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SHORT COMMUNICATION

A note on "Jacobi elliptic function solutions for the modified Korteweg–de Vries equation"

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Abstract The recently published paper "Jacobi elliptic function solutions for the modified Korteweg–de Vries equation" [J. King Saud Univ. Sci. 25 (2013) 271–274] is analyzed. We show that these Jacobi elliptic function solutions obtained by the authors do not satisfy the original modified Korteweg–de Vries equation.

Recently, Wang and Xiang (2013) studied the modified Korteweg–de Vries (mKdV) equation in the form

$$\lambda u_t - \mu u^2 u_x + w u_{xxx} = 0, \tag{1}$$

where λ, μ and w are constant parameters. They sought solutions by taking the traveling waves into account

$$u(x,t) = u(\xi), \xi = \alpha x + kt. \tag{2}$$

As a result they transform (1) to an ordinary differential equation (ODE) in the form

$$c + k\lambda u - \frac{\mu\alpha}{3}u^3 + w\alpha^3 u_{\xi\xi} = 0, \tag{3}$$

where c is an integral constant. They expressed u as

$$u = u_0 + pu_1 + p^2 u_2, \tag{4}$$

where p is a small parameter, and u_0, u_1 and u_2 are ansatz functions to be determined.

Substituting (4) into (3), we have

$$\begin{aligned} &c + k\lambda u_0 - \frac{\mu\alpha}{3}u_0^3 + w\alpha^3 u_{0\xi\xi} + p\cdot(k\lambda u_1 - \mu\alpha u_0^2 u_1 + w\alpha^3 u_{1\xi\xi}) \\ &\quad + p^2\cdot(k\lambda u_2 - \mu\alpha u_0^2 u_2 - \mu\alpha u_0 u_1^2 + w\alpha^3 u_{2\xi\xi}) \\ &\quad - p^3\cdot\left(2\mu\alpha u_0 u_1 u_2 + \frac{\mu\alpha}{3}u_1^3\right) - p^4\cdot\mu\alpha u_2(u_0u_2 + u_1^2) \\ &\quad - p^5\cdot\mu\alpha u_1 u_2^2 - p^6\cdot\frac{\mu\alpha}{3}u_2^3 = 0. \end{aligned} \tag{5}$$

Since (4) is a solution of (3), (5) must hold for all values of p. Then each coefficient of p must vanish independently. Thus

$$c + k\lambda u_0 - \frac{\mu\alpha}{3}u_0^3 + w\alpha^3 u_{0\xi\xi} = 0, \tag{6}$$

$$k\lambda u_1 - \mu\alpha u_0^2 u_1 + w\alpha^3 u_{1\xi\xi} = 0, \tag{7}$$

$$k\lambda u_2 - \mu\alpha u_0^2 u_2 - \mu\alpha u_0 u_1^2 + w\alpha^3 u_{2\xi\xi} = 0, \tag{8}$$

$$2\mu\alpha u_0 u_1 u_2 + \frac{\mu\alpha}{3}u_1^3 = 0, \tag{9}$$

$$\mu\alpha u_2(u_0u_2 + u_1^2) = 0, \quad \mu\alpha u_1 u_2^2 = 0, \quad \frac{\mu\alpha}{3}u_2^3 = 0. \tag{10}$$

The solution of (9) and (10) is $u_2 = u_1 = 0$. In this case, the left hand side of (7) and (8) vanishes. So $u = u_0$, and both of u

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and u_0 satisfy the same ordinary differential equation, which means nothing can be obtained from above approach.

It is a pity that Wang and Xiang (2013) obtained u_0, u_1 and u_2 through solving only three equations, namely (6)–(8) and neglected the rest of the equations. Then they claimed "The Jacobi elliptic function solutions, the trigonometric solutions and hyperbolic solutions are obtained". It is not difficult to find that these solutions, namely (22)–(24) in their paper, are not admitted by the original mKdV equation.

In particular, the solution (24) in their paper, namely,

$$u(x,t) = a_1 \tanh\xi + pg_0 \mathrm{sech}^2\xi + p^2 b_1 \tanh\xi + p^2 b_3 \tanh^3\xi \tag{11}$$

can also be checked by the tanh method (see, for example, Malfliet (2004)) easily. In fact, by replacing $\mathrm{sech}^2\xi$ with $1 - \tanh^2\xi$, the solution (11) is transformed into the following

$$u(x,t) = pg_0 + (a_1 + p^2 b_1) Y - pg_0 Y^2 + p^2 b_3 Y^3, \tag{12}$$

where $Y = \tanh\xi$ is an introduced variable.

The tanh method proposes the following solution admitted by (3)

$$u = a_0 + a_1 Y + \cdots + a_N Y^N = \sum_{n=0}^{n=N} a_n Y^n, \tag{13}$$

where N is a positive integer to be determined. Balancing the highest order linear term $u_{\xi\xi}$ in (3) with the highest order nonlinear term u^3 gives $N + 2 = 3N$, so $N = 1$, which means the proposed solution will be

$$u = a_0 + a_1 Y. \tag{14}$$

However, the solution (12) indicates $N = 3$. This is a contradiction.

Finally, we notice that $c = 0$ in the results of Wang and Xiang (2013). And in this case, (3) can be changed to an ODE of degree four in the form

$$u_\xi^2 = h_0 + h_2 u^2 + h_4 u^4, \tag{15}$$

where h_0 is an arbitrary constant, and h_2 and h_4 are certain constants. Many exact solutions of (15) have been presented and have played an important role in recently published papers, for example, Shang (2010); Alofi and Abdelkawy (2012); Ebaid and Aly (2012); Li et al. (2012); Ma et al. (2012); and Malik et al. (2012b,a). And its general solutions can be found in some literature, for example, Whittaker and Watson (1996) and Liu et al. (2014). Consequently, we can obtain general solutions of (3) in the case of $c = 0$ directly. Due to limited space, here we omit its discussion.

Remark. *It is worth to mention that if we regard the results obtained by* Wang and Xiang (2013) *as approximate solutions (in contrast to exact solutions), we can find that the derivations and results are correct. However, in this sense, these authors should express them in a proper way, and relevant calculated precisions for these solutions should be discussed as well, for example, see* Holmes (2013).

In summary, solutions obtained by these authors do not satisfy the original mKdV equation. We have to point out that similar concerns are discussed in some other published papers as well (see, for examples, Kudryashov (2009) and Kudryashov and Shilnikov (2012)). We believe that our work will help people have a good understanding of the results obtained by Wang and Xiang (2013).

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