Journal of King Saud University – Science 34 (2022) 101741

Contents lists available at ScienceDirect

Journal of King Saud University – Science

journal homepage: www.sciencedirect.com

Original article

A generalized class of estimators for sensitive variable in the presence of measurement error and non-response under stratified random sampling

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ARTICLE INFO

Article history:
Received 29 August 2020
Revised 23 November 2021
Accepted 25 November 2021
Available online 4 December 2021

Keywords:
Auxiliary variable
Measurement error
Non-response
Randomized response
Stratified random sampling

ABSTRACT

In survey sampling an investigator may be unable to get the complete and correct information at the same time. So non-response and measurement error occur simultaneously and consequently may effect the estimator. Considering this problem, a generalized class of estimators is proposed for estimating the finite population mean for sensitive variable in the presence of measurement error and non-response under stratified random sampling. We conducted a study based on real data set at Quaid-i-Azam University, Islamabad. Simulation and real life data sets are used to observe the performances of the estimators. Bias and MSE values are given for the comparison of estimators.

1. Introduction

In survey sampling, certain surveys cause some problems for the researchers due to the fact that the respondents are reluctant to discuss sensitive topics such as drug use, abortion, sexually transmitted diseases etc. When surveying on those topics, measurement error and non-response can occur since the respondent may choose, not to respond some specific questions, not to give the accurate answers, or not to take part in the survey. The problem of measurement error is usually ignored during the sensitive surveys and the assumption is made that the information obtained through surveys is free from error. Another important factor in surveys is non-response, which may arises due to refusal of respondents to give the information or not at home or lack of interest due to some sensitive issues. Usually measurement error and non-response are studied separately for the sensitive variable using the known auxiliary or additional information. In reality, when the variable of interest is sensitive, the respondents hesitate to provide the personal information, which give rise to measurement error. In most of the cases, the information is not obtained from all units in surveys, specially when the variable of interest is stigmatizing in nature. Many researchers studied the problem of non-response, including (Hansen and Hurwitz, 1946; Cochran, 1977; Rao, 1986; Khare and Srivastava, 2010; Andridge and Little, 2010; Singh et al., 2011; Khare et al., 2013; Shabbir and Khan, 2013; Shabbir et al., 2018 and Singh and Khalid, 2020). In survey sampling, when the variable under study contains social stigma, then the respondents are not comfortable to provide their personal information. Direct survey on sensitive question increases the relative bias. Warner (1965) introduced the randomized response technique (RRT), which reduces the possible bias and is used to obtain the true information while insuring the privacy of the respondents. For estimation of mean of a sensitive quantitative variable the Randomized Response model (RRM) is extended by Greenberg et al. (1971). Further work is done by Eichhorn and Hayre (1983); Gupta and Shabbir (2004), Kim and Warde (2004); Singh and Mathur (2005), Gjestvang and Singh (2006); Diana and Perri (2010), Gupta et al. (2010); Chaudhuri and Pal (2015), Gupta et al. (2016) and Bouza et al. (2018).

The researchers dealt with the problem of measurement error for estimating the population mean. For more details, see Cochran (1968); Fuller (1995); Shalabh (1997); Biemer et al. (2011); Shukla et al. (2012), etc. Recently few researchers studied the problem of measurement error and non-response together like Kumar et al. (2015); Singh and Sharma (2015); Azeem and Hanif

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Peer review under responsibility of King Saud University.

Production and hosting by Elsevier

https://doi.org/10.1016/j.jksus.2021.101741
1018-3647/

(2017) and Kumar (2016). Zahid and Shabbir (2018); Khalil et al. (2018) and Zahid and Shabbir (2019) have discussed the problem of measurement error and non-response under stratified random sampling.

In practice, the researchers who have studied measurement error, have ignored the presence of non-response and randomized response at the same time. In this study, we have proposed a class of estimators for estimating the population mean of a sensitive variable in the presence of measurement error and non-response simultaneously under stratified random sampling. The efficiency of the suggested class of estimators over the existing estimators is shown through simulation study and real data sets.

Consider a finite population of N identifiable units which are partitioned into L homogeneous subgroups called strata, such that the h^{th} strata consist of N_h units, where $h = 1, 2, \ldots, L$ and $\sum_{h=1}^{L} N_h = N$. It is assumed that N consists of two mutually exclusive groups called response and non-response groups. Let N_{1h} and N_{2h} are the responding and non-responding units in the h^{th} stratum respectively. We select a sample of size n_h from N_h by using simple random sampling without replacement (SRSWOR) and assume that n_{1h} units respond and n_{2h} units do not respond. We select a sub-sample of size $k_h, \left(k_h = \frac{n_{2h}}{g_h}, g_h > 1\right)$ from n_{2h} non-responding units in the h^{th} stratum.

Let $\left(z^*_{hi}, y^*_{hi}, x^*_{hi}, r^*_{x_{hi}}\right)$ be the observed values and $\left(Z^*_{hi}, Y^*_{hi}, X^*_{hi}, R^*_{x_{hi}}\right)$ be the actual values of the $i^{th}(i = 1, 2, \ldots, n)$ sampled units in the h^{th} stratum. Let $r^*_{x_{hi}}$, and $R^*_{x_{hi}}$ be the corresponding ranks of x^*_{hi} and X^*_{hi} respectively, then the measurement errors be.

$Q^*_{hi} = z^*_{hi} - Z^*_{hi}, V^*_{hi} = x^*_{hi} - X^*_{hi}$ and $T_{hi} = r_{x_{hi}} - R_{x_{hi}}$.

Let S^2_{hZ}, S^2_{hX} and $S^2_{hR_x}$ be the population variances for the responding units and $S^2_{hZ(2)}, S^2_{hX(2)}$ and $S^2_{hR_x(2)}$ be the population variances for non-responding units. Let S^2_{hQ}, S^2_{hV} and S^2_{hT} be the population variances associated with the measurement error for responding units. Let $S^2_{hQ(2)}, S^2_{hV(2)}$ and $S^2_{hT(2)}$ be the population variances associated with measurement error for the non-responding part of the population. Let $\rho_{hZX}, \rho_{hZR_x}, \rho_{hXR_x}$ be the coefficients of correlation, between their subscripts for respondents and $\rho_{hZX(2)}, \rho_{hZR_x(2)}, \rho_{hXR_x(2)}$ be the coefficients of correlation, between their subscripts for non-respondents in the population.

In Section 2, some existing estimators of the finite population mean are given. In Section 3, a generalized class of estimators is suggested for estimating the finite population mean by incorporating both measurement error and non-response information simultaneously. Numerical results and simulation study are presented in Section 4. Conclusion is given in Section 5.

2. Existing Estimators in Literature

In this section we consider the following existing estimators.

2.1. Hansen and Hurwitz (1946) Estimator

In stratified random sampling, the Hansen and Hurwitz (1946) estimator for population mean $\overline{Y}$, is given by

$$\bar{y}^{*\prime}_{S(HH)} = \sum_{h=1}^{L} P_h \bar{z}^*_h, \tag{1}$$

where $\bar{z}^*_h = \left(\frac{n_{1h}}{n_h}\right)\bar{z}_{n_{1h}} + \left(\frac{n_{2h}}{n_h}\right)\bar{z}_{k_h}$ and $P_h = \frac{N_h}{N}$.

Here $\bar{z}_{n_{1h}} = \frac{1}{n_{1h}} \sum_{i=1}^{n_{1h}} z_{hi}$ and $\bar{z}_{k_h} = \frac{1}{k_h} \sum_{i=1}^{k_h} y_{h_i}$ are the sample means based on n_{1h} of responding and k_h units of sub-samples from n_{2h} non-responding groups, respectively.

The variance of $\bar{y}^{*\prime}_{S(HH)}$, is given by

$$Var\left(\bar{y}^{*\prime}_{S(HH)}\right) = \sum_{h=1}^{L} P_h^2 A^{*\prime}_h, \tag{2}$$

where $A^{*\prime}_h = \lambda_{2h}\left(S^2_{hZ} + S^2_{hQ}\right) + \theta_h\left(S^2_{hZ(2)} + S^2_{hQ(2)}\right)$, $\theta_h = \frac{P_{2h}(g_h - 1)}{n_h}$, $P_{2h} = \frac{N_{2h}}{N_h}$,

$\lambda_{2h} = \left(n_h^{-1} - N_h^{-1}\right)$.

2.2. Ratio Estimator

The usual ratio estimator under stratified random sampling, is given by

$$\bar{y}^{*\prime}_{S(R)} = \sum_{h=1}^{L} P_h \frac{\bar{z}^*_h}{\bar{x}^*_h} \overline{X}_h, \tag{3}$$

where $\overline{X}_h = \frac{1}{N_h} \sum_{i=1}^{N_h} x_{hi}$ is known population mean and $\bar{x}^*_h = \overline{X}_h + \frac{1}{n_h}\left(\delta^*_{hX} + \delta^*_{hV}\right)$ be the sample mean, given in Eq. (22).

The bias and MSE of $\bar{y}^{*\prime}_{S(R)}$, are given by

$$B\left(\bar{y}^{*\prime}_{S(R)}\right) \cong \sum_{h=1}^{L} \frac{P_h}{\overline{X}_h}\left[R'_h B^{*\prime}_h - C^{*\prime}_h\right] \tag{4}$$

and

$$MSE\left(\bar{y}^{*\prime}_{S(R)}\right) \cong \sum_{h=1}^{L} P_h^2\left[A^{*\prime}_h + R'^2_h B^{*\prime}_h - 2R'_h C^{*\prime}_h\right], \tag{5}$$

where

$R'_h = \frac{\overline{Z}_h}{\overline{X}_h}$,

$B^{*\prime}_h = \lambda_{2h}\left(S^2_{hX} + S^2_{hV}\right) + \theta_h\left(S^2_{hX(2)} + S^2_{hV(2)}\right)$,

$C^{*\prime}_h = \lambda_{2h}\rho_{hYX}S_{hY}S_{hX} + \theta_h\rho_{hYX(2)}S_{hY(2)}S_{hX(2)}$.

2.3. Product Estimator

The product estimator under stratified random sampling, is given by

$$\bar{y}^{*\prime}_{S(Pr)} = \sum_{h=1}^{L} P_h\left[\bar{z}^*_h \frac{\bar{x}^*_h}{\overline{X}_h}\right]. \tag{6}$$

The bias and MSE of $\bar{y}^*_{S(Pr)}$, are given by

$$B\left(\bar{y}^{*\prime}_{S(Pr)}\right) \cong \sum_{h=1}^{L} P_h\left[\frac{C^{*\prime}_h}{\overline{X}_h}\right] \tag{7}$$

and

$$MSE\left(\bar{y}^{*\prime}_{S(Pr)}\right) \cong \sum_{h=1}^{L} P_h^2\left[A^{*\prime}_h + R'^2_h B^{*\prime}_h + 2R'_h C^{*\prime}_h\right]. \tag{8}$$

2.4. Bahl and Tuteja, 1991 Estimator

Bahl and Tuteja, 1991 estimator under stratified random sampling, is given by

$$\bar{y}^{*\prime}_{S(BT)} = \sum_{h=1}^{L} P_h\left[\bar{z}^*_h \exp\left(\frac{\overline{X}_h - \bar{x}^*_h}{\overline{X}_h + \bar{x}^*_h}\right)\right]. \tag{9}$$

The bias and MSE of $\bar{y}^*_{S(BT)}$, are given by

$$B\left(\bar{y}^{*\prime}_{S(BT)}\right) \cong \sum_{h=1}^{L} P_h\left[\frac{1}{\overline{X}_h}\left(\frac{3R'_h B^{*\prime}_h}{8} - \frac{C^{*\prime}_h}{2}\right)\right] \tag{10}$$

and

$$MSE\left(\bar{y}^{*\prime}_{S(BT)}\right) \cong \sum_{h=1}^{L} P_h^2 \left[A_h^{*\prime} + \frac{R_h^{\prime 2} B_h^{*\prime}}{4} - R_h^{\prime} C_h^{*\prime}\right]. \tag{11}$$

2.5. Singh and Kumar, 2010 Estimator

Singh and Kumar, 2010 estimator under stratified random sampling, is given by

$$\bar{y}^{*\prime}_{S(SK)} = \sum_{h=1}^{L} P_h \left[\bar{z}_h^{*} \left(\frac{\bar{X}_h}{\bar{x}_h^{*}}\right)^2\right]. \tag{12}$$

The bias and MSE of $\bar{y}^{*}_{S(SK)}$, are given by

$$B\left(\bar{y}^{*\prime}_{S(SK)}\right) \cong \sum_{h=1}^{L} P_h \left[\frac{1}{\bar{X}_h}\left(3R_h^{\prime} B_h^{*\prime} - 2C_h^{*\prime}\right)\right] \tag{13}$$

and

$$MSE\left(\bar{y}^{*\prime}_{S(SK)}\right) \cong \sum_{h=1}^{L} P_h^2 \left[A_h^{*\prime} + 4R_h^{\prime 2} B_h^{*\prime} - 4R_h^{\prime} C_h^{*\prime}\right]. \tag{14}$$

2.6. Difference Estimator

The usual difference estimator under stratified random sampling, is given by

$$\bar{y}^{*\prime}_{S(D)} = \sum_{h=1}^{L} P_h \left[\bar{y}_h^{*} + d_h\left(\bar{X}_h - \bar{x}_h^{*\prime}\right)\right], \tag{15}$$

where $\bar{x}_h^{*\prime} = \frac{N_h \bar{X}_h - n_h \bar{x}_h^{*}}{N_h - n_h}$ and d_h is the constant.

The minimum variance of $\bar{y}^{*}_{S(D)}$, is given by

$$Var\left(\bar{y}^{*\prime}_{S(D)}\right)_{min} = \sum_{h=1}^{L} P_h^2 \left[A_h^{*\prime} - \frac{C_h^{*\prime 2}}{B_h^{*\prime}}\right]. \tag{16}$$

The optimum value of d_h is $d_{h(opt)} = -\frac{C_h^{*\prime}}{t_h B_h^{*\prime}}$, where $t_h = \frac{nh}{N_h - n_h}$.

2.7. Azeem and Hanif (2017) Estimator

Azeem and Hanif (2016) estimator under stratified random sampling, is given by

$$\bar{y}^{*\prime}_{S(AH)} = \sum_{h=1}^{L} P_h \bar{y}_h^{*} \left(\frac{\bar{x}_h^{*\prime}}{\bar{X}_h}\right)\left(\frac{\bar{x}_h^{*\prime}}{\bar{x}_h^{*}}\right). \tag{17}$$

The bias and MSE of $\bar{y}^{*\prime}_{S(AH)}$, are given by

$$B\left(\bar{y}^{*\prime}_{S(AH)}\right) \cong \sum_{h=1}^{L} \frac{P_h}{\bar{X}_h}\left[t_h^2 R_h B_h^{*\prime} - q_h C_h^{*\prime}\right] \tag{18}$$

and

$$MSE\left(\bar{y}^{*\prime}_{S(AH)}\right)) \cong \sum_{h=1}^{L} P_h^2 \left[A_h^{*\prime} + q_h^2 R_h^{\prime 2} B_h^{*\prime} - 2q_h R_h^{\prime} C_h^{*\prime}\right], \tag{19}$$

where $q_h = \frac{N_h + nh}{N_h - n_h}$.

3. Proposed Generalized Class of Estimators

We suggest a generalized class of estimators for estimating the finite population mean for a sensitive variable, considering the problem of measurement error and non-response simultaneously in stratified random sampling. Measurement error and non-response are present on both the study variable and the auxiliary variable. The suggested estimator, is given by

$$\bar{y}^{*\prime}_{S(GP)} = \sum_{h=1}^{L} P_h \left\{ m_{1h} \bar{z}_h^{*} \left\{\frac{\bar{X}_h}{\bar{x}_h^{*\prime}}\right\}^{\alpha_1} + m_{2h}\left(\bar{X}_h - \bar{x}_h^{*\prime}\right)\left\{\frac{\bar{X}_h}{\bar{x}_h^{*\prime}}\right\}^{\alpha_2} + m_{3h}\left(\bar{R}_{xh} - \bar{r}_{xh}^{*}\right)\left\{\frac{\bar{X}_h}{\bar{x}_h^{*\prime}}\right\}^{\alpha_3} \right\}$$
$$exp(1-\alpha_0)\left(\frac{\bar{X}_h - \bar{x}_h^{*\prime}}{\bar{X}_h + \bar{x}_h^{*\prime}}\right), \tag{20}$$

where, m_{1h}, m_{2h} and m_{3h} are constants whose values are to be determined, and $\alpha_r (r = 0, 1, 2, 3)$ are the scalars, chosen arbitrary. For obtaining the bias and MSE, we assume that

$\delta_{hZ}^{*} = \sum_{i=1}^{n_h}\left(Y_{hi}^{*} - \bar{Y}_h\right)$, $\delta_{hU}^{*} = \sum_{i=1}^{n_{h_h}} U_{hi}^{*}$,

$\delta_{hX}^{*} = \sum_{i=1}^{n_h}\left(X_{hi}^{*} - \bar{X}_h\right)$, $\delta_{hV}^{*} = \sum_{i=1}^{n_{h_h}} V_{hi}^{*}$,

$\delta_{R_{hx}}^{*} = \sum_{i=1}^{n}\left(R_{x_{hi}}^{*} - \bar{R}_{x_h}\right)$, $\delta_{T}^{*} = \sum_{i=1}^{n_h} T_{hi}^{*}$.

Adding $\delta_{hY}^{*}$and δ_{hU}^{*}, we get.

$\delta_{hZ}^{*} + \delta_{hU}^{*} = \sum_{i=1}^{n_h}\left(Z_{hi}^{*} - \bar{Z}_h\right) + \sum_{i=1}^{n_h} U_{hi}^{*}$.

Dividing both sides by n_h, and then simplifying, we get

$$\bar{z}_h^{*} = \bar{Z}_h + \frac{1}{n_h}\left(\delta_{hY}^{*} + \delta_{hU}^{*}\right). \tag{21}$$

Similarly, we can get

$$\bar{x}_h^{*} = \bar{X}_h + \frac{1}{n_h}\left(\delta_{hX}^{*} + \delta_{hV}^{*}\right) \tag{22}$$

and

$$\bar{r}_{xh}^{*} = \bar{R}_{x_h} + \frac{1}{n_h}\left(\delta_{hR_x}^{*} + \delta_{hT}^{*}\right). \tag{23}$$

Further

$E\left(\frac{\delta_{hZ}^{*} + \delta_{hQ}^{*}}{n_h}\right)^2 = \lambda_{2h}\left(S_{hZ}^2 + S_{hQ}^2\right) + \theta_h\left(S_{hZ(2)}^2 + S_{hQ(2)}^2\right) = A_h^{*\prime}$,

$E\left(\frac{\delta_{hX}^{*} + \delta_{hV}^{*}}{n_h}\right)^2 = \lambda_{2h}\left(S_{hX}^2 + S_{hV}^2\right) + \theta_h\left(S_{hX(2)}^2 + S_{hV(2)}^2\right) = B_h^{*\prime}$,

$E\left(\frac{\delta_{hR_x}^{*} + \delta_{hT}^{*}}{n_h}\right)^2 = \lambda_{2h}\left(S_{hR_x}^2 + S_{hT}^2\right) + \theta_h\left(S_{hR_x(2)}^2 + S_{hT(2)}^2\right) = D_h^{*\prime}$,

$E\left(\frac{\delta_{hZ}^{*} + \delta_{hQ}^{*}}{n_h}\right)\left(\frac{\delta_{hX}^{*} + \delta_{hV}^{*}}{n}\right) = \lambda_{2h}\rho_{hZX} S_{hZ} S_{hX} + \theta_h \rho_{hZX(2)} S_{hZ(2)} S_{hX(2)} = C_h^{*\prime}$,

$E\left(\frac{\delta_{hZ}^{*} + \delta_{hQ}^{*}}{n_h}\right)\left(\frac{\delta_{hR_x}^{*} + \delta_{hT}^{*}}{n}\right) = \lambda_{2h}\rho_{hZR_x} S_{hZ} S_{hR_x} + \theta_h \rho_{hZR_x(2)} S_{hZ(2)} S_{hR_x(2)} = E_h^{*\prime}$,

$E\left(\frac{\delta_{hX}^{*} + \delta_{hV}^{*}}{n_h}\right)\left(\frac{\delta_{hR_x}^{*} + \delta_{hT}^{*}}{n_h}\right) = \lambda_{2h}\rho_{hXR_x} S_{hX} S_{hR_x} + \theta_h \rho_{hXR_x(2)} S_{hX(2)} S_{hR_x(2)} = F_h^{*\prime}$.

On simplifying, we get

$$\bar{y}^{*\prime}_{S(GP)} = \sum_{h=1}^{L} P_h \left[m_{1h}\left(\bar{Z}_h + W_{hZ} + e^{*\prime} R_h^{\prime} t_h W_{hX} + \frac{f^{*\prime} t_h^2 R_h^{\prime} W_{hX}^2}{\bar{X}_h} + e^{*\prime} t_h \frac{W_{hX} W_{hZ}}{\bar{X}_h}\right)\right.$$
$$\left. + m_{2h}\left(t_h W_{hX} + d^{*\prime} t_h^2 \frac{W_{hX}^2}{\bar{X}_h}\right) + m_{3h}\left(t_h W_{hR_x} + c^{*\prime} t_h \frac{W_{hX} W_{hR_x}}{\bar{X}_h} + b^{*\prime} t_h^2 \frac{W_{hR_x}^2}{\bar{R}_{xh}}\right)\right], \tag{24}$$

where

$b^{*\prime} = \alpha_3$,

$c^{*\prime} = \frac{1-\alpha_0}{2}$,

$d^{*\prime} = \alpha_2 + \frac{1-\alpha_0}{2}$,

$e^{*\prime} = \alpha_1 + \frac{1-\alpha_0}{2}$, and.

$f^{*\prime} = \frac{\alpha_0^2 - 4\alpha_0 + 3}{8} + \frac{\alpha_1(2-\alpha_0+\alpha_1)}{2}$.

$W_{hZ} = \frac{\delta_Z^{*} + \delta_Q^{*}}{n}$, $W_{hX} = \frac{\delta_X^{*} + \delta_V^{*}}{n}$ and $W_{hR_x} = \frac{\delta_{R_x}^{*} + \delta_T^{*}}{n}$.

Further simplifying, and ignoring error terms greater than two, we have

$$\bar{y}^{*\prime}_{S(GP)} - \bar{Z} = \sum_{h=1}^{L} P_h \left[(m_{1h} - 1)\bar{Z}_h + m_{2h}\left(t_h W_{hX} + d^{*\prime} t_h^2 \frac{W_{hX}^2}{\bar{X}_h}\right)\right.$$
$$+ m_{1h}\left(W_{hY} + e^{*\prime} R_h^{\prime} t_h W_{hX} + f^{*\prime} t_h^2 R_h^{\prime} W_{hX}^2 + e^{*\prime} t_h \frac{W_{hX} W_{hY}}{\bar{X}_h}\right) \tag{25}$$
$$\left. + m_{3h}\left(t_h W_{hR_x} + c^{*\prime} t_h \frac{W_{hX} W_{hR_x}}{\bar{X}_h} + b^{*\prime} t_h^2 \frac{W_{hR_x}^2}{\bar{R}_{xh}}\right)\right].$$

Using Eq. (25), the bias and MSE of $\bar{y}^{*\prime}_{S(GP)}$ to first order of approximation, is given by

$$B\left(\bar{y}^{*\prime}_{S(GP)}\right) \cong \sum_{h=1}^{L} P_h \left[(m_{1h}-1)\bar{Z}_h + m_{1h}\left(\frac{f^{*\prime} t_h^2 R_h B_h}{\bar{X}_h} + \frac{e^{*\prime} t_h C_h}{\bar{X}_h}\right) + m_{2h}\left(\frac{d^{*\prime} t_h^2 B_h}{\bar{X}_h}\right) + m_{3h}\left(\frac{c^{*\prime} t_h F_h}{\bar{X}_h} + \frac{b^{*\prime} t_h^2 D_h}{\bar{R}_{xh}}\right)\right] \quad (26)$$

and

$$MSE\left(\bar{y}^{*\prime}_{S(GP)}\right) \cong \sum_{h=1}^{L} P_h^2 \left[\bar{Z}_h^2 + m_{1h}^2 A^{*\prime}_{h1} + m_{2h}^2 B^{*\prime}_{h1} + 2m_{1h}m_{2h}C^{*\prime}_{h1} - 2m_{1h}D^{*\prime}_{h1} - 2m_{2h}E^{*\prime}_{h1} + m_{3h}^2 F^{*\prime}_{h1} + 2m_{1h}m_{3h}G^{*\prime}_{h1} + 2m_{2h}m_{3h}H^{*\prime}_{h1} - 2m_{3h}I^{*\prime}_{h1}\right] \quad (27)$$

where,

$A^{*\prime}_{h1} = \bar{Z}_h^2 + A_h + e^{*\prime 2} t_h^2 R_h'^2 B_h + 4e^{*\prime} t_h R_h' C_h + 2f^{*\prime} t_h^2 R_h'^2 B_h$,
$B^{*\prime}_{h1} = t_h^2 B_h$,
$C^{*\prime}_{h1} = t_h C_h + t_h^2 R_h' B_h (e^{*\prime} + d^{*\prime})$,
$D^{*\prime}_{h1} = \bar{Z}_h^2 + e^{*\prime} t_h R_h' C_h + f^{*\prime} t_h^2 R_h'^2 B_h$,
$E^{*\prime}_{h1} = d^{*\prime} t_h^2 R_h' B_h$,
$F^{*\prime}_{h1} = t_h^2 D_h$,
$G^{*\prime}_{h1} = c^{*\prime} t_h R_h' F_h + e^{*\prime} t_h^2 R_h' F_h + t_h E_h + b^{*\prime} t_h^2 R_{1h}' D_h$,
$H^{*\prime}_{h1} = t_h^2 F_h$,
$I^{*\prime}_{h1} = c^{*\prime} t_h R_h' F_h + b^{*\prime} t_h^2 R_{1h}' D_h$.

For finding the optimal values of m_{1h}, m_{2h} and m_{3h}, we differentiate Eq. (27) with respect to m_{1h}, m_{2h} and m_{3h} respectively. The optimal values, are given by.

$$m_{1h(opt)} = \frac{B^{*\prime}_{h1}D^{*\prime}_{h1}F^{*\prime}_{h1} - C^{*\prime}_{h1}E^{*\prime}_{h1}F^{*\prime}_{h1} + E^{*\prime}_{h1}G^{*\prime}_{h1}H^{*\prime}_{h1} - D^{*\prime}_{h1}H^{*2}_{h1} - B^{*\prime}_{h1}G^{*\prime}_{h1}I^{*\prime}_{h1} + C^{*\prime}_{h1}H^{*\prime}_{h1}I^{*\prime}_{h1}}{A^{*\prime}_{h1}B^{*\prime}_{h1}F^{*\prime}_{h1} - C^{*2}_{h1}F^{*\prime}_{h1} + 2C^{*\prime}_{h1}G^{*\prime}_{h1}H^{*\prime}_{h1} - A^{*\prime}_{h1}H^{*2}_{h1}},$$

Table 1
Mean squared error and |*Bias*| (in brackets) values of different estimators for Population I with and without measurement error.

Estimators with Measurement Error	10% non-response			20% non-response		
	g_h			g_h		
	2	4	8	2	4	8
$\bar{y}^{*\prime}_{S(HH)}$	0.107125	0.128072	0.169966	0.117775	0.160022	0.244517
$\bar{y}^{*\prime}_{S(R)}$	0.030971	0.037839	0.051576	0.036732	0.055122	0.091904
	(0.026764)	(0.034223)	(0.049143)	(0.034202)	0.056537)	(0.101206)
$\bar{y}^{*\prime}_{S(Pr)}$	0.458469	0.555397	0.749251	0.517367	0.732089	1.161533
	(0.087794)	(0.106418)	(0.143665)	(0.098841)	(0.139558)	(0.220992)
$\bar{y}^{*\prime}_{S(BT)}$	0.034649	0.040819	0.053159	0.037435	0.049176	0.072660
	(0.065539)	(0.069173)	(0.076441)	(0.053816)	(0.034004)	(0.005620)
$\bar{y}^{*\prime}_{S(SK)}$	1.085004	1.319813	1.789432	1.235507	1.771322	2.842953
	(0.157364)	(0.193539)	(0.265890)	(0.183396)	(0.271635)	(0.448113)
$\bar{y}^{*\prime}_{S(D)}$	0.022166	0.026426	0.034858	0.024615	0.033426	0.050598
$\bar{y}^{*\prime}_{S(AH)}$	0.096090	0.119132	0.165217	0.115666	0.177860	0.302249
	(0.103052)	(0.124466)	(0.167295)	(0.115001)	(0.160313)	(0.250939)
$\alpha = 0, \bar{y}^{*\prime}_{S(P1)}$	0.022067	0.026286	0.034616	0.024492	0.033201	0.050087
	(0.018672)	(0.022202)	(0.029160)	(0.020721)	(0.028004)	(0.042072)
$\alpha = 1, \bar{y}^{*\prime}_{S(P1)}$	0.022075	0.026298	0.034638	0.024503	0.033223	0.050141
	(0.018679)	(0.022213)	(0.029180)	(0.020731)	(0.028023)	(0.042119)
$\alpha = -1, \bar{y}^{*\prime}_{S(P1)}$	0.022076	0.026299	0.034638	0.024503	0.033223	0.050143
	(0.018679)	(0.022213)	(0.029180)	(0.020731)	(0.028024)	(0.042121)
$\alpha_r = 1, r = 0,1,2,3 \bar{y}^{*\prime}_{S(GP)}$	0.021993	0.026179	0.034423	0.024394	0.033006	0.049605
	(0.018608)	(0.022109)	(0.028993)	(0.020636)	(0.027834)	(0.041651)
$\alpha_0 = 0, \alpha_{1,2,3} = 1 \bar{y}^{*\prime}_{S(GP)}$	0.022057	0.026230	0.034607	0.024428	0.033176	0.050079
	(0.018698)	(0.022240)	(0.029231)	(0.020752)	(0.028070)	(0.042240)
Estimators without Measurement Error	10% non-response			20% non-response		
	g_h			g_h		
	2	4	8	2	4	8
$\bar{y}^{*\prime}_{S(HH)}$	0.096942	0.115829	0.153602	0.106425	0.144277	0.219980
$\bar{y}^{*\prime}_{S(R)}$	0.004814	0.006475	0.009796	0.007385	0.014187	0.027792
	(0.013029)	(0.017786)	(0.027300)	(0.018722)	(0.034866)	(0.067153)
$\bar{y}^{*\prime}_{S(Pr)}$	0.432313	0.524032	0.707471	0.488020	0.691154	1.097421
	(0.087794)	(0.106418)	(0.143665)	(0.098841)	(0.139558)	(0.220992)
$\bar{y}^{*\prime}_{S(BT)}$	0.020473	0.023796	0.030441	0.021586	0.027134	0.038230
	(0.141554)	(0.160285)	(0.197747)	(0.139441)	(0.153945)	(0.182954)
$\bar{y}^{*\prime}_{S(SK)}$	1.010927	1.231085	1.671403	1.152171	1.654819	2.660115
	(0.138073)	(0.170463)	(0.235244)	(0.161654)	(0.241207)	(0.400314)
$\bar{y}^{*\prime}_{S(D)}$	0.001424	0.001860	0.002574	0.002043	0.003129	0.004569
$\bar{y}^{*\prime}_{S(AH)}$	0.049966	0.063866	0.091666	0.063824	0.105438	0.188667
	(0.106485)	(0.128576)	(0.172756)	(0.118871)	(0.165731)	(0.259452)
$\alpha = 0, \bar{y}^{*\prime}_{S(P1)}$	0.001423	0.001858	0.002570	0.002040	0.003124	0.004558
	(0.001219)	(0.001607)	(0.002228)	(0.001801)	(0.002785)	(0.004044)
$\alpha = 1, \bar{y}^{*\prime}_{S(P1)}$	0.001424	0.001859	0.002572	0.002042	0.003127	0.004564
	(0.001220)	(0.001608)	(0.002230)	(0.001802)	(0.002787)	(0.004050)
$\alpha = -1, \bar{y}^{*\prime}_{S(P1)}$	0.001424	0.001859	0.002572	0.002042	0.003127	0.004564
	(0.001220)	(0.001608)	(0.00223)	(0.001802)	(0.002787)	(0.004050)
$\alpha_r = 1, r = 0,1,2,3 \bar{y}^{*\prime}_{S(GP)}$	0.001409	0.001837	0.002530	0.002019	0.003079	0.004440
	(0.001207)	(0.001589)	(0.002193)	(0.001783)	(0.002745)	(0.003941)
$\alpha_0 = 0, \alpha_{1,2,3} = 1 \bar{y}^{*\prime}_{S(GP)}$	0.001422	0.001858	0.002571	0.002040	0.003126	0.004564
	(0.001219)	(0.001607)	(0.002229)	(0.001801)	(0.002787)	(0.004051)

$m_{2h(opt)} = \frac{A^{*\prime}_{h1}E^{*\prime}_{h1}F^{*\prime}_{h1} - C^{*\prime}_{h1}D^{*\prime}_{h1}F^{*\prime}_{h1} - E^{*\prime}_{h1}G^{*2}_{h1} + D^{*\prime}_{h1}G^{*\prime}_{h1}H^{*\prime}_{h1} + C^{*\prime}_{h1}G^{*\prime}_{h1}I^{*\prime}_{h1} - A^{*\prime}_{h1}H^{*\prime}_{h1}I^{*\prime}_{h1}}{A^{*\prime}_{h1}B^{*\prime}_{h1}F^{*\prime}_{h1} - C^{*2}_{h1}F^{*\prime}_{h1} + 2C^{*\prime}_{h1}G^{*\prime}_{h1}H^{*\prime}_{h1} - A^{*\prime}_{h1}H^{*2}_{h1}}$, and.

$m_{3(opt)} = \frac{C^{*\prime}_{h1}E^{*\prime}_{h1}G^{*\prime}_{h1} - B^{*\prime}_{h1}D^{*\prime}_{h1}G^{*\prime}_{h1} + C^{*\prime}_{h1}D^{*\prime}_{h1}H^{*\prime}_{h1} - A^{*\prime}_{h1}E^{*\prime}_{h1}H^{*\prime}_{h1} + A^{*\prime}_{h1}B^{*\prime}_{h1}I^{*\prime}_{h1} - C^{*2}_{h1}I^{*\prime}_{h1}}{A^{*\prime}_{h1}B^{*\prime}_{h1}F^{*\prime}_{h1} - C^{*2}_{h1}F^{*\prime}_{h1} + 2C^{*\prime}_{h1}G^{*\prime}_{h1}H^{*\prime}_{h1} - A^{*\prime}_{h1}H^{*2}_{h1}}$.

Substituting these optimum values in Eq. (27), we get the minimum MSE of $\bar{y}^{*\prime}_{S(GP)}$ as:

$$MSE\left(\bar{y}^{*\prime}_{S(GP)}\right)_{min} \cong \sum_{h=1}^{L} P_h^2 \left[\bar{Z}_h^2 - \frac{L^{*\prime}_{h1}}{L^{*\prime}_{h2}}\right], \quad (28)$$

where

$L^{*\prime}_{h1} = A^{*\prime}_{h1}E^{*2}_{h1}F^{*\prime}_{h1} - 2C^{*\prime}_{h1}D^{*\prime}_{h1}E^{*\prime}_{h1}F^{*\prime}_{h1} - E^{*\prime}_{h1}2G^{*\prime}_{h1}2 + 2D^{*\prime}_{h1}E^{*\prime}_{h1}G^{*\prime}_{h1}H^{*\prime}_{h1} - D_{h1} * 2 H^{*2}_{h1} + 2C^{*\prime}_{h1}E^{*\prime}_{h1}G^{*\prime}_{h1}I^{*\prime}_{h1} + 2C^{*\prime}_{h1}D^{*\prime}_{h1}H^{*\prime}_{h1}I^{*\prime}_{h1} - 2A^{*\prime}_{h1}E^{*\prime}_{h1}H^{*\prime}_{h1}I^{*\prime}_{h1} - C^{*2}_{h1}I^{*2}_{h1} + B^{*\prime}_{h1} D^{*2}_{h1}F^{*\prime}_{h1} - 2B^{*\prime}_{h1}D^{*\prime}_{h1}G^{*\prime}_{h1}I^{*\prime}_{h1} + B^{*\prime}_{h1}A^{*\prime}_{h1}I^{*2}_{h1}$ and.

$L^{*\prime}_{h2} = A^{*\prime}_{h1}B^{*\prime}_{h1}F^{*\prime}_{h1} - C^{*2}_{h1}F^{*\prime}_{h1} + 2C^{*\prime}_{h1}G^{*\prime}_{h1}H^{*\prime}_{h1} - A^{*\prime}_{h1}H^{*2}_{h1} - B^{*\prime}_{h1}G^{*2}_{h1}$.

4. Numerical Results

In this section simulated data and two real data sets are used to show the performance of the generalized class of proposed estimator. The results are given in Tables 1, 2 (simulation) and 5, 6 (real data).

4.1. Simulation Study

We have generated two populations (Population I and II) from normal distribution by using R language program, which are given in Appendix A. The results based on these population are given in Tables 1 and 2.

Table 2
Mean squared error and |*Bias*| (in brackets) values of different estimators for Population II with and without measurement error

Estimators	10% non-response			20% non-response		
	g_h			g_h		
	2	4	8	2	4	8
$\bar{y}^{*\prime}_{S(HH)}$	0.089797	0.108185	0.144961	0.098465	0.135429	0.209356
$\bar{y}^{*\prime}_{S(R)}$	0.013224	0.016604	0.023364	0.014699	0.021331	0.034594
	(0.000313)	(0.000378)	(0.001761)	(0.003640)	(0.004433)	(0.006020)
$\bar{y}^{*\prime}_{S(Pr)}$	0.320038	0.387672	0.522939	0.334810	0.460273	0.711199
	(0.074086)	(0.089385)	(0.119983)	(0.081194)	(0.111309)	(0.171537)
$\bar{y}^{*\prime}_{S(BT)}$	0.032302	0.038906	0.052115	0.037509	0.052036	0.081090
	(0.180648)	(0.209399)	(0.266902)	(0.235166)	(0.319488)	(0.488131)
$\bar{y}^{*\prime}_{S(SK)}$	0.703946	0.855063	1.157297	0.723734	0.995864	1.540122
	(0.067164)	(0.081340)	(0.109694)	(0.063355)	(0.087213)	(0.134930)
$\bar{y}^{*\prime}_{S(D)}$	0.012522	0.015889	0.022418	0.014072	0.020597	0.033554
$\bar{y}^{*\prime}_{S(AH)}$	0.027173	0.034466	0.049053	0.024482	0.035358	0.057109
	(0.089356)	(0.107659)	(0.144267)	(0.098718)	(0.135291)	(0.208437)
$\alpha = 0, \bar{y}^{*\prime}_{S(P1)}$	0.012476	0.015816	0.022274	0.014014	0.020474	0.033231
	(0.012576)	(0.015866)	(0.022247)	(0.014418)	(0.020942)	(0.033839)
$\alpha = 1, \bar{y}^{*\prime}_{S(P1)}$	0.012477	0.015820	0.022281	0.014016	0.020480	0.033245
	(0.012578)	(0.015868)	(0.022253)	(0.014420)	(0.020948)	(0.033853)
$\alpha = -1, \bar{y}^{*\prime}_{S(P1)}$	0.012478	0.015821	0.022282	0.014017	0.020481	0.033246
	(0.012579)	(0.015869)	(0.022254)	(0.014421)	(0.020949)	(0.033854)
$\alpha_r = 1, r = 0,1,2,3 \bar{y}^{*\prime}_{S(GP)}$	0.012455	0.015786	0.022218	0.013989	0.020426	0.033115
	(0.012556)	(0.015835)	(0.022191)	(0.014392)	(0.020892)	(0.033718)
$\alpha_0 = 0, \alpha_{1,2,3} = 1 \bar{y}^{*\prime}_{S(GP)}$	0.012474	0.015809	0.022230	0.014013	0.020464	0.033224
	(0.012585)	(0.015879)	(0.022273)	(0.014429)	(0.020964)	(0.033895)
Estimators without Measurement Error	10% non-response			20% non-response		
	g_h			g_h		
	2	4	8	2	4	8
$\bar{y}^{*\prime}_{S(HH)}$	0.079750	0.095863	0.128091	0.087280	0.119695	0.184526
$\bar{y}^{*\prime}_{S(R)}$	0.001954	0.002369	0.003199	0.002038	0.002794	0.004305
	(0.001402)	(0.001226)	(0.005081)	(0.007060)	(0.011019)	
$\bar{y}^{*\prime}_{S(Pr)}$	0.308768	0.373437	0.502774	0.322149	0.441736	0.680909
	(0.089385)	(0.119983)	(0.081194)	(0.111309)	(0.171537)	
$\bar{y}^{*\prime}_{S(BT)}$	0.021949	0.026106	0.034421	0.025956	0.035602	0.054895
	(0.222047)	(0.288625)	(0.245101)	(0.338267)	(0.524598)	
$\bar{y}^{*\prime}_{S(SK)}$	0.689010	0.835090	1.127248	0.706644	0.968914	1.493455
(0.066129)	(0.079822)	(0.107208)	(0.062190)	(0.085148)	(0.131064)	
	0.001194	0.001576	0.002205	0.001210	0.001686	0.002635
$\bar{y}^{*\prime}_{S(D)}$	0.014726	0.018444	0.025880	0.010445	0.014281	0.021953
	(0.108009)	(0.144843)	(0.099002)	(0.135796)	(0.209385)	
$\bar{y}^{*\prime}_{S(AH)}$	0.001190	0.001572	0.002203	0.001205	0.001682	0.002625
	(0.001574)	(0.002202)	(0.001256)	(0.001760)	(0.002759)	
$\alpha = 0, \bar{y}^{*\prime}_{S(P1)}$	0.001191	0.001573	0.002204	0.001206	0.001683	0.002626
	(0.001575)	(0.002203)	(0.001257)	(0.001761)	(0.002760)	
$\alpha = 1, \bar{y}^{*\prime}_{S(P1)}$	0.001192	0.001574	0.002205	0.001207	0.001684	0.002627
	(0.001576)	(0.002204)	(0.001258)	(0.001762)	(0.002760)	
$\alpha_r = 1, r = 0,1,2,3 \bar{y}^{*\prime}_{S(GP)}$	0.001184	0.001564	0.002189	0.001200	0.001673	0.002604
	(0.001566)	(0.002189)	(0.001251)	(0.001751)	(0.002737)	
$\alpha_0 = 0, \alpha_{1,2,3} = 1 \bar{y}^{*\prime}_{S(GP)}$	0.001189	0.001570	0.002202	0.001204	0.001680	0.002622
	(0.001573)	(0.002200)	(0.001254)	(0.001759)	(0.002760)	

Table 3
Data summary of Strata I.

Variable	Mean	st.Dev	Min	Med	Max
Forced expiratory volume (Y_1)	2.45	0.65	0.79	2.48	3.83
Age (X_1)	9.84	2.93	3.00	10.00	19.00
Smoke (S_1) 0,1	0.12	0.32	0.00	0.00	1.00

Table 4
Data summary of Strata II.

Variable	Mean	st.Dev	Min	Med	Max
Forced expiratory volume (Y_2)	2.68	1.00	0.79	2.61	5.79
Age (X_2)	10.01	2.97	3.00	10.00	19.00
Sex (S_2)0,1	0.07	0.27	0.00	0.00	1.00

Table 5
Mean squared error and $|Bias|$ (in brackets) values of different estimators for Population III with and without measurement error

Estimators	10% non-response			20% non-response		
	g_h			g_h		
	2	4	8	2	4	8
$\bar{y}^{*\prime}_{S(HH)}$	0.009864	0.011976	0.016201	0.012941	0.017808	0.027542
$\bar{y}^{*\prime}_{S(R)}$	0.009932	0.012202	0.016742	0.009053	0.012444	0.019226
	(0.003997)	(0.004906)	(0.006722)	(0.003688)	(0.005117)	(0.007975)
$\bar{y}^{*\prime}_{S(Pr)}$	0.037387	0.045484	0.061677	0.044100	0.060841	0.094323
	(0.004802)	(0.005828)	(0.007881)	(0.006228)	(0.008600)	(0.013344)
$\bar{y}^{*\prime}_{S(BT)}$	0.006449	0.007872	0.010719	0.007588	0.010418	0.016076
	(0.005316)	(0.006551)	(0.009022)	(0.003434)	(0.004720)	(0.007291)
$\bar{y}^{*\prime}_{S(SK)}$	0.092504	0.112726	0.153171	0.102530	0.141543	0.219569
	(0.009534)	(0.011631)	(0.015826)	(0.010642)	(0.014727)	(0.022896)
$\bar{y}^{*\prime}_{S(D)}$	0.006262	0.007627	0.010355	0.006917	0.009419	0.014419
$\bar{y}^{*\prime}_{S(AH)}$	0.016593	0.020397	0.028004	0.014191	0.019546	0.030256
	(0.004941)	(0.005987)	(0.008080)	(0.006670)	(0.009207)	(0.014279)
$\alpha = 0, \bar{y}^{*\prime}_{S(P1)}$	0.006252	0.007612	0.010329	0.006904	0.009395	0.014362
	(0.004152)	(0.005037)	(0.006804)	(0.004963)	(0.006760)	(0.010341)
$\alpha = 1, \bar{y}^{*\prime}_{S(P1)}$	0.006252	0.007613	0.010329	0.006904	0.009395	0.014362
	(0.004153)	(0.005038)	(0.006804)	(0.004963)	(0.006760)	(0.010342)
$\alpha = -1, \bar{y}^{*\prime}_{S(P1)}$	0.006252	0.007613	0.010329	0.006904	0.009396	0.014363
	(0.004153)	(0.005038)	(0.006804)	(0.004963)	(0.006760)	(0.010342)
$\alpha_r = 1, r = 0,1,2,3\bar{y}^{*\prime}_{S(GP)}$	0.004734	0.005749	0.007771	0.005848	0.007960	0.012160
	(0.003277)	(0.003966)	(0.005342)	(0.004165)	(0.005672)	(0.008669)
$\alpha_0 = 0, \alpha_{1,2,3} = 1\bar{y}_{S(GP)^{*\prime}}$	0.004737	0.005752	0.007777	0.005851	0.007966	0.012176
	(0.003278)	(0.003969)	(0.005346)	(0.004168)	(0.005677)	(0.008681)
Estimators without Measurement Error	10% non-response			20% non-response		
	g_h			g_h		
	2	4	8	2	4	8
$\bar{y}^{*\prime}_{S(HH)}$	0.008595	0.010446	0.014149	0.010151	0.014012	0.021732
$\bar{y}^{*\prime}_{S(R)}$	0.001193	0.001468	0.002018	0.000905	0.001243	0.001920
	(0.000333)	(0.000407)	(0.000556)	(0.000353)	(0.000485)	(0.000750)
$\bar{y}^{*\prime}_{S(Pr)}$	0.028648	0.034750	0.046953	0.035952	0.049640	0.077016
	(0.004802)	(0.005828)	(0.007881)	(0.006228)	(0.008600)	(0.013344)
$\bar{y}^{*\prime}_{S(BT)}$	0.003313	0.004041	0.005499	0.003459	0.004770	0.007392
	(0.004884)	(0.005936)	(0.008042)	(0.006588)	(0.009102)	(0.014129)
$\bar{y}^{*\prime}_{S(SK)}$	0.061352	0.074378	0.100428	0.078308	0.108130	0.167773
	(0.004815)	(0.005841)	(0.007893)	(0.006268)	(0.008659)	(0.013441)
$\bar{y}^{*\prime}_{S(D)}$	0.001144	0.001407	0.001934	0.0008737	0.001201	0.001856
$\bar{y}^{*\prime}_{S(AH)}$	0.001604	0.001962	0.002677	0.001586	0.002183	0.003379
	(0.005709)	(0.006930)	(0.009371)	(0.007385)	(0.010198)	(0.015823)
$\alpha = 0, \bar{y}^{*\prime}_{S(P1)}$	0.001143	0.001407	0.001933	0.000873	0.001200	0.001855
	(0.000677)	(0.000833)	(0.001145)	(0.000651)	(0.000893)	(0.001378)
$\alpha = 1, \bar{y}^{*\prime}_{S(P1)}$	0.001143	0.001407	0.001933	0.000873	0.001201	0.001855
	(0.000677)	(0.000833)	(0.001145)	(0.000651)	(0.000893)	(0.001378)
$\alpha = -1, \bar{y}^{*\prime}_{S(P1)}$	0.001143	0.001407	0.001933	0.000873	0.001201	0.001855
	(0.000677)	(0.000833)	(0.001145)	(0.000651)	(0.000893)	(0.001378)
$\alpha_r = 1, r = 0,1,2,3\bar{y}^{*\prime}_{S(GP)}$	0.001060	0.001307	0.001800	0.000822	0.001128	0.001742
	(0.000631)	(0.000777)	(0.001069)	(0.000611)	(0.000839)	(0.001293)
$\alpha_0 = 0, \alpha_{1,2,3} = 1\left(\bar{y}^{*\prime}_{S(GP)}\right.$	0.001063	0.001309	0.001801	0.000824	0.001129	0.001743
	(0.000631)	(0.000777)	(0.001070)	(0.000611)	(0.000839)	(0.001294)

Tables 1 and 2 show that the generalized class of proposed estimators $\bar{y}^{*\prime}_{S(GP)}$ perform better than other existing estimators for both with and without measurement errors. The values of the absolute biases are given in brackets. In Table 1 the MSE for the generalized proposed estimator, when $\alpha_r = 1, r = 0, 1, 2, 3$ is 0.021993 for 10% of non-response rate. When the non-response rate increases to 20%, the MSE for generalized proposed estimator increases to 0.024394. It is also observed that $\bar{y}^{*\prime}_{S(P1)}$ is less biased and $\bar{y}^{*\prime}_{S(SK)}$ is highly biased among all other considered estimator. Table 1 shows the same pattern of results for the case of no measurement error.

In Table 2 the MSE for the generalized proposed estimator, when $\alpha_r = 1, r = 0, 1, 2, 3$ is 0.012455 for 10% non-response rate. When the non-response rate increases to 20%, the MSE for generalized proposed estimator increases to 0.013989. It is also observed that $\bar{y}^{*\prime}_{S(R)}$ is less biased and $\bar{y}^{*\prime}_{S(BT)}$ is highly biased among all other considered estimator. Table 2 shows the same pattern of results for the case of no measurement error.

Through the simulation study it is concluded that the generalized proposed class of estimators perform better as compared to the all other existing estimators. For 10% non-response rate, the MSE is minimum as compared to 20% of the non-response rate. The MSE also increases as the value of constant g_h increases.

4.2. Application to Real Data Set

In this section we consider two real life data sets for numerical comparisons, Population III is taken from Rosner, 2015, Population IV is obtained by conducting a survey at Quaid-i-Azam University, Islamabad 4.2.1. The results based on these data sets are given in Tables 5 and 6.

Table 6
Mean squared error and $|Bias|$ (in brackets) values of different estimators for Population IV with and without measurement error.

Estimators	10% non-response			20% non-response		
	g_h			g_h		
	2	4	8	2	4	8
$\bar{y}^{*\prime}_{S(HH)}$	0.630021	0.837808	1.045595	0.630686	0.839804	1.048922
$\bar{y}^{*\prime}_{S(R)}$	1.921722	2.793240	3.464758	2.121722	3.170594	4.119467
	(0.130760)	(0.157226)	(0.104294)	(0.148446)	(0.192598)	
$\bar{y}^{*\prime}_{S(Pr)}$	2.506471	3.404393	4.302315	2.735328	4.090965	5.446601
	(0.013928)	(0.017060)	(0.011940)	(0.017360)	(0.022779)	
$\bar{y}^{*\prime}_{S(BT)}$	0.906671	1.228910	1.551149	0.945071	1.344110	1.743148
	(307.1653)	(395.3818)	(248.3003)	(395.2198)	(542.1394)	
$\bar{y}^{*\prime}_{S(SK)}$	7.622591	10.43603	13.24947	8.484520	13.02182	17.55912
	(0.567979)	(0.727644)	(0.461703)	(0.728147)	(0.994592)	
$\bar{y}^{*\prime}_{S(D)}$	0.619716	0.824887	1.029976	0.619579	0.824426	1.029194
	4.905149	6.754218	8.603288	5.446953	8.379632	11.31231
(0.030936)	(0.044995)	(0.059054)	(0.035498)	(0.058680)	(0.081861)	
$\alpha = 0, \bar{y}^{*\prime}_{S(P1)}$	0.617825	0.821560	1.024813	0.617755	0.821341	1.024502
	(0.076580)	(0.095337)	(0.056926)	(0.074051)	(0.091135)	
$\alpha = 1, \bar{y}^{*\prime}_{S(P1)}$	0.617948	0.821788	1.025177	0.617894	0.821628	1.024985
	(0.076603)	(0.096939)	(0.064079)	(0.091183)	(0.145374)	
$\alpha = -1, \bar{y}^{*\prime}_{S(P1)}$	0.617950	0.821792	1.025185	0.617896	0.821632	1.024994
	(0.076603)	(0.095375)	(0.056940)	(0.074080)	(0.091184)	
$\alpha_r = 1, r = 0, 1, 2, 3\left(\bar{y}^{*\prime}_{S(GP)}\right.$	0.334101	0.444233	0.552328	0.335842	0.447262	0.554985
	(0.040907)	(0.050769)	(0.030738)	(0.040354)	(0.049608)	
$\alpha_0 = 0, \alpha_{1,2,3} = 1\bar{y}^{*\prime}_{S(GP)}$	0.336942	0.449357	0.560385	0.339090	0.453985	0.566332
	(0.041422)	(0.051582)	(0.031060)	(0.041023)	(0.050737)	
Estimators without Measurement Error	10% non-response			20% non-response		
	g_h			g_h		
	2	4	8	2	4	8
$\bar{y}^{*\prime}_{S(HH)}$	0.531888	0.706251	0.880614	0.534662	0.714573	0.894483
$\bar{y}^{*\prime}_{S(R)}$	0.847220	1.129459	1.411698	0.893275	1.267623	1.641971
	(0.043594)	(0.054407)	(0.037339)	(0.057267)	(0.077194)	
$\bar{y}^{*\prime}_{S(Pr)}$	1.360452	1.797576	2.234700	1.458010	2.090249	2.722487
	(0.013928)	(0.017060)	(0.011940)	(0.017360)	(0.022779)	
$\bar{y}^{*\prime}_{S(BT)}$	0.546567	0.728539	0.910510	0.553723	0.750007	0.946291
	(70.46203)	(88.11975)	(60.15501)	(92.51417)	(124.8733)	
$\bar{y}^{*\prime}_{S(SK)}$	3.332912	4.403433	5.473954	3.663318	5.394650	7.125983
	(0.172948)	(0.214997)	(0.147613)	(0.223090)	(0.298568)	
$\bar{y}^{*\prime}_{S(D)}$	0.499049	0.664792	0.830500	0.499692	0.666329	0.832772
$\bar{y}^{*\prime}_{S(AH)}$	1.780605	2.369676	2.958748	1.941566	2.852558	3.763550
	(0.003230)	(0.003599)	(0.002798)	(0.003040)	(0.003283)	
$\alpha = 0, \bar{y}^{*\prime}_{S(P1)}$	0.497680	0.662389	0.826776	0.498415	0.664262	0.829706
	(0.064500)	(0.080376)	(0.047678)	(0.061733)	(0.075745)	
$\alpha = 1, \bar{y}^{*\prime}_{S(P1)}$	0.497707	0.662436	0.826849	0.498447	0.664327	0.829816
	(0.064504)	(0.080382)	(0.047681)	(0.061739)	(0.075755)	
$\alpha = -1, \bar{y}^{*\prime}_{S(P1)}$	0.497708	0.662437	0.826850	0.498447	0.664328	0.829818
	(0.064505)	(0.080382)	(0.047681)	(0.061739)	(0.075755)	
$\alpha_r = 1, r = 0, 1, 2, 3\bar{y}^{*\prime}_{S(GP)}$	0.222332	0.293731	0.363450	0.223663	0.296212	0.366048
	(0.030276)	(0.037404)	(0.022712)	(0.029331)	(0.035698)	
$\alpha_0 = 0, \alpha_{1,2,3} = 1\bar{y}^{*\prime}_{S(GP)}$	0.223172	0.295152	0.365602	0.224612	0.298063	0.369087
	(0.030408)	(0.037603)	(0.022800)	(0.029499)	(0.035973)	

Population III. [Source:Rosner, 2015].

Strata I consist of 318 observations and strata II contains 336 observations. The data summary is given in Tables 3 and 4.

$\rho_{1XY} = 0.7564, \rho_{1XR_x} = 0.7831$ and $\rho_{1YR_x} = 0.6151$.

$\rho_{2XY} = 0.8109, \rho_{2XR_x} = 0.7765$ and $\rho_{2YR_x} = 0.6575$.

4.2.1. Data Collection

To see the practical implication of measurement error, we conducted a study based on real data set at Quaid-i-Azam University, Islamabad. We distributed 55 questionnaires to the students of BS Statistics (5th Semester Fall, 2018) and M.Phil Statistics (1st and 2nd Semesters, Fall 2018) of Quaid-i-Azam University, Islamabad. We consider our population of those students who gave the false response, which comes out to be 23. As we already have the true response from their academic record. In question (i) we asked for Y = Age, X = Marks in A level or Intermediate (in percentage). In question (ii) S = Social media effects the academic result is asked, where Y is the study variable, X is the auxiliary variable and S is the scrambling response variable. We have 23 students ($N = 23$), including 8 male students and 15 female students who gave the false response.

Population IV. [Source: Section 4.2.1].

Let, Y: Age of BS 5^{th} and Mphil Students of Statistics department, X: Marks in A level or Intermediate, S:Social media effects on the academic result

NumberofStrata = 2 (Male and Female).

$N_1 = 8, N_2 = 15, \overline{Z}_1 = 24.25, \overline{Z}_2 = 23.53, \overline{X}_1 = 54.25, \overline{X}_2 = 63.67, \overline{R}_{x1} = 4.5, \overline{R}_{x2} = 8, S^2_{1Z} = 6.39, S^2_{ZY} = 3.75, S^2_{1X} = 73.36, S^2_{2X} = 76.95238, S^2_{1R_x} = 6, S^2_{2R_x} = 20, \rho_{1ZX} = 0.37740, \rho_{2ZX} = 0.15107, \rho_{1XR_x} = -0.25875, \rho_{2XR_x} = -0.10014, \rho_{1ZR_x} = -0.69314, \rho_{2ZR_x} = -0.62315$.

Tables 5 and 6 show that the generalized class of proposed estimators $\overline{y}^{*\prime}_{S(GP)}$ perform better than other existing estimators for both with and without measurement error. The values of the absolute biases are given in brackets in the tables. In Table 5 the MSE for the generalized proposed estimator, when $\alpha_r = 1, r = 0, 1, 2, 3$ is 0.004734 for 10% non-response rate. When the non-response rate becomes 20%, the MSE for generalized proposed estimator increases to 0.005848. It is also observed that $\overline{y}^{*\prime}_{S(GP)}$ is less biased and $\overline{y}^{*\prime}_{S(SK)}$ is highly biased among all other considered estimator. Table 5 shows the same pattern of results in case for no measurement error.

In Table 6 the MSE for the generalized proposed estimator, when $\alpha_r = 1, r = 0, 1, 2, 3$ is 0.334101 for 10% non-response rate. When the non-response rate becomes 20%, the MSE for generalized proposed estimator increases to 0.335842. It is also observed that $\overline{y}^{*\prime}_{S(R)}$ is less biased and $\overline{y}^{*\prime}_{S(BT)}$ is most biased among all other considered estimator. Table 6 shows the same pattern of results in case for no measurement error.

Through real data sets it is concluded that the generalized proposed estimator performs better as compared to the other existing estimators. For 10% non-response rate the MSE is minimum. The MSE also increases as the value of constant g_h increases.

5. Conclusion

In the present study, we proposed a generalized class of estimators in estimating the finite population mean for the sensitive variable in the presence of measurement error and non-response under stratified random sampling. Through simulation study and real life data sets it is observed that the proposed class of estimators perform better than the existing estimators. The MSE values are generally smaller under 10% of non-response as compared to 20% of non-response, which are expected results. Generally as the non-response rate increases, MSE also increases. Based on numerical findings, it turns out that the generalized proposed class of estimators is more efficient as compared to the other existing estimators, under certain situations.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A

Simplification of MSE

Squaring both sides of Eq. (25), and keeping the terms up to power two in errors, and then taking expectations, the MSE of $\overline{y}^{*\prime}_{S(GP)}$ is given by

$$MSE\left(\overline{y}^{*\prime}_{S(GP)}\right) \cong \sum_{h=1}^{L} P_h^2 \Big[\overline{Z}_h^2 + m_{1h}^2\left(\overline{Z}_h^2 + A_h + e^{*2} t_h^2 R_h'^2 B_h + 4e^{*\prime} t_h R_h' C_h + 2f^{*\prime} t_h^2 R_h'^2 B_h\right)$$
$$+ m_{2h}^2 t_h^2 B_h + 2m_{1h} m_{2h}\left(t_h C_h + t^2 R_h' B_h\left(e^{*\prime} + d^{*\prime}\right)\right) - 2m_{h1}\left(\overline{Z}_h^2 + e^{*\prime} t_h R_h' C_h + f^{*\prime} t_h^2 R_h'^2 B_h\right)$$
$$- 2m_{2h} d^{*\prime} t_h^2 R_h' B_h + m_{3h}^2 t_h^2 D_h + 2m_{1h} m_{3h}\left(c^{*\prime} t_h R_h' F_h + e^{*\prime} t_h^2 R_h' F_h + t_h E_h + b^{*\prime} t_h^2 R_{1h}' D_h\right)$$
$$+ 2m_{2h} m_{3h} t^2 F_h - 2m_{3h}\left(c^{*\prime} t_h R_h' F_h + b^{*\prime} t_h^2 R_{1h}' D_h\right)\Big].$$

where $R'_{1h} = \frac{\overline{Z}_h}{\overline{R}_{xh}}$.

Population

Population I.

$X_1 = rnorm(1000, 5, 10), Y_1 = X_1 + rnorm(1000, 0, 1), y_1 = Y_1 + rnorm(1000, 1, 3), x_1 = X_1 + rnorm(1000, 1, 3)$.

$X_2 = rnorm(1000, 4, 8), Y_2 = X_2 + rnorm(1000, 0, 1), y_2 = Y_2 + rnorm(1000, 1, 3), x_2 = X_2 + rnorm(1000, 1, 3)$.

$X_3 = rnorm(1000, 4, 9), Y_3 = X_3 + rnorm(1000, 0, 1), y_3 = Y_3 + rnorm(1000, 1, 3), x_3 = X_3 + rnorm(1000, 1, 3)$.

$X_4 = rnorm(1000, 3, 7), Y_4 = X_4 + rnorm(1000, 0, 1), y_4 = Y_4 + rnorm(1000, 1, 3), x_4 = X_4 + rnorm(1000, 1, 3)$.

NumberofStrata = 4

$N_1 = 1000, N_2 = 1000, N_3 = 1000, N_4 = 1000$,
$n_1 = 200, n_2 = 200, n_3 = 200, n_4 = 200$,
$\overline{Z}_1 = 5.719824, \overline{Z}_2 = 4.985474, \overline{Z}_3 = 4.85276, \overline{Z}_4 = 3.835371$,
$\overline{X}_1 = 5.666893, \overline{X}_2 = 3.643237, \overline{X}_3 = 3.968049, \overline{X}_4 = 2.918596$,
$\overline{R}_{xi} = 500.5, i = 1, 2, 3, 4$,
$S^2_{1Z} = 124.6685, S^2_{2Z} = 73.65976, S^2_{3Z} = 90.66835, S^2_{4Z} = 61.00159$,
$S^2_{1X} = 104.2774, S^2_{2X} = 66.19725, S^2_{3X} = 81.06883, S^2_{4X} = 45.99937$,
$S^2_{iR_x} = 83416.67, i = 1, 2, 3, 4, \quad \rho_{1ZX} = 0.9953966, \rho_{2ZX} = 0.9927347$,
$\rho_{3ZX} = 0.9940606, \rho_{4ZX} = 0.9891463$,
$\rho_{1ZR_x} = -0.003890, \rho_{2ZR_x} = 0.016016, \rho_{3ZR_x} = 0.062953$,
$\rho_{4ZR_x} = -0.031585$,
$\rho_{1XR_x} = -0.044153, \rho_{2XR_x} = -0.011336, \rho_{3XR_x} = 0.022509$,
$\rho_{4XR_x} = 0.033215$.

Population II.

$X_1 = rnorm(1000, 5, 10), Y_1 = X_1 + rnorm(1000, 0, 1), y_1 = Y_1 + rnorm(1000, 1, 3), x_1 = X_1 + rnorm(1000, 0, 1)$.

$X_2 = rnorm(1200, 4, 8), Y_2 = X_2 + rnorm(1200, 0, 1), y_2 = Y_2 + rnorm(1200, 1, 3), x_2 = X_2 + rnorm(1200, 0, 1)$.

$X_3 = rnorm(1300, 4, 9), Y_3 = X_3 + rnorm(1300, 0, 1), y_3 = Y_3 + rnorm(1300, 1, 3), x_3 = X_3 + rnorm(1300, 0, 1)$.

$X_4 = rnorm(1500, 3, 7), Y_4 = X_4 + rnorm(1500, 0, 1), y_4 = Y_4 + rnorm(1500, 1, 3), x_4 = X_4 + rnorm(1500, 1, 3)$.

NumberofStrata = 4

$N_1 = 1000, N_2 = 1200, N_3 = 1300, N_4 = 1500$,
$n_1 = 200, n_2 = 210, n_3 = 220, n_4 = 215$,
$\overline{Z}_1 = 4.648022, \overline{Z}_2 = 4.036113, \overline{Z}_3 = 4.032501, \overline{Z}_4 = 2.969091$,
$\overline{X}_1 = 5.666893, \overline{X}_2 = 3.807569, \overline{X}_3 = 4.627208, \overline{X}_4 = 3.241139$,
$\overline{R}_{x1} = 500.5, \overline{R}_{x2} = 600.5, \overline{R}_{x3} = 650.5, \overline{R}_{x4} = 750.5$,

$S^2_{1Z} = 94.19621, S^2_{2Z} = 66.76728, S^2_{3Z} = 80.80177, S^2_{4Z} = 51.14123,$
$S^2_{1X} = 104.2774, S^2_{2X} = 65.46337, S^2_{3X} = 82.98812, S^2_{4X} = 52.84269,$
$S^2_{1R_x} = 83416.67, S^2_{2R_x} = 120100, S^2_{3R_x} = 140941.7, S^2_{4R_x} = 187625,$
$\rho_{1ZX} = 0.984077, \rho_{2ZX} = 0.987461, \rho_{3ZX} = 0.991750,$
$\rho_{4ZX} = 0.989362,$
$\rho_{1ZR_x} = 0.015173, \rho_{2ZR_x} = -0.008185, \rho_{3ZR_x} = 0.009465,$
$\rho_{4ZR_x} = 0.006319,$
$\rho_{1XR_x} = -0.105091, \rho_{2XR_x} = -0.124294, \rho_{3XR_x} = 0.002547,$
$\rho_{4XR_x} = -0.013688.$

Members of Generalized Proposed Class of Estimators $\bar{y}^{*\prime}_{S(GP)}$ for Different Choices of $(\alpha_0, \alpha_1, \alpha_2, \alpha_3, m_{1h}, m_{2h}, m_{3h})$ Members of the class of estimators $\bar{y}^{*\prime}_{S(GP)}$ by choosing different values of $\alpha_0, \alpha_1, \alpha_2, \alpha_3, m_{1h}, m_{2h}$ and m_{3h} are given below

1. For $\alpha_1 = m_{2h} = m_{3h} = 0$ and $\alpha_0 = m_{1h} = 1$ in Eq. (20), the generalized proposed class of estimators $\bar{y}^{*\prime}_{S(GP)}$ reduces to usual mean estimator as:

$$\bar{y}^{*\prime}_{S(0)} = \sum_{h=1}^{L} P_h \bar{z}^*_h.$$

2. For $\alpha_0 = \alpha_1 = m_{1h} = 1$ and $m_{2h} = m_{3h} = 0$ in Eq. (20), the generalized proposed class of estimators $\bar{y}^{*\prime}_{S(GP)}$ reduces to usual ratio estimator:

$$\bar{y}^{*\prime}_{S(R)} = \sum_{h=1}^{L} P_h \left(\frac{\bar{z}^*_h}{\bar{x}^*_h} \bar{X}_h \right).$$

3. For $\alpha_0 = m_{1h} = 1, \alpha_1 = -1$ and $m_{2h} = m_{3h} = 0$ in Eq. (20), the generalized proposed class of estimators $\bar{y}^{*\prime}_{S(GP)}$ reduces to usual product estimator:

$$\bar{y}^{*\prime}_{S(Pr)} = \sum_{h=1}^{L} P_h \left(\bar{z}^*_h \frac{\bar{x}^*_h}{\bar{X}_h} \right).$$

4. For $\alpha_0 = \alpha_1 = m_{2h} = m_{3h} = 0$ and $m_{1h} = 1$ in Eq. (20), the generalized proposed class of estimators $\bar{y}^{*\prime}_{S(GP)}$ reduces to Bahl and Tuteja, 1991 estimator:

$$\bar{y}^{*\prime}_{S(BT)} = \sum_{h=1}^{L} P_h \bar{z}^* \exp\left(\frac{\bar{X}_h - \bar{x}^*_h}{\bar{X}_h + \bar{x}^*_h} \right).$$

5. For $\alpha_0 = \alpha_1 = \alpha_2 = 1, m_{3h} = 0, m_{1h} = m_{4h}$ and $m_{2h} = m_{5h}$ in Eq. (20), the generalized proposed class of estimators $\bar{y}^{*\prime}_{S(GP)}$ reduces to Gupta and Shabbir, 2008 estimator:

$$\bar{y}^{*\prime}_{S(GS)} = \sum_{h=1}^{L} P_h \left[m_{4h} \bar{z}^*_h \left(\frac{\bar{X}_h}{\bar{x}^*_h} \right) + m_{5h} (\bar{X}_h - \bar{x}^*_h) \left(\frac{\bar{X}_h}{\bar{x}^*_h} \right) \right].$$

6. For $\alpha_0 = m_{2h} = m_{3h} = 0, \alpha_1 = 2$ and $m_{1h} = 1$ in Eq. (20), the generalized proposed class of estimators $\bar{y}^{*\prime}_{S(GP)}$ reduces to Singh and Kumar, 2010 estimator:

$$\bar{y}^{*\prime}_{S(SK)} = \sum_{h=1}^{L} P_h \left[\bar{z}^*_h \left(\frac{\bar{X}_h}{\bar{x}^*_h} \right)^2 \right].$$

7. For $\alpha_0 = \alpha_1 = \alpha_2 = 0, m_{3h} = 0, m_{1h} = m_{6h}$ and $m_{2h} = m_{7h}$ in Eq. (20), the generalized proposed class of estimators $\bar{y}^{*\prime}_{S(GP)}$ reduces to Grover and Kaur, 2011 estimator:

$$\bar{y}^{*\prime}_{S(GK)} = \sum_{h=1}^{L} P_h \left[m_{6h} \bar{z}^*_h + m_{7h} (\bar{X}_h - \bar{x}^*_h) \exp\left(\frac{\bar{X}_h - \bar{x}^*_h}{\bar{X}_h + \bar{x}^*_h} \right) \right].$$

8. For $\alpha_1 = \alpha_2 = m_{3h} = 0, \alpha_0 = m_{1h} = 1$ and $m_{2h} = d^{*\prime}_h$ in Eq. (20), the generalized proposed class of estimators $\bar{y}^{*\prime}_{S(GP)}$ reduces to difference estimator:

$$\bar{y}^{*\prime}_{S(D)} = \sum_{h=1}^{L} P_h \left[\bar{z}^*_h + d^{*\prime}_h (\bar{X}_h - \bar{x}^{*\prime}_h) \right].$$

9. For $\alpha_1 = \alpha_2 = g, m_{1h} = 1, m_{2h} = k_h$ and $m_{3h} = 0$ in Eq. (20), the generalized proposed class of estimators $\bar{y}^{*\prime}_{S(GP)}$ reduces to Khalil et al., 2018 estimator given by,

$$\bar{y}^{*\prime}_{S(K)} = \sum_{h=1}^{L} P_h \left[\bar{z}^*_h + k_h (\bar{X}^*_h - \bar{x}^*_h) \right] \left(\frac{\overline{W}^*}{\overline{w}^*} \right)^g. \quad (29)$$

10. For $\alpha_0 = \alpha_1 = \alpha_2 = \alpha, m_{1h} = m_{8h}, m_{2h} = m_{9h}$ and $m_{3h} = 0$ in Eq. (20), the generalized proposed estimator $\bar{y}^{*\prime}_{S(GP)}$ reduces to the proposed estimator.

$$\bar{y}^{*\prime}_{S(P1)} = \sum_{h=1}^{L} P_h \left[\left\{ m_{8h} \bar{z}^*_h + m_{9h} (\bar{X}_h - \bar{x}^{*\prime}_h) \right\} \left\{ \frac{\bar{X}_h}{\bar{x}^{*\prime}_h} \right\}^{\alpha} exp(1-\alpha) \left(\frac{\bar{X}_h - \bar{x}^{*\prime}_h}{\bar{X}_h + \bar{x}^{*\prime}_h} \right) \right]. \quad (30)$$

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