



# Lie group analysis of thermophoresis on a vertical surface in a porous medium

Hossam A. Nabwey<sup>a,b,\*</sup>, Hamed A. EL-Mky<sup>c</sup>

<sup>a</sup> Department of Mathematics, Prince Sattam bin Abdulaziz University, College of Science and Humanity Studies, Al-Kharj, Saudi Arabia

<sup>b</sup> Department of Basic Engineering Science, Faculty of Engineering, Menoufia University, Shebin El-Kom 32511, Egypt

<sup>c</sup> Department of Mathematics, Aswan University, Faculty of Science, 81528, Egypt

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## ABSTRACT

The magnetohydrodynamic and thermophoretic effects on a vertical surface in a porous medium are investigated. The fascinating aspects of thermo diffusion and diffusion-thermo impacts are given. Mathematical modelling via Lie group method was applied. Thereafter, the infinitesimal generators of governing equations are computed. Using suitable similarity variables the existing system which was non-linear is converted into expressions having no dimensions. The resulting expressions were solved numerically using shooting method and the characteristics of embedded parameters such as temperature, velocity, concentration profiles have been displayed graphically. We have compared our findings with those of previous ones to assure the affirmity of our analysis. The change in Sherwood number for progressive Dufour solutal values are also analyzed.

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## 1. Introduction

Since last few decades' fluid flow in porous media has been intensively investigated by various mathematicians. In many of these investigations, emphasis has been given to insulation systems - both granular as well as fibrous, which are used to contain the movement of radio nuclides from nuclear waste material deposits. Since then many papers were published on boundary-layer flow past surfaces of diverse flow configuration models. For some applications which require further theoretical and experimental developments, see [Nield and Bejan \(2013\)](#), [Vafai \(2000\)](#), [Pop and Ingham \(2001\)](#), and [Ingham and Pop \(1998\)](#).

Magnetohydrodynamic (MHD) is a momentous and riveting region of science which delineates with the movement of electrically conducting fluid. The paramount factuality on the backside of MHD is to beget current due to applying magnetic field; the

impact of this process induces Lorentz force which significantly impacts the fluid motion. The multifarious MHD fluids exist in nature as salt water, plasmas and electrolysis; see [Ingham and Pop \(1998\)](#), [Pai \(1962\)](#). Lately, MHD is a content of intensive era of study because of its numerous industrial applications like glass manufacturing, MHD electrical generation process and procedure for magnetic materials. Additionally, it has alluring characteristics in the field of geophysics and astrophysics i.e. it is habituated in energy extraction and radio propagation. Engineering products like MHD fluid flow meters and MHD pumps which exploited the MHD phenomenon. However, in view of such motivation, authors have investigations in MHD flow for several geometries (see refs. [Liuta and Larachi, 2003](#); [Chamkha et al., 2013](#); [Rashad, 2008](#); [EL-Kabeir et al., 2010](#); [Hayat et al., 2013](#)).

Heat and mass transfer flows with Dufour and Soret impacts are of substantial attraction among numerous authors owing to several grown engineering applications like manufacture for rubber and plastic sheets, Catalytic process, chemical production engineering, geophysical procedures, compact heat insulation exchangers and layout of nuclear reactor. A considerable amount of investigation has been performed to demonstrate the significance of these two impacts for various aspects - For instance, [Dursunkaya and Worek \(1992\)](#) and [Anghel et al. \(2000\)](#) investigated for fluids, the effect on convection flow of fluid over a stretchable surface porous medium by [Hayat et al. \(2010\)](#), [Makinde and Olanrewaju \(2011\)](#) analyzed the combined convective with Dufour and Soret impacts over a

\* Corresponding author at: Department of Mathematics, Prince Sattam bin Abdulaziz University, College of Science and Humanity Studies, Al-Kharj, Saudi Arabia.

E-mail address: [eng\\_hossam21@yahoo.com](mailto:eng_hossam21@yahoo.com) (H.A. Nabwey).

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**Nomenclature**

$x, y$	Coordinates along and normal to the surface (m)	$kn$	Knudsen number
$\tilde{u}, \tilde{v}$	Velocity components in x and y directions (m/s)	$Z_1, Z_2, Z_3, Z_4, Z_5, Z_6$	Constants
$T_s$	Temperature at the surface (k)	$Z_7, Z_8, Z_9, Z_{10}, k_1, k_2, k_3, k_4$	Arbitrary constants
$C_s$	Concentration at the surface (kg/m <sup>2</sup> )	$Ra$	Rayleigh number
$\tilde{T}$	Temperature (k)	$L$	Characteristic length of the plate (m)
$C$	Concentration (kg/m <sup>2</sup> )	$Pr$	Prandtl number
$T_{am}$	Ambient temperature (k)	$Le$	Lewis number
$C_{am}$	Ambient concentration (kg/m <sup>2</sup> )	$V(x)$	Transpiration velocity (m/s)
$K$	Permeability of the porous medium (m <sup>2</sup> )	$Df$	Dufour number
$\rho$	Fluid density (kg/m <sup>3</sup> )	$Sr$	Soret number
$\mu$	Viscosity of the base fluid (kg m <sup>-1</sup> s <sup>-1</sup> )	$B$	The parameter of buoyancy
$H_0$	Magnetic field (T)	$CT$	The parameter for thermophoresis
$g$	Gravitational acceleration (m/s <sup>2</sup> )	$D$	Chemical molecular diffusivity
$\nu$	Kinematic viscosity (m <sup>2</sup> s <sup>-1</sup> )	$\theta$	Dimensionless temperature function
$\beta_T$	Volumetric thermal coefficient of the base fluid (1/k)	$\phi$	Dimensionless species concentration function
$\beta_c$	Solutal expansion coefficient of the base fluid (m <sup>2</sup> /kg)	$\psi$	Stream function (m <sup>2</sup> ·s <sup>-1</sup> )
$C_p$	Specific heat due to constant pressure (J·kg <sup>-1</sup> ·K <sup>-1</sup> )	$\eta$	Non-dimensional pseudo-similarity variable
$\alpha$	Thermal diffusivity (m <sup>2</sup> s <sup>-1</sup> )	$F$	Dimensionless stream function
$V_T$	Thermophoretic velocity (m/s)	$\epsilon$	Lie group parameter
$Kv$	The thermophoretic diffusivity (m <sup>-2</sup> s <sup>-1</sup> )	$F, \Theta, \Phi$	Arbitrary functions
$\lambda g \lambda p$	Thermal conductivity of gas and diffused particles, respectively (W m <sup>-1</sup> k <sup>-1</sup> )		

permeable surface roving through a mixture of fluid. And for more details see [Beg et al. \(2009\)](#), [Aziz \(2008\)](#) and [Chamkha and Rashad \(2014; Rashad and Chamkha, 2014\)](#).

Many mathematicians were illustrated and explained in their research papers the importance of applying the group theory in the field of fluid mechanics. See [\(Bakier et al., 2009; Nabwey et al., 2017, 2015; EL-Kabeir et al., 2008\)](#)

The prime purpose of the current investigation is to analyse the influence of Soret and Dufour effects on heat and mass transfer by MHD flow through a vertical surface in a porous medium with thermophoresis. The mathematical model which represent the case study is transformed to couple non-linear ODEs by impose the Lie group method. The system is solved numerically by shooting method. Finally, the effect of the associated physical fluid dynamical parameters on the flow is exhibited and analyzed using the graphical aid and tabular forms.

**2. Analysis**

Here we consider the flow of electrically conducting fluid over a heated vertical surface passing through a porous medium. The surface is maintained at constant temperature  $T_s$  and the concentration  $C_s$ , while their ambient values are denoted by  $T_{am}$  and  $C_{am}$  respectively. From the above basic assumptions and following [Chiou \(1998\)](#) and [Chamkha, and Pop \(2004\)](#) the governing equations for an unsteady flow are:

$$\frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} = 0 \tag{1}$$

$$\tilde{u} \left( 1 + \frac{K\sigma\mu_e^2 H_0^2}{\mu} \right) = \frac{gK}{\nu} \left( \beta_T(\tilde{T} - T_{am}) + \beta_c(\tilde{C} - C_{am}) \right) \tag{2}$$

$$\tilde{u} \frac{\partial \tilde{T}}{\partial x} + \tilde{v} \frac{\partial \tilde{T}}{\partial y} = \alpha_m \frac{\partial^2 \tilde{T}}{\partial y^2} + \frac{D_m k_t}{z_1 c_p} \frac{\partial^2 \tilde{C}}{\partial y^2} \tag{3}$$

$$\tilde{u} \frac{\partial \tilde{C}}{\partial x} + \tilde{v} \frac{\partial \tilde{C}}{\partial y} + \frac{\partial}{\partial y} (V_T \tilde{C}) = D_m \frac{\partial^2 \tilde{C}}{\partial y^2} + \frac{D_m k_t}{T_m} \frac{\partial^2 \tilde{T}}{\partial y^2} \tag{4}$$

In this work, the boundary conditions were taken as:

$$\begin{aligned} \text{at } y = 0 : \tilde{T} &= T_s, \tilde{C} = C_s, \tilde{v} = V(x) \\ \text{as } y \rightarrow \infty : \tilde{T} &\rightarrow T_{am}, \tilde{C} \rightarrow C_{am}, \tilde{u} \rightarrow 0, \end{aligned} \tag{5}$$

where  $V(x)$  is the permeability of the porous surface (negative sign refer to suction and a positive sign indicates injection). Since the temperature gradient is higher along y-axis, the thermophoretic velocity  $V_T$  is considered along y-axis and is given by:

$$V_T = -\frac{\kappa\nu}{\tilde{T}} \frac{\partial \tilde{T}}{\partial y} \tag{6}$$

where  $\kappa$  represent the thermophoretic coefficient and its range from 0.2 to 1.2 (see [Batchelor and Shen, 1985](#)) and is defined by [\(Talbot et al., 1980\)](#):

$$\kappa = \frac{2z_1(\lambda_g/\lambda_p + z_2 kn)[1 + kn(z_3 + z_4 e^{-z_5/kn})]}{(1 + 3z_6 kn)(1 + 2\lambda_g/\lambda_p + 2z_2 kn)} \tag{7}$$

The next step is to define the dimensionless variables as:

$$\begin{aligned} X = \frac{x}{L}, Y = Ra^{1/2} \left( \frac{y}{L} \right), \tilde{U} = \frac{\tilde{u}}{U_c}, \tilde{V} = Ra^{1/2} \left( \frac{\tilde{v}}{U_c} \right) V_t \\ = Ra^{1/2} \left( \frac{v_t}{U_c} \right), \theta = \frac{\tilde{T} - T_{am}}{T_s - T_{am}}, \phi = \frac{\tilde{C} - C_{am}}{C_s - C_{am}} \end{aligned} \tag{8}$$

where,

$U_c = \frac{g\beta_r K(T_s - T_{am})}{\nu}$  represent the characteristic velocity

$Ra = \frac{gK\beta_r(T_s - T_{am})L}{z_m \nu}$  represent the Rayleigh number,

$L$  is a characteristic length of the plate.

From the above Eqs. (1)-(5) becomes:

$$\frac{\partial \tilde{U}}{\partial X} + \frac{\partial \tilde{V}}{\partial Y} = 0 \tag{9}$$

$$\tilde{U} (1 + M) = \theta + B\phi \tag{10}$$

$$\tilde{U} \frac{\partial \theta}{\partial X} + \tilde{V} \frac{\partial \theta}{\partial Y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2} + Df \frac{\partial^2 \phi}{\partial Y^2} \tag{11}$$

$$\tilde{U} \frac{\partial \phi}{\partial X} + \tilde{V} \frac{\partial \phi}{\partial Y} + \frac{\partial}{\partial Y}(V_T \phi) = \frac{1}{Le} \frac{\partial^2 \phi}{\partial Y^2} + Sr \frac{\partial^2 \theta}{\partial Y^2} \tag{12}$$

$$V_T = -\frac{\kappa Pr}{(C_T + \theta)} \frac{\partial \theta}{\partial Y} \tag{13}$$

where  $C_T$  is the parameter for thermophoresis while  $B$  being parameter of buoyancy. Dufour and Soret numbers respectively, are defined as:

$$Pr = \frac{\nu}{\alpha_m}, Le = \frac{\alpha_m}{D}, M = \frac{K\sigma\mu_e^2 H_0^2}{\mu}, C_T = \frac{(T_s - T_{am})}{T_{am}},$$

$$B = \frac{\beta_c(Z_1 - C_{am})}{\beta_T(T_s - T_{am})}, Df = \frac{D_m k_t (C_s - C_{am})}{Z_1 c_p \alpha_m (T_s - T_{am})}, Sr = \frac{D_m k_t (T_s - T_{am})}{Z_1 c_p \alpha_m (C_s - C_{am})} \tag{14}$$

Then the new boundary conditions are:

$$\text{at } Y = 0 : \tilde{V} = V(X), \theta = 1, \phi = 1 \tag{15}$$

$$\text{as } Y \rightarrow \infty : \tilde{U} \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0$$

Using the standard definition of the stream function  $\psi(x, y)$  and putting  $\tilde{U} = \frac{\partial \psi}{\partial Y}, \tilde{V} = -\frac{\partial \psi}{\partial X}$ , into Eqs. (9)-(12), we have

$$(1 + M) \frac{\partial \psi}{\partial Y} = \theta + B\phi \tag{16}$$

$$\frac{\partial \psi}{\partial Y} \frac{\partial \theta}{\partial X} - \frac{\partial \psi}{\partial X} \frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial Y^2} + Df \frac{\partial^2 \phi}{\partial Y^2} \tag{17}$$

$$\frac{\partial \psi}{\partial Y} \frac{\partial \phi}{\partial X} - \frac{\partial \psi}{\partial X} \frac{\partial \phi}{\partial Y} + \frac{\partial}{\partial Y}(V_T \phi) = \frac{1}{Le} \frac{\partial^2 \phi}{\partial Y^2} + Sr \frac{\partial^2 \theta}{\partial Y^2} \tag{18}$$

And the boundary conditions (5) become:

$$\text{at } Y = 0 : -\psi_X = V(X), \theta = 1, \phi = 1 \tag{19}$$

$$\text{as } Y \rightarrow \infty : \psi_X \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0$$

### 3. Determination of the symmetry groups

#### 3.1. Lie-point symmetries equations

For symmetry of a differential equations and some methods of its calculations we refer to (Ovsiannikov, 1982; Olver, 1986; Bluman and Kumei, 1989; Ibragimov, 1999; EL-Kabeir et al., 2007, 2008; Nabwey et al., 2015, 2017); consider the one-parameter Lie group of infinitesimal transformations in  $(X, Y, \psi, \theta, \phi)$  given by:

$$X^* = X + \varepsilon \xi^1(X, Y, \psi, \theta, \phi) + O(\varepsilon^2)$$

$$Y^* = Y + \varepsilon \xi^2(X, Y, \psi, \theta, \phi) + O(\varepsilon^2)$$

$$\psi^* = \psi + \varepsilon \mu^1(X, Y, \psi, \theta, \phi) + O(\varepsilon^2)$$

$$\theta^* = \theta + \varepsilon \mu^2(X, Y, \psi, \theta, \phi) + O(\varepsilon^2)$$

$$\phi^* = \phi + \varepsilon \mu^3(X, Y, \psi, \theta, \phi) + O(\varepsilon^2), \tag{20}$$

where  $\varepsilon$  is the Lie group parameter. Since Eqs. (16)-(18) are not affected by these transformations we get an over-determined, linear system of equations for the infinitesimals  $\xi^1(X, Y, \psi, \theta, \phi), \xi^2(X, Y, \psi, \theta, \phi), \mu^1(X, Y, \psi, \theta, \phi), \mu^2(X, Y, \psi, \theta, \phi)$  and  $\mu^3(X, Y, \psi, \theta, \phi)$ .

The associated lie algebra of these infinitesimal symmetries take the form:

$$\aleph = \xi^1(X, Y, \psi, \theta, \phi) \frac{\partial}{\partial X} + \xi^2(X, Y, \psi, \theta, \phi) \frac{\partial}{\partial Y} + \mu^1(X, Y, \psi, \theta, \phi) \frac{\partial}{\partial \psi} + \mu^2(X, Y, \psi, \theta, \phi) \frac{\partial}{\partial \theta} + \mu^3(X, Y, \psi, \theta, \phi) \frac{\partial}{\partial \phi} \tag{21}$$

The action of  $\aleph$  is extended to all derivatives appearing in (16)-(18) through the second prolongation  $\aleph^{(2)}$ ,

$$\aleph^{(2)} = \aleph + \mu_X^1 \frac{\partial}{\partial \psi_X} + \mu_Y^1 \frac{\partial}{\partial \psi_Y} + \mu_X^2 \frac{\partial}{\partial \theta_X} + \mu_Y^2 \frac{\partial}{\partial \theta_Y} + \mu_X^3 \frac{\partial}{\partial \phi_X} + \mu_Y^3 \frac{\partial}{\partial \phi_Y} + \mu_{XY}^2 \frac{\partial}{\partial \psi_{XY}} + \mu_{YY}^3 \frac{\partial}{\partial \phi_{YY}} \tag{22}$$

where,

$$\mu_X^1 = D_X(\mu^1) - \psi_X D_X(\xi^1) - \psi_Y D_Y(\xi^2)$$

$$\mu_Y^1 = D_Y(\mu^1) - \psi_X D_Y(\xi^1) - \psi_Y D_Y(\xi^2)$$

$$\mu_X^2 = D_X(\mu^2) - \theta_X D_X(\xi^1) - \theta_Y D_X(\xi^2)$$

$$\mu_Y^2 = D_Y(\mu^2) - \theta_X D_Y(\xi^1) - \theta_Y D_Y(\xi^2)$$

$$\mu_X^3 = D_X(\mu^3) - \phi_X D_X(\xi^1) - \phi_Y D_X(\xi^2)$$

$$\mu_Y^3 = D_Y(\mu^3) - \phi_X D_Y(\xi^1) - \phi_Y D_Y(\xi^2)$$

$$\mu_{XY}^2 = D_Y(\mu_Y^2) - \theta_{XY} D_Y(\xi^1) - \theta_{YY} D_Y(\xi^2),$$

$$\mu_{YY}^3 = D_{YY}(\mu_Y^3) - \phi_{XY} D_Y(\xi^1) - \phi_{YY} D_Y(\xi^2). \tag{23}$$

and  $D_X$  and  $D_Y$  are the of total differentiation operators w.r.t  $X$  and  $Y$ , respectively. The operator  $\aleph$  by (21) is a point symmetry of (16)-(18) if:

$$\aleph^{(1)}((1 + M)\psi_Y - \theta - B\phi) = 0 \tag{24}$$

$$\aleph^{(2)}(\psi_Y \theta_X - \psi_X \theta_Y - \theta_{YY} - Df \phi'') = 0 \tag{25}$$

$$\aleph^{(2)}(\psi_Y \phi_X - \psi_X \phi_Y + (V_T \phi)_Y - Le^{-1} \phi_{YY} - Le^{-1} Sr \theta'') = 0 \tag{26}$$

Proper algebraic calculations gives the following infinitesimals:

$$\xi^1 = (2Z_7 - Z_8)X$$

$$\xi^2 = Z_7Y + Z_9$$

$$\mu^1 = Z_7\psi$$

$$\mu^2 = Z_8\theta - BZ_{10}$$

$$\mu^3 = Z_7\phi + Z_{10} \tag{27}$$

where  $Z_7, Z_8, Z_9, Z_{10}$  arbitrary constants. In order that the data held on the boundary surfaces must be invariant, we have:

$$\aleph^{(1)}[\psi_Y - \Psi_1(X)] = 0 \text{ when } \psi_Y(X, 0) = \Psi_1(X) = 0,$$

$$\aleph^{(1)}[-\psi_X - \Psi_2(X)] = 0 \text{ when } \psi_X(X, 0) = -\Psi_2(X) = V(X),$$

$$\aleph^{(1)}[\theta - \theta_0(X)] = 0 \text{ when } \theta(X, 0) = \theta_0(X) = 1,$$

$$\aleph^{(1)}[\phi - \phi_0(X)] = 0 \text{ when } \phi(X, 0) = \phi_0(X) = 1 \tag{28}$$

From the above we get:

$$(2Z_7 - Z_8)X\Psi_1' - (Z_7 - Z_8)\Psi_1 = 0$$

$$(2Z_7 - Z_8)X\Psi_2' + (Z_7 - Z_8)\Psi_2 = 0$$

$$(2Z_7 - Z_8)X\theta_0' - Z_8\theta_0 = -BZ_{10}$$

$$(2Z_7 - Z_8)X\phi_0' - Z_8\phi_0 = Z_{10} \tag{29}$$

which gives the admissible form for  $\psi_1, \psi_2, \psi_3, \psi_3, \theta_w$  as follows:

$$\begin{aligned} \Psi_1(x) &= k_1 |X|^{(Z_7-Z_8)/(2Z_7-Z_8)} = 0 \\ \Psi_2(x) &= k_2 |X|^{-(Z_7-Z_8)/(2Z_7-Z_8)} = V(X) \\ \theta_0(X) &= k_3 |X|^{Z_7/(2Z_7-Z_8)} + BZ_{11} = 1 \\ \phi_0(X) &= k_4 |X|^{Z_8/(2Z_7-Z_8)} - Z_{11} = 1. \end{aligned} \tag{30}$$

where  $k_1, k_2, k_3, k_4$  are arbitrary constants and  $Z_{11} = Z_{10}/Z_8$ .

Thus the boundary conditions complying with symmetries (19) are given by

$$\begin{aligned} \psi_Y(X, 0) &= k_1 |X|^{(Z_7-Z_8)/(2Z_7-Z_8)}, y = 0 \\ \psi_X(X, 0) &= -k_2 |X|^{-(Z_7-Z_8)/(2Z_7-Z_8)}, X > 0 \\ \theta(X, 0) &= k_4 |X|^{Z_8/(2Z_7-Z_8)} + BZ_{11} \\ \phi(X, 0) &= k_4 |X|^{Z_8/(2Z_7-Z_8)} - Z_{11} \end{aligned} \tag{31}$$

and

$$Y \rightarrow \infty : \psi_Y \rightarrow 0 : \theta \rightarrow 0 : \phi \rightarrow 0 \tag{32}$$

3.2. Proposition

The BVP given in Eqs. (16)–(18) and (19), have the following multi-parameter group of symmetries:

$$\begin{aligned} X^* &= X + \varepsilon((2Z_7 - Z_8)X) + O(\varepsilon^2) \\ Y^* &= Y + \varepsilon(Z_7Y + Z_9) + O(\varepsilon^2) \\ \psi^* &= \psi + \varepsilon(Z_7\psi) + O(\varepsilon^2) \\ \theta^* &= \theta + \varepsilon(Z_8\theta - BZ_{10}) + O(\varepsilon^2) \\ \phi^* &= \phi + \varepsilon(Z_7\phi + Z_{10}) + O(\varepsilon^2). \end{aligned} \tag{33}$$

But Eq. (29) is the admissible form of data on the boundaries. From (27), we can find the symmetry corresponding to those problems studied in (Nabwey et al., 2015, 2017). By neglecting the parameters  $Z_8, Z_9$  and  $Z_{10}$ , the scaling group is parameterized by  $Z_7$ .

4. Group-Invariant solutions

Here, we examine the group-invariant solutions of the symmetry group gained in the last sections. For details on group-invariant solutions see (Ovsiannikov, 1982; Olver, 1986). Suppose  $(\psi, \theta, \phi)$  is solution of the problem (16)–(18). This solution will remain the same under the transformation group in Eq. (33) if the following hold:

Or

$$\xi^1 \frac{\partial \psi}{\partial X} + \xi^2 \frac{\partial \psi}{\partial Y} = \mu^1, \xi^1 \frac{\partial \theta}{\partial X} + \xi^2 \frac{\partial \theta}{\partial Y} = \mu^2, \text{ and } \xi^1 \frac{\partial \phi}{\partial X} + \xi^2 \frac{\partial \phi}{\partial Y} = \mu^3,$$

$$(2Z_7 - Z_8)X \frac{\partial \psi}{\partial X} + (Z_7Y + Z_9) \frac{\partial \psi}{\partial Y} = Z_7\psi \tag{34.1}$$

$$(2Z_7 - Z_8)X \frac{\partial \theta}{\partial X} + (Z_7Y + Z_9) \frac{\partial \theta}{\partial Y} = Z_7\theta - BZ_{10} \tag{34.2}$$

$$(2Z_7 - Z_8)X \frac{\partial \phi}{\partial X} + (Z_7Y + Z_9) \frac{\partial \phi}{\partial Y} = Z_7\phi + Z_{10} \tag{34.3}$$

The following system can be solved with the method of characteristics.

$$\psi(X, Y) = k_1 |X|^{Z_7/(2Z_7-Z_8)} F(\eta) \tag{35.1}$$

$$\theta(X, Y) = k_2 |X|^{Z_7/(2Z_7-Z_8)} \Theta(\eta) \tag{35.2}$$

$$\phi(X, Y) = k_3 |X|^{Z_7/(2Z_7-Z_8)} \Phi(\eta) \tag{35.3}$$

where  $F, \Theta, \Phi$  are arbitrary functions and  $\eta$  is the similarity variable given by the relation:

$$\eta = k_4 YX^{-Z_7/(2Z_7-Z_8)} \tag{35.4}$$

The group-invariant solution of the system is given by Eqs. (35.1)–(35.4). It can be observed that the IBVP have been transformed into BVP of non-similar transient equations, which are easily solvable and it is noted that our solution make a reduction to the number of independent variables

5. Scaling symmetry

We restrict our study with regard to Scaling Symmetry and accordingly we have chosen  $Z_7 = 1$  and  $Z_8 = Z_{11} = 0$ . We revisit the problem presented in (Cheng and Mynkowycz, 1977; Bejan and Khair, 1985) without considering the Dufour and Soret effects with thermophoresis impact, whose transformation equations are given by:

$$X^* = e^{2\varepsilon} X, Y^* = e^\varepsilon Y, \psi^* = e^\varepsilon \psi, \theta^* = \theta, \phi^* = \phi \tag{36}$$

With above parameters, the similarity solutions (35.1)–(35.4) become:

$$\begin{aligned} \psi(X, Y) &= k_1 X^{1/2} F(\eta) \theta(X, Y) = k_2 \Theta(\eta) \\ \phi(X, Y) &= k_3 \Phi(\eta) \eta = k_4 YX^{-1/2} \end{aligned} \tag{37}$$

By Putting the value of (37) in Eqs. (16)–(18), we notice that the number of independent variables get reduced by one and accordingly, the transformed mathematical model is:

$$\begin{aligned} (1 + M)F' &= \Theta + B\Phi \\ \Theta'' + \frac{1}{2}F\Theta' + Df\Phi'' &= 0 \\ \frac{1}{Le}\Phi'' + \frac{1}{2}F\Phi' + \kappa \frac{Pr}{C_T + \Theta} \left[ \Theta'\Phi' + \Phi\Theta'' - \frac{\Theta^2\Phi'}{(\Theta + C_T)} \right] + Sr\Theta'' &= 0 \end{aligned} \tag{38}$$

The new boundary conditions are:

$$\begin{aligned} \text{at } \eta = 0 : F = F_w, \Theta = 1, \Phi = 1 \\ \text{as } \eta \rightarrow \infty : F' \rightarrow 0, \Theta \rightarrow 0, \Phi \rightarrow 0 \end{aligned} \tag{39}$$

where the derivatives in system (38) are done with respect to  $\eta$ .

Here the parameter  $F_w = k_1 X^{-1/2} V(X)$  denotes the suction/injection, such that the permeability of the porous surface  $V(X)$  varies inversely to square root of  $x$ , in order to make  $F_w$  independent of  $x$ . Four new arbitrary constants i.e.,  $k_1 = \dots = k_4 = 1$  have been introduced; so the coefficients in the above equations do not reflect the fluid properties explicitly.

For  $\kappa = 0$  (absence of thermophoresis) and  $B = 0$ , we notice that the Eqs. (38)–(39) reduce to the system represented by Cheng and Minkowycz (Pai, 1962) while at  $B \neq 0$  the system reduce to the one given by Bejan and Khair (EL-Kabeir et al., 2007) respectively.

6. Results and discussion

The convenient similarity transformations in (37) are applied to gain identical ordinary differential equations from the flow, heat and transfer arising governing equations (38) which are highly nonlinear in nature. Shooting technique is accustomed along with Runge-Kutta method to find the solution of these equations. The impacts of various dimensionless quantities on velocity, tempera-

ture and concentration distributions are exhibited and analyzed using the graphical aid (see Fig. 1–12). Shooting technique is one of the techniques that are used to solve the initial value problems. The behavior of suction/injection parameter,  $F_w$  and magnetic parameter  $M$  on velocity is exposed in Fig. 1. It is manifested that velocity regime appears to be reduced with the decrease of the suction/injection parameter  $F_w$  and the magnetic parameter  $M$ . Figs. 2 and 3 portray the effect of suction/injection parameter,  $F_w$  on the temperature and concentration distributions. It is illustrated that for decreasing values of  $F_w$  temperature and concentration distribution become more uniform and the thermal boundary layer thickness increases. Figs. 4–6 is exhibited to visualize the behavior of involved sundry such as Dufour and Soret numbers ( $Df$  and  $Sr$ ) on the of profiles velocity, temperature and concentration. It is perceived that the velocity of the fluid declines as  $Df$  and  $Sr$  grow but the temperature reduces (or concentration enhances) with the incerment in  $Df$  and increase in  $Sr$ . Figs. 7–12 demonstrate the effects of thermophroesis parameter and Schmidt number on velocity, temperature, and concentration profiles. These graphs clarify that the temperature declines as the thermophroesis parameter and Schmidt number decrease, while the velocity and concentration decrease as the thermophroesis parameter and Sch-

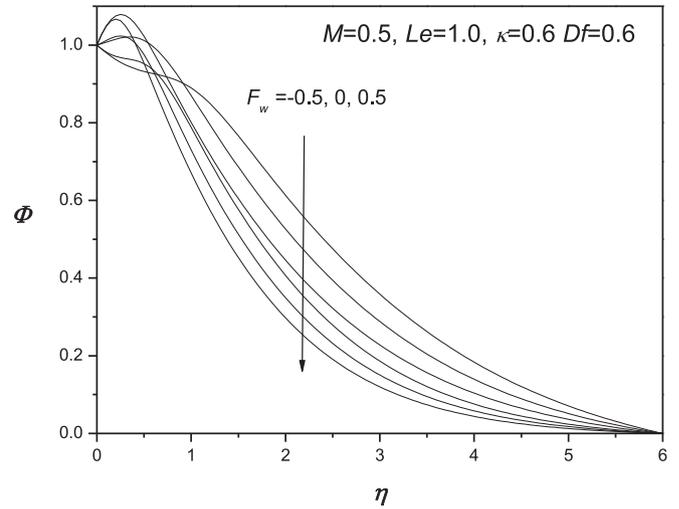


Fig. 3. Distribution of concentration.

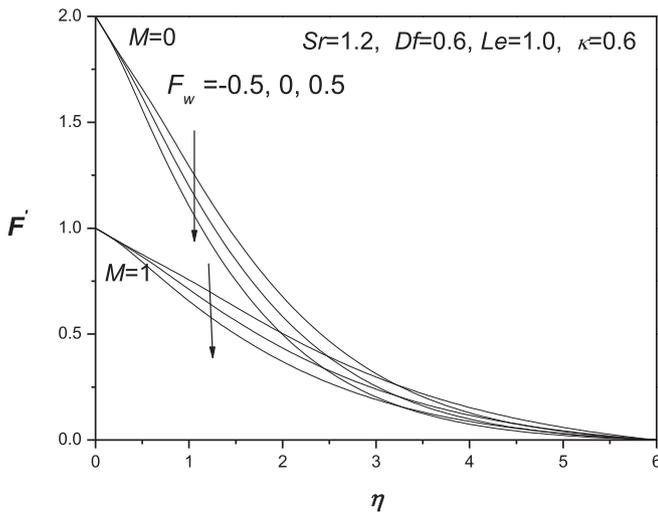


Fig. 1. Distribution of velocity.

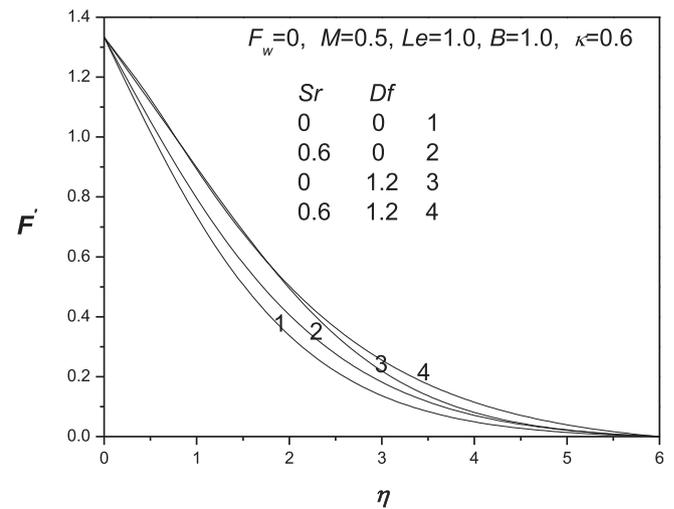


Fig. 4. Distribution of velocity.

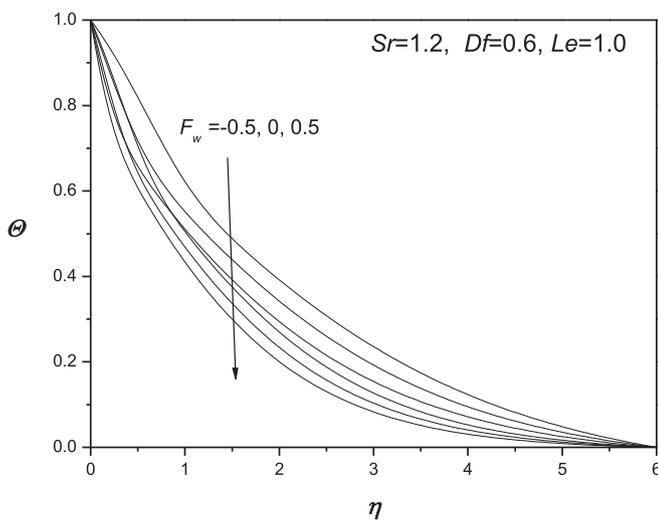


Fig. 2. Distribution of temperature.

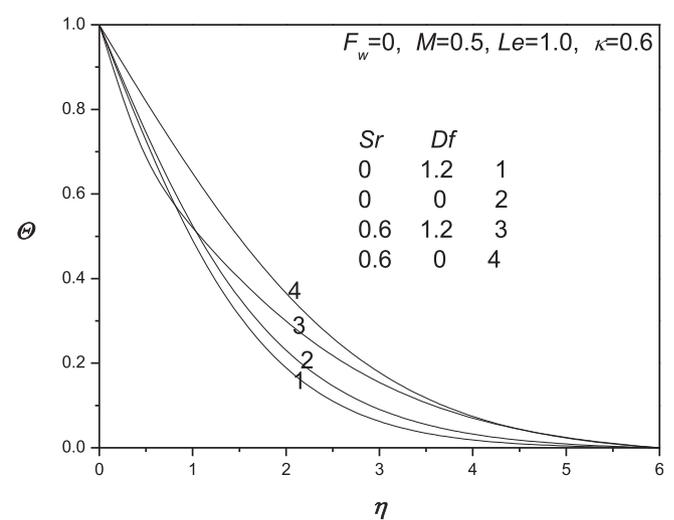


Fig. 5. Distribution of temperature.

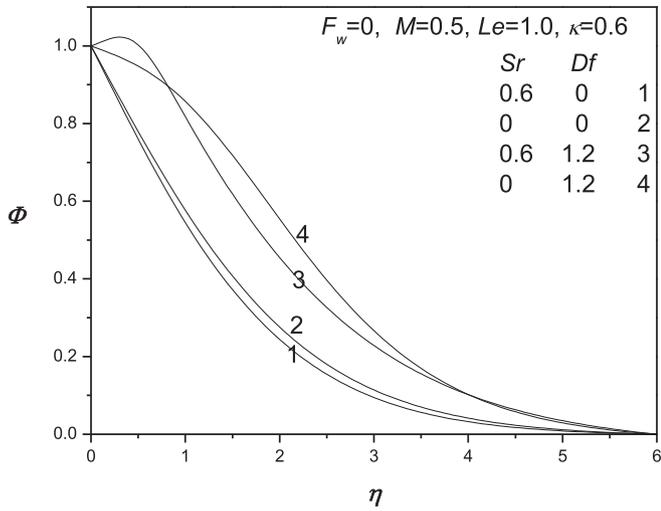


Fig. 6. Distribution of concentration.

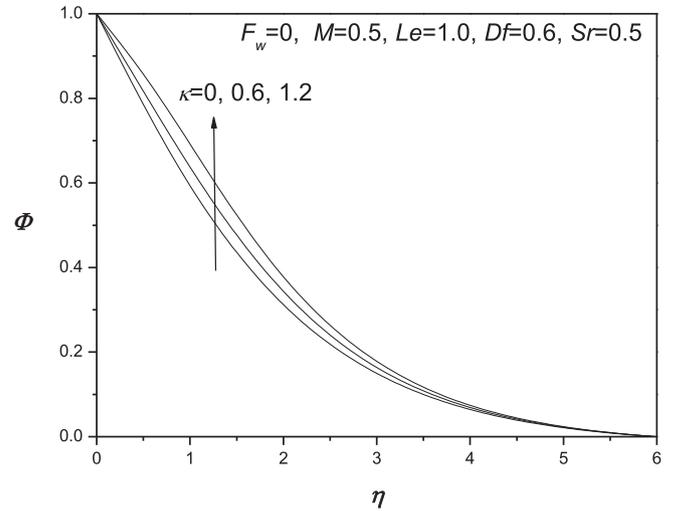


Fig. 9. Distribution of concentration.

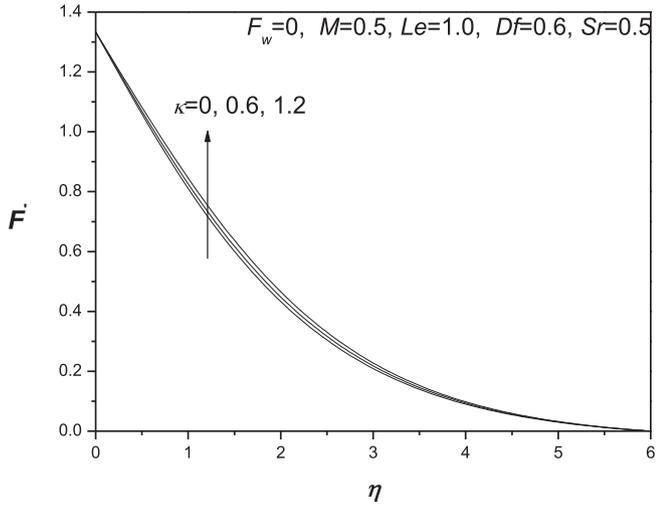


Fig. 7. Distribution of velocity.

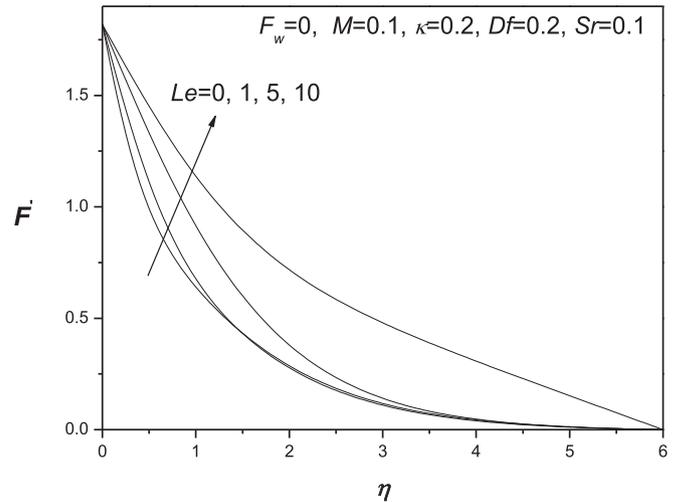


Fig. 10. Distribution of velocity.

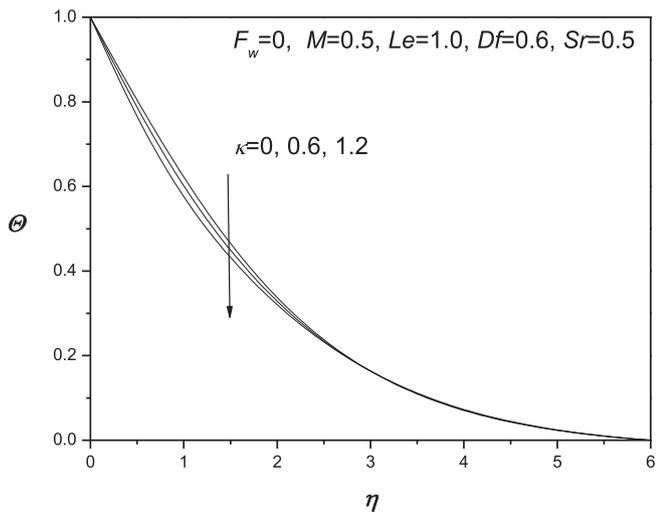


Fig. 8. Distribution of temperature.

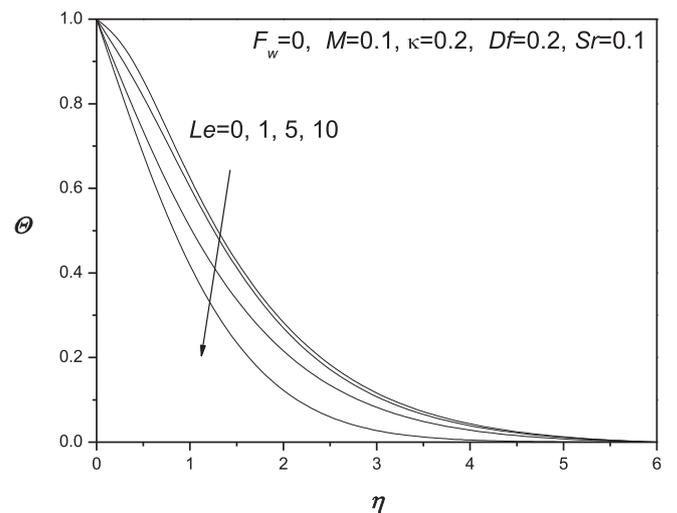


Fig. 11. Distribution of temperature.

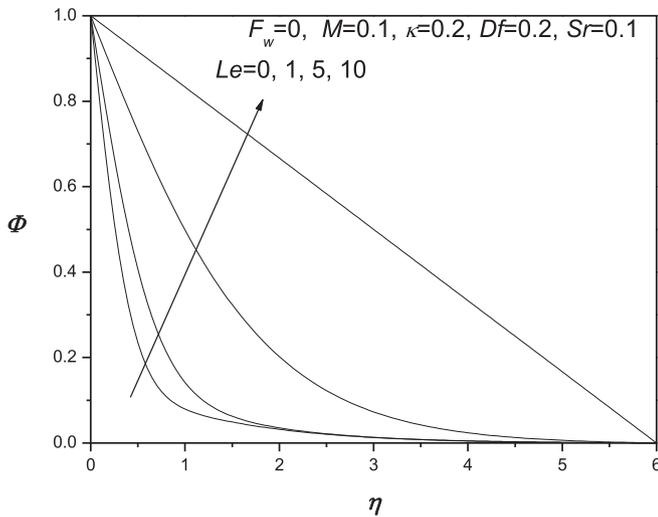


Fig. 12. Distribution of concentration.

Table 1  
Variation of dimensionless heat transfer and concentration rates with M, D<sub>f</sub> and S<sub>r</sub>.

M	D <sub>f</sub>	S <sub>r</sub>	−θ'(0)	−φ'(0)
0	0.05	0.2	0.6374	0.4991
1	0.05	0.2	0.4673	0.3765
2	0.05	0.2	0.3999	0.3301
0	0.075	0.8	0.6665	0.2896
1	0.075	0.8	0.4839	0.2447
2	0.075	0.8	0.4107	0.2351
0	0.6	0.1	0.4698	0.5831
1	0.6	0.1	0.3688	0.4253
2	0.6	0.1	0.3331	0.3631

Table 2  
Variation of dimensionless heat transfer and concentration rates with, κ D<sub>f</sub> and S<sub>r</sub>.

κ	D <sub>f</sub>	S <sub>r</sub>	−θ'(0)	−φ'(0)
0.05	0.05	0.2	0.4620	0.4217
0.5	0.05	0.2	0.4673	0.3765
0.8	0.05	0.2	0.4709	0.3471
1.2	0.05	0.2	0.4756	0.3089
0.05	0.075	0.8	0.4773	0.2907
0.5	0.075	0.8	0.4839	0.2447
0.8	0.075	0.8	0.4883	0.2148
1.2	0.075	0.8	0.4941	0.1763
0.05	0.6	0.1	0.3461	0.4632
0.5	0.6	0.1	0.3688	0.4253
0.8	0.6	0.1	0.3856	0.3973
1.2	0.6	0.1	0.4100	0.3562

midt number increase. The behaviors of magnetic parameter M, Dufour and Soret numbers (D<sub>f</sub> and S<sub>r</sub>) and thermophoresis parameter κ on the rate of heat transfer −θ'(0) and the rate of mass transfer −φ'(0) are exposed in Tables 1–2. It is manifested that local Nusselt numbers Nu and Sherwood number Sh reduce as M increases. Furthermore, From Table 2, It is engrossed that by increasing the the thermophoresis parameter κ increases, the rate of heat transfer −θ'(0) increases whereas rate of mass transfer −φ'(0) decreases.

7. Conclusion

In this study we analyze phenomenon of thermophoresis on a vertical surface in a porous medium. The fascinating aspects of

thermo diffusion and diffusion-thermo impacts are accounted with magnetic field and thermophoresis influence. In this communication, it can be seen that after incorporating these impacts and utilizing Lie group method, the mathematical model has been designed using PDE which are non-linear and subsequently transforming into the system of ODE by using suitable similarity transformations. The numerical solutions of the system have been arrived and the following results are arrived:

- Velocity profiles are growing due to acceleration in the value of suction/injection parameter and magnetic parameter but declination happens by increasing the Dufour and Soret numbers.
- Increase in the value of thermophoresis parameter results in reduction of both velocity and concentration profiles and enhancement of temperature profiles.
- Considerable reduction in local Nusselt and Sherwood numbers is exhibited by promoting the values of magnetic parameter.
- Massive in reduction in local Nusselt number and declination in Sherwood number for elevating the values of thermophoresis parameter.
- Both Local Nusselt Number and Sherwood number varies inversely with Soret Number but with Dufour number it is vice-versa.

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