) \\ = \frac{(1-\alpha)}{\mathscr{B}(\alpha)}(\mathscr{G}_3(t,z_{n-1}) - \mathscr{G}_3(t,z_{n-2})) + \frac{\alpha}{\mathscr{B}(\alpha)\Gamma(\alpha)}\int_0^t \mathscr{G}_3(\zeta,z_{n-1})(t-\zeta)^{\alpha-1}d\zeta. \end{cases} \tag{25}$$

Clearly

$$\begin{cases} x_n(t) = \sum_{i=1}^n \phi_{1i}(t), \\ y_n(t) = \sum_{i=1}^n \phi_{2i}(t), \\ y_z(t) = \sum_{i=1}^n \phi_{3i}(t). \end{cases} \tag{26}$$

By using Eq. (21) after employing the norm on the $x_n(t)$, we get

$$\|\phi_{1n}(t)\| \le \frac{(1-\alpha)}{\mathscr{B}(\alpha)}\eta_1\|\phi_{1(n-1)}(t)\| + \frac{\alpha}{\mathscr{B}(\alpha)\Gamma(\alpha)}\eta_1 \times \int_0^t \|\phi_{1(n-1)}(\zeta)\|d\zeta. \tag{27}$$

In the same manner, we have

$$\begin{cases} \|\phi_{2n}(t)\| \le \frac{(1-\alpha)}{\mathscr{B}(\alpha)}\eta_2\|\phi_{2(n-1)}(t)\| + \frac{\alpha}{\mathscr{B}(\alpha)\Gamma(\alpha)}\eta_2\int_0^t \|\phi_{2(n-1)}(\zeta)\|d\zeta, \\ \|\phi_{3n}(t)\| \le \frac{(1-\alpha)}{\mathscr{B}(\alpha)}\eta_3\|\phi_{3(n-1)}(t)\| + \frac{\alpha}{\mathscr{B}(\alpha)\Gamma(\alpha)}\eta_3\int_0^t \|\phi_{3(n-1)}(\zeta)\|d\zeta. \end{cases} \tag{28}$$

With the assist of forgoing theorem, we find the following result:

Theorem 4. If we have particular t_0, then the solution for the model (10) exist and unique, and

$$\frac{(1-\alpha)}{\mathscr{B}(\alpha)}\eta_i + \frac{\alpha}{\mathscr{B}(\alpha)\Gamma(\alpha)}\eta_i < 1,$$

for $i = 1, 2$ and 3.

Proof. *Let $x(t), y(t)$ and $z(t)$ be the bounded functions and admits the Lipschitz condition. With the help of Eqs.* (26) and (28), *one can get*

$$\|\phi_{1i}(t)\| \le \|x_n(0)\|\left[\frac{(1-\alpha)}{\mathscr{B}(\alpha)}\eta_1 + \frac{\alpha}{\mathscr{B}(\alpha)\Gamma(\alpha)}\eta_1\right]^n,$$

$$\|\phi_{2i}(t)\| \le \|y_n(0)\|\left[\frac{(1-\alpha)}{\mathscr{B}(\alpha)}\eta_2 + \frac{\alpha}{\mathscr{B}(\alpha)\Gamma(\alpha)}\eta_2\right]^n,$$

$$\|\phi_{3i}(t)\| \le \|z_n(0)\|\left[\frac{(1-\alpha)}{\mathscr{B}(\alpha)}\eta_3 + \frac{\alpha}{\mathscr{B}(\alpha)\Gamma(\alpha)}\eta_3\right]^n. \tag{29}$$

Therefore, which show the existence and continuity for the attained solutions. In order to verify the system (29) is a solution for the model (10), we begin with

$$x(t) - x(0) = x_n(t) - \mathscr{K}_{1n}(t),$$

$$y(t) - y(0) = y_n(t) - \mathscr{K}_{2n}(t),$$

$$z(t) - z(0) = z_n(t) - \mathscr{K}_{3n}(t). \tag{30}$$

Now, we consider

$$\|\mathscr{K}_{1n}(t)\| = \|\frac{(1-\alpha)}{\mathscr{B}(\alpha)}(\mathscr{G}_1(t,x) - \mathscr{G}_1(t,x_{n-1}))$$

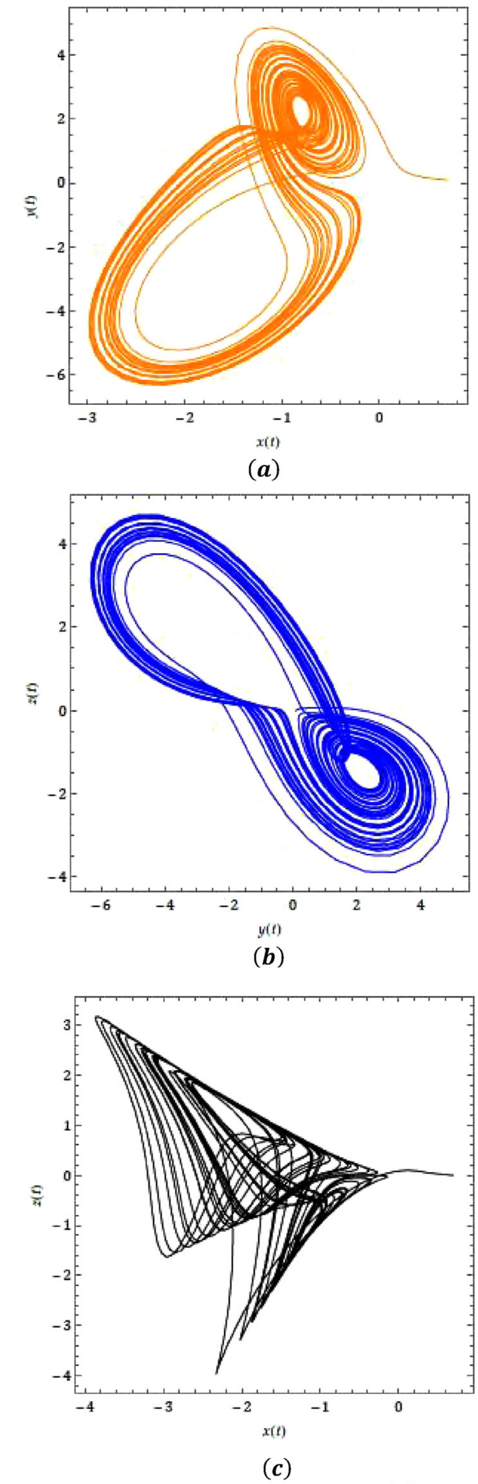

Fig. 2. Chaotic nature of the suggested model (***a***) $x-y$, (***b***) $y-z$ and (*c*) $x-z$ at $\hbar = -1, n = 1$ and $\alpha = 0.95$.

$$+\frac{\alpha}{\mathscr{B}(\alpha)\Gamma(\alpha)}\int_0^t (t-\zeta)^{\mu-1}(\mathscr{G}_1(\zeta,x) - \mathscr{G}_1(\zeta,x_{n-1}))d\zeta\|$$

$$\le \frac{(1-\alpha)}{\mathscr{B}(\alpha)}\|(\mathscr{G}_1(t,x) - \mathscr{G}_1(t,x_{n-1}))\|$$

$$+\frac{\alpha}{\mathscr{B}(\alpha)\Gamma(\alpha)}\int_0^t \|(\mathscr{G}_1(\zeta,x)-\mathscr{G}_1(\zeta,x_{n-1}))\|d\zeta$$

$$\leq \frac{(1-\alpha)}{\mathscr{B}(\alpha)}\eta_1\|x-x_{n-1}\|+\frac{\alpha}{\mathscr{B}(\alpha)\Gamma(\alpha)}\eta_1\|x-x_{n-1}\|t. \tag{31}$$

Similarly at t_0, we get

$$\|\mathscr{K}_{1n}(t)\| \leq \left(\frac{(1-\alpha)}{\mathscr{B}(\alpha)}+\frac{\alpha t_0}{\mathscr{B}(\alpha)\Gamma(\alpha)}\right)^{n+1}\eta_1^{n+1}M. \tag{32}$$

From Eq. (32) we can observer that, as $n\to\infty$, $\|\mathscr{K}_{1n}(t)\|\to 0$. Similarly, we can verify for $\|\mathscr{K}_{2n}(t)\|$ and $\|\mathscr{K}_{3n}(t)\|$.

Now, we present the uniqueness of the obtained solution. Suppose $x^*(t), y^*(t)$ and $z^*(t)$ be the set of other solutions, then one can get

$$x(t)-x^*(t)=\frac{(1-\alpha)}{\mathscr{B}(\alpha)}(\mathscr{G}_1(t,x)-\mathscr{G}_1(t,x^*))$$

$$+\frac{\alpha}{\mathscr{B}(\alpha)\Gamma(\alpha)}\int_0^t (\mathscr{G}_1(\zeta,x)-\mathscr{G}_1(\zeta,x^*))d\zeta. \tag{33}$$

the Eq. (33) reduces with the assist of the norm, to

$$\|x(t)-x^*(t)\| = \|\frac{(1-\alpha)}{\mathscr{B}(\alpha)}(\mathscr{G}_1(t,x)-\mathscr{G}_1(t,x^*)) +\frac{\alpha}{\mathscr{B}(\alpha)\Gamma(\alpha)}\int_0^t (\mathscr{G}_1(\zeta,x)-\mathscr{G}_1(\zeta,x^*))d\zeta\|$$

$$\leq \frac{(1-\alpha)}{\mathscr{B}(\alpha)}\eta_1\|x(t)-x^*(t)\|+\frac{\alpha}{\mathscr{B}(\alpha)\Gamma(\alpha)}\eta_1 t\|x(t)-x^*(t)\|. \tag{34}$$

With the assist of the above relation, one can get

$$\|x(t)-x^*(t)\|\left(1-\frac{(1-\alpha)}{\mathscr{B}(\alpha)}\eta_1-\frac{\alpha}{\mathscr{B}(\alpha)\Gamma(\alpha)}\eta_1 t\right)\leq 0. \tag{35}$$

By the aid of forgoing relation, we can see that $x(t)=x^*(t)$, if

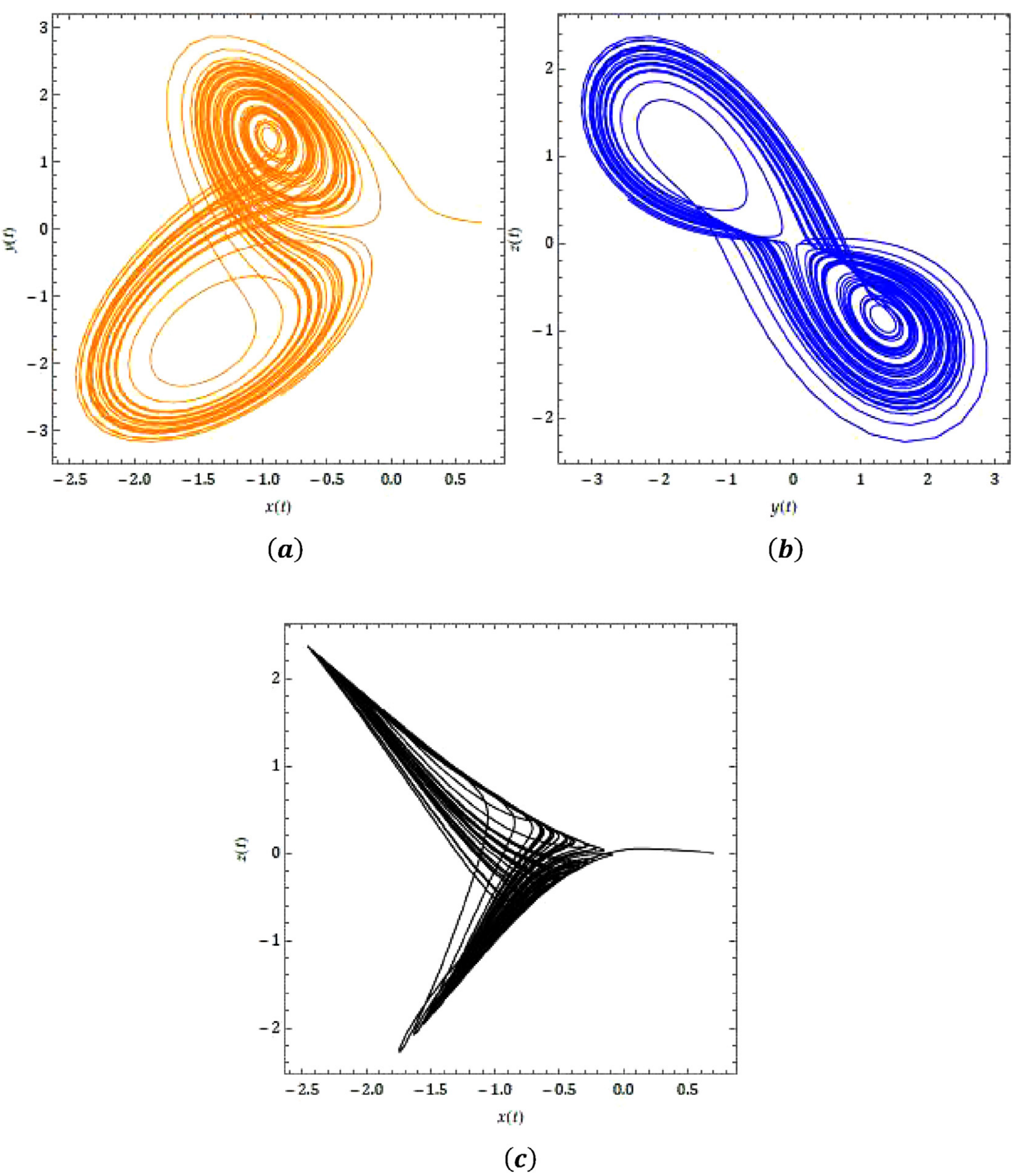

Fig. 3. Chaotic nature for the suggested model (***a***) $x-y$, (***b***) $y-z$ and (c) $x-z$ at $\hbar=-1, n=1$ and $\alpha=1$.

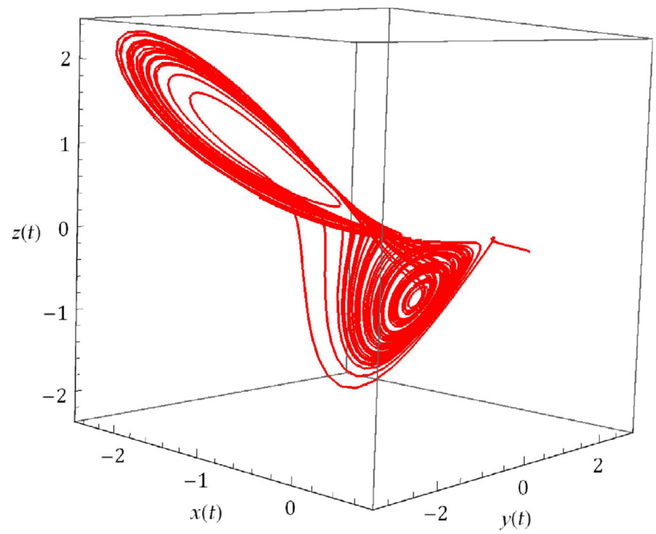

Fig. 4. Chaotic nature of the suggested model at $\hbar = -1, n = 1$ and $\alpha = 1$.

$$\left(1 - \frac{(1-\alpha)}{\mathscr{B}(\alpha)}\eta_1 - \frac{\alpha}{\mathscr{B}(\alpha)\Gamma(\alpha)}\eta_1 t\right) \geq 0. \tag{36}$$

Hence, Eq. (53) evidences our essential result.

5. Results and discussion

In this paper, we applied an analytical technique in order to capture the chaotic nature of the projected fractional dynamical system using q-HATM. The chaotic natures of the system (10) with $(x_0, y_0, z_0) = (0.7, 0.1, 0)$ for different fractional order (*i.e.*, $\alpha = 0.90, 0.95, 1$) have been respectively cited in Figs. 1 to 3 in the form of 2D plots. In Fig. 4, we present the 3D chaotic attractor for the future model at $\alpha = 1$. In Fig. 5, the behaviour of the time series for the system is demonstrated. The responses of the obtained solution for the diverse value of α are presented in Fig. 6. To demonstrate the nature of achieved results with the homotopy parameter, the $\hbar$-curves are schemed with different fractional-order and cited in Fig. 6. These curves help us to control and adjust the convergence region. For a proper choice of $\hbar$, the acquired result quickly converges to the analytical solution. Further, with the aid of all figures one can observe that the procedure is exact and very efficient to exemplify the projected fractional chaotic system (Fig. 7).

6. Conclusion

In this paper, we examined and capture the chaotic nature of the projected arbitrary order model. In the present framework, we illuminate the effeteness of the projected AB derivative and since this derivative proposed by the assist of generalized Mittag-Leffler function. We presented the existence and uniqueness for the achieved results with the help of fixed point theorem. More preciously, the considered scheme offered the solution for the considered model without necessitating any discretization, conversion or perturbation. As associated with consequences accessible in the literature, the results acquired by the projected solution procedure are more stimulating. The present investigation illuminates, the projected chaotic system highly be contingent on the time history and the time instant, and these phenomena can be effectively exemplified will be aid of the concept of fractional calculus. Lastly, the present study ensures considered solution procedure is very accurate, more effective and extremely methodical, and it can be employed to describe the distinct classes of the dynamical system.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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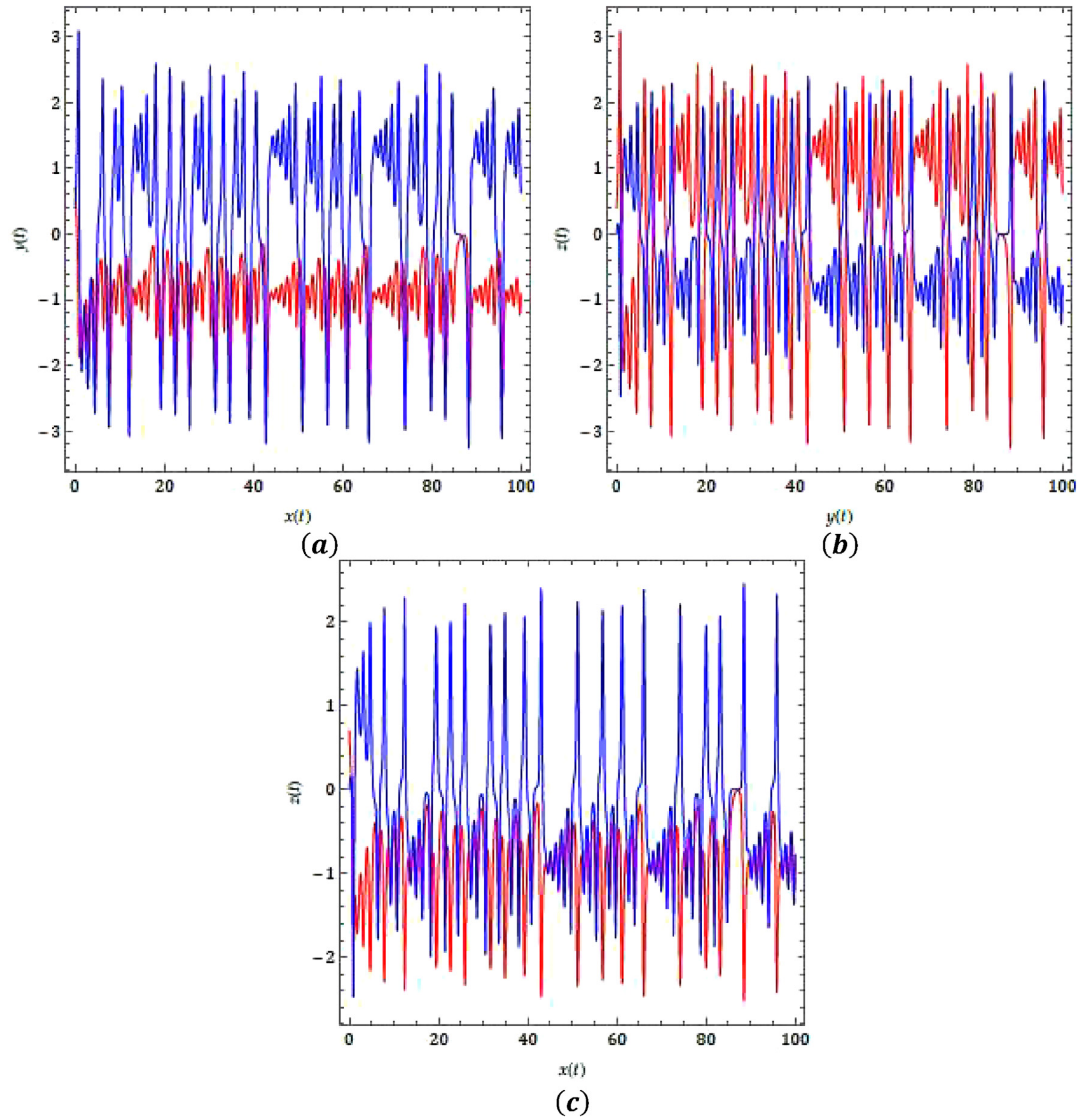

Fig. 5. Behaviour of the time series for the suggested model (***a***) $x - y$, (***b***) $y - z$ and (c) $x - z$ at $\hbar = -1, n = 1$ and $\alpha = 1$.

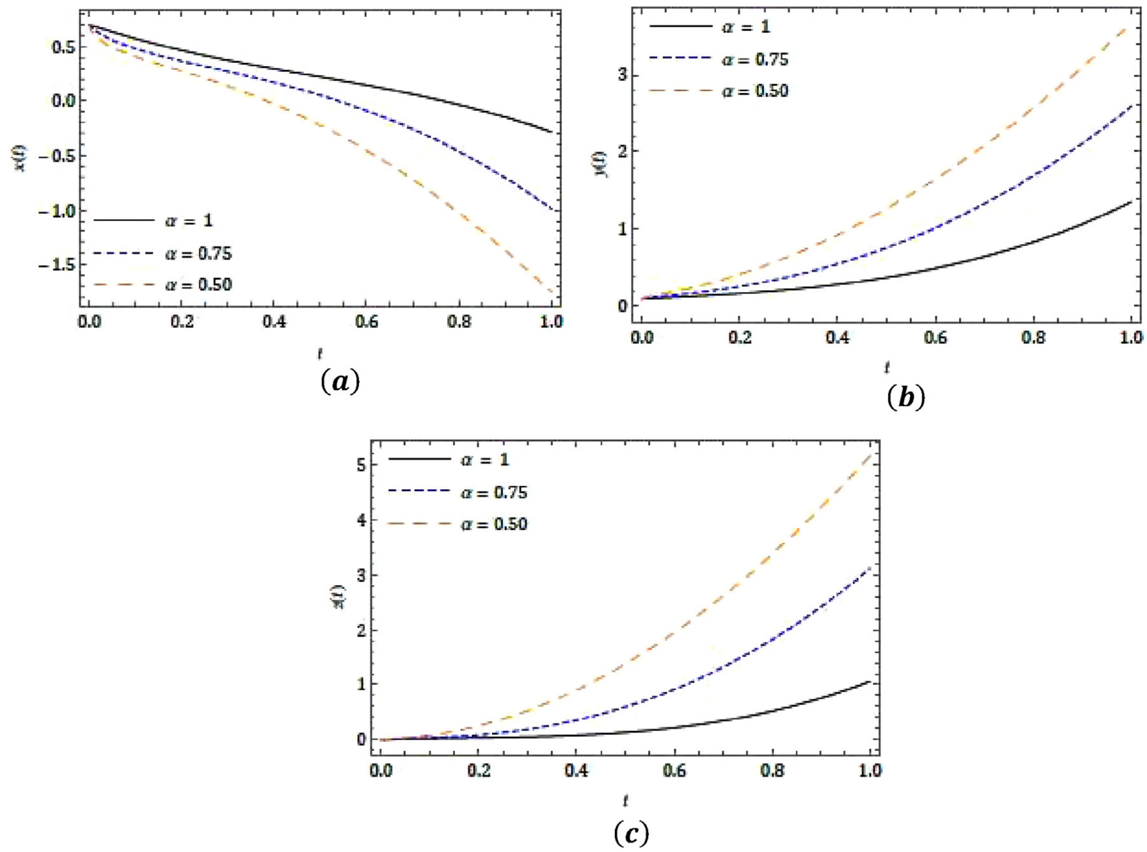

Fig. 6. Nature of the attained result for $(a)x(t), (b)y(t)$ and $(c)z(t)$ with distinct α at $\hbar = -1$ and $n = 1$.

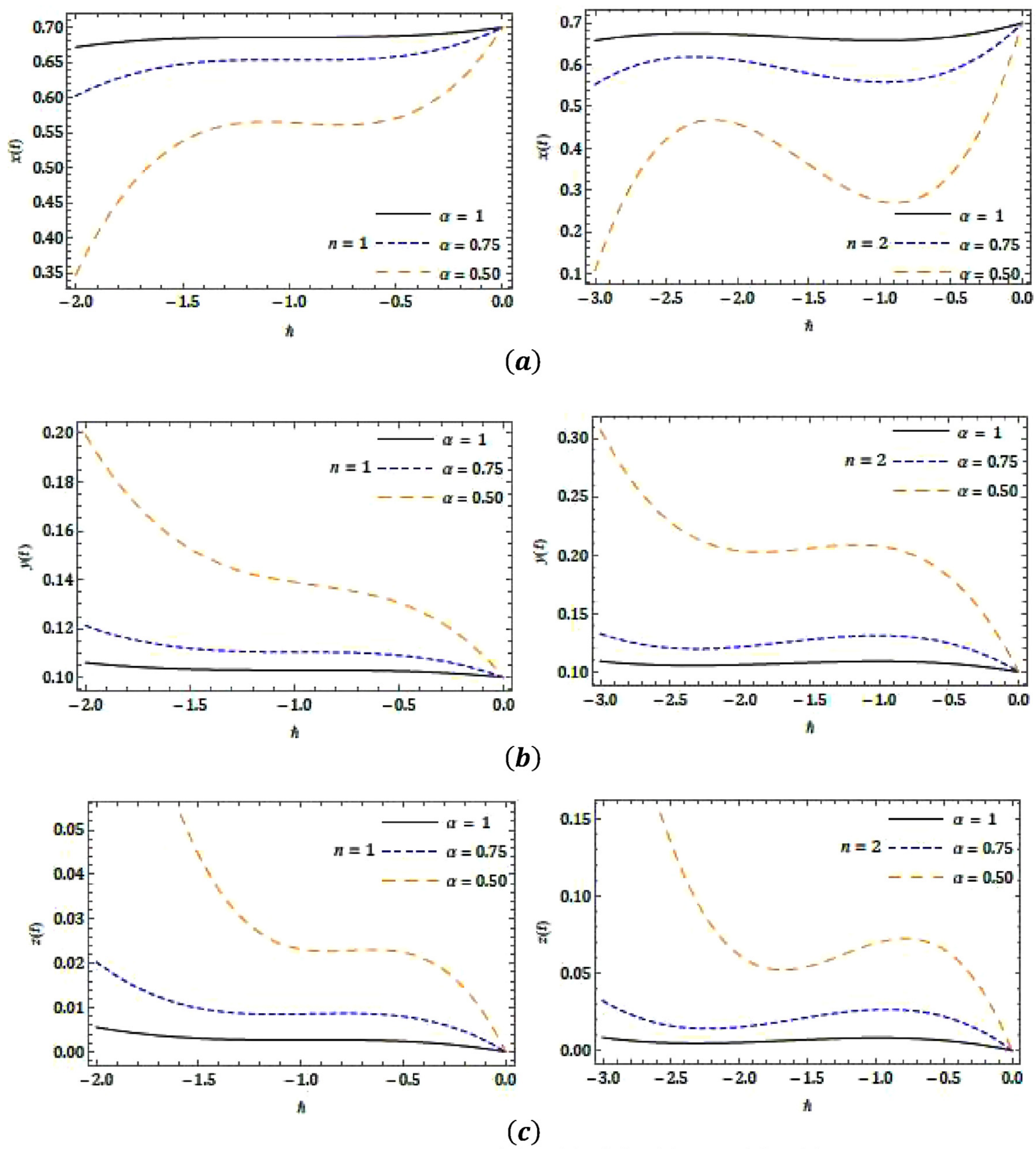

Fig. 7. ħ-curves for the attained result of $(a)x(t), (b)y(t)$ and $(c)z(t)$ with different α at $t = 0.01$ for $n = 1$ and 2.

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