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Original article

An approach to multi-attribute decision making based on intuitionistic fuzzy rough Aczel-Alsina aggregation operators

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ABSTRACT

A concept of intuitionistic fuzzy rough set based on approximations plays a vital role in coping with uncertainty. Aczel-Alsina t-norm and t-conorm are the most flexible operational laws based on the parameter for fuzzy frameworks. In multi-attribute group decision-making (MAGDM), the aim of this article is to develop some tools based on norm operations for the information fusion. In this article, intuitionistic fuzzy rough Aczel-Alsina weighted geometric operators are developed and studied their properties. Based on these operators, MAGDM algorithm is presented. The developed aggregation operators (AOs) are compared with some existing AOs, and their significance is discussed.

1. Introduction

MAGDM (multi-attribute group decision-making) is a procedure to select the most optimized alternative among the list of the alternatives based on different types of the criteria. Due to its significance, several approaches have already been investigated to cope with ambiguity and uncertainty. The roughness in the data was also fixed with the introduction of the rough set (RS). But the major problem is dealing with the information extracted from the real-life phenomenon. IFRS (intuitionistic fuzzy rough set) is a tool to cope with inexact and imprecise information because IFRS can aggregate the information based on approximations and reduce unimportant attributes at the initial stages. The AOs are the fundamental basic of the MAGDM process. In the literature, there are several AOs which are helpful in MAGDM process and most of them are based on triangular norm (TNrM) and triangular conorm (TCNrM). In terms of IFRS, many researchers have developed AOs to solve MAGDM problems based on different TNrMs and TCNrMs. Aczel-Alsina t-norm (AATNrM) and Aczel-Alsina t-conorm (AATCNrM) are the most flexible operational laws based on the parameter for fuzzy frameworks which plays a significant role in the fusion of the information. Figueroa-García (2020) compared the several types of TNrMs and TCNrMs and found that the AATNrM and AATCNrM as the most useful and flexible in the fusion of information. The AATNrM consists of a parameter and genializes all the TNrMs and TCNrMs mentioned above. For example, it converts to the minimum TNrM and TCNrM when the value of the parameter is very large and it converts to the Drastic TNrM and TCNrM when the value of the parameter is zero.

The formulization of this study is very significant because it covers two important aspects. First, it provides a flexible technique to solve the MAGDM problem based on AATNrM and AATCNrM. Second, the AOs introduced in this article are helpful to aggregate the information which is extracted from the real-life scenario. The extracted information is full of the uncertainty and ambiguity. With the help of the IFRS, the ambiguity and uncertainty are reduced by using the idea of roughness first and then the AOs are applied to aggregate the information. Hence, the major goal of this article is to develop the AOs for the aggregation of the information in the form of the IFRS based on the AATNrM and AATCNrM to obtain flex-

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ibility in the results. In the nutshells, the major contribution of this article is to introduce AOs based on the AATNrM and AATCNrM for the aggregation in the form of IFRS. In this article, IFRAAWG, IFRAAOWG, and IFRAAHWG operators are introduced.

2. Literature review

In 1965, a fuzzy set (FS) was introduced by Zadeh (1965) as the generalization of the crisp set with the help of the membership grade (MrG) $\mathfrak{r} \in [0,1]$. However, Atanassov (1986) introduced an IFS by adding a non-membership grade (NMrD) with the condition $sum(\text{MrG}, \text{NMrG}) \in [0,1]$. Several AOs based on IFS have been introduced, which play a significant role in information aggregation. Chen (2014) developed prioritized AOs based on IFS and applied them in MAGDM. Liu et al. (2017) introduced AOs for IFS based on the Dombi operational laws. Akram et al. (2021) introduced IF AOs based on the Hamacher operational laws. Büyüközkan and Güleryüz (2015) introduced AOs for IFS and used them to the MAGDM in phone selection.

Another generalization of the crisp set based on the approximations known as the rough set (RS) was introduced by Pawlak (1982) to cover the inaccurate information. Several researchers applied the RS to develop new techniques and ideas. For example, authors in (Yazdani et al., 2020; Zhu et al., 2015) applied the rough numbers to solve the MADM problems. Pamučar et al. (2018, 2019) developed Bonferroni AOs based on the rough numbers. Deveci et al. (2021) developed the AOs based on the rough numbers and applied them for the site selection. Pamučar et al. (2017) introduced AOs based on the rough numbers to solve the problem of MADM. The bridge between the IFS and RS was stated by Chinram et al. (2021) by introducing the IFRS using fuzzy relations instead of crisp relations. Yahya et al. (2021) introduced AOs based on the Frank operational laws for aggregating the information. Das (2017) utilized IFRS to solve the MADM problem. Mishra et al. (2022) applied IFRS to solve the MADM problem. Mukherjee and Mukherjee (2022) defined the interval-valued IFRS to solve the MADM problem. Zhou and Wachs (2017) implemented the IFRS to the problem of MADM in the medical field.

MAGDM is a very complex procedure to deal with the decision-making process. Mahmood (2020, 2022) introduced an approach to solving the problem of MAGDM problem. Garg (2021) solved the MAGDM problem by introducing the AOs. In (Sahu et al., 2021; Sivaprakasam and Angamuthu, 2023), authors introduced the methodology to deal with the MAGDM problem. Xiao et al. (2022) investigated AOs to solve the MAGDM problem. Xiao (2019) solved MAGDM problems by introducing the AOs for different frameworks. The role of the t-norm (TNrM) and t-conorm (TCNrM) (Butnariu and Klement, 1991) is very significant in the aggregation of the information. AOs in (Li et al., 2018) are based on the Dombi TNrM and TCNrM. AOs in (Iancu, 2014) are based on the Frank TNrM and TCNrM. AOs in (Wang and Liu, 2011) are based on the Einstein TNrM and TCNrM, while in (Kamacı et al., 2021), authors utilized Einstein TNrM and TCNrM. AOs in (Ali Khan et al., 2019) are also based on Einstein TNrM and TCNrM. AOs in (De Baets and De Meyer, 2001; Seikh, 2021) are based on Frank TNrM and TCNrM. Operational laws introduced by Aczél and Alsina (1982) known as AATNrM and AATCNrM, are significant operational laws that are helpful in the fusion of information based on parameters.

3. Preliminaries

This section consists of the basic concepts used in this article.

Definition 1. (Atanassov, 1986) *Let* P *denotes the universe then the IFS Z is defined as*

$$Z = \{z, (\mathfrak{r}_z, \mathfrak{s}_z) : z \in P\}$$

Where $\mathfrak{s}_z, \mathfrak{r}_z : P \to [0,1]$ are the MrG and NMrG with condition $0 \le \mathfrak{r}_z + \mathfrak{s}_z \le 1$. Consider $Z = (\mathfrak{r}_z, \mathfrak{s}_z), Z_p = (\mathfrak{r}_{zp}, \mathfrak{s}_{zp})$ for $p = 1, 2$ be two IFVs and $\sigma > 0$ be any real number.

1. $Z_1 \cup Z_2 = (\max(\mathfrak{r}_{z1}, \mathfrak{r}_{z2}), \min(\mathfrak{s}_{z1}, \mathfrak{s}_{z2}))$
2. $Z_1 \cap Z_2 = (\min(\mathfrak{r}_{z1}, \mathfrak{r}_{z2}), \max(\mathfrak{s}_{z1}, \mathfrak{s}_{z2}))$
3. $Z_1 \oplus Z_2 = (\mathfrak{r}_{z1} + \mathfrak{r}_{z2} - \mathfrak{r}_{z1}\mathfrak{r}_{z2}, \mathfrak{s}_{z1}\mathfrak{s}_{z2})$
4. $Z_1 \otimes Z_2 = (\mathfrak{r}_{z1}\mathfrak{r}_{z2}, \mathfrak{s}_{z1} + \mathfrak{s}_{z2} - \mathfrak{s}_{z1}\mathfrak{s}_{z2})$
5. $Z^c = (\mathfrak{s}_z, \mathfrak{r}_z)$ where Z^c is the complement of the IFV Z
6. $\sigma Z = \left(1 - (1 - \mathfrak{r}_Z)^\sigma, \mathfrak{s}_Z^\sigma\right)$
7. $Z^\sigma = \left(\mathfrak{r}_Z^\sigma, 1 - (1 - \mathfrak{s}_Z)^\sigma\right)$

Definition 2. (Pawlak, 1982) *Let* P *be the universe and* $\Omega \in P \times P$ *be a relation. Then.*

1. Ω is reflexive iff $(\kappa, \kappa) \forall \kappa \in P$
2. Ω is symmetric if $(\kappa, \tau) \in \Omega$then $(\tau, \kappa) \in \Omega \forall \kappa, \tau \in P$
3. Ω is transitive if $\forall \kappa, \tau, \omega \in P$ if $(\tau, \omega) \in \Omega$ and $(\omega, \kappa) \in \Omega$ then $(\tau, \kappa) \in \Omega$

Definition 3. (Pawlak, 1982) *Let* P *be the universal set and* Ω *be the relation. Now we assume a mapping* $\Omega^* : P \to \mathscr{A}(P)$ *as.*

$$\Omega^*(a) = \{\kappa \in P : (a, \kappa) \in \Omega\}, for\, a \in P$$

Where $\Omega^*(a)$ is named as the successor neighborhood of an element a concerning Ω and (P, Ω) is said to be the crisp space of the approximation. For any set $\mathfrak{W} \subseteq P$ the definition of the LA and UA are stated below.

$$\Omega^{LA}(\mathfrak{W}) = \{\kappa \in \Omega | \Omega^*(\kappa) \subseteq \mathfrak{W}\}$$

$$\Omega^{UA}(\mathfrak{W}) = \{\kappa \in \Omega | \Omega^*(\kappa) \cap \mathfrak{W} \neq \phi\}$$

The set $\left\{\left(\Omega^{LA}(\mathfrak{W}), \Omega^{UA}(\mathfrak{W})\right)\right\}$ is said to be an RS based on LA and UA.

Definition 4. (Pawlak, 1982) *Let* P *be the universal set and* Ω *be the relation from IFS*$(P \times P)$. *Then.*

1. Ω is reflexive iff $\mathfrak{r}_\Omega(\kappa, \kappa) = 1$ and $\mathfrak{s}_\Omega(\kappa, \kappa) = 0 \forall \kappa \in P$
2. Ω is symmetric if $\forall (\kappa, \tau) \in P \times P$ then $\mathfrak{r}_\Omega(\tau, \kappa) = \mathfrak{r}_\Omega(\kappa, \tau) \forall \kappa, \tau \in P$ and $\mathfrak{s}_\Omega(\tau, \kappa) = \mathfrak{s}_\Omega(\kappa, \tau)$
3. Ω is transitive if $\forall \kappa, \tau, \omega \in P$ if $(\tau, \omega) \in \Omega$ and $(\omega, \kappa) \in \Omega$ then $\mathfrak{r}_\Omega(\tau, \kappa) \ge \bigvee [\mathfrak{r}_\Omega(\tau, \omega) \wedge \mathfrak{r}_\Omega(\omega, \kappa)]$ and $\mathfrak{s}_\Omega(\tau, \kappa) \ge \bigwedge [\mathfrak{s}_\Omega(\tau, \omega) \wedge \mathfrak{s}_\Omega(\omega, \kappa)]$.

Definition 5. *Let* P *be the universal set, and* $\Omega \in P \times P$ *be the IF relation, then* (P, Ω) *is said to be the IF space of the approximation. For any set* $\mathfrak{W} \subseteq$ *IFS*(P) *the definition of the IFLA and IFUA are stated below.*

$$\Omega^{IFUA}(\mathfrak{W}) = \left\{\kappa, \mathfrak{r}_{\Omega^{IFUA}(\mathfrak{W})}(\kappa), \mathfrak{s}_{\Omega^{IFUA}(\mathfrak{W})}(\kappa) | \kappa \in P\right\}$$

$$\Omega^{IFLA}(\mathfrak{W}) = \left\{\kappa, \mathfrak{r}_{\Omega^{IFLA}(\mathfrak{W})}(\kappa), \mathfrak{s}_{\Omega^{IFLA}(\mathfrak{W})}(\kappa) | \kappa \in P\right\}$$

Where,

$$\mathfrak{r}_{\Omega^{IFUA}(\mathfrak{W})}(\kappa) = \bigvee\nolimits_{l \in P}\left[\mathfrak{r}_{\Omega(\kappa)}(\kappa, l) \vee \mathfrak{r}_{\mathfrak{W}(\kappa)}\right]$$

$$\mathfrak{s}_{\Omega^{IFUA}(\mathfrak{W})}(\kappa) = \bigwedge\nolimits_{l \in P}\left[\mathfrak{s}_{\Omega(\kappa)}(\kappa, l) \wedge \mathfrak{s}_{\mathfrak{W}(\kappa)}\right]$$

$$\mathfrak{r}_{\Omega^{\mathrm{IFLA}}(\mathfrak{W})}(\mathcal{K}) = \bigwedge\nolimits_{l\in \mathrm{P}}\left[\mathfrak{r}_{\Omega(\mathcal{K})}(\mathcal{K},l) \wedge \mathfrak{r}_{\mathfrak{W}(\mathcal{K})}\right]$$

$$\mathfrak{s}_{\Omega^{\mathrm{IFLA}}(\mathfrak{W})}(\mathcal{K}) = \bigvee\nolimits_{l\in \mathrm{P}}\left[\mathfrak{s}_{\Omega(\mathcal{K})}(\mathcal{K},l) \vee \mathfrak{s}_{\mathfrak{W}(\mathcal{K})}\right]$$

With condition $0 \le \mathfrak{r}_{\Omega^{\mathrm{IFUA}}(\mathfrak{W})}(\mathcal{K}) + \mathfrak{s}_{\Omega^{\mathrm{IFUA}}(\mathfrak{W})}(\mathcal{K}) \le 1$ and $0 \le \mathfrak{r}_{\Omega^{\mathrm{IFLA}}(\mathfrak{W})}(\mathcal{K}) + \mathfrak{s}_{\Omega^{\mathrm{IFLA}}(\mathfrak{W})}(\mathcal{K}) \le 1$. The set $\left\{\left(\Omega^{\mathrm{IFLA}}(\mathfrak{W}), \Omega^{\mathrm{IFUA}}(\mathfrak{W})\right)\right\}$ is said to be an IFRS based on IFLA and IFUA.

Definition 6. (Aczél and Alsina, 1982) The definition of the *AATNrM and.*

$$\mathbb{T}^{\mathbb{Z}}_{\mathbb{X}}(\mathscr{U},\mathrm{F}) = \begin{cases} \mathbb{T}_{\mathrm{C}}(\mathscr{U},\mathrm{F}) \mathit{if Z} = 0 \\ \min(\mathscr{U},\mathrm{F}) \quad \text{if } \mathbb{Z}\to\infty \\ \mathrm{e}^{-\left((-\ln\mathscr{U})^{\mathbb{Z}}+(-\ln\mathscr{U})^{\mathbb{Z}}\right)^{1/\mathbb{Z}}} \mathit{otherwise}. \end{cases}$$

And AATCNrM is defined as

$$\mathrm{S}^{\mathbb{Z}}_{\mathbb{X}}(\mathscr{U},\mathrm{F}) = \begin{cases} \mathbb{T}_{\mathrm{C}}(\mathscr{U},\mathrm{F}) \mathit{if Z} = 0 \\ \max(\mathscr{U},\mathrm{F}) \quad \text{if } \mathbb{Z}\to\infty \\ 1-\mathrm{e}^{-\left((-\ln(1-\mathscr{U}))^{\mathbb{Z}}+(-\ln(1-\mathrm{F}))^{\mathbb{Z}}\right)^{1/\mathbb{Z}}} \mathit{otherwise}. \end{cases}$$

$\mathbb{Z} \in [0,\infty)$

4. Aggregation operators based on AATNrM and AATCNrM

This section consists of the development of the operational laws for IFRVs based on the AATNrM and AATCNrM. Let ω_p is the weight of the *pth* IFRV such that.$\sum_{p=1}^{k}\omega_p = 1$

Definition 7. *Let* $\mathfrak{R}_p = \left(\left(\mathfrak{r}^{la}_p,\mathfrak{s}^{la}_p\right),\left(\mathfrak{r}^{ua}_p,\mathfrak{s}^{ua}_p\right)\right), p = 1,2$ *be the collection of IFRVs. Then.*

$$\mathfrak{R}_1 \oplus \mathfrak{R}_2 = \begin{pmatrix} \left(1-e^{-\left(\left(-ln\left(1-\mathfrak{r}^{la}_1\right)\right)^{\mathbb{Z}}+\left(-ln\mathfrak{r}^{la}_2\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}, e^{-\left(\left(-ln\mathfrak{s}^{la}_1\right)^{\mathbb{Z}}+\left(-ln\mathfrak{s}^{la}_2\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}\right) \\ \left(1-e^{-\left(\left(-ln\left(1-\mathfrak{r}^{ua}_1\right)\right)^{\mathbb{Z}}+\left(-ln\mathfrak{r}^{ua}_2\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}, e^{-\left(\left(-ln\mathfrak{s}^{ua}_1\right)^{\mathbb{Z}}+\left(-ln\mathfrak{s}^{ua}_2\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}\right) \end{pmatrix}$$

$$\mathfrak{R}_1 \otimes \mathfrak{R}_2 = \begin{pmatrix} \left(1-e^{-\left(\left(-ln\left(1-\mathfrak{r}^{la}_1\right)\right)^{\mathbb{Z}}+\left(-ln\mathfrak{r}^{la}_2\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}, e^{-\left(\left(-ln\mathfrak{s}^{la}_1\right)^{\mathbb{Z}}+\left(-ln\mathfrak{s}^{la}_2\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}\right) \\ \left(1-e^{-\left(\left(-ln\left(1-\mathfrak{r}^{ua}_1\right)\right)^{\mathbb{Z}}+\left(-ln\mathfrak{r}^{ua}_2\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}, e^{-\left(\left(-ln\mathfrak{s}^{ua}_1\right)^{\mathbb{Z}}+\left(-ln\mathfrak{s}^{ua}_2\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}\right) \end{pmatrix} \quad (1)$$

Now, weighted GAO for IFRS based on the operational laws defined above are defined as below.

Definition 8. *Let* $\mathfrak{R}_p = \left(\left(\mathfrak{r}^{la}_p,\mathfrak{s}^{la}_p\right),\left(\mathfrak{r}^{ua}_p,\mathfrak{s}^{ua}_p\right)\right), p = 1,2,\cdots,k$ *be the collection of the IFRVs. Then.*

$$IFRAAWG(\mathfrak{R}_1,\mathfrak{R}_2,\cdots,\mathfrak{R}_k) = \bigotimes_{p=1}^{k} \mathfrak{R}_p^{\omega_p} \quad (2)$$

Theorem 1. *Let* $\mathfrak{R}_p = \left(\left(\mathfrak{r}^{la}_p,\mathfrak{s}^{la}_p\right),\left(\mathfrak{r}^{ua}_p,\mathfrak{s}^{ua}_p\right)\right), p = 1,2,\cdots,k$ *be the collection of the IFRVs. Then.*

$$IFRAAWG(\mathfrak{R}_1,\mathfrak{R}_2,\cdots,\mathfrak{R}_k) = \begin{pmatrix} \left(e^{-\left(\sum_{p=1}^{k}\left(-ln\left(\mathfrak{r}^{la}_p\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}, 1-e^{-\left(\sum_{p=1}^{k}\left(-ln\left(1-\mathfrak{s}^{la}_p\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}\right) \\ \left(e^{-\left(\sum_{p=1}^{k}\left(-ln\left(\mathfrak{r}^{ua}_p\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}, 1-e^{-\left(\sum_{p=1}^{k}\left(-ln\left(1-\mathfrak{s}^{ua}_p\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}\right) \end{pmatrix}$$

Proof:
For $k = 2$

$$IFRAAWG(\mathfrak{R}_1,\mathfrak{R}_2) = \begin{pmatrix} \left(e^{-\left(\sum_{p=1}^{2}\left(-ln\left(\mathfrak{r}^{la}_p\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}, 1-e^{-\left(\sum_{p=1}^{2}\left(-ln\left(1-\mathfrak{s}^{la}_p\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}\right) \\ \left(e^{-\left(\sum_{p=1}^{2}\left(-ln\left(\mathfrak{r}^{ua}_p\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}, 1-e^{-\left(\sum_{p=1}^{2}\left(-ln\left(1-\mathfrak{s}^{ua}_p\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}\right) \end{pmatrix}$$

Which is an IFRV.
Assume Eqn. (2) true for $k = n$

$$IFRAAWG(\mathfrak{R}_1,\mathfrak{R}_2,\cdots,\mathfrak{R}_n) = \begin{pmatrix} \left(e^{-\left(\sum_{p=1}^{n}\left(-ln\left(\mathfrak{r}^{la}_p\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}, 1-e^{-\left(\sum_{p=1}^{n}\left(-ln\left(1-\mathfrak{s}^{la}_p\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}\right) \\ \left(e^{-\left(\sum_{p=1}^{n}\left(-ln\left(\mathfrak{r}^{ua}_p\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}, 1-e^{-\left(\sum_{p=1}^{n}\left(-ln\left(1-\mathfrak{s}^{ua}_p\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}\right) \end{pmatrix}$$

We have to prove Eqn. (2) true for $k = n + 1$, which is as follows.

$$IFRAAWG(\mathfrak{R}_1,\mathfrak{R}_2,\cdots,\mathfrak{R}_n,\mathfrak{R}_{n+1}) = \begin{pmatrix} \left(e^{-\left(\sum_{p=1}^{n}\left(-ln\left(\mathfrak{r}^{la}_p\right)\right)^{\mathbb{Z}}+\left(-ln\left(\mathfrak{r}^{la}_{n+1}\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}, 1-e^{-\left(\sum_{p=1}^{n}\left(-ln\left(1-\mathfrak{s}^{la}_p\right)\right)^{\mathbb{Z}}+\left(-ln\left(1-\mathfrak{s}^{la}_{n+1}\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}\right) \\ \left(e^{-\left(\sum_{p=1}^{n}\left(-ln\left(\mathfrak{r}^{ua}_p\right)\right)^{\mathbb{Z}}+\left(-ln\left(\mathfrak{r}^{ua}_{n+1}\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}, 1-e^{-\left(\sum_{p=1}^{n}\left(-ln\left(1-\mathfrak{s}^{ua}_p\right)\right)^{\mathbb{Z}}+\left(-ln\left(1-\mathfrak{s}^{ua}_{n+1}\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}\right) \end{pmatrix}$$

Next, we have,

$$IFRAAWG(\mathfrak{R}_1,\mathfrak{R}_2,\cdots,\mathfrak{R}_{n+1}) = \begin{pmatrix} \left(e^{-\left(\sum_{p=1}^{n+1}\left(-ln\left(\mathfrak{r}^{la}_p\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}, 1-e^{-\left(\sum_{p=1}^{n+1}\left(-ln\left(1-\mathfrak{s}^{la}_p\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}\right) \\ \left(e^{-\left(\sum_{p=1}^{n+1}\left(-ln\left(\mathfrak{r}^{ua}_p\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}, 1-e^{-\left(\sum_{p=1}^{n+1}\left(-ln\left(1-\mathfrak{s}^{ua}_p\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}\right) \end{pmatrix}$$

Theorem 2. ***(Idempotency)*** *Let* $\mathfrak{R}_p = \left(\left(\mathfrak{r}^{la}_p,\mathfrak{s}^{la}_p\right),\left(\mathfrak{r}^{ua}_p,\mathfrak{s}^{ua}_p\right)\right) = \left(\left(\mathfrak{r}^{la},\mathfrak{s}^{la}\right),\left(\mathfrak{r}^{ua},\mathfrak{s}^{ua}\right)\right) = \mathfrak{R}, \forall p = 1,2,\cdots,k.$ *Then.*

$$IFRAAWG(\mathfrak{R}_1,\mathfrak{R}_2,\cdots,\mathfrak{R}_k) = \left(\left(\mathfrak{r}^{la},\mathfrak{s}^{la}\right),\left(\mathfrak{r}^{ua},\mathfrak{s}^{ua}\right)\right) = \mathfrak{R}$$

Proof: As $\mathfrak{R}_p = \left(\left(\mathfrak{r}^{la}_p,\mathfrak{s}^{la}_p\right),\left(\mathfrak{r}^{ua}_p,\mathfrak{s}^{ua}_p\right)\right) = \left(\left(\mathfrak{r}^{la},\mathfrak{s}^{la}\right),\left(\mathfrak{r}^{ua},\mathfrak{s}^{ua}\right)\right)$, so we have

$$IFRAAWG(\mathfrak{R}_1,\mathfrak{R}_2,\cdots,\mathfrak{R}_k) = (\mathfrak{R},\mathfrak{R},\cdots,\mathfrak{R})$$

$$= \begin{pmatrix} \left(e^{-\left(\sum_{p=1}^{k}\left(-ln\left(\mathfrak{r}^{la}_p\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}, 1-e^{-\left(\sum_{p=1}^{k}\left(-ln\left(1-\mathfrak{s}^{la}_p\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}\right) \\ \left(e^{-\left(\sum_{p=1}^{k}\left(-ln\left(\mathfrak{r}^{ua}_p\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}, 1-e^{-\left(\sum_{p=1}^{k}\left(-ln\left(1-\mathfrak{s}^{ua}_p\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}\right) \end{pmatrix}$$

$$=\left(\begin{array}{c}\left(e^{-\left(\sum_{p=1}^{k}\left(-ln(\mathfrak{r}^{la})\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}, 1-e^{-\left(\sum_{p=1}^{k}\left(-ln\left(1-\mathfrak{s}^{la}\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}\right)\\ \left(e^{-\left(\sum_{p=1}^{k}\left(-ln(\mathfrak{r}^{ua})\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}, 1-e^{-\left(\sum_{p=1}^{k}\left(-ln\left(1-\mathfrak{s}^{ua}\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}\right)\end{array}\right)$$

$$=\left(\left(\mathfrak{r}^{la},\mathfrak{s}^{la}\right),\left(\mathfrak{r}^{ua},\mathfrak{s}^{ua}\right)\right)=\mathfrak{R}$$

Theorem 3. ***(Boundedness)*** *Let $\mathfrak{R}_p^s$ is the smallest and $\mathfrak{R}_p^g$ is the greatest IFRV. Then.*

$$\mathfrak{R}_p^s \le IFRAAWG(\mathfrak{R}_1,\mathfrak{R}_2,\cdots,\mathfrak{R}_k) \le \mathfrak{R}_p^g$$

Theorem 4. ***(Monotonicity)*** *Let $\mathfrak{R}_p=\left(\left(\mathfrak{r}_p^{la},\mathfrak{s}_p^{la}\right),\left(\mathfrak{r}_p^{ua},\mathfrak{s}_p^{ua}\right)\right), p=1,2,\cdots,k$ and $\mathfrak{R}_p^{v}=\left(\left(\mathfrak{r}_p^{vla},\mathfrak{s}_p^{vla}\right),\left(\mathfrak{r}_p^{vua},\mathfrak{s}_p^{vua}\right)\right)$ be the collections of the IFRVs such that $\mathfrak{R}_p \le \mathfrak{R}_p^{v}$. Then.*

$$IFRAAWG(\mathfrak{R}_1,\mathfrak{R}_2,\cdots,\mathfrak{R}_k) \le IFRAAWG(\mathfrak{R}_1^{v},\mathfrak{R}_2^{v},\cdots,\mathfrak{R}_k^{v})$$

Definition 9. *Let $\mathfrak{R}_p=\left(\left(\mathfrak{r}_p^{la},\mathfrak{s}_p^{la}\right),\left(\mathfrak{r}_p^{ua},\mathfrak{s}_p^{ua}\right)\right), p=1,2,\cdots,k$ be the collection of the IFRVs. Then.*

$$IFRAAOWG(\mathfrak{R}_1,\mathfrak{R}_2,\cdots,\mathfrak{R}_k)=\bigotimes_{p=1}^{k}\mathfrak{R}_{\mathcal{T}(p)}^{\omega_p} \tag{3}$$

Where, $\mathcal{T}(p)$ is the permutation of the IFRVs $(p=1,2,\cdots,k)$ such that $\mathcal{T}(p-1)>\mathcal{T}(p)$.

Theorem 5. *Let $\mathfrak{R}_p=\left(\left(\mathfrak{r}_p^{la},\mathfrak{s}_p^{la}\right),\left(\mathfrak{r}_p^{ua},\mathfrak{s}_p^{ua}\right)\right), p=1,2,\cdots,k$ be the collection of the IFRVs. Then.*

$$IFRAAOWG(\mathfrak{R}_1,\mathfrak{R}_2,\cdots,\mathfrak{R}_k)$$
$$=\left(\begin{array}{c}\left(e^{-\left(\sum_{p=1}^{k}\left(-ln\left(\mathfrak{r}_{\mathcal{T}(p)}^{la}\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}, 1-e^{-\left(\sum_{p=1}^{k}\left(-ln\left(1-\mathfrak{s}_{\mathcal{T}(p)}^{la}\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}\right)\\ \left(e^{-\left(\sum_{p=1}^{k}\left(-ln\left(\mathfrak{r}_{\mathcal{T}(p)}^{ua}\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}, 1-e^{-\left(\sum_{p=1}^{k}\left(-ln\left(1-\mathfrak{s}_{\mathcal{T}(p)}^{ua}\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}\right)\end{array}\right)$$

Definition 10. *Let $\mathfrak{R}_p=\left(\left(\mathfrak{r}_p^{la},\mathfrak{s}_p^{la}\right),\left(\mathfrak{r}_p^{ua},\mathfrak{s}_p^{ua}\right)\right), p=1,2,\cdots,k$ be the collection of the IFRVs and ω_p is the weight of the pth IFRV. Then.*

$$IFRAAHWG(\mathfrak{R}_1,\mathfrak{R}_2,\cdots,\mathfrak{R}_k)=\bigotimes_{p=1}^{k}\widetilde{\mathfrak{R}}_{\mathcal{T}(p)}^{\omega_p} \tag{4}$$

Where, $\mathcal{T}(p)$ is the permutation of the IFRVs $(p=1,2,\cdots,k)$ such that $\mathcal{T}(p-1)>\mathcal{T}(p)$ and $\widetilde{\mathfrak{R}}=z\omega\mathfrak{R}$ with significant balancing coefficient z.

Theorem 9. *Let $\mathfrak{R}_p=\left(\left(\mathfrak{r}_p^{la},\mathfrak{s}_p^{la}\right),\left(\mathfrak{r}_p^{ua},\mathfrak{s}_p^{ua}\right)\right), p=1,2,\cdots,k$ be the collection of the IFRVs. Then.*

$$IFRAAHWG(\mathfrak{R}_1,\mathfrak{R}_2,\cdots,\mathfrak{R}_k)$$
$$=\left(\begin{array}{c}\left(e^{-\left(\sum_{p=1}^{k}\left(-ln\left(\tilde{\mathfrak{r}}_{\mathcal{T}(p)}^{la}\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}, 1-e^{-\left(\sum_{p=1}^{k}\left(-ln\left(1-\tilde{\mathfrak{s}}_{\mathcal{T}(p)}^{la}\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}\right)\\ \left(e^{-\left(\sum_{p=1}^{k}\left(-ln\left(\tilde{\mathfrak{r}}_{\mathcal{T}(p)}^{ua}\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}, 1-e^{-\left(\sum_{p=1}^{k}\left(-ln\left(1-\tilde{\mathfrak{s}}_{\mathcal{T}(p)}^{ua}\right)\right)^{\mathbb{Z}}\right)^{\frac{1}{\mathbb{Z}}}}\right)\end{array}\right)$$

5. Multi-Attribute Decision-Making approach

Consider $\{\mathcal{Q}_1,\mathcal{Q}_2,\cdots,\mathcal{Q}_q\}$ be the set of q alternatives from which one alternative is to be selected based on the set of attributes $\{\mathcal{G}_1,\mathcal{G}_2,\cdots,\mathcal{G}_r\}$ be the set of r attributes. Consider $\{\mathcal{X}_1,\mathcal{X}_2,\cdots,\mathcal{X}_h\}$ be the set of h experts having weights $\mathscr{W}_n\in[0,1]n=1,2,\cdots,h$ such that $\sum_{n=1}^{h}\mathscr{W}_n=1$. The steps involve to select an alternative are given below.

Step 1: Represent the information in the form of the IFRVs $\left(\left(\mathfrak{r}_p^{la},\mathfrak{s}_p^{la}\right),\left(\mathfrak{r}_p^{ua},\mathfrak{s}_p^{ua}\right)\right)$. Change the rating of cost type to benefits type, if any by taking their complement.

Step 2: Aggregate the information of each expert by using defined GAOs.

Step 3: Aggregate the information with the GAOs operator to get the collective value for each alternative.

Step 4: Obtain the score values of each collective IFRVs.

Step 5: Rank the alternative using obtain score values.

The flowchart of the steps is shown in Fig. 1.

Example 1. *Assume that the owner of a company wants to evaluate the progress of the managers. The evaluation can be done based on some criteria or attributes. So, the owner decides some attributes on which he evaluates. Based on these attributes, the experts $e_\ell(\ell=1,2,3)$having weight $(0.37,0.32,0.31)^T$ assign the values to each manager in the form of the IFRVs. After the initial screening, we assume four managers $\mathbb{M}_\ell(\ell=1,2,3,4)$ in the list of alternatives. Among these four managers, the best one is to be selected. The attributes based on which the IFRVs are assigned are given in the following with weights $(0.24,0.18,0.21,0.19,0.18)^T$.*

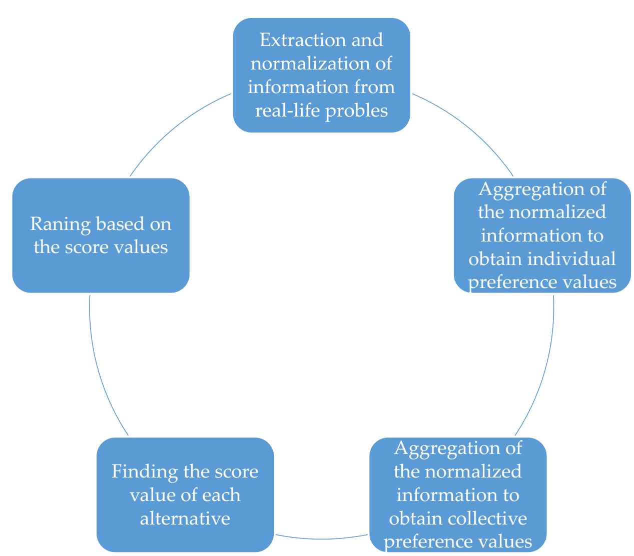

Fig. 1. Flowchart of stepwise procedure of MAGDM.

i. Organization skills $\mathbb{C}_1$
ii. Communication skills $\mathbb{C}_2$
iii. Knowledge $\mathbb{C}_3$
iv. Confidence $\mathbb{C}_4$
v. Time management skills $\mathbb{C}_4$.

Step 1: Each expert provides their ratings to the given alternatives in the form of IFRV and listed in Table 1.

Step 2: Utilize IFRAAWG operator to aggregate each expert information. The result obtained is listed in Table 2.

Step 3: Utilize IFRAAWG operator to aggregate the data and result is listed in Table 3.

Step 4: The score value of each alternative is computed as

$$\mathfrak{s}(\mathbb{M}_1) = -0.111, \mathfrak{s}(\mathbb{M}_2) = -0.105, \mathfrak{s}(\mathbb{M}_3) = -0.127,$$
$$\mathfrak{s}(\mathbb{M}_4) = -0.128$$

Step 5: The ordering is $\mathbb{M}_2 \succ \mathbb{M}_1 \succ \mathbb{M}_3 \succ \mathbb{M}_4$ and get alternative 2 is the best choice for the given problem.

Table 1
Rating in the form of IFRVS by expert $\mathfrak{e}_1, \mathfrak{e}_2$, and $\mathfrak{e}_3$.

$\mathfrak{e}_1$	$\mathbb{M}_1$				$\mathbb{M}_2$				$\mathbb{M}_3$				$\mathbb{M}_4$			
	$\mathfrak{r}^{la}$	$\mathfrak{r}^{ua}$	$\mathfrak{s}^{la}$	$\mathfrak{s}^{ua}$	$\mathfrak{r}^{la}$	$\mathfrak{r}^{ua}$	$\mathfrak{s}^{la}$	$\mathfrak{s}^{ua}$	$\mathfrak{r}^{la}$	$\mathfrak{r}^{ua}$	$\mathfrak{s}^{la}$	$\mathfrak{s}^{ua}$	$\mathfrak{r}^{la}$	$\mathfrak{r}^{ua}$	$\mathfrak{s}^{la}$	$\mathfrak{s}^{ua}$
$\mathbb{C}_1$	0.54	0.45	0.43	0.5	0.5	0.54	0.35	0.56	0.54	0.29	0.2	0.4	0.64	0.2	0.34	0.65
$\mathbb{C}_2$	0.39	0.33	0.38	0.44	0.3	0.39	0.55	0.15	0.39	0.56	0.3	0.7	0.44	0.19	0.43	0.46
$\mathbb{C}_3$	0.52	0.29	0.34	0.39	0.43	0.52	0.2	0.45	0.52	0.54	0.5	0.25	0.4	0.38	0.53	0.19
$\mathbb{C}_4$	0.38	0.43	0.23	0.23	0.38	0.23	0.23	0.23	0.23	0.38	0.23	0.23	0.23	0.23	0.23	0.38
$\mathbb{C}_5$	0.33	0.51	0.34	0.43	0.33	0.39	0.25	0.6	0.5	0.33	0.4	0.6	0.45	0.45	0.4	0.33
$\mathfrak{e}_2$	$\mathbb{M}_1$				$\mathbb{M}_2$				$\mathbb{M}_3$				$\mathbb{M}_4$			
	$\mathfrak{r}^{la}$	$\mathfrak{r}^{ua}$	$\mathfrak{s}^{la}$	$\mathfrak{s}^{ua}$	$\mathfrak{r}^{la}$	$\mathfrak{r}^{ua}$	$\mathfrak{s}^{la}$	$\mathfrak{s}^{ua}$	$\mathfrak{r}^{la}$	$\mathfrak{r}^{ua}$	$\mathfrak{s}^{la}$	$\mathfrak{s}^{ua}$	$\mathfrak{r}^{la}$	$\mathfrak{r}^{ua}$	$\mathfrak{s}^{la}$	$\mathfrak{s}^{ua}$
$\mathbb{C}_1$	0.34	0.54	0.2	0.32	0.5	0.4	0.2	0.34	0.4	0.5	0.54	0.44	0.2	0.52	0.4	0.45
$\mathbb{C}_2$	0.3	0.35	0.3	0.32	0.5	0.19	0.4	0.45	0.2	0.45	0.39	0.53	0.45	0.19	0.35	0.54
$\mathbb{C}_3$	0.54	0.38	0.39	0.43	0.29	0.54	0.54	0.38	0.54	0.19	0.52	0.45	0.54	0.39	0.54	0.19
$\mathbb{C}_4$	0.2	0.4	0.35	0.36	0.35	0.45	0.45	0.35	0.7	0.19	0.45	0.42	0.4	0.52	0.34	0.54
$\mathbb{C}_5$	0.4	0.5	0.4	0.47	0.35	0.44	0.3	0.35	0.5	0.45	0.53	0.19	0.15	0.42	0.36	0.56
$\mathfrak{e}_3$	$\mathbb{M}_1$				$\mathbb{M}_2$				$\mathbb{M}_3$				$\mathbb{M}_4$			
	$\mathfrak{r}^{la}$	$\mathfrak{r}^{ua}$	$\mathfrak{s}^{la}$	$\mathfrak{s}^{ua}$	$\mathfrak{r}^{la}$	$\mathfrak{r}^{ua}$	$\mathfrak{s}^{la}$	$\mathfrak{s}^{ua}$	$\mathfrak{r}^{la}$	$\mathfrak{r}^{ua}$	$\mathfrak{s}^{la}$	$\mathfrak{s}^{ua}$	$\mathfrak{r}^{la}$	$\mathfrak{r}^{ua}$	$\mathfrak{s}^{la}$	$\mathfrak{s}^{ua}$
$\mathbb{C}_1$	0.39	0.48	0.3	0.55	0.4	0.49	0.54	0.43	0.4	0.55	0.2	0.35	0.3	0.54	0.26	0.5
$\mathbb{C}_2$	0.3	0.43	0.4	0.53	0.5	0.3	0.39	0.12	0.2	0.45	0.25	0.45	0.23	0.39	0.36	0.29
$\mathbb{C}_3$	0.3	0.38	0.3	0.38	0.3	0.3	0.52	0.3	0.38	0.3	0.3	0.38	0.3	0.52	0.38	0.3
$\mathbb{C}_4$	0.38	0.33	0.29	0.33	0.42	0.45	0.6	0.35	0.33	0.17	0.52	0.33	0.42	0.56	0.33	0.45
$\mathbb{C}_5$	0.33	0.5	0.34	0.46	0.5	0.36	0.3	0.6	0.39	0.46	0.29	0.44	0.32	0.51	0.33	0.39

Table 2
Expert aggregated value by IFRAAWG operator.

	$\mathbb{M}_1$				$\mathbb{M}_2$			
	$\mathfrak{r}^{la}$	$\mathfrak{r}^{ua}$	$\mathfrak{s}^{la}$	$\mathfrak{s}^{ua}$	$\mathfrak{r}^{la}$	$\mathfrak{r}^{ua}$	$\mathfrak{s}^{la}$	$\mathfrak{s}^{ua}$
$\mathbb{C}_1$	0.404	0.494	0.278	0.488	0.460	0.492	0.294	0.481
$\mathbb{C}_2$	0.326	0.376	0.353	0.455	0.387	0.326	0.433	0.339
$\mathbb{C}_3$	0.408	0.354	0.338	0.402	0.329	0.492	0.303	0.396
$\mathbb{C}_4$	0.287	0.396	0.276	0.316	0.380	0.406	0.329	0.319
$\mathbb{C}_5$	0.348	0.504	0.356	0.453	0.371	0.401	0.279	0.559
	$\mathbb{M}_3$				$\mathbb{M}_4$			
	$\mathfrak{r}^{la}$	$\mathfrak{r}^{ua}$	$\mathfrak{s}^{la}$	$\mathfrak{s}^{ua}$	$\mathfrak{r}^{la}$	$\mathfrak{r}^{ua}$	$\mathfrak{s}^{la}$	$\mathfrak{s}^{ua}$
$\mathbb{C}_1$	0.436	0.480	0.240	0.402	0.289	0.478	0.321	0.570
$\mathbb{C}_2$	0.239	0.502	0.300	0.612	0.331	0.297	0.378	0.467
$\mathbb{C}_3$	0.462	0.437	0.403	0.382	0.381	0.444	0.465	0.240
$\mathbb{C}_4$	0.310	0.300	0.325	0.348	0.309	0.490	0.284	0.470
$\mathbb{C}_5$	0.455	0.422	0.374	0.505	0.243	0.464	0.362	0.462

Table 3
Aggregated values by IFRAAWG operator.

$\mathbb{M}_1$				$\mathbb{M}_2$			
$\boldsymbol{\mathfrak{r}^{la}}$	$\mathfrak{r}^{ua}$	$\mathfrak{s}^{la}$	$\mathfrak{s}^{ua}$	$\mathfrak{r}^{la}$	$\mathfrak{r}^{ua}$	$\mathfrak{s}^{la}$	$\mathfrak{s}^{ua}$
0.289	0.496	0.256	0.492	0.322	0.494	0.259	0.507
$\mathbb{M}_3$				$\mathbb{M}_4$			
$\boldsymbol{\mathfrak{r}^{la}}$	$\mathfrak{r}^{ua}$	$\mathfrak{s}^{la}$	$\mathfrak{s}^{ua}$	$\mathfrak{r}^{la}$	$\mathfrak{r}^{ua}$	$\mathfrak{s}^{la}$	$\mathfrak{s}^{ua}$
0.287	0.501	0.253	0.545	0.244	0.505	0.288	0.538

Table 4
Impact of parameter $\mathbb{Z}$.

$\mathbb{Z}$	Ranking order
2	$\mathbb{M}_2 \succ \mathbb{M}_1 \succ \mathbb{M}_3 \succ \mathbb{M}_4$
3	$\mathbb{M}_2 \succ \mathbb{M}_1 \succ \mathbb{M}_3 \succ \mathbb{M}_4$
4	$\mathbb{M}_2 \succ \mathbb{M}_1 \succ \mathbb{M}_3 \succ \mathbb{M}_4$
5	$\mathbb{M}_2 \succ \mathbb{M}_1 \succ \mathbb{M}_3 \succ \mathbb{M}_4$
6	$\mathbb{M}_2 \succ \mathbb{M}_1 \succ \mathbb{M}_3 \succ \mathbb{M}_4$
7	$\mathbb{M}_2 \succ \mathbb{M}_1 \succ \mathbb{M}_3 \succ \mathbb{M}_4$
8	$\mathbb{M}_2 \succ \mathbb{M}_1 \succ \mathbb{M}_3 \succ \mathbb{M}_4$
9	$\mathbb{M}_2 \succ \mathbb{M}_1 \succ \mathbb{M}_3 \succ \mathbb{M}_4$
10	$\mathbb{M}_2 \succ \mathbb{M}_1 \succ \mathbb{M}_3 \succ \mathbb{M}_4$
15	$\mathbb{M}_2 \succ \mathbb{M}_1 \succ \mathbb{M}_3 \succ \mathbb{M}_4$
30	$\mathbb{M}_2 \succ \mathbb{M}_1 \succ \mathbb{M}_3 \succ \mathbb{M}_4$
40	$\mathbb{M}_2 \succ \mathbb{M}_1 \succ \mathbb{M}_3 \succ \mathbb{M}_4$
50	$\mathbb{M}_2 \succ \mathbb{M}_1 \succ \mathbb{M}_3 \succ \mathbb{M}_4$

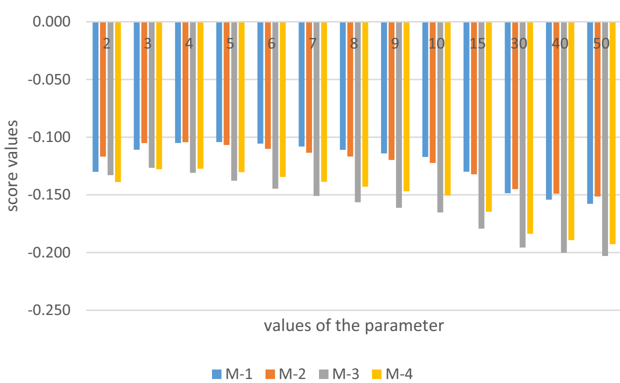

Fig. 2. Sensitivity analysis of IRAAWG operator.

5.1. Sensitivity analysis

The proposed approach involves the parameter $\mathbb{Z}$. In this section, we study the change in the ranking results obtained at different values of the parameter $\mathbb{Z}$. Table 4 represents the impact of its parameter on the ranking order, while Fig. 2 depicts its range values and conclude that $\mathbb{M}_2$ is the most efficient among all the managers.

5.2. Comparative analysis

In this section, we compare the performance of proposed algorithm to some of the existing approaches (Chinram et al., 2021; Yahya et al., 2021; Ali et al., 2021; Liu et al., 2017; Ahmmad et al., 2022). The result of it is listed in Table 5. It is seen that approaches (Ali et al., 2021; Liu et al., 2017) cannot aggregate the information in the form of the IFRVs, while approach given in (Ahmmad et al., 2022) indefinity $\mathbb{M}_3$ as the best one. On the other hand, approaches by IFRAAWG, IFRWA and IFRWG operator produce $\mathbb{M}_2$ is the best alternative. The variation of their score values is presented in Figs. 3 and 4.

Table 5
Comparative study with exsisting approaches.

Operator	Score Values
IFRAAWA (Ahmmad et al., 2022)	$\mathbb{M}_3 \succ \mathbb{M}_2 \succ \mathbb{M}_4 \succ \mathbb{M}_1$
IFRAAWG	$\mathbb{M}_2 \succ \mathbb{M}_1 \succ \mathbb{M}_3 \succ \mathbb{M}_4$
IFRWA	$\mathbb{M}_2 \succ \mathbb{M}_3 \succ \mathbb{M}_4 \succ \mathbb{M}_1$
IFRWG	$\mathbb{M}_2 \succ \mathbb{M}_3 \succ \mathbb{M}_4 \succ \mathbb{M}_1$
IFRFWA	$\mathbb{M}_3 \succ \mathbb{M}_2 \succ \mathbb{M}_4 \succ \mathbb{M}_1$
IFRFWG	$\mathbb{M}_3 \succ \mathbb{M}_2 \succ \mathbb{M}_4 \succ \mathbb{M}_1$
AOs in (Ali et al., 2021)	Not applicable
AOs in (Liu et al., 2017)	Not applicable

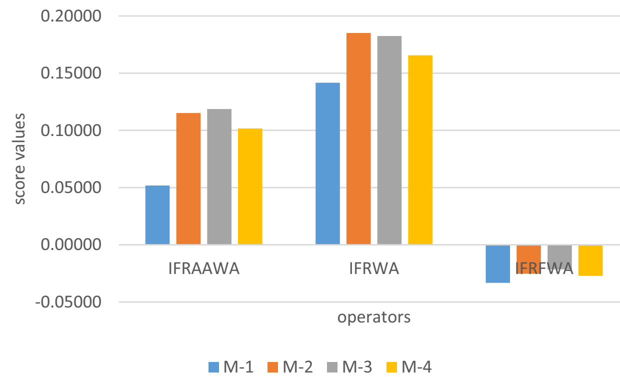

Fig. 3. Comparison of IFRAAWA operator.

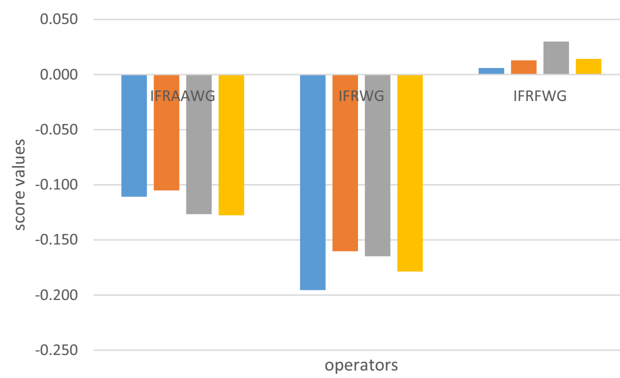

Fig. 4. Comparison of IFRAAWG operator.

The following are some advantages of the proposed approach.

1. The IFRAAWG operator is more reliable than the IFRAAWA (Ahmmad et al., 2022) operator due to its nature of aggregation.
2. IFRAAWA (Ahmmad et al., 2022) and IFRAAWG operators are more reliable AOs than the existing AOs because these AOs are based on the IFRS which can cover the maximum information in the form of the IFRVs.
3. As compared to the IFS the IFRS is the more reliable framework because IFRS is based on approximations that are further fuzzified instead of crisp.
4. AOs developed in (Ali et al., 2021; Liu et al., 2017), cannot deal with the information in the form of the IFRVs.

6. Conclusion

In this study, some basic operations for the IFRVs are developed based on the AATNrM and AATCNrM and study basic properties. The developed approach is applied to the real-life problem of the MAGDM. The results obtained are compared with some existing AOs. The results at different values of the parameter involved in the AATNrM and AATCNrM are investigated. The developed approaches, IFRAAWG, OFRAAOWG, and IFRAAHWG operators are based on the AATNrM and AATCNrM which are very flexible and helpful in the fusion of information. The most suitable manager obtained from IFRAAWG operators is $\mathbb{M}_3$. No doubt the developed model is a good technique to cover the uncertainty and roughness in information. However, there are some limitations to this model. The proposed model is unable to give the value at the even value of the parameter involved. Moreover, the proposed model can also cause a loss of information in case of the presence of an additional degree that cannot be covered by the IFRS. Hence, we aim to extend the developed approach to address such kinds of the problems in the future work.

Ethical approval

The article does not contain any studies with human participants or animal performed by any of the authors.

Informed consent

Informed consent was obtained from all individual participants included in the study.

Authors' contributions

All the authors equally contributed.

Data availability statement

No data were used to support this study.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.jksus.2023.102760.

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