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Original article

New approach of soft M-open sets in soft topological spaces

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ABSTRACT

The goal of this article is to introduce, investigate and prove several of the properties of M-open and M-closed soft sets in soft topological structure ($\mathscr{S}\tau s$). Furthermore, we prove that the collection of soft M-open ($\mathscr{S}$Mo) sets is a soft topology by stating and proving the conditions. Finally, we define and study some characteristics of soft M ($\mathscr{S}$M)-continuous function, soft M-irresolute function, soft M-compactness, soft M-connectedness and soft M-separation axioms.

1. Introduction

Soft set was created by Molodtsov (1999) in order to deal with challenges arising from incomplete information. He had indicated a few applications of soft theory for finding solutions of problems in economics, medical science and so on. Recently, there has been a significant increase in the number of papers published on soft sets and their applications in various fields (Al-Shami and Abo-Tabl, 2021, Alzahrani et al., 2022). Soft sets and their applications have advanced significantly in recent years (Alzahrani et al., 2022; Fatimah and Alcantud, 2021). The concept of S τs was introduced by Shabir and Naz (2011), who study them as existing in an initial universe with a fixed set of parameters. Furthermore, (Maji et al., 2003) presented various operations on soft sets, and so far, several of the fundamental features of these operations have been exposed. Soft open ($\mathscr{S}$o) sets, soft interior, soft closed sets, soft closure, and soft separation axioms were defined by the authors. M-open sets were introduced into general topology by (EL-Maghrabi and AL-Juhani, 2011). The purpose of this article is to conduct a theoretical investigation of the new set termed $\mathscr{S}$Mo and $\mathscr{S}$Mc sets over $\mathscr{S}\tau$s and to analysis some of their properties. Also, in this research, we express soft operations by '~', soft closed ($\mathscr{S}$c) set, soft open cover by $\mathscr{S}$o cover.

Throughout this entire article, we will refer to U_1 as an initial universal set, (U_1, τ, H) is a $\mathscr{S}\tau$s and (F_1, H_1) is a soft set over U_1.

2. Preliminaries

Except where else stated, U_1 and W denoted a $\mathscr{S}\tau$s with (U_1, τ, S) and (W,ν, T). Additionally, a soft mapping *f*: $U_1 \to$ W, since *f*: (U_1, τ, S) →(W,ν, T), *u*: $U_1 \to$ W and p:S → T denote assumed mappings.

Definition 2.1 Maji et al., 2003. *If* (F_1, H_1) *and* (F_2, D) *are two soft sets over universe* U_1, *then* $(F_1, H_1) \tilde{\cup} (F_2, D) = (V,J)$ *is a soft set, where* $J = H_1 \tilde{\cup} D$.

$$V(s) = \begin{cases} F_1(s) & if \quad s \in H_1 - D \\ F_2(s) & if \quad s \in D - H_1 \\ F_1(s) \cup F_2(s) & if \quad s \in H_1 \tilde{\cap} D \end{cases}$$

And$(F_1, H_1) \tilde{\cap} (F_2, D)$ =(V,J) is a soft sets defined as J = $H_1 \tilde{\cap}$D, and V(s) = F_1 (s) $\tilde{\cap}$ F_2 (s), ∀ s ∈ J.

Theorem 2.1 (*Shabir and Naz, 2011*). *Any union of* $\mathscr{S}$*o sets is* $\mathscr{S}$*o set and finite intersection of* $\mathscr{S}$*c sets is* $\mathscr{S}$*c set.*

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Definition 2.2 (*Zorlutuna et. al, 2012*). *If* (U_1, τ, H_1) *be a* $\mathscr{S}\tau s$ *and* (F_2, H_1) *be a soft set, then*:

(i) The soft closure of (F_2, H_1) is $\tilde{s}$ cl(F_2, H_1) = $\tilde{\cap}\{(D, H_1)$: (D, H_1) is $\mathscr{S}$c set and $(F_2, H_1)\ \tilde{\subseteq}\ (D, H_1)\}$.

(ii) The soft interior of (F_2, H_1) is s~int(F_2, H_1) =$\tilde{\cup}\{(D, H_1)$: (D, H_1) is $\mathscr{S}$o set and $(D, H_1)\ \tilde{\subseteq}\ (F_2, H_1)\}$.

Theorem 2.2 Zorlutuna et al., 2012. *Consider* (U_1, τ, H_1) *is a* $\mathscr{S}\tau s$, *(*(F_1, H_1) *and* (F_2, H_1)*) are soft sets, then:*

(i) (F_1, H_1) is $\mathscr{S}$c set iff (F_1, H_1) = s~cl(F_1, H_1).
(ii) (F_2, H_1) is $\mathscr{S}$o set iff (F_2, H_1) = s~int(F_2, H_1).

Remark 2.1 Li, 2011. *If* (F_1, D) *and* (V, D) *are arbitrary-two soft sets in* (U_1, τ, D), *then* $U_D - ((F_1, D)\ \tilde{\cap}(V, D)) = (U_D - (F_1, D))\ \tilde{\cup}\ (U_D - (V, D))$.

Akdağ and Özkan (2014a,b) defined soft (pre-open(closed), α-open(closed), Chen (2013) defined soft (semi-open(closed)) and Arockiarani and Arokialancy (2013) defined soft (regular open (closed), β-open(closed)) sets, for example.

Definition 2.3. *A soft set* (F_1, K) *in a* $\mathscr{S}\tau s$ (U_1, τ, K) *is called a soft regular open set* (*resp. soft regular closed*) *set if* $(F_1, K) = int(cl(F_1, K))$ [*resp.*, $(F_1, K) = cl(int(F_1, K))$].

3. M-open and M-closed soft sets

We define soft θ-semi-open and $\mathscr{S}$Mo sets in $\mathscr{S}\tau s$ and study some of their characteristics.

Mukherjee and Debnath (2017) defined soft (δ-pre open (closed), δ-pre interior and δ-pre closure) sets, for example.

Definition 3.1. *A soft* (F_1, H) *in a* $\mathscr{S}\tau s$ (U_1, τ, S) *is called soft δ-pre open* ($\mathscr{S}$*δ-pre*) *open set iff* $(F_1, H)\ \tilde{\subseteq}\ int(cl_\delta(F_1, H))$.

Definition 3.2. *A soft* (F_1, H) *in a* $\mathscr{S}\tau s$ (U_1, τ, S) *is called*:

(i) soft θ-semi ($\mathscr{S}$θ-semi) open set iff $(F_1, H)\ \tilde{\subseteq}$ cl(int$_\theta$ (F_1, H)).
(ii) $\mathscr{S}$θ-semi ($\mathscr{S}$θ-semi) closed set iff $(F_1, H)\ \tilde{\supseteq}$ int(cl$_\theta$ (F_1, H)).

Definition 3.3. *In a* $\mathscr{S}\tau s$ (U_1, τ, H) *a soft set* (F_1, H) *is called*:

(i) soft M-open (briefly, $\mathscr{S}$Mo) set if $(F_1, H)\ \tilde{\subseteq}$cl(int$_\theta(F_1, H)$) $\tilde{\cup}$ int(cl$_\delta(F_1, H)$),

(ii) soft M-closed (briefly, $\mathscr{S}$Mc) set if ($(F_1, H)\ \tilde{\supseteq}$ int(cl$_\theta(F_1, H)$) $\tilde{\cap}$cl(int$_\delta(F_1, H)$).

Remark 3.1. *According to Definitions* 3.3, *soft near open sets can be linked as in* Fig. 1 *as*:

In the next example, we will prove that $\mathscr{S}$Mo se is not $\mathscr{S}$δ-preopen.

Example 3.1. *Let* U_1={u_1, u_2, u_3},*S*={s_1, s_2} *and* τ ={$\tilde{\phi}$, $\widetilde{U_1}$, (F_1,S),(F_2, S), (F_3,S), …, (F_7,S)} *where,*

$F_1(s_1)$ = {u_1, u_2} $F_1(s_2)$ = {u_1, u_2}.
$F_2(s_1)$ = {u_2} $F_2(s_2)$ = {u_1, u_3}.
$F_3(s_1)$ = {u_2, u_3} $F_3(s_2)$ = {u_1}.
$F_4(s_1)$ = {u_2} $F_4(s_2)$ = {u_1}.
$F_5(s_1)$ = {u_1, u_2} $F_5(s_2)$ = U.
$F_6(s_1)$ = U $F_6(s_2)$ = {u_1, u_2}.
$F_7(s_1)$ = {u_2, u_3} $F_7(s_2)$ = {u_1, u_3}.

(U_1, τ, S) is a $\mathscr{S}\tau s$, and $\mathscr{S}$c sets are $\widetilde{U_1}$, $\tilde{\phi}$, $(F_1, S)^c$, $(F_2, S)^c$, $(F_3, S)^c$, …., $(F_7, S)^c$.

Let (F_1, H_1)= {(h_1, {u_1, u_2}, (h_2, {u_2}), (h_3, {u_1, u_3})}. Then.

cl(int$_\theta$ (F_1, H_1)) $\tilde{\cup}$ int(cl$_\delta$ (F_1, H_1)) = $\widetilde{U_1}$ and $(F_1, H_1)\ \tilde{\subseteq}$cl(int$_\theta$($F_1, H_1$)) $\tilde{\cup}$ int(cl$_\delta(F_1, H_1)$), therefore, (F_1, H_1) is $\mathscr{S}$Mo set but not $\mathscr{S}$δ-preopen.

Theorem 3.1. *For a soft set* (F_1, K) *in a* $\mathscr{S}\tau s$ (U_1, τ, E)*, then.*

(i) (F_1, K) is a $\mathscr{S}$Mo set iff $(F_1, K)^c$ is a $\mathscr{S}$Mc set.
(ii) (F_1, K) is a $\mathscr{S}$Mc set iff $(F_1, K)^c$ is a $\mathscr{S}$Mo set.
Proof. Obvious.

Definition 3.4. *If* (U_1, τ, S) *is a* $\mathscr{S}\tau s$ *and* (F_1, H_1) *is a soft set, then.*

(i) $\mathscr{S}$M-closure of a soft set is defined as $\mathscr{S}$Mcl(F_1, H_1) = $\tilde{\cap}$\{(V, H_1)$\tilde{\supseteq}$ (F_1, H): (V,H) is a $\mathscr{S}$Mc set of U_1\}.
(ii) $\mathscr{S}$M-interior of a soft set is defined as $\mathscr{S}$Mint(F_1, H_1) = $\tilde{\cup}$\{(F_2, H_1) $\tilde{\subseteq}$$(F_1, H_1)$:($F_2$, H_1) is a $\mathscr{S}$Mo set of U_1\}.

Theorem 3.2. *Let* (F_1, H_1) *be an arbitrary soft set in a* $\mathscr{S}\tau s$*. Then,*

(i) $\mathscr{S}$Mcl$(F_1, H_1)^c$ = $\widetilde{U_1}$-$\mathscr{S}$Mint(F_1, H_1),
(ii) $\mathscr{S}$Mint$(F_1, H_1)^c$ = $\widetilde{U_1}$ -$\mathscr{S}$Mcl(F_1, H_1).

Proof. *(i) Let* $\mathscr{S}$*Mo set* $(F_2, H_1)\ \tilde{\subseteq}\ (F_1, H_1)$ *and* $\mathscr{S}$*Mc set* $(F_1, H)^c\tilde{\subseteq}$ (V, H_1)*. Then* $\mathscr{S}$*Mint*(F_1, H_1) =$\tilde{\cup}$\{(V, H_1)^c: (V, H_1) *is* $\mathscr{S}$*Mc set and* $(F_1, H_1)^c\tilde{\subseteq}$ (Q, H_1)\} =$\widetilde{U_1}$ $\tilde{\cap}$ \{(Q, H_1): (Q, H_1) *is* $\mathscr{S}$*Mc set and* $(F_1, H_1)^c\tilde{\subseteq}$ (Q, H_1)\}= $\widetilde{U_1}$ - $\mathscr{S}$*Mcl*$(F_1, H_1)^c$. *So,* $\mathscr{S}$*Mcl*$(F_1, H_1)^c$ = $\widetilde{U_1}$ - $\mathscr{S}$*Mint* (F_1, H_1).

(ii) Let (F_2, H_1) be a $\mathscr{S}$Mo set. Then for a $\mathscr{S}$Mc set $(F_1, H_1)\ \tilde{\subseteq}$ $(F_2, H_1)^c$,

$(F_2, H_1)\ \tilde{\subseteq}\ (F_1, H_1)^c$. Now, $\mathscr{S}$Mcl(F_1, H_1) = $\tilde{\cap}$ \{$(F_2, H_1)^c$: (F_2, H_1) is a $\mathscr{S}$Mo set ,$(F_2, H_1)\ \tilde{\subseteq}\ (F_1, H_1)^c$ = $\widetilde{U_1}$ - $\tilde{\cup}$\{(F_2, H_1): (F_2, H_1) is a $\mathscr{S}$Mo set and $(F_2, H_1)\ \tilde{\subseteq}\ (F_1, H_1)^c$\}=$\widetilde{U_1}$ - $\mathscr{S}$Mint$(F_1, H_1)^c$. So, $\mathscr{S}$Mint $(F_1, H_1)^c$ = $\widetilde{U_1}$ -$\mathscr{S}$Mcl(F_1, H_1).

Theorem 3.3. *In a* $\mathscr{S}\tau s$ (U_1, τ, S)*, a soft set* (F_1, H_1) *is a* $\mathscr{S}$*Mc (resp.* $\mathscr{S}$*Mo) iff* (F_1, H_1) = $\mathscr{S}$*Mcl*(F_1, H_1) *(resp.* (F_1, H_1) = $\mathscr{S}$*Mint*(F_1, H_1)*).*

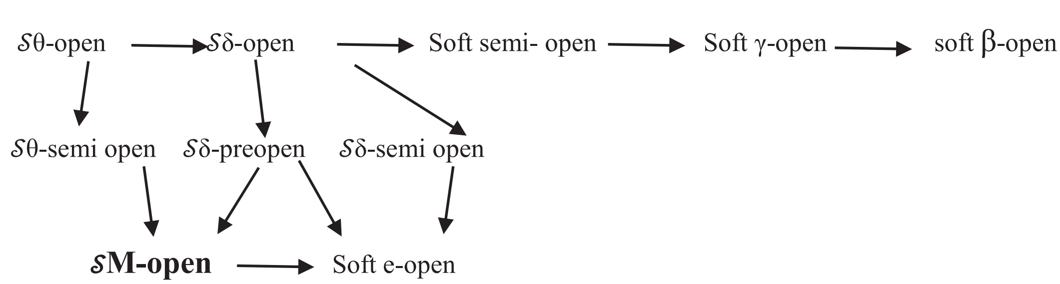

Fig. 1.

Proof. Obvious.

Theorem 3.4. *In a $\mathscr{S}\tau s$, the next are hold:*

(i) $\mathscr{S}\mathrm{Mcl}(\widetilde{\phi}) = \widetilde{\phi}$,
(ii) $\mathscr{S}\mathrm{Mint}(\widetilde{\phi}) = \widetilde{\phi}$,
(iii) $\mathscr{S}\mathrm{Mcl}(F_1, H)$ is a $\mathscr{S}$Mc set,
(iv) $\mathscr{S}\mathrm{Mint}(F_1, H)$ is a $\mathscr{S}$Mo set,
(v) $\mathscr{S}Mcl(F_1,H) \widetilde{\subseteq} \mathscr{S}Mcl(F_2,H) if (F_1,H) \widetilde{\subseteq} (F_2,H)$,
(vi) $\mathscr{S}Mint(F_1,H) \widetilde{\subseteq} \mathscr{S}Mint(F_2,H) if (F_1,H) \widetilde{\subseteq} (F_2,H)$,
(vii) $\mathscr{S}\mathrm{Mcl}(\mathscr{S}\mathrm{Mcl}(F_1,H)) = \mathscr{S}\mathrm{Mcl}(F_1,H)$,
(viii) $\mathscr{S}\mathrm{Mint}(\mathscr{S}\mathrm{Mint}(F_1,H)) = \mathscr{S}\mathrm{Mint}(F_1,H)$.

Theorem 3.5. *In a $\mathscr{S}\tau s$ (U_1, τ,S), we have.*

(i) $\mathscr{S}\mathrm{Mcl}\ ((G_1,K_1)\ \widetilde{\cup}\ (G_2,K_1))\ \widetilde{\supseteq}\ \mathscr{S}\mathrm{Mcl}(G_1,K_1)\ \widetilde{\cup}\ \mathscr{S}\mathrm{Mcl}(G_2,K_1)$.
(ii) $\mathscr{S}\mathrm{Mcl}((G_1,K_1)\ \widetilde{\cap}\ (G_2,K_1))\ \widetilde{\subseteq}\ \mathscr{S}\mathrm{Mcl}(G_1,K_1)\ \widetilde{\cap}\ \mathscr{S}\mathrm{Mcl}(G_2,K_1)$.

Proof. *(i) Since $(G_1,K_1)\ \widetilde{\subseteq}\ ((G_1,K_1)\ \widetilde{\cup}\ (G_2,K_1))$ or $(G_2,K_1)\ \widetilde{\subseteq}\ ((G_1,K_1)\ \widetilde{\cup}\ (G_2,K_1))$, then $\mathscr{S}Mcl(G_1,K_1)\ \widetilde{\subseteq}\ \mathscr{S}Mcl((G_1,K_1)\ \widetilde{\cup}\ (G_2,K_1))$ or $\mathscr{S}Mcl(G_2,K_1)\ \widetilde{\subseteq}\ \mathscr{S}Mcl(G_1,K_1)\ \widetilde{\cup}\ (G_2,K_1))$.*

Therefore, $\mathscr{S}\mathrm{Mcl}((G_1,K_1)\ \widetilde{\cup}\ (G_2,K_1))\ \widetilde{\supseteq}\ \mathscr{S}\mathrm{Mcl}(G_1,K_1)\ \widetilde{\cup}\ \mathscr{S}\mathrm{Mcl}(G_2,K_1)$.

(ii)Similar to (i).

Theorem 3.6. *In a $\mathscr{S}\tau s$ (U_1, τ,S), we have.*

(i) $\mathscr{S}\mathrm{Mint}(L_1,H)\ \widetilde{\cup}\ (L_2,\mathrm{H}))\ \widetilde{\supseteq}\ \mathscr{S}\mathrm{Mint}(L_1,H)\ \widetilde{\cup}\ \mathscr{S}\mathrm{Mint}(L_1,H)$
(ii) $\mathscr{S}\mathrm{Mint}((L_1,H)\ \widetilde{\cap}\ (L_2,\mathrm{H}))\ \widetilde{\subseteq}\ \mathscr{S}\mathrm{Mint}(L_1,H)\ \widetilde{\cap}\ \mathscr{S}\mathrm{Mint}(L_2,\mathrm{H})$

Proof. Obvious.

Theorem 3.7. *If (F_1,H) is a $\mathscr{S}$Mc set, then the next are hold:*

(i) If $\mathrm{int}_\theta(F_1,H) = \widetilde{\phi}$, then (F_1,H) is a $\mathscr{S}\delta$-preopen set.
(ii) If $\mathrm{cl}_\delta(F_1,H) = \widetilde{\phi}$, then (F_1,H) is a $\mathscr{S}\theta$-semiopen set.
Proof. Obvious.

Theorem 3.8. *In a $\mathscr{S}\tau s$ (U_1, τ,S), we have.*

(i) The union of any $\mathscr{S}$Mo sets is a $\mathscr{S}$Mo set,
(ii) The intersection of any $\mathscr{S}$Mc sets is a $\mathscr{S}$Mc set.

Proof. *(i) Consider $\{(F_1,H)_\alpha:\alpha\in\Lambda$, an index set$\}$ is a collection of $\mathscr{S}$Mo sets, hence for each α, $(F_1,H)_\alpha\ \widetilde{\subseteq}[cl(int_\theta(((F_1,H)_\alpha))\ \widetilde{\cup} int(cl_\delta((F_1,H)_\alpha))]$.Taking the union of all such relations we get,$\widetilde{\cup}\{(F_1,H)_\alpha\}\ \widetilde{\subseteq}\ \widetilde{\cup}[cl(int_\theta((F_1,H)_\alpha))\ \widetilde{\cup} int(cl_\delta((F_1,H)_\alpha))]\ \widetilde{\subseteq}$.*

$[\mathrm{cl}(\mathrm{int}_\theta(\widetilde{\cup}(F_1,H)_\alpha))\ \widetilde{\cup}\mathrm{int}(\mathrm{cl}_\delta(\widetilde{\cup}(F_1,H)_\alpha))]$. Thus $\widetilde{\cup}(F_1,H)_\alpha$ is $\mathscr{S}$Mo set.

(ii) As (i) by taking the complements.

Remark 3.9. *The finite union (resp. intersection) of $\mathscr{S}$Mc (resp. $\mathscr{S}$Mo) sets need not be a $\mathscr{S}$Mc set.*

Example 3.10. *ConsiderU_1={u_1, u_2, u_3},S={s_1, s_2} and τ ={$\widetilde{\phi}$, $\widetilde{U_1}$, (F_1, S),(F_2,S), (F_3,S)}, where*

(F_1, S) = {(s_1,{u_1}), (s_2, {u_2})}, (F_2, S) = {(s, U), (s_2, {u_2})}.

(F_3, S) = {(s_1,{u_1}), (s_2, U_1)}.

(U_1, τ, S) is a $\mathscr{S}\tau$s. So, the soft sets (G_1, S), (G_2, S) which defines as (G_1, S) = {(s_1,{u_2}), (s_2, {u_1})} and (G_2, S) ={(s_1,{u_1, u_2}), (s_2, {u_1})} are $\mathscr{S}$Mo sets, but (G_1, S) $\widetilde{\cap}$ (G_2, S)=(K, S) is not a $\mathscr{S}$Mo set.

And, the soft sets (G_3, S) and (G_4, S) which defines as(G_3, S) = {(s_1,{u_1}), (s_2, {u_1})} and (G_4, S) ={((s_2, {u_2})} are $\mathscr{S}$Mc sets, but (G_3, S) $\widetilde{\cup}$ (G_4, S)= {(s_1,{u_1}), (s_2, {u_1, u_2})} =(M, S) is not a $\mathscr{S}$Mc set.

Theorem 3.11. *In a $\mathscr{S}\tau s$ (U_1, τ,H), we have:*

(i) Each $\mathscr{S}\delta$-preopen set is $\mathscr{S}$Mo.
(ii) Every $\mathscr{S}\theta$-semi-open set is $\mathscr{S}$Mo.

Proof. *(i) Consider (F_1,H) is a soft δ-preopen set in a $\mathscr{S}\tau s$ (U_1, τ,H). Thus,*

$(F_1,H)\ \widetilde{\subseteq}\mathrm{int}(\mathrm{cl}_\delta\ (F_1,H))$, therefore $(F_1,H)\ \widetilde{\subseteq}\ [\mathrm{int}(\mathrm{cl}_\delta\ (F_1,H))\widetilde{\cup}\ \mathrm{int}_\delta\ (F_1,H))]\ \widetilde{\subseteq}\ [\mathrm{int}(\mathrm{cl}_\delta\ (F_1,H))\widetilde{\cup}\ \mathrm{cl}(\mathrm{int}_\delta\ (F_1,H))]$. Hence, (F_1,H) is a $\mathscr{S}$Mo set.

(ii) Suppose that (F_1,H) is a $\mathscr{S}\theta$-semio-penset in a $\mathscr{S}\tau$s. Then, $(F_1,H)\ \widetilde{\subseteq}\mathrm{cl}(\mathrm{int}_\theta(F_1,H))$ which implies that $(F_1,H)\ \widetilde{\subseteq}[\mathrm{cl}(\mathrm{int}_\theta(F_1,H))\widetilde{\cup}\mathrm{int}(F_1,H)]\ \widetilde{\subseteq}[\mathrm{cl}(\mathrm{int}_\theta(F_1,H))\widetilde{\cup}\mathrm{int}(\mathrm{cl}_\delta\ (F_1,H))]$. Thus (F_1,H) is a $\mathscr{S}$Mo set.

4. Soft M-continuity and soft M-functions

In this section, we define $\mathscr{S}$M-continuous functions, $\mathscr{S}$M-irresolute functions, $\mathscr{S}$M-open function ($\mathscr{S}$Mof) and $\mathscr{S}$M- closed function ($\mathscr{S}$Mcf). Also, we study some of their characteristics and separation axioms by using $\mathscr{S}$Mo sets.

Theorem 4.1 Kharal and Ahmad, 2011. *If f: (U_1, S) →(W,T);*

u: U_1 → W, p: S → T are functions, for soft sets (F_1, H), (F_2, B) and a family of soft sets {($F_{1\alpha}$, H $_\alpha$):$\alpha\in\Lambda$, an index set} in the soft class (U_1, S), then:

(1) $f(\widetilde{\phi}) = \widetilde{\phi}$,
(2) $f(\widetilde{U_1}) = \widetilde{W}$ and $f^{-1}(\widetilde{W}) = \widetilde{U_1}$,
(3) $f(\mathrm{U}_{\alpha\in\Lambda}\ (F_{1\alpha},\mathrm{H}_\alpha)) = (\mathrm{U}_{\alpha\in\Lambda}\ f\ (F_{1\alpha},\ \mathrm{H}_\alpha))$,
(4) $f(\cap_{\alpha\in\Lambda}\ (F_{1\alpha},\ \mathrm{H}_\alpha))\ \widetilde{\subseteq}\ (\cap_{\alpha\in\Lambda}\ f\ (F_{1\alpha\alpha},\ \mathrm{H}_\alpha))$,
(5) If (F_1, H) $\widetilde{\subseteq}$ (F_2, B), then f (F_1, H) $\widetilde{\subseteq}$ f (F_2, B),

Definition 4.1. *A function f:(U_1, τ_1,S) → (W,τ_2,T) is called*:

(i) a $\mathscr{S}\delta$-precontinuous (Anjan Mukherjee and Bishnupada Debnath, 2017).

if f^{-1}(F_1, H) is $\mathscr{S}\delta$-preopen in U_1, for each $\mathscr{S}$o set (F_1, H) in W,

(ii) a $\mathscr{S}\theta$-semi continuous if f^{-1}(F_1, H) is $\mathscr{S}\theta$-semi-open in U_1, for each $\mathscr{S}$o set (F_1, H) in W.

Definition 4.2. *A mapping f:(U_1, τ_1,E) → (W,τ_2,T) is called*:

(i) a $\mathscr{S}$M-continuous if f^{-1}(F_1, H) is a $\mathscr{S}$Mo in U_1, for each $\mathscr{S}$o set (F_1, H) in W.
(ii) a $\mathscr{S}$M-irresolute if f^{-1}(F_1, H) is a $\mathscr{S}$Mo set in U_1, for each $\mathscr{S}$Mo set (F_1, H) in W.

Remark 4.1. *According to the above discussion, every $\mathscr{S}$ δ -pre continuous mapping and $\mathscr{S}$-semi continuous mapping is clearly $\mathscr{S}$M-continuous.*

Theorem 4.2. *For a mapping f:(U_1, τ 1, S) →(W, τ 2, T), the statements that follow are equivalent.:*

(i) f is a $\mathscr{S}$M-continuous.
(ii) For each soft singleton $P_\lambda^{F_1} \in U_1$ and each $\mathscr{S}$o set (F_1, H) in W, where $f(P_\lambda^{F_1}) \widetilde{\subseteq} (F_1, H)$ ∃ a $\mathscr{S}$Mo set (F_2, H) in U, since $P_\lambda^{F_1} \in (F_2, H)$ and $f((F_2, H)) \widetilde{\subseteq} (F_1, H)$
(iii) $f^{-1}(F_1, H) = cl(int_\theta(f^{-1}(F_1, H)))\widetilde{\cup}int(cl_\delta(f^{-1}(F_1, H)))$, for each $\mathscr{S}$o set (F_1, H) in E.
(iv) The inverse image of every $\mathscr{S}$c set in E is $\mathscr{S}$Mc set.
(v) $cl(int_\theta(f^{-1}(F_1, H)))\widetilde{\cap}int(cl_\delta(f^{-1}(F_1, H))) \widetilde{\subseteq} f^{-1}((cl(F_1, H)))$ for every soft set $(F_1, H) \widetilde{\subseteq}$ W.
(vi) $f[cl(int_\theta(F_2, H))\widetilde{\cap}int(cl_\delta(F_2, H))] \widetilde{\subseteq} cl(f(F_2, H))$, for each soft set (F_2, H) in U_1.

Proof. *(i) ⇒(ii). Suppose that the singleton set $P_\lambda^{F_1}$ in U and each $\mathscr{S}$o set (F_1, H) in W since $f(P_\lambda^{F_1}) \widetilde{\subseteq} (F_1, H)$. Where f is $\mathscr{S}$M-continuous, hence $P_\lambda^{F_1} \in f^{-1}(f(P_\lambda^{F_1}))\widetilde{\subseteq}$.*

$f^{-1}(F_1, H)$. Suppose that $(F_2, H) = f^{-1}(F_1, H)$ which is a $\mathscr{S}$Mo set in U_1. So, we have $P_\lambda^{F_1} \in (F_2, H)$. Now, $f(F_2, H) = f(f^{-1}(F_1, H)) \subseteq \sim (F_1, H)$.

(ii)⇒(iii). Consider (F_1, H) is an arbitrary $\mathscr{S}$o set in W, let $P_\lambda^{F_1}$ be an arbitrary soft point in U_1, since $f(P_\lambda^{F})\widetilde{\subseteq} (F_1, H)$, then $P_\lambda^{F_1} \in f^{-1}(F_1, H)$. By (ii), there is a $\mathscr{S}$Mo set(F_2, H) in U_1, where $P_\lambda^{F_1} \in (F_2, H)$ and $f((F_2, H)) \widetilde{\subseteq} (F_1, H)$. Therefore, $P_\lambda^{F_1} \in (F_2, H) \widetilde{\subseteq} f^{-1}(f((F_2, H)))\widetilde{\subseteq} f^{-1}(F_1, H) \widetilde{\subseteq} cl(int_\theta(f^{-1}(F_1, H)))\widetilde{\cup}int(cl_\delta(f^{-1}(F_1, H)))$.

(iii)⇒(iv). Suppose that (F_1, H) is an arbitrary $\mathscr{S}$c set in W. Then $\widetilde{W}$- (F_1, H) is a $\mathscr{S}$o set in W. By (iii), $(f^{-1}(\widetilde{W}- (F_1, H)))\widetilde{\subseteq} cl(int_\theta(- f^{-1}(\widetilde{W}- (F_1, H))))\widetilde{\cup}int(cl_\delta(f^{-1}(\widetilde{W}- (F_1, H))))$. This implies $\widetilde{U_1}$- $(f^{-1}(F_1, H))\widetilde{\subseteq} cl(int_\theta(\widetilde{U}- f^{-1}(F_1, H)))\widetilde{\cup}int(cl_\delta(\widetilde{U_1}-f^{-1}(F_1, H)))\widetilde{\subseteq}$ cl $(\widetilde{U_1}-int_\theta(f^{-1}(F_1, H)))\widetilde{\cup}int(\widetilde{U_1}- cl_\delta(f^{-1}(F_1, H)))\widetilde{\subseteq} [\widetilde{U_1}-cl(int_\theta(f^{-1}(F_1, H)))] \widetilde{\cup}$.

$[\widetilde{U_1}-int(cl_\delta(f^{-1}(F_1, H)))]$ and hence $\widetilde{U_1}- f^{-1}(F_1, H) \widetilde{\subseteq} \widetilde{U_1}-$ [cl $(int_\theta(f^{-1}(F_1, H)))\widetilde{\cap}$.

$int(cl_\delta(f^{-1}(F_1, H)))]$. Hence, $f^{-1}(F_1, H) \widetilde{\supseteq} [cl(int_\theta(f^{-1}(F_1, H)))\widetilde{\cap}int(cl_\delta(f^{-1}(F_1, H)))]$ then $f^{-1}(F_1, H)$ is $\mathscr{S}$Mc in U_1.

(iv)⇒(v). Consider $(F_1, H) \widetilde{\subseteq}$W, therefore $f^{-1}(cl(F_1, H))$ is $\mathscr{S}$Mc in U_1. Now,

$[cl(int_\theta(f^{-1}(F_1, H)))\widetilde{\cap}int(cl_\delta(f^{-1}(F_1, H)))]\widetilde{\subseteq}[cl(int_\theta(f^{-1}(cl(F_1, H))))\widetilde{\cap}int(cl_\delta(f^{-1}(cl(F_1, H))))]\widetilde{\subseteq} f^{-1}(cl(F_1, H))$.

(v)⇒(vi). Consider $(F_2, H) \widetilde{\subseteq} U_1$. Put, $(F_1, H) = f(F_2, H)$ in (v) implies that $[cl(int_\theta(f^{-1}(f(F_2, H))))\widetilde{\cap}int(cl_\delta(f^{-1}(f(F_2, H))))]\widetilde{\subseteq} f^{-1}(cl(f(F_2, H)))$. This is implies that $[cl(int_\theta)(F_2, H)\widetilde{\cap}int(cl_\delta(F_2, H))]\widetilde{\subseteq} f^{-1}(cl(f(F_2, H)))$, hence, $f[cl(int_\theta(F_2, H))\widetilde{\cap}int(cl_\delta(F_2, H))]\widetilde{\subseteq} cl(f(F_2, H))$.

(vi)⇒(i). Consider $(F_2, H)\widetilde{\subseteq}$W is a $\mathscr{S}$o set, let $(F_2, H) = f^{-1}(F_1, H)$ and $(F_1, H) = \widetilde{W}-(F_1, H)$. Then $f[cl(int_\theta(f^{-1}(F_1, H)))\widetilde{\cap}int(cl_\delta(f^{-1}(F_1, H)))]\widetilde{\subseteq} cl(f(f^{-1}(F_1, H))) \widetilde{\subseteq} cl(F_1, H) = (F_1, H)$.Thus, $f^{-1}(F_1, H)$ is $\mathscr{S}$Mc in U_1, so f is $\mathscr{S}$M-continuous.

Theorem 4.3. *Each $\mathscr{S}$M-irresolute mapping is $\mathscr{S}$M-continuous.*

Proof. Clearly by Definition 4.2.

Theorem 4.4. *If f: $(U_1, \tau 1,S) \to (W, \tau 2,T)$ is a $\mathscr{S}$M-continuous function and.*

g: (W, τ 2,T) →(E, τ 3,J) is soft continuous function, then gof: $(U_1, \tau 1,S) \to (E, \tau 3,J)$ is.
a $\mathscr{S}$M-continuous function.

Proof. Suppose that (F_1, H) is a $\mathscr{S}$o set in E. Now, $(gof)^{-1}(F_1, H) = (f^{-1}og^{-1})(F_1, H) =$.

$(f^{-1}(g^{-1}(F_1, H))$. Where, g is soft continuous, $g^{-1}(F_1, H)$ is a $\mathscr{S}$o set, then $(gof)^{-1}(F_1, H)$ is $\mathscr{S}$M-open in W. But f being $\mathscr{S}$M-continuous, $(gof)^{-1}(F_1, H)$ is a $\mathscr{S}$Mo set in U_1. Thus g*of* is a $\mathscr{S}$M-continuous mapping.

Theorem 4.5. *If f:$(U_1, \tau 1,S) \to (W, \tau 2,T)$ is a $\mathscr{S}$M-irresolute function and.*

g: (W, τ 2,T) → (W, τ 3,J) is a $\mathscr{S}$M-continuous function., then g*of*: $(U_1, \tau 1,S) \to (E, \tau 3,J)$ is also $\mathscr{S}$M-continuous function.

Proof. Suppose that (F_1, H) is a $\mathscr{S}$o set in J. Now, $(gof)^{-1}(F_1, H) = (f^{-1}og^{-1})(F_1, H) =$.

$(f^{-1}(g^{-1}(F_1, H))$, where g is $\mathscr{S}$M-continuous, $g^{-1}(F_1, H)$ is a $\mathscr{S}$Mo set and hence.

$(gof)^{-1}(F_1, H)$ is $\mathscr{S}$Mo in W. But f being M-irresolute, $(gof)^{-1}(F_1, H)$ is a $\mathscr{S}$Mo set in U_1. Thus g*of* is a $\mathscr{S}$M-continuous function.

Theorem 4.6. *Composition of two $\mathscr{S}$M-irresolute functions is again $\mathscr{S}$M-irresolute.*

Proof. **Clear**.

Definition 4.3. *A function f: $U_1 \to W$ is called*:

(i) $\mathscr{S}$M-open function (briefly, $\mathscr{S}$Mof) if the image of each $\mathscr{S}$o set in U_1 is a $\mathscr{S}$Mo set in W,
(ii) $\mathscr{S}$M-closed function (briefly, $\mathscr{S}$Mcf) if the image of each $\mathscr{S}$c set in U_1 is a $\mathscr{S}$Mc set in W.

Theorem 4.7. *Consider f: $U_1 \to W$ is a soft closed function and g: $W \to E$ is $\mathscr{S}$Mcf, then gof is $\mathscr{S}$Mcf.*

Proof. For a $\mathscr{S}$c set (F_1, H) in U_1, $f(F_1, H)$ is $\mathscr{S}$c set in W, where g: $W \to E$ is $\mathscr{S}$Mcf, $g(f(F_1,H))$ is a $\mathscr{S}$Mc set in E ,$g(f(F_1,H)) = (gof)(F_1,H)$ is a $\mathscr{S}$Mc set in E. Hence,gof is $\mathscr{S}$Mcf.

5. Soft M-separation axioms

In this section, soft M-separation axioms has been introduced and investigated with the help of $\mathscr{S}$Mo sets. Also, some properties of $\mathscr{S}$M- separation axioms are studied.

Definition 5.1. *A $\mathscr{S}\tau s$ (U_1, τ,S) is called a $\mathscr{S}$M-T_0- space if for every pair of soft points $P_\lambda^{F_1}, P_\mu^{F_2}$ of U_1 and $P_\lambda^{F_1}, \neq P_\mu^{F_2}$, there exists a $\mathscr{S}$Mo set (J,G) since $P_\lambda^{F_1} \in (J,D)$ and $P_\mu^{F_2} \notin (J,D)$ or $P_\lambda^{F_1} \notin (J,D)$ and $P_\mu^{F_2} \in (J,D)$.*

Theorem 5.1. *A $\mathscr{S}\tau s$ (U_1, τ,S) is a $\mathscr{S}$M-T_0-space, if the $\mathscr{S}$M-closure of two distinct soft points are distinct.*

Proof. Consider $P_\lambda^{F_1}$ and $P_\mu^{F_2}$ *are* two soft points *and* $P_\lambda^{F_1} \neq P_\mu^{F_2}$ with distinct $\mathscr{S}$M-closure in a $\mathscr{S}\tau s$ (U_1, τ,S). If possible, let $P_\lambda^{F_1} \in \mathscr{S}$Mcl $\{P_\mu^{F_2}\}$, then $\mathscr{S}$Mcl$\{P_\lambda^{F_1}\} \subseteq \mathscr{S}Mcl\{P_\mu^{F_2}\}$ which is a contradiction. So, $P_\lambda^{F_1} \notin \mathscr{S}Mcl\{P_\lambda^{F_1}\}$which implies $(\mathscr{S}$Mcl$\{P_\lambda^{F_1}\})^c$ is a $\mathscr{S}$Mo set containing $P_\lambda^{F_1}$ but not $P_\mu^{F_2}$, therefore (U_1, τ, E) is a $\mathscr{S}$M-T_0 structure.

Definition 5.2. *A $\mathscr{S}\tau s$ (U_1, τ,S) is called a $\mathscr{S}$M-T_1if for arbitrary-two soft points $P_\lambda^{F_1}, P_\mu^{F_2}$ o f U_1 and $P_\lambda^{F_1}, \neq P_\mu^{F_2}$, there exist $\mathscr{S}$Mo sets (L,H) and (Q,D) such that $P_\lambda^{F_1} \in (L,H)$, $P_\mu^{F_2} \notin (L,H)$ and $P_\mu^{F_2} \in (Q,D)$,$P_\lambda^{F_1} \notin (Q,D)$.*

Theorem 5.2. *Consider f: $U_1 \to W$ is an injective $\mathscr{S}$M-continuous mapping and W is a soft T_1. Then U_1 is $\mathscr{S}$M-T_1.*

Proof. Suppose that W is a soft T_1. For arbitrary-two soft points $P_\lambda^{F_1}$, $P_\mu^{F_2}$o f U_1 and $P_\lambda^{F_1} \neq P_\mu^{F_2}$, there exist $\mathscr{S}$o sets (L,H) and (Q,H) in W since, $f(P_\lambda^{F_1}) \in$ (L,H), $P_\mu^{F_2} \notin$ (L, H) and $f(P_\mu^{F_2}) \in$ (Q,H),$f(P_\lambda^{F_1}) \notin$ (Q,H). Where f is an injective $\mathscr{S}$M-continuous function, we have f^{-1} (L, H) and f^{-1} (Q,H) are $\mathscr{S}$Mo sets in U_1. Hence U_1 is a $\mathscr{S}$M-T_1.

Definition 5.3. *A $\mathscr{S}\tau s$ (U_1, τ, S) is called a $\mathscr{S}$M-T_2 ($\mathscr{S}$M-.*

Hausdorff) if For every-two soft points $P_\lambda^{F_1}$, $P_\mu^{F_2}$o f U_1 and $P_\lambda^{F_1}$, $\neq P_\mu^{F_2}$, there exist disjoint $\mathscr{S}$Mo sets (L,H) and (A,D) where $P_\lambda^{F_1} \in$ (L, H) and $P_\mu^{F_2} \in$ (A,D).

Theorem 5.3. *If f: $(U_1, \tau\ 1, S) \rightarrow (W, \tau\ 2, S)$ is an injective $\mathscr{S}$M-continuous function and W is a soft T_2, then U_1 is a $\mathscr{S}$M-T_2.*

***Proof*. Obvious**

Definition 5.4. *A $\mathscr{S}\tau s$ (U_1, τ, S) is called a $\mathscr{S}$M-regular if for every $\mathscr{S}$c set (F_1, H) of U_1 and every soft point $P_\lambda^{F_1} \in U_1 - (F_1, H)$, there exist disjoint $\mathscr{S}$Mo sets (L,H) and (Q,B) where $P_\lambda^{F_1} \in$ (L,H) and $(F_1, H) \widetilde{\subseteq}$ (Q, B).*

Definition 5.5. *A $\mathscr{S}$M-regular* M-M-*T_1-space is called $\mathscr{S}$M-T_3.*

Theorem 5.4. *If f: $(U_1, \tau\ 1, S) \rightarrow (W, \tau\ 2, S)$ is a $\mathscr{S}$M-continuous closed injective function and W is soft regular, then U_1 is $\mathscr{S}$M-regular.*

***Proof*.** Consider $(F_1$, H) is a soft closed set in W with a soft point $P_\mu^{F_2} \notin (F_1$, H). Take.

$P_\mu^{F_2} = f(P_\lambda^{F_1})$. Since W is soft regular, there exists disjoint $\mathscr{S}$o sets (L,H) and (J,B) since , $P_\lambda^{F_1} \in$(L,H), $P_\mu^{F_2} = f(P_\lambda^{F_1}) \in f$(L,H) and $(F_1$, H) $\widetilde{\subseteq} f$ (J,B) such that f (L,H), f(J,B) and f(L,H) $\widetilde{\cap}$ f(J,B) = $\widetilde{\phi}$ are $\mathscr{S}$o sets. Thus, $f^{-1}(F_1$, H) $\widetilde{\subseteq}$(J,B). Where f is $\mathscr{S}$M-continuous, $f^{-1}(F_1$, H) is a $\mathscr{S}$Mc set in U_1 and $P_\lambda^{F_1} \notin f^{-1}(F_1$, H). Hence U_1 is $\mathscr{S}$M-regular.

Definition 5.6. *A $\mathscr{S}\tau s$ (U_1, τ, S) is called a $\mathscr{S}$M-normal if for each two disjoint $\mathscr{S}$c sets (F_1, H), (V,B) and $(F_1$, H) $\widetilde{\cap}$(V,B) = $\widetilde{\phi}$ of U_1, there exist pair of $\mathscr{S}$Mo sets (L,H) and (Q,B) where $(F_1$, H) $\widetilde{\subseteq}$(L,H) , (V,B)$\widetilde{\subseteq}$(Q,B) and (L,H) $\widetilde{\cap}$(Q,B) = $\widetilde{\phi}$.*

Definition 5.7. *A $\mathscr{S}$M-normal T_1-space is said to be $\mathscr{S}$M-M-T_4.*

Theorem 5.5. *If f: $(U_1, \tau\ 1, S) \rightarrow (W, \tau\ 2, S)$ is a $\mathscr{S}$M-continuous closed injective mapping and W is soft normal, then U_1 is $\mathscr{S}$M-normal.*

Proof. Let W be a soft normal space, (F_1, H) and (V,D) be $\mathscr{S}$c sets in U_1, where (F_1, H) $\widetilde{\cap}$ (V,D) = $\widetilde{\phi}$. Since f is soft closed injection, $f(F_1, H)$ and f(V,D) are $\mathscr{S}$c sets in W and $f(F_1, H)$ $\widetilde{\cap}$ $f(V, D) = \widetilde{\phi}$, but W is soft normal, there exist $\mathscr{S}$Mo sets (L,H) and (Q,B)in W where $f(F_1, H) \widetilde{\subseteq}$ $l, f(V, D) \widetilde{\subseteq}$ Q and L$\widetilde{\cap}$Q = $\widetilde{\phi}$. Thus we obtain, (F_1, H) $\widetilde{\subseteq}$ f^{-1} (L), (V,D)$\widetilde{\subseteq}$ f^{-1}(Q) and f^{-1}(L $\widetilde{\cap}$Q) = $\widetilde{\phi}$, where f is $\mathscr{S}$M-continuous, $f^{-1}(L)$ and f^{-1}(Q) are $\mathscr{S}$Mc sets. Hence U_1 is $\mathscr{S}$M-normal.

6. Soft M-connectedness and soft M-compactness

The investigation of compactness (which is based on open sets) for a $\mathscr{S}\tau s$ was started by Zorlutuna et al. (2012). Peyghan et al. (2012) defined and investigated the concept of soft connectedness in $\mathscr{S}\tau s$. This section is objective to present M-connectedness in $\mathscr{S}\tau s$ and to characterize it. Finally, we discuss the properties of M-compactness in a $\mathscr{S}\tau s$.

Definition 6.1. *A soft subset (F_1, H) of a $\mathscr{S}\tau s$ (U_1, τ, S) is $\mathscr{S}$M-connected iff (F_1, H) can't be written as the union of two non-empty disjoint $\mathscr{S}$Mo sets.*

Theorem 6.1. *Let $g : U_1 \rightarrow Y$ be a surjection $\mathscr{S}$M-continuous map. If (V, H) is $\mathscr{S}$M-connected, then $g(V, H)$ is soft connected.*

***Proof*.** Let $g(V, H)$ be not soft connected. Therefore, there is non empty $\mathscr{S}$o sets (F_1, H) and $(F_2$, H) in Y, where g(V,H)= $(F_1, H) \widetilde{\cup} (F_2$, H), where g is $\mathscr{S}$M-continuous, $g^{-1}(F_1, H)$, $g^{-1}(F_2$, H) are $\mathscr{S}$Mo sets in U_1 and (V,H) = $g^{-1}[(F_1, H) \widetilde{\cup} (F_2$, H)]= $g^{-1}(F_1, H) \widetilde{\cup} g^{-1}(F_2$, H). Hence, $g^{-1}(F_1, H)$ and $g^{-1}(F_2$, H) are $\mathscr{S}$Mo sets in U_1. Thus, (V, H) is not $\mathscr{S}$M-connected which is in the opposition to the proposed hypothesis. Hence ,$g(V, H)$ is soft connected.

Definition 6.2. *A cover of a soft set is a $\mathscr{S}$Mo cover (briefly, $\mathscr{S}$Moc) if each member of the cover is a $\mathscr{S}$Mo set.*

Definition 6.3. *A $\mathscr{S}\tau s$ (U_1, τ, E) is a $\mathscr{S}$M-compact if each $\mathscr{S}$Mo cover of U_1 has a finite subcover.*

Theorem 6.2. *$\mathscr{S}$M-continuous image of a $\mathscr{S}$M-compact space is soft compact.*

Proof. Consider f:$(U_1, \tau\ 1, S) \rightarrow (W, \tau\ 2, S)$ is a M-continuous function, where $(U_1, \tau\ 1, S)$ is M-compact $S\tau s$ and (W, τ 2, S) is another $S\tau s$. Let $\{(F_2, H)_\alpha : \alpha \in \Lambda\}$ be $\mathscr{S}$o cover of W, therefore,$\{f^{-1}(F_2, H)_\alpha : \alpha \in \Lambda\}$ is $\mathscr{S}$Mo cover of U_1, therefore there exists a finite subset Δ of Λ where $\{f^{-1}(F_2, H)_\alpha : \alpha \in \Delta\}$ is a Mo cover of U_1. Hence, $\{(F_2, H)_\alpha : \alpha \in \Delta\}$ is a finite $\mathscr{S}$o cover of W. Therefore, W is soft compact.

7. Conclusion

The authors introduce $\mathscr{S}$Mc sets to study various topological structures in $\mathscr{S}\tau ss$, including $\mathscr{S}$M-continuous function, $\mathscr{S}$M-irresolute function, $\mathscr{S}$M-compactness, $\mathscr{S}$M-connectedness and $\mathscr{S}$M-separation axioms. Also, soft sets are important in many disciplines of mathematics. In the future, these results can be applied to study the processes for nucleic acids "mutation, recombination and crossover.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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