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ORIGINAL ARTICLE

A reliable algorithm for solving nonlinear Jaulent–Miodek equation

Jafar Biazar, Mostafa Eslami *

Department of Mathematics, Faculty of Sciences, University of Guilan, P.O. Box 413351914, P.C. 4193833697, Rasht, Iran

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Abstract Homotopy perturbation method has been applied to solve many functional equations so far. In this work, we propose this method (HPM), for solving Jaulent–Miodek (JM) equation (Kaya and El-Sayed, 2003; Fan, 2003). Numerical solutions obtained by the homotopy perturbation method are compared with the exact solutions. The results for some values of the variables are shown in tables and the solutions are presented as plots as well, showing the ability of the method.
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1. Introduction

Large varieties of physical, chemical, and biological phenomena are governed by nonlinear evolution equations. Except a limited number of these problems, most of them do not have precise analytical solutions, so they have to be solved by other methods which lead to closed approximate solutions or numerical solutions. Homotopy perturbation method has been used by many mathematicians and engineers to solve various functional equations. This method continuously deforms the difficult problems under study into a simple problem (He, 2000, 1999). In recent years the application of homotopy perturbation theory has appeared in many researches (He, 2003, 2004, 2005a, b, 2006; Biazar et al., 2007; Ariel et al., 2006; Siddiqui et al., 2006; Odibat and Momani, 2008; Cveticanin, 2006; Ozis and Yildirim, 2007).

2. Solution of system of partial differential equations by homotopy perturbation method

We first consider the system of partial differential equations written in an operator form

$$\begin{aligned} &\frac{\partial u_1}{\partial t}+\frac{\partial u_2}{\partial x_1}+\ldots+\frac{\partial u_n}{\partial x_{n-1}}+N_1=g_1,\\ &\frac{\partial u_2}{\partial t}+\frac{\partial u_1}{\partial x_1}+\ldots+\frac{\partial u_n}{\partial x_{n-1}}+N_2=g_2,\\ &\vdots\\ &\frac{\partial u_n}{\partial t}+\frac{\partial u_2}{\partial x_1}+\ldots+\frac{\partial u_1}{\partial x_{n-1}}+N_n=g_n. \end{aligned} \tag{1}$$

with initial conditions

* Corresponding author.
E-mail address: mostafa.eslami@hotmail.com (M. Eslami).

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$$
\begin{aligned}
&u_1(x_1, x_2, \dots x_{n-1}, 0) = f_1(x_1, x_2, \dots x_{n-1}),\\
&u_2(x_1, x_2, \dots x_{n-1}, 0) = f_2(x_1, x_2, \dots x_{n-1}),\\
&\vdots\\
&u_n(x_1, x_2, \dots x_{n-1}, 0) = f_n(x_1, x_2, \dots x_{n-1}).
\end{aligned} \tag{2}
$$

Where $N_1, N_2, \dots N_n$ are nonlinear operators, and $g_1, g_2, \dots g_n$ are inhomogeneous terms.

To solve system (1) by homotopy perturbation method, we construct the following homotopies:

$$
\begin{aligned}
&(1-p)\left(\frac{\partial U_1}{\partial t} - \frac{\partial u_{10}}{\partial t}\right)\\
&\quad + p\left(\frac{\partial U_1}{\partial t} + \frac{\partial U_2}{\partial x_1} + \dots + \frac{\partial U_n}{\partial x_{n-1}} + N_1 - g_1\right) = 0,\\
&(1-p)\left(\frac{\partial U_2}{\partial t} - \frac{\partial u_{20}}{\partial t}\right)\\
&\quad + p\left(\frac{\partial U_2}{\partial t} + \frac{\partial U_1}{\partial x_1} + \dots + \frac{\partial U_n}{\partial x_{n-1}} + N_2 - g_2\right) = 0,\\
&\vdots\\
&(1-p)\left(\frac{\partial U_n}{\partial t} - \frac{\partial u_{n0}}{\partial t}\right)\\
&\quad + p\left(\frac{\partial U_n}{\partial t} + \frac{\partial U_2}{\partial x_1} + \dots + \frac{\partial U_1}{\partial x_{n-1}} + N_n - g_n\right) = 0.
\end{aligned} \tag{3}
$$

consider the solution of the system (3) as the following

$$
\begin{aligned}
&U_1 = U_{10} + pU_{11} + p^2 U_{12} + \dots,\\
&U_2 = U_{20} + pU_{21} + p^2 U_{22} + \dots,\\
&U_3 = U_{30} + pU_{31} + p^2 U_{32} + \dots,\\
&\vdots\\
&U_n = U_{n0} + pU_{n1} + p^2 U_{n2} + \dots.
\end{aligned} \tag{4}
$$

Equating the coefficients of the terms with the identical powers of p leads to

$$
p^0 : \begin{cases}
\frac{\partial U_{10}}{\partial t} - \frac{\partial u_{10}}{\partial t} = 0,\\
\frac{\partial U_{20}}{\partial t} - \frac{\partial u_{20}}{\partial t} = 0,\\
\vdots\\
\frac{\partial U_{n0}}{\partial t} - \frac{\partial u_{n0}}{\partial t} = 0,
\end{cases}
$$

$$
p^1 : \begin{cases}
\frac{\partial U_{11}}{\partial t} + \frac{\partial u_{10}}{\partial t} + \frac{\partial U_{20}}{\partial x_1} + \dots + \frac{\partial U_{n0}}{\partial x_{n-1}} + M_{10} - g_1 = 0,\\
\frac{\partial U_{22}}{\partial t} + \frac{\partial u_{20}}{\partial t} + \frac{\partial U_{10}}{\partial x_1} + \dots + \frac{\partial U_{n0}}{\partial x_{n-1}} + M_{20} - g_2 = 0,\\
\vdots\\
\frac{\partial U_{n1}}{\partial t} + \frac{\partial u_{10}}{\partial t} + \frac{\partial U_{20}}{\partial x_1} + \dots + \frac{\partial U_{n0}}{\partial x_{n-1}} + M_{n0} - g_n = 0,
\end{cases}
$$

$$
p^2 : \begin{cases}
\frac{\partial U_{12}}{\partial t} + \frac{\partial U_{21}}{\partial x_1} + \dots + \frac{\partial U_{n1}}{\partial x_{n-1}} + M_{11} = 0,\\
\frac{\partial U_{22}}{\partial t} + \frac{\partial U_{11}}{\partial x_1} + \dots + \frac{\partial U_{n1}}{\partial x_{n-1}} + M_{21} = 0,\\
\vdots\\
\frac{\partial U_{n2}}{\partial t} + \frac{\partial U_{21}}{\partial x_1} + \dots + \frac{\partial U_{11}}{\partial x_{n-1}} + M_{n1} = 0,
\end{cases}
$$

$\vdots$

$$
p^j : \begin{cases}
\frac{\partial U_{1j}}{\partial t} + \frac{\partial U_{2j-1}}{\partial x_1} + \dots + \frac{\partial U_{nj-1}}{\partial x_{n-1}} + M_{1j-1} = 0,\\
\frac{\partial U_{2j}}{\partial t} + \frac{\partial U_{1j-1}}{\partial x_1} + \dots + \frac{\partial U_{nj-1}}{\partial x_{n-1}} + M_{2j-1} = 0,\\
\vdots\\
\frac{\partial U_{nj}}{\partial t} + \frac{\partial U_{2j-1}}{\partial x_1} + \dots + \frac{\partial U_{1j-1}}{\partial x_{n-1}} + M_{nj-1} = 0.
\end{cases}
$$

$\vdots$

Where $M_{ij},\ i = 1, 2, \dots n,\ j = 0, 1, 2, \dots n-1,$ are terms that are obtained with equating the coefficients of the nonlinear operators $N_{ij},\ i = 1, 2, \dots n,\ j = 0, 1, 2, \dots n-1$, with the identical powers of p.

For simplicity we take

$$
\begin{aligned}
&U_{10} = u_{10} = f_1(x_1, x_1, \dots, x_{n-1}),\\
&U_{20} = u_{20} = f_2(x_1, x_1, \dots, x_{n-1}),\\
&\vdots\\
&U_{n0} = u_{n0} = f_n(x_1, x_1, \dots, x_{n-1}).
\end{aligned} \tag{5}
$$

So we have

$$
\begin{aligned}
&U_{11}(x, t) = -\int_0^t \left(\frac{\partial U_{20}}{\partial x_1} + \dots + \frac{\partial U_{n0}}{\partial x_{n-1}} + M_{10} - g_1\right) dt,\\
&U_{21}(x, t) = -\int_0^t \left(\frac{\partial U_{10}}{\partial x_1} + \dots + \frac{\partial U_{n0}}{\partial x_{n-1}} + M_{20} - g_2\right) dt,\\
&\vdots\\
&U_{n1}(x, t) = -\int_0^t \left(\frac{\partial U_{20}}{\partial x_1} + \dots + \frac{\partial U_{10}}{\partial x_{n-1}} + M_{n0} - g_n\right) dt.
\end{aligned}
$$

For $j > 1$, we derive the following recurrent relation

$$
U_{1j}(x, t) = -\int_0^t \left(\frac{\partial U_{2j-1}}{\partial x_1} + \dots + \frac{\partial U_{n0}}{\partial x_{n-1}} + M_{1j-1}\right) dt,
$$
$j = 2, 3, \dots,$

$$
U_{2j}(x, t) = -\int_0^t \left(\frac{\partial U_{1j-1}}{\partial x_1} + \dots + \frac{\partial U_{nj-1}}{\partial x_{n-1}} + M_{2j-1}\right) dt,
$$
$j = 2, 3, \dots,$

$\vdots$

$$
U_{nj}(x, t) = -\int_0^t \left(\frac{\partial U_{2j-1}}{\partial x_1} + \dots + \frac{\partial U_{1j-1}}{\partial x_{n-1}} + M_{nj-1}\right) dt,
$$
$j = 2, 3, \dots.$

The approximate solutions of (1) can be obtained by letting p tend to one

$$
\begin{aligned}
&u_1 = \lim_{p \to 1} U_1 = U_{10} + U_{11} + U_{12} + \dots,\\
&u_2 = \lim_{p \to 1} U_2 = U_{20} + U_{21} + U_{22} + \dots,\\
&u_3 = \lim_{p \to 1} U_3 = U_{30} + U_{31} + U_{32} + \dots,\\
&\vdots\\
&u_n = \lim_{p \to 1} U_n = U_{n0} + U_{n1} + U_{n2} + \dots.
\end{aligned} \tag{6}
$$

3. Applications

Consider the following Jaulent–Miodek equation

$$\frac{\partial u}{\partial t}+\frac{\partial^3 u}{\partial x^3}+\frac{3}{2}v\frac{\partial^3 v}{\partial x^3}+\frac{9}{2}\frac{\partial v}{\partial x}\frac{\partial^2 v}{\partial x^2}-6u\frac{\partial u}{\partial x}-6uv\frac{\partial v}{\partial x}-\frac{3}{2}\frac{\partial u}{\partial x}v^2=0,$$
$$\frac{\partial v}{\partial t}+\frac{\partial^3 v}{\partial x^3}-6\frac{\partial u}{\partial x}v-6u\frac{\partial v}{\partial x}-\frac{15}{2}\frac{\partial v}{\partial x}v^2=0. \quad (7)$$

Subject to initial conditions

$$u(x,0)=\frac{1}{4}c_2-\frac{1}{4}b_0^2-\frac{1}{2}b_0\sqrt{c_2}\sec h(\sqrt{c_2}x)-\frac{3}{4}c_2\sec h^2(\sqrt{c_2}x),$$
$$v(x,0)=b_0+\sqrt{c_2}\sec h(\sqrt{c_2}x). \quad (8)$$

With the exact solutions

$$u=\frac{1}{4}(c_2-b_0^2)-\frac{1}{2}b_0\sqrt{c_2}\sec h\left(\sqrt{c_2}(x+\frac{1}{2}(6b_0^2+c_2)t)\right)-\frac{3}{4}c_2 \times \sec h^2\left(\sqrt{c_2}(x+\frac{1}{2}(6b_0^2+c_2)t)\right),$$

$$v=b_0+\sqrt{c_2}\sec h\left(\sqrt{c_2}(x+\frac{1}{2}(6b_0^2+c_2)t)\right).$$

Where b_0, c_2 are arbitrary constants.

For solving Eq. (7) with initial conditions (8) according to the homotopy perturbation method the following homotopy can be constructed

$$(1-p)\left(\frac{\partial U}{\partial t}-\frac{\partial u_0}{\partial t}\right)+p\left(\frac{\partial U}{\partial t}+\frac{\partial^3 U}{\partial x^3}+\frac{3}{2}V\frac{\partial^3 V}{\partial x^3}+\frac{9}{2}\frac{\partial V}{\partial x}\frac{\partial^2 V}{\partial x^2} -6U\frac{\partial U}{\partial x}-6UV\frac{\partial V}{\partial x}-\frac{3}{2}\frac{\partial U}{\partial x}V^2\right)=0,$$

$$(1-p)\left(\frac{\partial V}{\partial t}-\frac{\partial v_0}{\partial t}\right)+p\left(\frac{\partial V}{\partial t}+\frac{\partial^3 V}{\partial x^3}-6\frac{\partial U}{\partial x}V -6U\frac{\partial V}{\partial x}-\frac{15}{2}\frac{\partial V}{\partial x}V^2\right)=0,$$

or

$$\frac{\partial U}{\partial t}-\frac{\partial u_0}{\partial t}+p\left(\frac{\partial u_0}{\partial t}+\frac{\partial^3 U}{\partial x^3}+\frac{3}{2}V\frac{\partial^3 V}{\partial x^3}+\frac{9}{2}\frac{\partial V}{\partial x}\frac{\partial^2 V}{\partial x^2} -6U\frac{\partial U}{\partial x}-6UV\frac{\partial V}{\partial x}-\frac{3}{2}\frac{\partial U}{\partial x}V^2\right)=0,$$

$$\frac{\partial V}{\partial t}-\frac{\partial v_0}{\partial t}+p\left(\frac{\partial v_0}{\partial t}+\frac{\partial^3 V}{\partial x^3}-6\frac{\partial U}{\partial x}V-6U\frac{\partial V}{\partial x}-\frac{15}{2}\frac{\partial V}{\partial x}V^2\right)=0. \quad (9)$$

Suppose the solutions of Eq. (9) has the following form

$$U=U_0+pU_1+p^2U_2+\dots$$
$$V=V_0+pV_1+p^2V_2+\dots. \quad (10)$$

Substituting (10) into (9) and equating the coefficients of the terms with the identical powers of p leads to

$$P^0:\begin{cases}\frac{\partial U_0}{\partial t}-\frac{\partial u_0}{\partial t}=0,\\ \frac{\partial V_0}{\partial t}-\frac{\partial v_0}{\partial t}=0,\end{cases}$$

$$P^1:\begin{cases}\frac{\partial U_1}{\partial t}+\frac{\partial u_0}{\partial t}+\frac{\partial^3 U_0}{\partial x^3}+\frac{3}{2}V_0\frac{\partial^3 V_0}{\partial x^3}+\frac{9}{2}\frac{\partial V_0}{\partial x}\frac{\partial^2 V_0}{\partial x^2}-6U_0\frac{\partial U_0}{\partial x}\\ \quad -6U_0V_0\frac{\partial V_0}{\partial x}-\frac{3}{2}\frac{\partial U_0}{\partial x}V_0^2=0,\\ \frac{\partial V_1}{\partial t}+\frac{\partial v_0}{\partial t}+\frac{\partial^3 V_0}{\partial x^3}-6\frac{\partial U_0}{\partial x}V_0-6U_0\frac{\partial V_0}{\partial x}-\frac{15}{2}\frac{\partial V_0}{\partial x}V_0^2=0,\end{cases}$$

$$P^2:\begin{cases}\frac{\partial U_2}{\partial t}+\frac{\partial^3 U_1}{\partial x^3}+\frac{3}{2}V_0\frac{\partial^3 V_1}{\partial x^3}+\frac{3}{2}V_1\frac{\partial^3 V_0}{\partial x^3}+\frac{9}{2}\frac{\partial V_0}{\partial x}\frac{\partial^2 V_1}{\partial x^2}+\frac{9}{2}\frac{\partial V_1}{\partial x}\frac{\partial^2 V_0}{\partial x^2}\\ -6U_1\frac{\partial U_0}{\partial x}-6U_0\frac{\partial U_1}{\partial x}-6U_1V_0\frac{\partial V_0}{\partial x}-6U_0V_1\frac{\partial V_0}{\partial x}\\ \quad -6U_0V_0\frac{\partial V_1}{\partial x}-\frac{3}{2}\frac{\partial U_1}{\partial x}V_0^2-3\frac{\partial U_0}{\partial x}V_1V_0=0,\\ \frac{\partial V_2}{\partial t}+\frac{\partial^3 V_1}{\partial x^3}-6\frac{\partial U_1}{\partial x}V_0-6\frac{\partial U_0}{\partial x}V_1-6U_1\frac{\partial V_0}{\partial x}\\ \quad -6U_0\frac{\partial V_1}{\partial x}-\frac{15}{2}\frac{\partial V_1}{\partial x}V_0^2-15\frac{\partial V_0}{\partial x}V_1V_0=0,\end{cases}$$

$\vdots$

$$P^j:\begin{cases}\frac{\partial U_j}{\partial t}+\frac{\partial^3 U_{j-1}}{\partial x^3}+\frac{3}{2}\sum_0^{j-1}V_k\frac{\partial^3 V_{j-1-k}}{\partial x^3}+\frac{9}{2}\sum_{k=0}^{j-1}\frac{\partial V_k}{\partial x}\frac{\partial^2 V_{j-1-k}}{\partial x^2}-6\sum_{k=0}^{j-1}U_k\frac{\partial U_{j-1-k}}{\partial x}\\ -6\sum_{i=0}^{j-1}\sum_{k=0}^{j-i-1}U_iV_k\frac{\partial V_{j-k-i-1}}{\partial x}-\frac{3}{2}\sum_{i=0}^{j-1}\sum_{k=0}^{j-i-1}\frac{\partial U_i}{\partial x}V_kV_{j-k-i-1}=0,\\ \frac{\partial V_j}{\partial t}+\frac{\partial^3 V_{j-1}}{\partial x^3}-6\sum_{k=0}^{j-1}\frac{\partial U_k}{\partial x}V_{j-1-k}-6\sum_{k=0}^{j-1}U_k\frac{\partial V_{j-1-k}}{\partial x}\\ \quad -\frac{15}{2}\sum_{i=0}^{j-1}\sum_{k=0}^{j-i-1}\frac{\partial V_i}{\partial x}V_kV_{j-k-i-1}=0,\end{cases}$$

$\vdots$

Let us start with initial approximations $u(x,0)$ and $v(x,0)$ given by Eq. (8)

$$U_0=u_0=\frac{1}{4}c_2-\frac{1}{4}b_0^2-\frac{1}{2}b_0\sqrt{c_2}\sec h(\sqrt{c_2}x)-\frac{3}{4}c_2\sec h^2(\sqrt{c_2}x),$$
$$V_0=v_0=b_0+\sqrt{c_2}\sec h(\sqrt{c_2}x). \quad (11)$$

And we have the following recurrent equations for $j=1,2,3,\dots$.

$$U_j=-\int_0^t\left(\begin{array}{l}\frac{\partial^3 U_{j-1}}{\partial x^3}+\frac{3}{2}\sum_{k=0}^{j-1}V_k\frac{\partial^3 V_{j-1-k}}{\partial x^3}+\frac{9}{2}\sum_{k=0}^{j-1}\frac{\partial V_k}{\partial x}\frac{\partial^2 V_{j-1-k}}{\partial x^2}\\ -6\sum_{k=0}^{j-1}U_k\frac{\partial U_{j-1-k}}{\partial x}-6\sum_{i=0}^{j-1}\sum_{k=0}^{j-i-1}U_iV_k\frac{\partial V_{j-k-i-1}}{\partial x}\\ -\frac{3}{2}\sum_{i=0}^{j-1}\sum_{k=0}^{j-i-1}\frac{\partial U_i}{\partial x}V_kV_{j-k-i-1}\end{array}\right)dt,$$

$$V_j=-\int_0^t\left(\frac{\partial^3 V_{j-1}}{\partial x^3}-6\sum_{k=0}^{j-1}\frac{\partial U_k}{\partial x}V_{j-1-k}-6\sum_{k=0}^{j-1}U_k\frac{\partial V_{j-1-k}}{\partial x} -\frac{15}{2}\sum_{i=0}^{j-1}\sum_{k=0}^{j-i-1}\frac{\partial V_i}{\partial x}V_kV_{j-k-i-1}\right)dt. \quad (12)$$

All terms of the expansions (10) can be obtained via iterative formula (12)

Table 1 The numerical results, when $b_0 = c_2 = 0.01$ for solutions of Eq. (7) for initial conditions (8).

x	t	$u^*(x,t)$	$\lvert u^* - u\rvert$	$v^*(x,t)$	$\lvert v^* - v\rvert$
0.1	0.1	−0.00552421681535966474	4×10^{-20}	0.109994947072326492	1.6×10^{-19}
0.1	0.15	−0.00552421268119929705	2.1×10^{-19}	0.109994920399016371	8.3×10^{-19}
0.15	0.1	−0.00552324416776666749	4×10^{-20}	0.109988671429124380	1.6×10^{-19}
0.2	0.3	−0.00552185135682729130	3.26×10^{-18}	0.109979684175985571	1.329×10^{-17}
0.35	0.25	−0.00551544200337122719	1.55×10^{-18}	0.109938317095391529	6.37×10^{-18}
0.45	0.45	−0.00550916080832543826	1.622×10^{-17}	0.109897761088575401	6.657×10^{-17}
0.5	0.3	−0.00550553384209772092	3.19×10^{-18}	0.109874335473663468	1.311×10^{-17}
0.45	0.7	−0.00550906801481426520	9.504×10^{-17}	0.109897161826108341	3.8981×10^{-16}
0.4	0.8	−0.00551234934376313588	1.6273×10^{-16}	0.109918350605131830	6.6674×10^{-16}
0.8	0.8	−0.00547508560544371429	1.5616×10^{-16}	0.109677468293014595	6.4730×10^{-16}
0.85	0.95	−0.00546861261467419606	3.0831×10^{-16}	0.109635567784157087	1.28058×10^{-15}
0.9	0.9	−0.00546189829138478970	2.4645×10^{-16}	0.109592087075285587	1.02594×10^{-15}
0.95	0.9	−0.00545477408645114569	2.4448×10^{-16}	0.109545931955545074	1.02005×10^{-15}
1	1	−0.00544719469399408060	3.6944×10^{-16}	0.109496805097819525	1.54524×10^{-15}

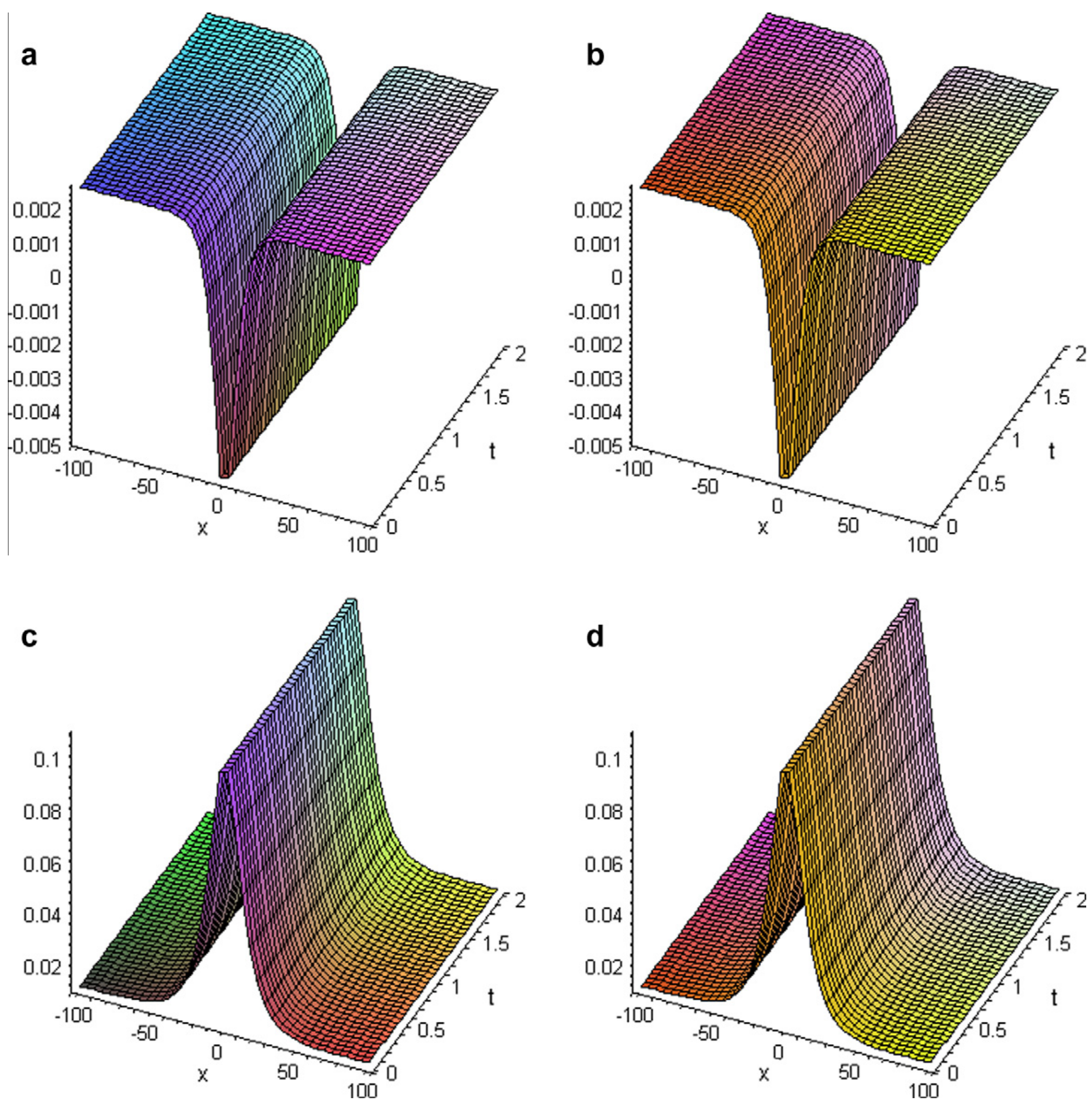

Figure 1 The numerical results for $u^*(x,t)$, $v^*(x,t)$ are, respectively, (a) and (c) in comparison with the analytical solutions $u(x,t)$ and $v(x,t)$ are, respectively, (b) and (d) with the initial conditions (8) of Eq. (7), when $b_0 = 0.01$ and $c_2 = 0.01$.

$$U_1 = \frac{3}{4}\sqrt{c_2^5}\sec h^2(\sqrt{c_2}x)\tanh(\sqrt{c_2}x)t + 6b_0c_2^2 \times \sec h(\sqrt{c_2}x)\tanh^3(\sqrt{c_2}x)t - \frac{23}{4}b_0c_2^2\sec h(\sqrt{c_2}x) \times \tanh(\sqrt{c_2})x)t + \frac{3}{2}tb_0^3c_2\sec h(\sqrt{c_2}x)\tanh(\sqrt{c_2}x) + \frac{9}{2}tb_0^2 \times \sqrt{c_2^3}\sec h^2(\sqrt{c_2}x)\tanh(\sqrt{c_2}x) + 6tb_0c_2^2\sec h^3(\sqrt{c_2}x) \times \tanh(\sqrt{c_2}x),$$

$$V_1 = 6c_2^2\sec h(\sqrt{c_2}x)\tanh^3(\sqrt{c_2}x)t - \frac{13}{2}c_2^2\sec h(\sqrt{c_2}x) \times \tanh(\sqrt{c_2}x)t - 3tb_0^2c\sec h(\sqrt{c_2}x)\tanh(\sqrt{c_2}x) + 6tc_2^2 \times \sec h^3(\sqrt{c_2}x)\tanh(\sqrt{c_2}x),$$

$$\vdots$$

An approximation to the solution of (7) can be obtained by letting p tend to one.

For numerical study four terms approximations have been considered. Suppose $u^* = \sum_{j=0}^{3} U_j$, and $v^* = \sum_{j=0}^{3} V_j$, the results are presented in Table 1 and Fig. 1

4. Conclusions

In this article, we have applied homotopy perturbation method for solving the nonlinear Jaulent–Miodek (JM) equation (Kaya and El-Sayed, 2003; Fan, 2003). The approximate solutions obtained by the homotopy perturbation method are compared with the exact solutions. The results show that the homotopy perturbation method is a powerful mathematical tool for solving systems of nonlinear partial differential equations, which appears in mathematical modeling of different phenomena. This model has been solved by Adomian method, as well (Kaya and El-Sayed, 2003) Homotopy perturbation method in comparison with Adomian's decomposition method has the advantage of overcoming the difficulty arising in calculating Adomain polynomial in ADM. In our work, we use the maple package to carry the computations.

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