Journal of King Saud University - Science 36 (2024) 103059

HOSTED BY

ELSEVIER

Contents lists available at ScienceDirect

Journal of King Saud University - Science

journal homepage: www.sciencedirect.com

The Schwarzschild solution contains three problems, which can be easily solved

Firyuza Yanchilina[a,*], Vasily Yanchilin[b]

[a] Science Journalist and Independent Researcher, New York 11374, USA
[b] Independent Researcher, New York 11374, USA

ARTICLE INFO

Keywords:
Feynman problem
Mach's principle
Schwarzschild solution
Time component of the metric tensor
Event horizon
Gravitational redshift
Black holes

ABSTRACT

After examining the Schwarzschild solution, Nobel prize winner Richard Feynman concluded that it contains three problems: the constant 1, the negative sign, and the singularity. These three problems do not manifest themselves in the case of a weak gravitational field but can lead to misinterpretation of processes near black holes and in cosmology. As a working hypothesis, we consider a corrected Schwarzschild solution: the constant 1 is expressed in terms of the distribution of all masses in the universe, positive sign instead of a negative sign, and no singularity. In a weak gravitational field, the new solution leads to the same effects as the usual solution. The choice between the two solutions can be made if the accuracy of gravitational experiments in the solar system is increased by 1–2 orders of magnitude. The new solution implies that black holes do not have an event horizon. Therefore, the new solution can also be tested using future, more accurate observations of black holes. If it is confirmed, our ideas about cosmogonic processes can change in the most radical way.

1. Introduction: Three problems in the Schwarzschild solution

The Schwarzschild solution for the interval s in the gravitational field of a point mass m has the following form in spherical coordinates r, θ, ϕ (Schwarzschild, 1916; Landau and Lifshitz, 1975):

$$ds^2 = (1 - \frac{2Gm}{rc^2})c^2dt^2 - \frac{dr^2}{(1 - \frac{2Gm}{rc^2})} - r^2(sin^2\theta\, d\varphi^2 + d\theta^2) \quad (1)$$

Here $G \approx 6.67 \times 10^{-11}$ kg $^{-1}$m^{3}s^{-2} is the gravitational constant; c is the speed of light. The coefficient before c^2dt^2 is the time component g_{00} of the metric tensor g_{ik}, which determines the time scale near the massive body (Landau and Lifshitz, 1975):

$$\sqrt{g_{00}} = \sqrt{1 - \frac{2Gm}{rc^2}} = \frac{d\tau}{dt} \quad (2)$$

Here dt is a time interval that has passed at a great distance from mass m, $d\tau$ is a time interval that has passed at distance r from mass m: $d\tau < dt$. That is, atomic frequencies decrease in a gravitational field. The ratio of frequencies ν_1 and ν_2 of two identical atoms located at points x_1 and x_2 is (Weinberg, 1972):

$$\frac{\nu_2}{\nu_1} = \sqrt{\frac{g_{00}(x_2)}{g_{00}(x_1)}} \quad (3)$$

What is constant 1 in the time component of the metric tensor g_{00}? When deriving solution (1), Schwarzschild (Schwarzschild, 1916) proceeded from the fact that 1 is the scale for empty space with no gravity. Such an assumption was acceptable in 1916, when there was little information about the structure of the universe. The famous big dispute about the nature of nebulae took place 4 years later (Trimble, 1995). Only 10 years later, Edwin Hubble (Hubble, 1926) proved that nebulae are not clouds of gas, but giant clusters of stars like the Milky Way.

Gradually it became clear that the space of the universe is filled with stars and galaxies. Moreover, the galaxies are moving away from each other, and the universe is expanding. If some mass m, which is included in the Schwarzschild solution (1), affects the time, then the rest of the masses of the universe must also affect the time. Maybe the constant 1 is not a constant characterizing empty space, but some contribution from distant galaxies? After researching this topic Richard Feynman concluded: "Another possibility is that the 1 which appears in the formula of time dilation is a mistake in thought. We have written a formula which applies only when the potential differences φ are much smaller than 1, so that the constant 1 may somehow represent a normalized contribution of faraway nebulae." Based on this, he proposed a way to combine the theory of gravity, Mach's principle, and quantum mechanics in *Feynman Lectures on Gravitation* (Lecture 5.3 and 5.4) (Feynman et al., 1995).

* Corresponding author.

https://doi.org/10.1016/j.jksus.2023.103059
Received 15 November 2022; Received in revised form 3 December 2023; Accepted 8 December 2023
Available online 15 December 2023

Feynman considered a particle that moves far from gravitating masses. If a period and wavelength of the de Broglie wave associated with that particle are constant, then the lines of constant phase will be parallel. In this case, the particle will move in a straight line. If such a particle flies past a star, then the time scales are reduced in the direction of the star. In this case, the lines of constant phase will not be parallel, and the particle will move with an acceleration directed towards the star. If the star causes a shortening of time scales, then the rest of the stars and galaxies of the universe also cause a shortening of time scales. As a result, Feynman concluded that the space–time scale far from gravitating masses is determined by distant galaxies. In this case, the motion of the particle by inertia is also determined by distant galaxies. Feynman concluded (Feynman et al., 1995): "Mach's Principle is equivalent to the statement that the fundamental units of length and time at a point are the result of the influence of nebulae."

To verify this conclusion, we need to find the total amount of the contribution from all the masses in the universe, that is sum $\sum_{n=1}^{n=N} 2\frac{Gm_n}{r_n c^2}$ (m_n is the mass, r_n is the distance to it, N is the number of all masses in the universe). If we are on the right track, then this sum will be equal to 1 within the measurement error. Feynman did the calculations and, considering the error, obtained (Feynman et al., 1995):

$$\sum_{n=1}^{n=N} 2\frac{Gm_n}{r_n c^2} = 1 \tag{4}$$

It would seem, that the task of unifying the theory of gravity and Mach's principle has been solved. But then Feynman notes that the sum in Eq. (4) has a positive sign, but the small addition $2Gm/rc^2$ in Eq. (2) has a negative sign. If we assume that 1 is the contribution to the scale created by all the stars and galaxies, then this contribution must have the same sign as a small addition to it. However, this is not the case for Eq. (2): the signs of these quantities are different. Here is how Feynman commented such a contradiction (Feynman et al., 1995): "Except for the disastrous appearance of a (+) sign instead of a (−) sign, the result is identical to the "correct" arc length. We have succeeded in getting the correct sizes by juggling purely cosmological numbers."

Using modern astrophysical data, we can also obtain Eq. (4) and evaluate its accuracy. According to modern estimates, the age of the Universe is about $T_U \approx 13.8 \times 10^9$ years (Valcin et al., 2020). Considering that the speed of the most distant galaxies almost reaches the speed of light, the radius of our universe R_U can be estimated by multiplying the age of the universe by the speed of light:

$$R_U \approx cT_U \approx 1.3 \times 10^{26}\mathrm{m} \tag{5}$$

The average density of the universe is close to the critical density ρ_C:

$$\rho_c = \frac{3H^2}{8\pi G} \tag{6}$$

Here H is the Hubble constant, which, according to modern data, is: $H \approx 70$ km/s/Mpc (Chen et al., 2018; Freedman, 2021; and Pesce et al., 2020).

Substituting this value into Eq. (6), we get:

$$\rho_c = 9.2 \times 10^{-27}\mathrm{kg/m^3} \tag{7}$$

Knowing the Radius of the universe R_U Eq. (5) and its density ρ_C Eq. (7), we can estimate the mass of the universe M_U: $M_U \approx 7.8 \times 10^{52}$ kg. As a result, we find that the contribution of all the matter of the Universe to the time scale near the Earth is approximately equal to:

$$2G\frac{M_U}{c^2 R_U} \approx 0.89 \tag{8}$$

We should make three remarks about Eq. (8). First, all the astrophysical quantities that we used for the calculation have a high error. Second, it is assumed in Eq. (8) that the entire mass of the Universe M_U is located at distance R_U, which, of course, is not true. If we assume that the mass of the universe is uniformly distributed, then the numerical value in Eq. (8) should be multiplied by 1.5. Thirdly, the value of the Hubble constant is calculated from the assumption that the value of the redshift in the spectrum of some galaxy is caused only by the Doppler effect. However, the light from any galaxy comes to us from the past, when the density of the universe was higher and, therefore, the gravitational potential was lower. According to numerous experiments, it is known that the measured frequency of light will be lower if this light comes from a region of lower gravitational potential (Pound and Rebka, 1960; Chou et al., 2010). Therefore, it is possible that some of the observed redshift in the spectra of galaxies is caused by growth of the gravitational potential of the Universe due to its expansion (Yanchilina and Yanchilin, 2022).

Nevertheless, despite these remarks, we can conclude that the contribution of all matter in the Universe to the time scale near the Earth is close to 1. In this regard, the conclusion suggests itself that the distribution of all matter in the Universe completely determines the flow of time inside the Universe, including in near-Earth space. However, at the same time, the contribution made by the mass m in the Schwarzschild solution, Eq. (1) has the opposite sign than the contribution made by the rest of the mass.

This is very strange. The more you think about this, the weirder it looks. Indeed, each mass is a source of a gravitational field. Each mass contributes to changing the rate of time and affects the frequency of atomic radiation. For example, the Earth affects the rate of an atomic clock located on the Earth. The Sun also influences the rate of atomic clocks on the Earth. The Milky Way, the Virgo Supercluster of Galaxies, the Laniakea Supercluster, which includes the Virgo Supercluster, also affect the rate of atomic clocks on the Earth. It is easy to estimate that the contribution to the time scale on the Earth's surface made by the Sun ($m_S \approx 2 \times 10^{30}$ kg, $r \approx 1.5 \times 10^{11}$ m) is almost 15 times greater than the contribution made by the Earth ($m_E \approx 6 \times 10^{24}$ kg, $r \approx 6.4 \times 10^6$ m). The contribution made by the Milky Way ($m_{MW} \approx 4 \times 10^{12}$ m_S, $r \approx 40$ kps (Laura et al., 2019)) exceeds the contribution of the Earth by about 2 thousand times. The contribution made by the Virgo supercluster ($m_V \approx 10^{15}$ m_S, $r \approx 16.5$ Mps (Einasto et al., 2007)) exceeds the contribution of the Milky Way by about 7 times. The Laniakea Supercluster ($m_L \approx 10^{17}$ m_S, $r \approx 80$ Mps (Tully et al., 2014)), which includes the Virgo supercluster, contributes to the time scale near Earth about 140 times more than the Milky Way.

It is not difficult to guess that more distant galaxies contribute even more to the time scale near the Earth. We have the right to expect that the influence of all these masses on the time scale (or length) should have the same sign. Why is there a minus sign before mass m in Eq. (2)? It is because of this minus sign that a singularity arises in the Schwarzschild solution. It is worth noting that Feynman, like many other well-known scientists (references in Section 3), had a negative attitude towards the singularity, considering it to be a shortcoming of the Schwarzschild solution (Feynman et al., 1995).

Summing up, we list all three problems in the Schwarzschild solution.

Problem 1. Constant 1 cannot be constant because it contains a contribution to the time scale from distant galaxies, and this contribution changes due to the expansion of the universe.

Problem 2. If constant 1 contains a contribution to the time scale from distant masses, then we can expect that mass m in the Schwarzschild solution (1) will have the same sign as constant 1. However, it is not the case.

Problem 3. The singularity in the Schwarzschild solution appears because mass m and constant 1 have opposite signs. If the signs of these quantities were the same, then the singularities would disappear from the Schwarzschild solution.

So, Feynman proposed method to combine the theory of gravity, Mach's principle, and quantum mechanics. But this way led us to a dead end. However, Feynman retained the hope of unifying Mach's principle and the theory of gravity, since, initiating this topic, he wrote (Feynman et al., 1995): "The following discussion is purely qualitative and is meant only to stimulate wiser thoughts on this subject." Finishing this topic, he still hoped that the constant 1 in g_{00} of Eqs. (1) or (2) "is a meaningful number, not to be simply taken as 1."

To solve the problem posed by Feynman and introduce Mach's principle into the theory of gravity, in the next section we will consider as a working hypothesis an adjusted Schwarzschild solution in which: constant 1 is expressed in terms of the distribution of all the masses, mass m has a positive sign and therefore there is no singularity in the solution.

2. Solution of three problems in the Schwarzschild solution

Let us consider in the most general terms how the component of the metric tensor g_{00} is calculated in the field of point mass m at distance r from it. First, it is assumed that in empty space $g_{00} = 1$. Therefore, the solution is sought in the form (Landau and Lifshitz, 1975):

$$g_{00} = 1 + O(r) \tag{9}$$

Here $O(r)$ is some small function: $|O(r)| \ll 1$. Secondly, to find this function, the approximation of Newton's law of universal gravitation is used, according to which (Landau and Lifshitz, 1975):

$$O_N(r) = -2\frac{Gm}{rc^2} \tag{10}$$

We denoted the small function $O(r)$ as $O_N(r)$ because it was obtained by the Newtonian gravity approximation method. Considering Eqs. (9) and (10) we obtain the Schwarzschild solution for the time component of the metric tensor g_{00} (Landau and Lifshitz, 1975):

$$g_{00} = 1 + O_N(r) = 1 - 2\frac{Gm}{rc^2} \tag{11}$$

High precision experiments to verify Eqs. (11) and (1) with an accuracy of the order of hundredths and thousandths of a percent were carried out, as a rule, in the Solar System. However, this accuracy is not enough to exclude the possibility of the appearance in Eqs. (10) and (1) terms of the second order of smallness like $O_N^2(r)$ (Will, 2018). Therefore, if we add to the value of g_{00} in Eq. (11) some function of small order $O_N^2(r)$, then such a new equation will also satisfy all experiments carried out in the Solar System.

What is the physical meaning of constant 1 in Eq. (11)? If the Solar System were in an orbit located closer to the center of the Milky Way, then all atomic frequencies on the Earth's surface would be shifted to the red end of the spectrum. That is, the time component of the metric tensor on the Earth's surface would be less, Eq. (11). However, scientists could also take this component of the metric tensor as constant 1. Therefore, we can conclude that constant 1 is not a fundamental constant, but some variable that characterizes the time scale, which we take for constant 1 only for reasons of convenience.

Consider an extended area of empty space A inside a giant void. Let the gravitating mass m be in its central part. Let us assume that inside region A, at a sufficiently large distance from mass m, the atomic frequencies are constant with high accuracy. T_A is a duration of 1 s at a sufficiently large distance from mass m, T_r is a duration of 1 s at distance r from mass m. Considering (3), we get:

$$\frac{g_{00}(r)}{g_{00}(A)} = \frac{T_A^2}{T_r^2} \quad \Rightarrow \tag{12}$$

$$\frac{g_{00}(r)}{g_{00}(A)} = \frac{1}{1 + \frac{T_r^2 - T_A^2}{T_A^2}} \tag{13}$$

Following tradition, we assume that at a sufficiently large distance from mass m: $g_{00}(r) \to g_{00}(A) = 1$. Using the Newtonian gravity approximation in Eqs. (9) and (10), we get:

$$\frac{T_r^2 - T_A^2}{T_A^2} = 2\frac{Gm}{rc^2} \tag{14}$$

Considering Eq. (13) and Eq. (14) as a working hypothesis, we propose the following equation for the time component of the metric tensor:

$$g_{00}^M = \left[1 + \frac{2Gm}{rc^2}\right]^{-1} \quad \text{or} \tag{15}$$

$$g_{00}^M = [1 - O_N(r)]^{-1} \tag{16}$$

We introduced the notation g_{00}^M, emphasizing that such a "corrected" time component is introduced to unite the theory of gravity and Mach's principle. Considering Eq. (10), we get:

$$g_{00}^M = \left[1 + \frac{2Gm}{rc^2}\right]^{-1} \tag{17}$$

Considering equation, we can "correct" the Schwarzschild solution (1):

$$ds_M^2 = \left[1 + \frac{2Gm}{rc^2}\right]^{-1} c^2dt^2 - \left[1 + \frac{2Gm}{rc^2}\right] dr^2 - r^2(sin^2\theta\, d\varphi^2 + d\theta^2) \tag{18}$$

From Eqs. (11) and (16) we find the ratio of the usual g_{00} and new g_{00}^M time components:

$$\frac{g_{00}}{g_{00}^M} = 1 - O_N^2(r) < 1 \quad \Rightarrow \tag{19}$$

$$1 > g_{00}^M > g_{00} \tag{20}$$

It follows from Eq. (19):

$$g_{00}^M - g_{00} = g_{00}^M O_N^2(r) < O_N^2(r) \tag{21}$$

So, the difference between the time components is less than $O_N^2(r)$. For example, in the Solar System: $|O_N(r)| < 4.4 \times 10^{-6}$ and$g_{00}^M - g_{00} < 2 \times 10^{-11}$. To determine which of the two values (g_{00}^M or g_{00}) is correct, it is necessary to carry out measurements in the Solar System with an error lower than 5×10^{-6}. To do this, it is necessary to increase the accuracy of modern experiments by 1–2 orders of magnitude.

So, we have obtained a new, "corrected" Schwarzschild solution, in which the addition to 1, introduced by mass m, has the same positive sign as 1. Now we can represent the constant 1 as a contribution from all matter in the universe, as suggested by Feynman in Eq. (4). Therefore, we can also represent g_{00}^M as the contribution from all the matter in the universe:

$$g_{00}^M = \left[\sum_{n=1}^{n=N} 2\frac{Gm_n}{r_n\, c^2}\right]^{-1} \tag{22}$$

In the region located far from all the masses, the following equality holds with good accuracy: g_{00}^M= 1. In the region near mass m_1 Eq. (22) can be represented as:

$$g_{00}^M = \left[\sum_{n=2}^{n=N} 2\frac{Gm_n}{r_n\, c^2} + 2\frac{Gm_1}{r_1 c^2}\right]^{-1} \tag{23}$$

Feynman wrote about gravitation (Feynman et al., 1995): "The second amazing thing about gravitation is how weak it is." It is clear from Eq. (23) why gravity is weak. This is a consequence of the fact that the change in time scale introduced by mass m_1 (the second term in square brackets) is very small compared to the time scale created by the rest of the masses in the universe (the first term in square brackets).

According to Eq. (2), in empty space $g_{00} = 1$. According to Eq. (23) it is not the case. If all gravitating masses included in Eq. (23) are removed to infinite distances, then:

$$g_{00}^M = 0^{-1} \tag{24}$$

That is, the time component of the metric tensor will be indefinite. Consequently, in empty space, the metric also becomes indefinite, in full accordance with Mach's principle (Pauli, 1958). From the new point of view, space and time exist thanks to stars and galaxies. Stars and galaxies with their huge mass create a space–time metric and determine the time scale at every point in space, including those near the Earth. By changing the sign in the Schwarzschild solution, Eq. (1), we solved the problem posed by Feynman and introduced Mach's principle into the theory of gravity.

3. Gravitational radius and event horizon

According to the usual the Schwarzschild solution, Eq. (1), an event horizon can exist near a massive and compact object. The event horizon appears if the radius of a massive object is less than the gravitational radius, which is: $2Gm/c^2$. Upon reaching the gravitational radius, the coordinate speed of light tends to zero, and the gravitational redshift tends to infinity (Misner et al., 1973). A massive object with a radius less than the gravitational one is called a black hole. Thus, according to Eq. (1), even light cannot escape from a black hole. From the new point of view, this is not the case. Let us calculate the value of the coordinate speed of light and the value of the gravitational redshift using Eq. (18).

The propagation of light is determined by the following equation: $ds = 0$. Suppose light is moving towards mass m. In this case, the 3rd term on the right side of Eq. (18) will be equal to zero because $d\phi = 0$ and $d\theta = 0$. As a result, we get:

$$ds_M^2 = 0 = \left[1 + \frac{2Gm}{rc^2}\right]^{-1} c^2 dt^2 - \left[1 + \frac{2Gm}{rc^2}\right] dr^2 \tag{25}$$

Accordingly, the coordinate speed of light is:

$$\frac{dr}{dt} = c\left[1 + \frac{2Gm}{rc^2}\right]^{-1} \tag{26}$$

It follows from Eq. (26) that the coordinate speed of light is not equal to zero either near the gravitational radius or inside it.

According to general relativity, the frequency of atomic radiation decreases near a massive body. If ν_0 is the radiation frequency of an atom outside the gravitational field, then the measured radiation frequency of this atomν, located in the gravitational field, will be lower (Weinberg, 1972):

$$\nu = \nu_0\sqrt{g_{00}} \tag{27}$$

Therefore, the magnitude of the gravitational redshift z is:

$$z = \frac{\nu_0}{\nu} - 1 = \sqrt{g_{00}^{-1}} - 1 \tag{28}$$

According to the Schwarzschild solution, Eq. (1), the time component of the metric tensor g_{00} tends to zero and the gravitational redshift z tends to infinity near the gravitational radius. Inside the gravitational radius, the gravitational redshift becomes imaginary.

It is worth noting that many well-known scientists had a negative attitude towards the singularity in the Schwarzschild solution (the term black hole appeared later). Albert Einstein purposely wrote an article in which he tried to prove that the Schwarzschild singularity cannot exist (Einstein, 1939). Richard Feynman was negative about the fact that inside the gravitational radius the time component of the metric tensor becomes negative (Feynman et al., 1995). General relativity expert K. Möller concluded that the Schwarzschild solution loses its physical meaning in strong gravitational fields and is therefore incorrect (Möller, 1979). Nobel prize winner Steven Weinberg believed that the Schwarzschild singularity has nothing to do with the real world (Weinberg, 1972). When Chandrasekhar, relying on Schwarzschild's solution, concluded that a sufficiently massive star must collapse to a point, Sir Arthur Eddington argued that there must be a law of nature that would not allow such an absurd behavior of the star (Thorne, 1994).

According to Eq. (17) the gravitational redshift z_M is:

$$z_M = \sqrt{1 + \frac{2Gm}{rc^2}} - 1 \tag{29}$$

It follows from Eq. (29) that the gravitational redshift near the gravitational radius does not tend to infinity and is less than 1: $z_M = \sqrt{2} - 1 \approx 0.4$. Within the gravitational radius, the gravitational redshift also remains a finite value. We conclude from the new perspective that black holes do not have an event horizon. Therefore, light and even matter can overcome the attraction of a black hole and fly out of it.

Supermassive black holes are found at the centers of many galaxies. From some black holes, giant ejections of matter are visible, which sometimes move at relativistic speeds. According to the Schwarzschild solution (1), a single black hole cannot eject its own matter into the surrounding space. Therefore, it is assumed that a black hole becomes active only because of matter accreting onto it from outside. In general terms, the scheme of "work" of a black hole looks like this. Matter from outside falls into the black hole. The black hole absorbs most of this matter, the rest is ejected into the surrounding space. From this point of view, on average, much more matter should fall into black holes than is ejected from them. However, this is not a case. In the 1960s, astronomers discovered that black holes eject much more matter than falls on them. Here is what astronomer Fred Hoyle wrote about this (Hoyle, 1965):

"In the next chapter I shall be discussing many cases where gas comes out of the centers of galaxies, so there is confirmation that processes leading to the expulsion of material do occur. There can hardly any quarrel with this statement. Where the controversy comes in is in deciding whether gas must first fall into the middle from outside before it gets expelled. Common sense would tell us that this must be the case. However, infall has not so far been observed, whereas there is lots of evidence of expulsion, as we shall see later. Moreover, it is difficult to understand how material could have so little angular momentum as to permit it to fall into the very center."

More than half a century has passed since Hoyle wrote this, but the problem remains unresolved: ejections from black holes are observed in a wide variety and in huge quantities, but the fall of matter into black holes either does not occur, or these processes are so weak that they are difficult to observe (Massaro et al., 2015).

From the new point of view, even a lone black hole can be active and eject its matter into the surrounding space. If this is indeed the case, then our ideas about the direction of astrophysical processes may change radically.

4. Conclusion

One of the main reasons that prompted Einstein to construct general relativity was the desire to introduce Mach's principle into physics (Einstein, 1913a,b, 1914a,b,c). When Einstein created general relativity, he hoped that this theory would satisfy Mach's principle (Einstein, 1916). Unfortunately, when general relativity was constructed, it turned out that it did not satisfy Mach's principle (Pauli, 1958). Mach's principle and its experimental verification are discussed in the article (Yanchilin, 2018). Feynman, having studied the Schwarzschild solution, concluded that this solution can be joined with Mach's principle.

The Schwarzschild solution, Eq. (1) contains the time component of the metric tensor g_{00}, which determines the speed of an atomic clock near a massive body m Eq. (2). What is 1, which is included in g_{00} Eqs. (1) or (2)? It is generally accepted that 1 characterizes the scale of empty space with no gravity. But our space is not empty: it is filled with stars and galaxies, which make a significant contribution to the space metric. Therefore, Richard Feynman suggested that the constant 1 in Eqs. (1) or (2) is the normalized contribution to the time scale (length) from distant galaxies. This assumption looks quite plausible, and, moreover, allows us to combine the theory of gravity and Mach's principle. If it is the case, then the contribution to the scale from mass *m* must have the same sign as 1. However, the constant 1 has a positive sign (+) and the contribution from mass *m* has a negative sign (−).

We know from astrophysical observations that the contribution from all the galaxies into the time scale is close to 1 Eq. (4). That is, at least a significant part of the constant 1 is the contribution of distant galaxies. This contribution of distant galaxies has the same positive sign as 1. But why in Eqs. (1) or (2) does this contribution from the galaxies, which is included in the constant 1, have the opposite sign than the contribution from mass *m*? All galaxies consist of the same ordinary masses. If we assume that the nature of all the masses is the same, then the contribution (total or partial) of distant galaxies to 1 must have the same sign as the small correction to it, introduced by mass *m*. However, this is not the case in the Schwarzschild solution.

In our opinion, it looks strange and unnatural. In addition, the Schwarzschild solution is obtained by transition to the limit to Newton's Law of Universal Gravitation, which is an approximate one. Therefore, we proposed to change the sign in the Schwarzschild solution, Eq. (1) and obtained the new solution Eq. (18). In weak gravitational fields, the new equation, within the accuracy of modern gravitational experiments, leads to the same effects as Eq. (1). To make a choice between these two equations, it is necessary to increase the accuracy of measurements by about 1–2 orders of magnitude. In strong gravitational fields, an important consequence follows from the new equation: the absence of an event horizon for black holes. If this is true, then even a lone black hole can be active and eject its matter into the surrounding space. In this case, our ideas about the direction of astrophysical processes can change radically.

Finally, the most important thing is that in the new solution, the contribution from mass *m* has the same positive sign as the constant 1, so 1 can be represented as the contribution from all the masses that fill the universe. The new solution fully satisfies Mach's principle: in space with test bodies and no gravitating masses, the metric becomes indeterminate. Thus, by changing the sign in the Schwarzschild solution, we solved the problem posed by Feynman and introduced Mach's principle into the theory of gravitation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

Chen, H.-Y., Fishbach, M., Holz, D.E., 2018. A two per cent Hubble constant measurement from standard sirens within five years. Nature 562 (7728), 545–547.
Chou, C.W., Hume, D.B., Rosenband, T., Wineland, D.J., 2010. Optical clocks and relativity. Science 329 (5999), 1630–1633.
Einasto, M., et al., 2007. The richest superclusters. A&A 476 (2), 697–711.
Einstein, A., 1913a. Physical Foundations of a Theory of Gravitation. Naturforschende Gesellschaft, Zürich, Vierteljahrsschrift. 58, 284–290.
Einstein, A., 1913b. On the Present State of the Problem of Gravitation. Physikalische Zeitschrift. 14, 1249–1266.
Einstein, A., 1914a. On the Theory of Gravitation. Naturforschende Gesellschaft, Zürich, Vierteljahrsschrift 59, 4–6.
Einstein, A., 1914b. Formal Foundations of the General Theory of Relativity. Preussische Akademie Der Wissenschaften, Sitzungsberichte Part 2, 1030–1085.
Einstein, A., 1914c. On the Relativity Problem. Scientia Bologna 15, 337–348.
Einstein, A., 1916. The Foundation of the General Theory of Relativity. Annalen Der Physik Ser. 4 (49), 769–822.
Einstein, A., 1939. On a stationary system with spherical symmetry consisting of many gravitating masses. Ann. Math. 40, 922–936.
Feynman, R.P., Morinigo, F.B., Wagner, W.G., 1995. Feynman Lectures on Gravitation. Addison-Wesley, Boston.
Freedman, W.L., 2021. Measurements of the Hubble constant: tensions in perspective. Astrophys. J. 919, 16–27.
Hoyle, F., 1965. Galaxies, Nuclei, and Quasars. Harper & Row, Publishers, NY.
Hubble, E., 1926. Extra-Galactic Nebulae. Apj 64, 321–369.
Landau, L.D., Lifshitz, E.M., 1975. The Classical Theory of Fields, 4th ed. Pergamon Press, Oxford.
Laura, L.W., et al., 2019. Evidence for an intermediate-mass milky way from Gaia DR2 halo globular cluster motions. ApJ 873, 118–130.
Massaro, F., Cheung, C.C., Lopez, E., Siemiginowska, A., 2015. Extragalactic Jets from Every Angle. Proceedings of the IAU. Cambridge University Press, Cambridge.
Misner, C.W., Thorne, K.S., Wheeler, J.A., 1973. Gravitation. Freeman W N and Company, San Francisco.
Möller, C., 1979. in Astrofisica e Cosmologia, Gravitazione Quanti e Relativitá neglisviluppi del pensiero scientifico di Albert Einstein. GiuntiBarbera, Firenze.
Pauli, W., 1958. Theory of Relativity. Pergamon Press, Oxford.
Pesce, D.W., et al., 2020. The Megamaser Cosmology Project. XIII. Combined Hubble Constant Constraints. Apj Let. 891, 9–22.
Pound, R., Rebka, G., 1960. Apparent weight of photons. Phys. Rev. Lett. 4, 337–341.
Schwarzschild, K., 1916. On the gravitational field of a mass point according to Einstein's theory. Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.) 189–196.
Thorne, K.S., 1994. Black Holes and Time Warps. W.W. Norton Publishers, NY.
Trimble, V., 1995. The 1920 Shapley-Curtis discussion: background, issues, and aftermath. PASP 107, 1133–1144.
Tully, R., et al., 2014. The Laniakea supercluster of galaxies. Nature 513, 71–73.
Valcin, D., et al., 2020. Inferring the age of the universe with globular clusters. JCAP 12, 002–025.
Weinberg, S., 1972. Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity. John Wiley & Sons, New York.
Will, C.M., 2018. Theory and Experiment in Gravitational Physics. Cambridge University Press, Cambridge.
Yanchilin, V.L., 2018. The chaos formula and the experimental verification of Mach's principle. Phys Essays 31, 347–357.
Yanchilina, F., Yanchilin, V., 2022. Experiment with a laser and a mirror to accurately measure the Hubble constant. MPLA 37, 2250148.