Journal of King Saud University – Science 31 (2019) 252–256

Contents lists available at ScienceDirect

Journal of King Saud University – Science

journal homepage: www.sciencedirect.com

Original article

Derivation of quantum propagator for coupled harmonic oscillator with uniform electric field in a single harmonic oscillator environment using white noise functional approach

Norodin A. Rangaig [a,*], Caironesa T. Pada-Dulpina [a], Vernie C. Convicto [a,b]

[a] Department of Physics, Mindanao State University-Main Campus, 9700 Marawi City, Philippines
[b] Department of Physics, Mindanao State University-Iligan Institute of Technology, 9200 Iligan City, Philippines

ARTICLE INFO

Article history:
Received 6 December 2017
Accepted 11 April 2018
Available online 14 April 2018

Keywords:
Harmonic oscillator
Quantum propagator
Feynman path integral
White noise functional
Lagrangian

ABSTRACT

The use of white noise analysis is utilized in this study. We consider an open quantum system of coupled harmonic oscillator with uniform two dimensional electric field in a single harmonic oscillator environment. We obtain the quantum propagator for the said system by considering the Feynman path integral as a white noise functional. In addition, the resulting quantum propagator was consistent with the previous literature and also, this study gives more general form due to the inclusion of the electric field.

1. Introduction

In order to describe any physical object is to study its properties and its interaction with its environment. We already know that everything is made up of particles, so studying the dynamics of the particle gives meaning to everything that has physical manifestation. One way to describe the particle is to assume that it is a wave that propagates along a given space at a given time. Harmonic oscillation is a solid classical theory that describes the properties of the wave. Perhaps, the very difficult part in describing the particle as a wave is to know its position and what path it might take during its motion because of the randomness. One equation that describes the particle as a wave is the Schrodinger equation, although it may not give an exact position of the particle at a specific time, it gives us hint or idea where the particle might be, but still due to the probabilistic nature of the Scrodinger equation stems for the measurement which is one of the difficult aspects to understand.

In 20th century, solution in determining the final state of a particle throughout its motion was discovered by Feynman and Hibbs (1965) and developed a path integral that describes the path taken by the particle at a given range of time. The path integral includes spaces of the possible dynamics of the system between its initial state to final state. A very common form of the path integral is called the propagator which has a mathematical notation given by

$$K(r_1, r_2; t) = N \int \exp\left(\frac{i}{\hbar}\right) S[r] D_r$$

The propagator gives a probability amplitude for a particle to travel from initial to final position in a given time. It was named after Richard Feynman itself as Feynman integral. It is a more effective tool within the framework of an infinite dimensional analysis known as white noise analysis (Somerado and Convicto, 2017). Many authors have studied the use of white noise analysis formulated by Hida (1971) in different interaction. In Butanas and Caballar (2016) and Pabalay and Bornales (2008), it is shown that the propagator can be derived even if there is a coupled term of the systems.

In this paper, we will derive the quantum propagator of a coupled harmonic oscillator in uniform mass and frequency which is the same as proposed in Butanas and Caballar (2016) but this time, there is an inclusion effect of the electric field in the harmonic oscillator of the system coupled to multimode harmonic oscillator.

* Corresponding author at: Room 222, Math-Phys Building, Mindanao State University, 9700 Marawi City, Philippines.
E-mail address: azis.norodinp6@gmail.com (N.A. Rangaig).

Peer review under responsibility of King Saud University.

Production and hosting by Elsevier

https://doi.org/10.1016/j.jksus.2018.04.015
1018-3647/

Lastly, the obtained result is consistent with the obtained result in Somerado and Convicto (2017), Hida (1971), Butanas and Caballar (2016).

2. The white noise theory

A key feature of the White noise analysis is that it operates in the (Kou, 1996)

$$S(\mathbf{R}) \subset L^2 \subset S^*(\mathbf{R}) \tag{1}$$

linking the spaces of Hida distribution S^* and test function S through a Hilbert space of square integrable functions L^2. Via Minlo's theorem, we can formulate a Hida white noise space(S^*, B, μ) where μ is the probability measure and B is the σ-algebra generated on S and can be constructed by fixing the characteristic function in the following way,

$$C(\xi) = \int \exp(\langle \omega, \xi \rangle) d\mu(\omega) = \exp\left(-\frac{1}{2}\int \xi^2 d\tau\right) \tag{2}$$

where $\xi \in S$ and the white noise Gaussian measure $d\mu(\omega)$ is given by

$$d\mu(\omega) = N_\omega \exp\left(-\frac{1}{2}\int \omega(\tau)^2 d\tau\right) d^\infty \omega \tag{3}$$

with $N_\omega = \sqrt{\frac{1}{(2\pi)^n}}$ as a normalization constant. The exponential term in $d\mu(\omega)$ is responsible for the Gaussian fall-off of the propagator function. Formally, a white noise is a stochastic process $z(t)$ such that $z(t)$'s are independent for each t, has zero mean and has a covariant in the form $\mathbf{E}\langle z(t)z(s)\rangle = \delta(t-s)$ where $\mathbf{E}$ is the expectation value of the two variables. This expression shows that white noise is delta correlated and can be regarded as a time derivative of Brownian motion i,e $\omega = \frac{db}{dt}$, where ω is the Gaussian white noise. Furthermore, we can write the Gaussian white noise in terms of Wiener's Brownian motion as $B(t) = \int_{t_o}^t \omega(\tau)d\tau = \langle \omega, 1_{[t_o,t)}\rangle$.

White noise can also be considered as a generalized stochastic process which means that for each test function $\xi(x) = \xi(x_1, x_2, \ldots, x_n), X(\xi)$ is a gaussian random variable with zero mean and covariance $\int_{\mathbf{R}} \xi^2(t)dt$, we can write

$$X(\xi)\omega(\xi) = \langle \omega, \xi \rangle = \int \omega(\tau)\xi(\tau)d\tau \tag{4}$$

Now, a key feature of Hida's formulation is the treatment of the set $\omega(\tau)$ at different instants of time, $\omega(\tau); t \in \mathbf{R}$ as a continuum coordinate system. For the sum over all routes or histories in the path integral, paths starting from initial point x_o and propagating in Brownian fluctuations are parametrized within the white noise framework as

$$x(t) = x_o + \sqrt{\frac{\hbar}{m}}\int_o^t \omega(\tau)d\tau \tag{5}$$

Evaluating the Feynman integral in the context of white noise analysis is carried by the evaluation of the Gaussian white noise measure, where the white noise functionals can be described through their S- and T-Transforms. For the T-transform of a generalized white noise functional $H(\omega)$ has the form Bernido and Carpio-Berdino (2002)

$$TH(\xi : \xi \in S) = \int_{S^*} \exp(\langle \omega, \xi \rangle) H(\omega) d\mu(\omega) \tag{6}$$

Similarly, the S-Transform of the white noise functional is given by

$$SH(\xi : \xi \in S) = C(\xi)\int_{S^*} \exp(\langle \omega, \xi \rangle) H(\omega) d\mu(\omega) \tag{7}$$

This implies that S- and T-Transform are related in this manner (Butanas and Caballar, 2016)

$$SH(\xi) = C(\xi)TH(-i\xi) \tag{8}$$

where $C(\xi)$ is the characteristic functional given in Eq. (2).

3. Feynman path integral as a White noise functional

Hamilton's Principle states that the least action is the path of motion of object that it will take (Thornton and Marion, 2004) and the action can be represented by the expression

$$S = \int_{t_o}^t L(x, \dot{x}) d\tau \tag{9}$$

where $L(x, \dot{x})$ is the Lagrangian of the system, and as prescribed by Feynman and Hibbs (1965), the propagator can be expressed as

$$K(x, x_o; \tau) = \int \exp\left(\frac{i}{\hbar} S_x\right) D_x \tag{10}$$

which is the summation of all the possible paths of the particle to take from initial to final position where D_x is known as the infinite-dimensional Lebesgue measure. Now, parametrizing the path of the taken with the Wernier's Brownian motion in Eq. (5) and taking its derivative (Bernido and Carpio-Berdino, 2002; Butanas and Caballar, 2016), we can write the Gaussian integral in (10) as

$$\frac{i}{\hbar} S_x = \left(\frac{i}{2}\int_0^t \omega^2(\tau)d\tau - \frac{i}{\hbar}\int_0^t V(x)d\tau\right) \tag{11}$$

which is in the case for the Lagrangian having the form

$$L(x, \dot{x}) = \frac{1}{2}m\dot{x}^2 - V(x)$$

Now, evaluating the Lebesgue measure D_x will lead us to integrate it over the Gaussian White noise measure $d\mu(\omega)$ with the relation (Butanas and Caballar, 2016)

$$D_x = \lim_{N\to 0}\prod^N (A_j)\prod^{N-1}(dx_j) = Nd^\infty x \tag{12}$$

with

$$Nd^\infty x \to Nd^\infty \omega = \exp\left[\frac{1}{2}\int_0^t \omega^2(\tau)d\tau\right] d\mu(\omega) \tag{13}$$

where N is the normalization constant. Observe that in Eq. (5), only the initial point is fixed but throughout its position is random. So, to fix the final trajectory of the particle, we use the Fourier decomposition of the Donsker-Delta (Arfken, 1985) as

$$\delta(x(t) - x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \exp(i\lambda(x(t) - x))d\lambda \tag{14}$$

With this, the position of particle is located at time t. Thus, with the Eqs. (11), (13) and (14) we can now rewrite the Feynman propagator in the framework of white noise analysis as

$$K(x, x_o; \tau) = N\int \left(\exp\left[\frac{i+1}{2}\int_0^t \omega^2(\tau)d\tau\right] \times \exp\left[-\frac{i}{\hbar}\int_0^t V(x)d\tau\right]\delta(x(t) - x)\right)d\mu(\omega) \tag{15}$$

4. Evaluation of Feynman path integral for coupled harmonic oscillator with uniform electric field in a bath using white noise analysis

The Lagrangian for a coupled harmonic oscillator with two-dimensional electric field and coupled in bath with uniform mass and frequency is

$$L_1 = \frac{1}{2}m(\dot{x}^2+\dot{y}^2) - \frac{1}{2}m\Omega^2(x^2+y^2) + qE(x+y) - \sigma xy - \gamma z(x+y) \quad (16)$$

$$L_2 = \frac{1}{2}m\dot{z}^2 - \frac{1}{2}m\Omega_z^2 z^2 \quad (17)$$

Changing the coordinates $x \to X + \frac{qE}{m\Omega^2}$ and $y \to Y + \frac{qE}{m\Omega^2}$ (Somerado and Convicto, 2017) and we can now write L_1 as

$$L_1 = \frac{1}{2}m\left(\dot{X}^2+\dot{Y}^2\right) - \frac{1}{2}m\Omega^2\left(X^2+Y^2\right) + \frac{1}{2}\frac{q^2E^2}{m\Omega^2} - \sigma XY - \gamma z(X+Y) \quad (18)$$

Notice that we neglected the coupling of the coordinate and the term $\frac{qE}{m\Omega^2}$ to maintain the system-system coupling and system-bath coupling. Furthermore, we utilize a transformation of coordinates (Pabalay and Bornales, 2008) to decouple X and Y, by doing so, we need the relation

$$X = X_| \cos\phi + Y_| \sin\phi \quad (19)$$

$$Y = -X_| \sin\phi + Y_| \cos\phi \quad (20)$$

Substituting Eqs. (19) and (20) into Eq. (18) yields

$$\begin{aligned} L_1 &= \frac{1}{2}m\left(\dot{X}_|{}^2+\dot{Y}_|{}^2\right) - \frac{1}{2}m\Omega^2\left(X_|^2+Y_|^2\right) + \frac{1}{2}\frac{q^2E^2}{m\Omega^2} \\ &- \sigma\left[-X_|^2\cos\phi\sin\phi + X_|Y_|\cos 2\phi + Y_|^2 cos\phi\sin\phi\right] \\ &- \gamma z[X_|(\cos\phi - \sin\phi) + Y_|(\cos\phi - \sin\phi)] \end{aligned} \quad (21)$$

and imposing the condition $\phi = \frac{2n+1}{4}\pi$ to eliminate the system-system coupling will yield

$$L_1 = \frac{1}{2}m\left(\dot{X}_|{}^2+\dot{Y}_|{}^2\right) - \frac{1}{2}m\Omega_x^2X_|^2 - \frac{1}{2}m\Omega_y^2Y_|^2 + \frac{q^2E^2}{m\left(\Omega_x^2+\Omega_y^2\right)} - \sqrt{2}\gamma zY_| \quad (22)$$

$$L_2 = \frac{1}{2}m\dot{z}^2 - \frac{1}{2}m\Omega_z^2 z^2 \quad (23)$$

where

$\Omega_x^2 = \Omega^2 - \frac{\sigma}{m}$

$\Omega_y^2 = \Omega^2 + \frac{\sigma}{m}$

Eqs. (22) and (23) are our new Lagrangian, in the last term of Eq. (22) is the remaining coupling term by system-bath coupling. Following the previous step but this time $\phi \to \theta$. Thus, we can successfully decoupled the Lagrangian for harmonic oscillator as

$$L_1 = \frac{1}{2}m\left(\dot{X}_|{}^2+\dot{Y}_{||}{}^2\right) - \frac{1}{2}m\Omega_x^2X_|^2 - \frac{1}{2}m\Phi_1^2Y_{||}^2 + \frac{q^2E^2}{m\left(\Omega_x^2+\Omega_y^2\right)} \quad (24)$$

$$L_2 = \frac{1}{2}m\dot{z}_|{}^2 - \frac{1}{2}m\Phi_2^2 z_|^2 \quad (25)$$

where the new frequencies are

$\Omega_x^2 = \Omega^2 - \frac{\sigma}{m}$

$\Omega_y^2 = \Omega^2 + \frac{\sigma}{m}$

$\Phi_1^2 = \Omega^2 + \frac{\sigma-\sqrt{2}\gamma}{m}$

$\Phi_2^2 = \Omega^2 + \frac{\sigma+\sqrt{2}\gamma}{m}$

Eq. (24) is identical to the obtained Lagrangian of Somerado and Convicto (2017) which enabled us to solve for the Feynman path integral in the framework of white noise analysis. For simplicity, we will define another variable Q that generalizes the coordinate. Then parametrizing the possible path taken (as shown in Eq. (5)) we have

$$Q(t) = Q_0 + \sqrt{\frac{\hbar}{m}}\int_0^t \omega(\tau)d\tau \quad (26)$$

with this, we can write our propagator as

$$K(Q,Q_0;\tau) = \int \exp\left(\frac{i}{\hbar}S_Q\right)D_Q \quad (27)$$

where the Feynman integrand for the free particle can be written as

$$\exp\left(\frac{i}{\hbar}S_Q\right) = \exp\left[\int \frac{i}{2}\omega^2(\tau)d\tau\right] = \exp\left[\frac{i}{\hbar}\langle\omega,\omega\rangle\right] \quad (28)$$

which contains a second degree white noise and makes us difficult to deal with. Now, with the aid of Taylor series expansion (Arfken, 1985), we can write the classical action in (28) as

$$S_Q = S_{Q_0} + \frac{1}{1!}\int \omega(\tau)\frac{\partial S_{Q_0}}{\partial\omega(\tau)}d\tau + \frac{1}{2!}\int \omega(\tau_1)\omega(\tau_2)\frac{\partial^2 S_{Q_0}}{\partial\omega(\tau_1)\partial\omega(\tau_2)}d\tau_1 d\tau_2 \quad (29)$$

Consequently, the propagator takes the form

$$K(Q,Q_0;\tau) = \int \exp\left[\frac{i}{\hbar}\left(S_{Q_0} + \langle\omega,S'\rangle + \frac{1}{2}\langle\omega,S''\omega\rangle + \frac{q^2E^2}{m\left(\Omega_x^2+\Omega_y^2\right)}\right)\right]D_Q \quad (30)$$

where

$$S' = \frac{\partial S_{Q_0}}{\partial\omega} = \sqrt{\frac{\hbar}{m}}\int_\tau V'(Q_O)d\tau \quad (31)$$

$$S'' = \frac{\partial^2 S_{Q_0}}{\partial\omega_1\partial\omega_2} = \sqrt{\frac{\hbar}{m}}\int_{\tau_1\vee\tau_2} V''(Q_O)d\tau \quad (32)$$

For convenience, we choose $Q_O = 0$ which gives$S(Q_O) = 0$ and since path is extremal at $Q_O = 0$, it follows that

$$S' = \frac{\partial S_0}{\partial\omega} = \sqrt{\frac{\hbar}{m}}\int_\tau V'(0)d\tau \Rightarrow 0, \quad (33)$$

$$S'' = \frac{\partial^2 S_0}{\partial\omega_1\partial\omega_2} = \sqrt{\frac{\hbar}{m}}\int_{\tau_1\vee\tau_2} V''(0)d\tau \Rightarrow \hbar\omega_Q(t-\tau_1\vee\tau_2) \quad (34)$$

With these assumptions, we can now construct the Feynman Propagator to be

$$K(Q,0;\tau) = N\int\left[\exp\left(\frac{i+1}{2}\langle\omega,\omega,\rangle - \frac{i}{\hbar}\langle\omega,S''\omega\rangle + \frac{i}{\hbar}\frac{q^2E^2}{2m\Omega_Q}\right)\delta(Q(t)-Q)\right]d\mu(\omega) \quad (35)$$

Where the first,second and third term of Eq. (35) is the relation of the flat and Gaussian measure, due to the harmonic potential and the potential due to the uniform electric field. Now, the role of the Donsker-Delta is to specify the trajectory of the particle to any Q at time t and defined as

$$\delta(Q(t)-Q) = \frac{1}{2\pi}\int_\lambda \exp\left[i\lambda\frac{\hbar}{m}\int\omega(\tau)d\tau\right]\exp(-i\lambda Q)d\lambda \quad (36)$$

Now, we let

$$I_R = \exp\left(\frac{i+1}{2}\langle\omega,\omega,\rangle - \frac{i}{\hbar}\langle\omega,S''\omega\rangle + \frac{i}{\hbar}\frac{q^2E^2}{2m\Omega_Q}\right) \quad (37)$$

to become white noise functional and to simply write (35) as

$$K(Q,0;\tau) = N\int I_R\delta(Q(t)-Q)d\mu(\omega) \quad (38)$$

We can evaluate the Gaussian white noise measure where our white noise functional I_R can be described by its S- and

T-Transforms. Substituting the integrand of (38) to (7) we can have our S-Transform of our functional

$$SI_R(\xi) = \frac{1}{2\pi}C(\xi)N\exp\left(\frac{i}{\hbar}\frac{q^2E^2}{2m\Omega_Q^2}t\right)\left(\int\exp(-i\lambda Q)d\lambda\right) \times \int\left(\exp\left[\frac{1}{2}\langle\omega, -(i+1)\omega + \frac{i}{\hbar}S''\omega\rangle\right]\exp\left[\langle\omega, \xi + i\lambda\omega 1_{[0,t]}\rangle\right]\right)d\mu(\omega) \quad (39)$$

Imposing the definition of the Gaussian measure and letting the second integral of (39) as

$$R = \sqrt[n]{\frac{1}{2\pi}}\int\left(\exp\left[\frac{1}{2}\langle\omega, -(i+1)\omega + \frac{i}{\hbar}S''\omega\rangle\right]\exp\left[\langle\omega, \xi + i\lambda\omega 1_{[0,t]}\rangle\right]\right)d^\infty(\omega) \quad (40)$$

and consider a Gaussian Integral of the form

$$\int\exp\left(-\frac{ax^2}{2} + bx\right)dx = \sqrt{\frac{2\pi}{a}}\exp\left(\frac{b^2}{2a}\right) \quad (41)$$

then we can write Eq. (40) as

$$R = \frac{1}{(2\pi)^{\frac{n+1}{2}}}\sqrt{\frac{(2\pi)^n}{\det(1-\hbar^{-1}S'')}} \times \exp\left[\frac{1}{2}\langle(\xi + i\lambda 1_{[0,t]}), (1-\hbar^{-1}S'')^{-1}(\xi + i\lambda 1_{[0,t]})\rangle\right] \quad (42)$$

for $\xi = 0$, then we can write the S-Transform Eq. (39) as

$$SI(\xi = 0) = \frac{1}{2\pi}C(\xi)N\det(1-\hbar^{-1}S'')^{-\frac{1}{2}} \times \exp\left(\frac{i}{\hbar}\frac{q^2E^2}{2m\Omega_Q^2}t\right) \times \left(\int_\lambda \exp\left[-\frac{i\hbar}{2m}\lambda^2\langle 1_{[0,t]}, (1-\hbar^{-1}S'')^{-1}1_{[0,t]}\rangle\right]\exp(-i\lambda Q)d\lambda\right) \quad (43)$$

Since for $\xi = 0 \Rightarrow SI = TI$ and applying the Gaussian integral shown in (41) to (38). Thus we can have the propagator as

$$K(Q,0;\tau) = SI(\xi = 0) = \left[\det(1-\hbar^{-1}S'')\right]^{-\frac{1}{2}}\sqrt{\frac{m}{2\pi i\hbar t\langle e, (1-\hbar^{-1}S'')^{-1}e\rangle}} \times \exp\left[\frac{imQ^2}{2\hbar t\langle e, (1-\hbar^{-1}S'')^{-1}e\rangle}\right] \quad (44)$$

where the unit vector $e = t^{-\frac{1}{2}}1_{[0,t]}$ (Pabalay and Bornales, 2008) and in the diagonalization of $(1-\hbar^{-1}S'')$, then defined the

$$\langle e, (1-\hbar^{-1}S'')^{-1}e\rangle = \frac{1}{\Omega_Q t}\tan(\Omega_Q t) \quad (45)$$

Now, defining the eigenvalues of $(1-\hbar^{-1}S'')$ as

$$A_k = 1 - \left(\frac{\Omega_Q t}{(k-\frac{1}{2})\pi}\right)^2 \quad (46)$$

which solves

$$\left[\det(1-\hbar^{-1}S'')\right]^{-\frac{1}{2}} = \left(\frac{1}{\cos\Omega_Q t}\right)^{\frac{1}{2}} \quad (47)$$

Hence, for the Gaussian integral for shown in (41) we have the propagator

$$K(Q,0;\tau) = \sqrt{\frac{m\Omega_Q}{2\pi i\hbar\sin\Omega_Q t}}\exp\left[\frac{im\Omega_Q}{2\hbar}Q^2\cot(\Omega_Q t)\right] \quad (48)$$

Now, using (37) as Gaussian Integral and for $\xi = 0$, we have the S–transform

$$SI_R = \sqrt{\frac{m\Omega_Q}{2\pi i\hbar\sin\Omega_Q t}}\exp\left(\frac{i}{\hbar}\frac{q^2E^2}{2m\Omega_Q^2}t + \frac{imQ^2}{2\hbar t\langle e, (1-\hbar^{-1}S'')^{-1}e\rangle}\right) \quad (49)$$

Finally, we can have our propagator for $X_|$ and $Y_{||}$ as

$$K(X_|, Y_{||};\tau) = \frac{m}{2\pi i\hbar}\sqrt{\frac{\Omega_x\Phi_1}{(\sin\Omega_x t)(\sin\Phi_1 t)}}\exp\left(\frac{i}{\hbar}\frac{q^2E^2}{m\left(\Omega_x^2+\Omega_y^2\right)}t\right) \times \exp\left[\frac{im\Omega_x}{4\hbar}\cot(\Omega_x t)\left(X_| - \frac{qE}{m\left(\Omega_x^2+\Omega_y^2\right)}\right)^2\right] \times \exp\left[\frac{im\Phi_1}{4\hbar}\cot(\Phi_1 t)\left(Y_{||} + \frac{qE}{m\left(\Omega_x^2+\Omega_y^2\right)}\right)^2\right] \quad (50)$$

Our next step is to obtain the full propagator and by doing so, we need to transform back the transformed coordinate to their respective original coordinates and gives the following relations

$$Y_{||} = Y_|\cos\theta - z_|\sin\theta$$
$$z_| = Y_|\sin\theta + z_|\cos\theta$$

and imposing the condition $\theta = \frac{2n+1}{4}\pi$ we have the expressions

$$Y_{||} \to \frac{1}{\sqrt{2}}(Y_| - z)$$
$$z_| \to \frac{1}{\sqrt{2}}(Y_| + z)$$

Repeating the process above to get X and Y

$$X_| = X\cos\theta - Y\sin\theta$$
$$Y_| = X\sin\theta + Y\cos\theta$$

In Addition, imposing the condition $\phi = \frac{2n+1}{4}\pi$ to get

$$X_| \to \frac{1}{\sqrt{2}}(X - Y)$$
$$Y_| \to \frac{1}{\sqrt{2}}(X + Y)$$

Now we can write the Transformed coordinate as

$$X_| \to \frac{1}{\sqrt{2}}(X - Y)$$
$$Y_{||} \to \frac{1}{\sqrt{2}}(X + Y - \sqrt{2}z)$$
$$z_| \to \frac{1}{\sqrt{2}}(X + Y + \sqrt{2}z)$$

Substituting these relations in Eq. (50), therefore, the full propagator can be written as

$$K(X,Y,Z;t) = \left(\frac{m}{2\pi i\hbar}\right)^{\frac{3}{2}}\sqrt{\frac{\Omega_x\Phi_1\Phi_2}{(\sin\Omega_x t)(\sin\Phi_1 t)(\sin\Phi_2 t)}}\exp\left(\frac{i}{\hbar}\frac{q^2E^2}{m\left(\Omega_x^2+\Omega_y^2\right)}t\right) \times \exp\left[\frac{im\Omega_x}{4\hbar}\cot(\Omega_x t)\left(\frac{1}{\sqrt{2}}(X-Y) - \frac{qE}{m\left(\Omega_x^2+\Omega_y^2\right)}\right)^2\right] \times \exp\left[\frac{im\Phi_1}{4\hbar}\cot(\Phi_1 t)\left(\frac{1}{\sqrt{2}}(X+Y-\sqrt{2}z) + \frac{qE}{m\left(\Omega_x^2+\Omega_y^2\right)}\right)^2\right] \times \exp\left[\frac{im\Phi_2}{4\hbar}\cot(\Phi_2 t)\left(\frac{1}{\sqrt{2}}(X+Y+\sqrt{2}z)\right)^2\right] \quad (51)$$

Which is the exact propagator when there is an effect of uniform electric field on the coupled harmonic oscillators in a single multimode harmonic oscillator. Eq. (51) agrees with the result in Butanas and Caballar (2016) when $E = 0$; in Pabalay and Bornales (2008) when $E = 0$ and $z = 0$; and in Somerado and Convicto (2017) when $z = 0$.

5. Conclusion

We solved the Feynman propagator for a system of coupled harmonic oscillator with two dimensional electric field interacting with a bath consisting a single multimode harmonic oscillator in the framework of white noise analysis. By having a two successive transformation of coordinates, the full propagator was obtained by multiplying the propagator where there is a presence of electric field and the propagator along the bath coordinate. In addition, the obtained full propagator was consistent with some previous articles for different cases.

It has been shown that the method of white noise analysis (Hida, 2001) can be used. To evaluate the propagators for open quantum systems and quantum mechanical problems. Particularly, it can also be applied to systems with N coupled harmonic oscillators which are all coupled to an environment, which can then be used to model quantum transport of energy excitations in solid state and biological systems (Butanas and Caballar, 2017). Furthermore, an extension of this study is to explore (Butanas and Caballar, 2017) with an inclusion of electric and magnetic fields and *consider a non-quadratic form of the Lagrangian such as in the form of Virial theorem in thermodynamics*.

6. Conflict of interest

None.

Acknowledgements

The authors would like to thank the Department of Physics, Mindanao State University-Main Campus for the support extended on this paper. Norodin A. Rangaig wishes to thank and acknowledge the Department of Science and Technology of the Philippines (DOST-SEI), for the financial support throughout the years.

References

Arfken, G., 1985. Mathematical Methods for Physicist. Academic Press, New York.
Bernido, C.C., Carpio-Berdino, M.V., 2002. White noise analysis and the Feynman path integral. In: Proceedings of the 3rd Jagna International Workshop, pp. 18–31.
Butanas, B.M., Caballar, R.C.F., 2016. Coupled harmonic oscillator in a multimode harmonic oscillator bath: derivation of quantum propagator and master equation. J. Phys. Res. Gate Publ.
Butanas, B.M., Caballar, R.C.F., 2017. Quantum propagator dynamics of a harmonic oscillator in a multimode harmonic oscillators environment using white noise functional analysis. J. Phys. Res. Gate Publ.
Feynman, R., Hibbs, A.R., 1965. Quantum Mechanics and Path Integral. McGraw-Hill, New York.
Hida, T., 1971. Quadratic functional of Brownian motion. J. Multivariate Anal. 58–69.
Hida, T., 2001. White noise approach to feynman integrals, J. Kor. Math. Soc. 38(2) 275–281.
Kou, H., 1996. White Noise Theory. CRS Press Inc..
Pabalay, J.R., Bornales, J.B., 2008. Coupled harmonic oscillators: a white noise functional approach, research gate.
Somerado, S.A.M., Convicto, V.C., 2017. Analytical solution of perturbed two-dimensional harmonic oscillator using white noise path integral approach (Undergraduate thesis). Mindanao State University-Marawi City.
Thornton, S.T., Marion, J.B., 2004. Classical Dynamics of Particles and Systems. Academic Press.