Journal of King Saud University – Science 32 (2020) 563–566

Contents lists available at ScienceDirect

Journal of King Saud University – Science

journal homepage: www.sciencedirect.com

Original article

q-Laplace Type Transforms of q-Analogues of Bessel Functions

R.T. Al-Khairy

Imam Abdulrahman Bin Faisal University, Eastern region, Al- Dammam, Saudi Arabia

ARTICLE INFO

Article history:
Received 16 April 2018
Accepted 29 August 2018
Available online 3 September 2018

Keywords:
q-extensions of Bessel functions
q-Laplace Type Transforms
q-shift factorials

ABSTRACT

In this work, q- Laplace type integral transforms which are called qL_2-transform and $q\mathcal{L}_2$-transform are applied on three families of q^2-Bessel Functions. Moreover, we give some examples to show effectiveness of the proposed results.

1. Introduction

Bessel functions are series of solutions to a second order differential equation that arise in many diverse situations. In literature there are many q-extensions of Bessel functions. The first were introduced by Jackson (1905) and studied later by Hahn (1949), Exton and Srivastava (1994). These q-analogues have been studied extensively by Ismail in Ismail (1981,1982). Laplace transform is the most popular and widely used in applied mathematics. A certain type of Laplace transforms which is called $\mathcal{L}_2$-transform was introduced by Yürekli and Sadek (1991). Then, these transforms were studied in more details by Yürekli (1999a,1999b). The q-analogue of $\mathcal{L}_2$-transforms, which were called qL_2-transform and $q\mathcal{L}_2$-transforms were studied by Uçar and Albayrak (2011) and applied to some basic functions. Purohit and Kalla applied the q-Laplace transforms to a product of basic analogues of the Bessel functions Purohit and Kalla (2007). Uçar (2014) and Omari (2017) also applied Sumudu q-transforms and q-Natural transforms, respectively, to a product of q-analogues of the Bessel functions.

In this paper, we evaluate qL_2-transform and $q\mathcal{L}_2$-transforms presented in Uçar and Albayrak (2011) of a product of q^2-analogues of three families of Bessel functions. Finally, some examples of different parameters are presented in order to illustrate the accuracy and potentialities of the given theorems. The obtained limiting case results show good agreement with the previously obtained solutions.

1.1. Definitions and preliminaries

The mainly best known q-analogues of the remarkable Bessel function

$$J_\mu(x) = \sum_{k=0}^{\infty} \frac{(-1)^k (x/2)^{\mu+2k}}{k!\Gamma(\mu+k+1)}. \tag{1}$$

were given by Ismail (1982),

$$J_\mu^{(1)}(z;q) = \left(\frac{z}{2}\right)^\mu \sum_{n=0}^{\infty} \frac{\left(\frac{-z^2}{4}\right)^n}{(q;q)_{\mu+n}(q;q)_n}, \quad |z| < 2, \tag{2}$$

$$J_\mu^{(2)}(z;q) = \left(\frac{z}{2}\right)^\mu \sum_{n=0}^{\infty} \frac{q^{n(n+\mu)}\left(\frac{-z^2}{4}\right)^n}{(q;q)_{\mu+n}(q;q)_n}, \quad z \in C, \tag{3}$$

$$J_\mu^{(3)}(z;q) = z^\mu \sum_{n=0}^{\infty} \frac{(-1)^n q^{\frac{n(n-1)}{2}} (qz^2)^n}{(q;q)_{\mu+n}(q;q)_n}, \quad z \in C. \tag{4}$$

The q-shift factorials are defined, for fix $a \in C$, as

$$(a;q)_0 = 1; \quad (a;q)_n = \prod_{k=0}^{n-1}(1-aq^q), \quad n = 1,2,\ldots; \quad (a;q)_\infty = \lim_{n\to\infty}(a;q)_n. \tag{5}$$

We also denote by

Peer review under responsibility of King Saud University.

ELSEVIER Production and hosting by Elsevier

E-mail address: ralkhairy@uod.edu.sa

https://doi.org/10.1016/j.jksus.2018.08.012
1018-3647/

$$\left.\begin{array}{ll}[x]_q=\frac{1-q^x}{1-q}, & x\in C;\\ ([n]_q)!=\frac{(q;q)_n}{(1-q)^n}, & n\in N;\\ (a;q)_x=\frac{(a;q)_\infty}{(aq^x;a)_\infty}, & x\in R.\end{array}\right\} \tag{6}$$

The first type q-analogue of the exponential function was introduced as

$$E_q(x)=\sum_{n=0}^{\infty}\frac{(-1)^n q^{\frac{n(n-1)}{2}}x^n}{(q;q)_n}=(x;q)_\infty,\quad x\in C. \tag{7}$$

Whereas, the second type q-analogue of the exponential function was introduced as

$$e_q(x)=\sum_{n=0}^{\infty}\frac{x^n}{(q;q)_n}=\frac{1}{(x;q)_\infty},|x|<1. \tag{8}$$

The series representations of the q-gamma function are given by Kac and De Sole (2005)

$$\Gamma_q(\alpha)=\frac{(q;q)_\infty}{(1-q)^{\alpha-1}}\sum_{k=0}^{\infty}\frac{q^{k\alpha}}{(q;q)_k}, \tag{9}$$

$$\Gamma_q(\alpha)=\frac{K(A;\alpha)}{(1-q)^{\alpha-1}(\frac{-1}{A};q)_\infty}\sum_{k\in Z}\left(\frac{q^k}{A}\right)^{\alpha}\left(-\frac{1}{A};q\right)_k. \tag{10}$$

where $K(A;\alpha)$ is a remarkable function given by

$$K(A;\alpha)=A^{\alpha-1}\frac{(-q/\alpha;q)_\infty}{(-q^t/\alpha;q)_\infty}\frac{(-\alpha;q)_\infty}{(-\alpha q^{1-t};q)_\infty},\quad \alpha\in R. \tag{11}$$

$\mathcal{L}_2$-Laplace transform as defined by Yürekli and Sadek (1991) is written as

$$\mathcal{L}_2\{f(t);s\}=\int_0^\infty te^{-t^2s^2}f(t)dt. \tag{12}$$

The series form of the first type of the q-analogue of $\mathcal{L}_2$ -Laplace transform which is denoted by ${}_qL_2$ as given by Uçar and Albayrak (2011)

$${}_qL_2\{f(t);s\}=\frac{1}{[2]}\frac{(q^2;q^2)_\infty}{s^2}\sum_{n=0}^{\infty}\frac{q^{2n}}{(q^2;q^2)_n}f(q^ns^{-1}). \tag{13}$$

where, using (6), $[2]=[2]_q=1+q$.

While the series form of the second type of the q-analogue of $\mathcal{L}_2$-transform which is denoted by ${}_q\mathcal{L}_2$ as given by Uçar and Albayrak (2011)

$${}_q\mathcal{L}_2\{f(t);s\}=\frac{1}{[2]}\frac{1}{(-s^2;q^2)_\infty}\sum_{n\in Z}q^{2n}(-s^2;q^2)_nf(q^n). \tag{14}$$

There exists a relation between ${}_qL_2$-transform and L_q-transform as follows:

$${}_qL_2\{f(t);s\}=\frac{1}{[2]}L_{q^2}\{f(t^{1/2});s^2\}. \tag{15}$$

Also, similar relation exists between ${}_q\mathcal{L}_2$-transform and $\mathcal{L}_q$-transform as follows:

$${}_q\mathcal{L}_2\{f(t);s\}=\frac{1}{[2]}\mathcal{L}_{q^2}\{f(t^{1/2});s^2\}. \tag{16}$$

1.2. Main Theorems

In this section we evaluate qL_2-transform and $q\mathcal{L}_2$ -transform of $t^{2\Delta-2}$ weighted product of m different q^2-Bessel Functions. The q^2-Bessel Functions are more relevant than the original q-Bessel Functions because of the mathematical nature of qL_2-transform and $q\mathcal{L}_2$-transform which contain q^2-shift factorials.

Theorem 1. *(a) Let* $J^{(1)}_{2\mu_1}(a_1t;q^2),J^{(1)}_{2\mu_2}(a_2t;q^2),\ldots,J^{(1)}_{2\mu_m}(a_mt;q^2)$ *be a set of* q^2*-Bessel functions of the first kind and* $f(t)=t^{2\Delta-2}\prod_{k=1}^{m}J^{(1)}_{2\mu_k}(a_kt;q^2)$ *where* Δ,a_k *and* μ_k *where* $k=1,2,\ldots,m$ *are constants then* qL_2*-transform of* $f(t)$ *is,*

$${}_qL_2\{f(t);s\}=\prod_{k=1}^{m}B_k(s,2)\sum_{j_k=0}^{\infty}\left(-\frac{a_k^2}{4s^2}\right)^{j_k}H_{j_k}(q^2)\Gamma_{q^2}(\mu_k+\Delta+j_k). \tag{17}$$

(b) Let $J^{(2)}_{2\mu_1}(a_1t;q^2),J^{(2)}_{2\mu_2}(a_2t;q^2),\ldots,J^{(2)}_{2\mu_m}(a_mt;q^2)$ *be a set of* q^2*-Bessel functions of the second kind and* $f(t)=t^{2\Delta-2}\prod_{k=1}^{m}J^{(2)}_{2\mu_k}(a_kt;q^2)$ *where* Δ,a_k *and* μ_k *where* $k=1,2,\ldots,m$ *are constants, then* qL_2*-transform of* $f(t)$ *is,*

$${}_qL_2\{f(t);s\}=\prod_{k=1}^{m}B_k(s,2)\sum_{j_k=0}^{\infty}\left(-\frac{a_k^2}{4s^2}\right)^{j_k}(q^2)^{\frac{j_k(j_k+2\mu_k)}{2}}H_{j_k}(q^2)\Gamma_{q^2}(\mu_k+\Delta+j_k). \tag{18}$$

(c) Let $J^{(3)}_{2\mu_1}(a_1t;q^2),J^{(3)}_{2\mu_2}(a_2t;q^2),\ldots,J^{(3)}_{2\mu_m}(a_mt;q^2)$ *be a set of* q^2*-Bessel functions of the third kind and* $f(t)=t^{2\Delta-2}\prod_{k=1}^{m}J^{(3)}_{2\mu_k}(a_kt;q^2)$ *where* Δ,a_k *and* μ_k *where* $k=1,2,\ldots,m$*, then,* qL_2*-transform of* $f(t)$ *is,*

$${}_qL_2\{f(t);s\}=\prod_{k=1}^{m}B_k(s,1)\sum_{j_k=0}^{\infty}\left(-\frac{a_k^2q^2}{s^2}\right)^{j_k}(q^2)^{\frac{j_k(j_k-1)}{2}}H_{j_k}(q^2)\Gamma_{q^2}(\mu_k+\Delta+j_k). \tag{19}$$

where $Re(s)>0$, $Re(\Delta)>0$ *and*

$$B_k(s,\tau)=\frac{a_k^{2\mu_k}}{[2]2^{2\mu_k(\tau-1)}s^{2\Delta+2\mu_k}}, \tag{20}$$

$$H_{j_k}(q)=\frac{(1-q)^{\mu_k+\Delta+j_k-1}}{(q;q)_{j_k+2\mu_k}(q;q)_{j_k}}. \tag{21}$$

Proof. We will only give the proof of (17), because the proof of (18) and (19) is the same. We put

$$f(t)=t^{2\Delta-2}\prod_{k=1}^{m}J^{(1)}_{2\mu_k}(a_kt;q^2). \tag{22}$$

into the definition (13) yields

$$\begin{aligned}qL_2\{f(t);s\}&=\frac{(q^2;q^2)_\infty}{[2]s^2}\prod_{k=1}^{m}\sum_{n=0}^{\infty}\frac{q^{2n}}{(q^2;q^2)_n}(q^ns^{-1})^{2\Delta-2}J^{(1)}_{2\mu_k}(a_k(q^ns^{-1});q^2)\\&=\frac{(q^2;q^2)_\infty}{[2]s^{2\Delta}}\prod_{k=1}^{m}\sum_{n=0}^{\infty}\frac{q^{2n\Delta}}{(q^2;q^2)_n}J^{(1)}_{2\mu_k}(a_k(q^ns^{-1});q^2)\\&=\frac{(q^2;q^2)_\infty}{[2]s^{2\Delta}}\prod_{k=1}^{m}\sum_{n=0}^{\infty}\frac{q^{2n\Delta}}{(q^2;q^2)_n}\left[\left(\frac{a_k(q^ns^{-1})}{2}\right)^{2\mu_k}\sum_{j_k=0}^{\infty}\frac{\left(-\frac{(a_kq^ns^{-1})^2}{4}\right)^{j_k}}{(q^2;q^2)_{j_k+2\mu_k}(q^2;q^2)_{j_k}}\right]\\&=\prod_{k=1}^{m}\frac{(q^2;q^2)_\infty\left(\frac{a_k}{2}\right)^{2\mu_k}}{[2]s^{2\Delta+2\mu_k}}\sum_{\infty}^{n=0}\frac{q^{2n\Delta+2n\mu_k}}{(q^2;q^2)_n}\sum_{j_k=0}^{\infty}\frac{\left(-\frac{(a_kq^ns^{-1})^2}{4}\right)^{j_k}}{(q^2;q^2)_{j_k+2\mu_k}(q^2;q^2)_{j_k}}.\end{aligned}$$

On interchanging the order of summations, which is valid under the conditions given by the theorem, we obtain

$$qL_2\{f(t);s\} = \prod_{k=1}^{m} B_k(s,2)\sum_{j_k=0}^{\infty}\frac{\left(-\frac{a_k^2}{4s^2}\right)^{j_k}}{(q^2;q^2)_{j_k+2\mu_k}(q^2;q^2)_{j_k}}\sum_{n=0}^{\infty}(q^2;q^2)_\infty \times \frac{(q^2)^{n(\Delta+\mu_k+j_k)}}{(q^2;q^2)_n}$$

$$= \prod_{k=1}^{m} B_k(s,2)\sum_{j_k=0}^{\infty}\frac{\left(-\frac{a_k^2}{4s^2}\right)^{j_k}(1-q^2)^{\Delta+\mu_k+j_k-1}}{(q^2;q^2)_{j_k+2\mu_k}(q^2;q^2)_{j_k}}\Gamma_{q^2}(\mu_k+\Delta+j_k)$$

$$= \prod_{k=1}^{m} B_k(s,2)\sum_{j_k=0}^{\infty}\left(-\frac{a_k^2}{4s^2}\right)^{j_k} H_{j_k}(q^2)\Gamma_{q^2}(\mu_k+\Delta+j_k). \quad \square$$

Theorem 2. *(a) Let $J^{(1)}_{2\mu_1}(a_1t;q^2), J^{(1)}_{2\mu_2}(a_2t;q^2),\ldots,J^{(1)}_{2\mu_m}(a_mt;q^2)$ be a set of q^2-Bessel function of the first kind and $f(t) = t^{2\Delta-2}\prod_{k=1}^{m} J^{(1)}_{2\mu_k}(a_kt;q^2)$ where Δ, a_k and μ_k where $k = 1,2,\ldots,m$ are constants then $q\mathcal{L}_2$-transform of $f(t)$ is,*

$$q\mathcal{L}_2\{f(t);s\} = \prod_{k=1}^{m} B_k(s,2)\sum_{j_k=0}^{\infty}\left(-\frac{a_k^2}{4s^2}\right)^{j_k}\frac{H_{j_k}(q^2)}{K(\frac{1}{s^2},\mu_k+\Delta+j_k)}\Gamma_{q^2}(\mu_k+\Delta+j_k). \tag{23}$$

(b) Let $J^{(2)}_{2\mu_1}(a_1t;q^2), J^{(2)}_{2\mu_2}(a_2t;q^2),\ldots,J^{(2)}_{2\mu_m}(a_mt;q^2)$ be a set of q^2-Bessel functions of the second kind and $f(t) = t^{2\Delta-2}\prod_{k=1}^{m} J^{(2)}_{2\mu_k}(a_kt;q^2)$ where Δ, a_k and μ_k where $k = 1,2,\ldots,m$ are constants then $q\mathcal{L}_2$-transform of $f(t)$ is,

$$q\mathcal{L}_2\{f(t);s\} = \prod_{k=1}^{m} B_k(s,2)\sum_{j_k=0}^{\infty}\left(-\frac{a_k^2}{4s^2}\right)^{j_k}\frac{(q^2)^{\frac{j_k(j_k+2\mu_k)}{2}}H_{j_k}(q^2)}{K(\frac{1}{s^2},\mu_k+\Delta+j_k)}\Gamma_{q^2}(\mu_k+\Delta+j_k). \tag{24}$$

(c) Let $J^{(3)}_{2\mu_1}(a_1t;q^2), J^{(3)}_{2\mu_2}(a_2t;q^2),\ldots,J^{(3)}_{2\mu_m}(a_mt;q^2)$ be a set of q^2-Bessel function of the third kind and $f(t) = t^{2\Delta-2}\prod_{k=1}^{m} J^{(3)}_{2\mu_k}(a_kt;q^2)$ where Δ, a_k and μ_k where $k = 1,2,\ldots,m$, then, $q\mathcal{L}_2$-transform of $f(t)$ is,

$$q\mathcal{L}_2\{f(t);s\} = \prod_{k=1}^{m} B_k(s,1)\sum_{j_k=0}^{\infty}\left(-\frac{a_k^2q^2}{s^2}\right)^{j_k} \times \frac{(q^2)^{\frac{j_k(j_k-1)}{2}}H_{j_k}(q^2)}{K(\frac{1}{s^2},\mu_k+\Delta+j_k)}\Gamma_{q^2}(\mu_k+\Delta+j_k). \tag{25}$$

where $Re(s) > 0$, $Re(\Delta) > 0$.

Proof. We will only give the proof of (23), because the proof of (24) and (25) is the same. We put

$$f(t) = t^{2\Delta-2}\prod_{k=1}^{m} J^{(1)}_{2\mu_k}(a_kt;q^2). \tag{26}$$

into the definition (14) yields

$$q\mathcal{L}_2\{f(t);s\} = \frac{1}{[2](-s^2;q^2)_\infty}\prod_{k=1}^{m}\sum_{n\in z} q^{2n}(-s^2;q^2)_n (q^n)^{2\Delta-2} J^{(1)}_{2\mu_k}(a_kq^n;q^2)$$

$$= \frac{1}{[2](-s^2;q^2)_\infty}\prod_{k=1}^{m}\sum_{n\in z} q^{2n\Delta}(-s^2;q^2)_n J^{(1)}_{2\mu_k}(a_kq^n;q^2)$$

$$= \frac{1}{[2](-s^2;q^2)_\infty}\prod_{k=1}^{m}\sum_{n\in z} q^{2n\Delta}(-s^2;q^2)_n\left[\left(\frac{a_k(q^n)}{2}\right)^{2\mu_k}\sum_{j_k=0}^{\infty}\frac{\left(-\frac{(a_kq^n)^2}{4}\right)^{j_k}}{(q^2;q^2)_{j_k+2\mu_k}(q^2;q^2)_{j_k}}\right]$$

$$= \prod_{k=1}^{m}\frac{\left(\frac{a_k}{2}\right)^{2\mu_k}}{[2](-s^2;q^2)_\infty}\sum_{n\in z} q^{2n\Delta+2n\mu_k}(-s^2;q^2)_n\sum_{j_k=0}^{\infty}\frac{\left(-\frac{(a_kq^n)^2}{4}\right)^{j_k}}{(q^2;q^2)_{j_k+2\mu_k}(q^2;q^2)_{j_k}}.$$

On interchanging the order of summations, which is valid under the conditions given by the theorem, we obtain

$$q\mathcal{L}_2\{f(t);s\} = \prod_{k=1}^{m}\frac{\left(\frac{a_k}{2}\right)^{2\mu_k}}{[2]}\sum_{j_k=0}^{\infty}\frac{\left(-\frac{a_k^2}{4}\right)^{j_k}}{(q^2;q^2)_{j_k+2\mu_k}(q^2;q^2)_{j_k}}\sum_{n\in z}\frac{(-s^2;q^2)_n(q^2)^{n(\Delta+\mu_k+j_k)}}{(-s^2;q^2)_\infty}.$$

Now, use Eq. (10) with $A = \frac{1}{s^2}$ and $\alpha = \Delta+\mu_k+j_k$ then, the summation on n can be written as

$$\sum_{n\in z}\frac{(-s^2;q^2)_n(q^2)^{n(\Delta+\mu_k+j_k)}}{(-s^2;q^2)_\infty} = \frac{\Gamma_{q^2}(\mu_k+\Delta+j_k)(1-q^2)^{\Delta+\mu_k+j_k-1}}{K(\frac{1}{s^2},\mu_k+\Delta+j_k)s^{2(\mu_k+\Delta+j_k)}},$$

then

$$q\mathcal{L}_2\{f(t);s\} = \prod_{k=1}^{m} B_k(s,2)\sum_{j_k=0}^{\infty}\left(-\frac{a_k^2}{4s^2}\right)^{j_k}\frac{H_{j_k}(q^2)}{K(\frac{1}{s^2},\mu_k+\Delta+j_k)}\Gamma_{q^2}(\mu_k+\Delta+j_k). \quad \square$$

2. Illustrative Examples

In this final section we mainly give qL_2-transforms and $q\mathcal{L}_2$-transforms involving the q^2-Bessel Functions as applications of our main results.

Corollary 3. *If one takes $m = 1, \mu_1 = \mu$ and $a_1 = a$ in above theorems, respectively, one has*

$$qL_2\left\{t^{2\Delta-2}J^{(1)}_{2\mu}(at;q^2);s\right\} = B(s,2)\sum_{j=0}^{\infty}\left(-\frac{a^2}{4s^2}\right)^{j} H_j(q^2)\Gamma_{q^2}(\mu+\Delta+j),$$

$$qL_2\left\{t^{2\Delta-2}J^{(2)}_{2\mu}(at;q^2);s\right\} = B(s,2)\sum_{j=0}^{\infty}\left(-\frac{a^2}{4s^2}\right)^{j}(q^2)^{\frac{j(j+2\mu)}{2}} H_j(q^2)\Gamma_{q^2}(\mu+\Delta+j),$$

$$qL_2\left\{t^{2\Delta-2}J^{(3)}_{2\mu}(at;q^2);s\right\} = B(s,1)\sum_{j=0}^{\infty}\left(-\frac{a^2q^2}{s^2}\right)^{j}(q^2)^{\frac{j(j-1)}{2}} H_j(q^2)\Gamma_{q^2}(\mu+\Delta+j),$$

$$q\mathcal{L}_2\left\{t^{2\Delta-2}J^{(1)}_{2\mu}(at;q^2);s\right\} = B(s,2)\sum_{j=0}^{\infty}\left(-\frac{a^2}{4s^2}\right)^{j}\frac{H_j(q^2)}{K(\frac{1}{s^2},\mu+\Delta+j)}\Gamma_{q^2}(\mu+\Delta+j),$$

$$q\mathcal{L}_2\left\{t^{2\Delta-2}J^{(2)}_{2\mu}(at;q^2);s\right\} = B(s,2)\sum_{j=0}^{\infty}\left(-\frac{a^2}{4s^2}\right)^{j}\frac{(q^2)^{\frac{j(j+2\mu)}{2}}H_j(q^2)}{K(\frac{1}{s^2},\mu+\Delta+j)}\Gamma_{q^2}(\mu+\Delta+j),$$

$$q\mathcal{L}_2\left\{t^{2\Delta-2}J^{(3)}_{2\mu}(at;q^2);s\right\} = B(s,1)\sum_{j=0}^{\infty}\left(-\frac{a^2q^2}{s^2}\right)^{j} \times \frac{(q^2)^{\frac{j(j-1)}{2}}H_j(q^2)}{K(\frac{1}{s^2},\mu+\Delta+j)}\Gamma_{q^2}(\mu+\Delta+j). \tag{27}$$

Corollary 4. *If one takes $\Delta=\frac{n+2}{2}$ and $\mu=\frac{n}{2}$ in corollary (3), respectively, one has*

$$B(s,\tau)=\frac{a^n}{[2]2^{n(\tau-1)}s^{2n+2}},$$

$$H_j(q)=\frac{(1-q)^{n+j}}{(q;q)_{j+n}(q;q)_j}=\frac{1}{(q;q)_j\Gamma_q(n+j+1)}.$$

consequently

$$qL_2\left\{t^nJ_n^{(1)}(at;q^2);s\right\}=\frac{\left(\frac{a}{2}\right)^n}{[2]s^{2n+2}}e_{q^2}^{\left(\frac{-a^2}{4s^2}\right)},$$

$$qL_2\left\{t^nJ_n^{(2)}(at;q^2);s\right\}=\frac{\left(\frac{a}{2}\right)^n}{[2]s^{2n+2}}\sum_{j=0}^{\infty}\frac{(q^2)^{\frac{j(j+2\mu)}{2}}\left(-\frac{a^2}{4s^2}\right)^j}{(q^2;q^2)_j},$$

$$qL_2\left\{t^nJ_n^{(3)}(at;q^2);s\right\}=\frac{(a)^n}{[2]s^{2n+2}}E_{q^2}^{\left(\frac{q^2a^2}{s^2}\right)},$$

$$q\mathcal{L}_2\left\{t^nJ_n^{(1)}(at;q^2);s\right\}=\frac{\left(\frac{a}{2}\right)^n}{[2]s^{2n+2}}\sum_{j=0}^{\infty}\frac{\left(-\frac{a^2}{4s^2}\right)^j}{(q^2;q^2)_jK\left(\frac{1}{s^2},n+j+1\right)},$$

$$q\mathcal{L}_2\left\{t^nJ_n^{(2)}(at;q^2);s\right\}=\frac{\left(\frac{a}{2}\right)^n}{[2]s^{2n+2}}\sum_{j=0}^{\infty}\frac{(q^2)^{\frac{j(j+n)}{2}}\left(-\frac{a^2}{4s^2}\right)^j}{(q^2;q^2)_jK\left(\frac{1}{s^2},n+j+1\right)},$$

$$q\mathcal{L}_2\left\{t^nJ_n^{(3)}(at;q^2);s\right\}=\frac{a^n}{[2]s^{2n+2}}\sum_{j=0}^{\infty}\frac{(-1)^j(q^2)^{\frac{j(j-1)}{2}}\left(\frac{a^2q^2}{s^2}\right)^j}{(q^2;q^2)_jK\left(\frac{1}{s^2},n+j+1\right)}. \quad (28)$$

Corollary 5. *In corollary (4) if we use the relations (15), (16) and replace a with $2\sqrt{a}$ we obtain the results of Purohit and Kalla (2007), for example in the first equation of corollary (4)*

$$L_q\left\{t^{\frac{n}{2}}J_n^{(1)}(2\sqrt{at});s\right\}=\frac{a^{\frac{n}{2}}}{s^{n+1}}e_q^{\left(\frac{-a}{s}\right)}. \quad (29)$$

which is the same result given by Purohit and Kalla (2007).

Corollary 6. *In corollary (5) if we take the limit as $q\to 1$*

$$L_2\left\{t^nJ_n^{(1)}(at);s\right\}=\mathcal{L}_2\left\{t^nJ_n^{(1)}(at);s\right\}=\frac{\left(\frac{a}{2}\right)^n}{2s^{2n+2}}e^{\left(\frac{-a^2}{4s^2}\right)}. \quad (30)$$

which is the same result given in Yürekli and Sadek, 1991.

3. Concluding Remarks

The results proved in this paper give some contributions to the theory of the q-series especially q^2-Bessel functions. The above theorems and their various corollaries and consequences can be applied with different parameters to get different types of q-Laplace transforms of wide range q^2-Bessel and q-Bessel functions. Also, the results obtained here can be used in some q-difference equations.

Acknowledgment

The author would like to thank Professor S. K. Omari for his valuable suggestions.

References

Exton, H., Srivastava, H.M., 1994. A certain class of q-Bessel polynomials. Math. Comput. Model. 19 (2), 55–60.
Hahn, W., 1949. Beitrage Zur Theorie Der Heineschen Reihen, die 24 Integrale der hypergeometrischen q-diferenzengleichung, das q-Analog on der Laplace transformation. Mathematische Nachrichten 2, 340–379.
Ismail, M.E.H., 1981. The basic Bessel functions and polynomials. SIAM J. Math. Anal. 12 (3), 454–468.
Ismail, M.E.H., 1982. The zeros of basic Bessel functions, the functions Jv + ax(x), and associated orthogonal polynomials. J. Math. Anal. Appl. 86 (1), 1–19.
Jackson, F.H., 1905. The application of basic numbers to Bessel's and Legendre's functions. Proc. London Math. Soc. 2 (1), 192–220.
Kac, V.G., De Sole, A., 2005. On Integral representations of q-gamma and q-beta functions. Accademia Nazionale dei Lincei (9), 11–29.
Omari, S.K., 2017. On q-analogues of the natural transform of certain q-Bessel Functions and some application. Filomat 31 (9), 2587–2598.
Purohit, S.D., Kalla, S.L., 2007. On q-Laplace transforms of the q-Bessel functions. Fract. Calc. Appl. Anal. 10 (2), 189–196.
Uçar, F., 2014. q-Sumudu transforms of q-analogues of Bessel functions. Sci. World J. 2014, 1–7.
Uçar, F., Albayrak, D., 2011. On q-Laplace type integral operators and their applications. J. Diff. Equ. Appl., 1–14
Yürekli, O., 1999a. Theorems on L_2-transforms and its applications. Complex Var. Elliptic Equ. 38, 95–107.
Yürekli, O., 1999b. New identities involving the Laplace and the L_2-transforms and their applications. Appl. Math. Comput. 99, 141–151.
Yürekli, O., Sadek, I., 1991. A Parseval-Goldstein type theorem on the Widder potential transform and its applications. Int. J. Math. Math. Sci. 14, 517–524.