Journal of King Saud University – Science 30 (2018) 311–315

Contents lists available at ScienceDirect

Journal of King Saud University – Science

journal homepage: www.sciencedirect.com

Original article

Some important classes of neighbor balanced designs in linear blocks of small sizes

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ARTICLE INFO

Article history:
Received 24 October 2016
Accepted 3 October 2017
Available online 5 October 2017

MSC (2000):
05B05
62K10

Keywords:
Neighbor effects
Linear blocks
First order neighbor balanced designs
Second order neighbor balanced designs
Minimal designs

ABSTRACT

Neighbor balanced designs are useful to balance out the neighbor effects in field of agriculture, serology, agro forestry, industry, etc. In most of the agriculture experiments blocks are formed in a line and therefore, neighbor balanced designs are required in linear blocks. In this article some classes of first order neighbor balanced designs are presented in linear blocks of size three and four. A method to construct the second order neighbor balanced designs through two minimal first order neighbor balanced designs in linear binary blocks of size three is also developed here.

1. Introduction

If v treatments are arranged in b linear blocks of size k such that each unordered pair of adjacent treatments appears an equal number of times, say λ'_1, designs are called first order neighbor balanced designs in linear blocks. If $\lambda'_1 = 1$ then such designs are called minimal first order neighbor balanced designs in linear blocks. Kiefer and Wynn (1981) introduced an algorithm to construct the neighbor balanced designs (NBDs) in complete linear blocks. Cheng (1983) generated NBDs in linear blocks for different cases. Azais et al. (1993) constructed NBDs in complete blocks, in $k = v - 1$ and partially neighbor balanced designs in linear blocks. Jacroux (1998) constructed NBDs for all v having blocks of size 3 which are efficient under standard intrablock analysis as well as when experimental units adjacent within blocks are correlated. Tomar et al. (2005) constructed neighbor balanced block designs using Mutually Orthogonal Latin Squares (MOLS) and compared their designs with complete block designs balanced for neighbor effects. Ahmed (2010) constructed NBDs in linear blocks for k even, k odd &two different block sizes k_1 and k_2. Ahmed and Akhtar (2011) constructed NBDs in linear blocks of equal sizes for (i) $v = 4i + 1$, i integer, $k = 3$ with $\lambda' = 1$, (ii) $v = 2i + 1$, i (>1) odd, $k = 3$ with $\lambda' = 2$, and (iii) $v = 2i + 1$ (prime) and $k < v$. They also constructed these designs in linear blocks of unequal sizes for (i) $v = 4i–1$; in $k_1 = 3$ and $k_2 = 2$, $\lambda' = 1$, (ii) $v = 4i + 2$; in $k_1 = 3$ and $k_2 = 2$. Ahmed et al. (2013) developed some infinite series to generate minimal neighbor balanced designs for two and three different sizes in linear blocks. They also constructed generalized neighbor designs (GN_2-designs) in proper linear blocks. They developed following infinite series of minimal neighbor balanced designs in linear blocks. Shahid et al. (2017) constructed some important classes of generalized neighbor designs in linear blocks for four different cases. Minimal designs are always considered as the most economical. In this article, some infinite series are developed to generate the first order neighbor balanced designs for linear blocks of size 3 and 4. Catalogues are also presented of proposed designs. Two

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Peer review under responsibility of King Saud University.

ELSEVIER Production and hosting by Elsevier

https://doi.org/10.1016/j.jksus.2017.10.001
1018-3647/

series are also developed to generate second order neighbor balanced designs through two minimal first order neighbor balanced designs in linear binary blocks of size three. These designs are constructed using method of cyclic shifts which is explained in Section 2. It is important to note that all proposed neighbor designs, in this article, are assumed balanced for non-directional neighbor effect. Non-directional neighbor effect means same effect from right or left neighbor and direction of neighbor does not matter.

2. Models and notations

Consider a class of designs $\Omega_{(v,b,k)}$ with v treatments grouped in b circular blocks of k experimental units per block.

Definition 1. A NBD at distance 1 is a binary design in which each treatment appears as first order neighbor λ' times to all other treatments.

Definition 2. A NBD at distance 2 is a binary design in which each treatment appears as first order neighbor λ'_1 times to all other treatments and each treatment appears as second order neighborλ'_2. times to all other treatments.

NBD at distance 1 & 2 are constructed under the following models 1 & 2 respectively:

$$y_{ij} = \mu + \beta_i + \tau_{d(i,j)} + \lambda_{1d(i,j\pm1)} + \varepsilon_{ij} \quad i = 1,2,\cdots,b; j = 1,2,\cdots,k \tag{1}$$

$$y_{ij} = \mu + \beta_i + \tau_{d(i,j)} + \lambda_{1d(i,j\pm1)} + \lambda_{2d(i,j\pm1)} + \varepsilon_{ij} \quad i = 1,2,\cdots,b;\ j = 1,2,\cdots,k \tag{2}$$

Models (1) and (2) can be written in vector as

$$\mathbf{Y_d} = \mathbf{1}\mu + \mathbf{B}\beta + \mathbf{T_d}\tau + \mathbf{U_{1d}}\lambda_1 + \varepsilon \tag{3}$$

$$\mathbf{Y_d} = \mathbf{1}\mu + \mathbf{B}\beta + \mathbf{T_d}\tau + \mathbf{U_{1d}}\lambda_1 + \mathbf{U_{2d}}\lambda_2 + \varepsilon \tag{4}$$

where **Y** and **1** are vectors of observations and 1's each of order $(bk \times 1)$ respectively, $\mathbf{T_d}$, $\mathbf{U_{1d}}$ and $\mathbf{U_{2d}}$ are incidence matrices of order $(bk \times v)$ for treatment, first order neighbor and second order neighbor effects respectively, and **B** is the $(bk \times b)$ incidence matrix for block effects. The vectors $\beta, \tau, \lambda_1, \lambda_2$ are the parameters of respective effects. ε is the vector of error term with mean vector **0** and variance-covariance matrix $\sigma^2\mathbf{I}$. The complete information matrix for model (3) is

$$\mathbf{C}_d = \begin{bmatrix} \mathbf{1}'\mathbf{1} & \mathbf{1}'\mathbf{B} & \mathbf{1}'\mathbf{T}_d & \mathbf{1}'\mathbf{U}_{1d} \\ \mathbf{B}'\mathbf{1} & \mathbf{B}'\mathbf{B} & \mathbf{B}'\mathbf{T}_d & \mathbf{B}'\mathbf{U}_{1d} \\ \mathbf{T}'_d\mathbf{1} & \mathbf{T}'_d\mathbf{B} & \mathbf{T}'_d\mathbf{T}_d & \mathbf{T}'_d\mathbf{U}_{1d} \\ \mathbf{U}'_{1d}\mathbf{1} & \mathbf{U}'_{1d}\mathbf{B} & \mathbf{U}'_{1d}\mathbf{T}_d & \mathbf{U}'_{1d}\mathbf{U}_{1d} \end{bmatrix}$$

where $\mathbf{1}'\mathbf{1} = bk = rv = n$, For an equireplicate designs with constant block size k; $\mathbf{B}'\mathbf{B} = kI_b$ and $\mathbf{T}'_d\mathbf{T}_d = rI_v$, $\mathbf{T}'_d\mathbf{B} = N$, $\mathbf{U}'_{1d}\mathbf{B} = 2N$. Let $\mathbf{T}'_d\mathbf{U}_{1d} = L$ and $\mathbf{U}'_{1d}\mathbf{U}_{1d} = M$, the complete information matrix becomes

$$\mathbf{C}_d = \begin{bmatrix} n & \mathbf{1}'\mathbf{B} & \mathbf{1}'\mathbf{T}_d & \mathbf{1}'\mathbf{U}_{1d} \\ \mathbf{B}'\mathbf{1} & kI_b & N' & 2N' \\ \mathbf{T}'_d\mathbf{1} & N & rI_v & L \\ \mathbf{U}'_{1d}\mathbf{1} & 2N & L & M \end{bmatrix}$$

After imposing restrictions and simplification, the joint information matrix for treatment and neighbor effects is

$$\mathbf{C}_{\tau u} = \begin{bmatrix} \mathbf{T}'_d\mathbf{T}_d - (\mathbf{T}'_d\mathbf{B}_d\mathbf{K})^{-1}(\mathbf{B}'_d\mathbf{T}_d) & \mathbf{T}'_d\mathbf{U}_{1d} - (\mathbf{T}'_d\mathbf{B}_d\mathbf{K})^{-1}(\mathbf{B}'_d\mathbf{U}_{1d}) \\ \mathbf{U}'_{1d}\mathbf{T}_d - (\mathbf{U}'_{1d}\mathbf{B}_d)\mathbf{K}^{-1}(\mathbf{B}'_d\mathbf{T}_d) & \mathbf{U}'_{1d}\mathbf{U}_{1d} - (\mathbf{U}'_{1d}\mathbf{B}_d)\mathbf{K}^{-1}(\mathbf{B}'_d\mathbf{U}_{1d}) \end{bmatrix}$$

$$\mathbf{C}_{\tau u} = \begin{bmatrix} rI_v - (1/k)NN' & L - (2/k)NN' \\ L - (2/k)NN' & M - (4/k)NN' \end{bmatrix}$$

Then information matrix for treatment is

$$\mathbf{C}_\tau = \mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{B}'$$

where

$$\mathbf{A} = rI_v - (1/k)NN'$$

$$\mathbf{B} = L - (2/k)NN'$$

$$\mathbf{D} = M - (4/k)NN'$$

Here NN' is the treatment concurrence matrix whose diagonal elements are repetitions of each treatment and off-diagonal elements are the number of times two treatments appear together in same blocks. L is the incidence matrix of treatments versus neighbors (left and right). Diagonal elements of L matrix, for a design in which no treatment appears as neighbor to itself, are zero and off-diagonal matrix are the number of times a pair of treatments appear as neighbor to each other in same blocks. For further detail see Iqbal et al. (2006, 2009). To achieve a NBD, all off-diagonal elements of matrix L must be same. If off-diagonal elements of matrix L contain two or more distinct values, the design is known as generalized neighbor design. Similarly, if off-diagonal elements of concurrence matrix NN' are same then the design is BIBD otherwise PBIBD.

Information matrix for neighbor effect in model (3) and information matrices for model (4) can be derived accordingly.

According to Hinkelmann and Kempthorne (2005), average variance of treatment contrast is a function of information matrix as

$$\underset{i\neq i'}{av}.\mathrm{var}(\hat{\tau}_i - \hat{\tau}_{i'})_{IBD} = 2\sigma^2_{e(IBD)}(v-1)^{-1}\sum_{i=1}^{v-1} d_i^{-1}$$

where d_i is the ith eigenvalue of $\mathbf{C}_\tau$. Eigen values of $\mathbf{C}_\tau$ can be calculated using R-language.

3. Method of construction and efficiency factor

3.1. Method of cyclic shifts

Method of cyclic shifts introduced by Iqbal (1991) is simplified here to construct neighbor balanced designs only in linear blocks". v treatments are labeled as 0, 1, 2, ..., $v-1$ under rule I and II below.

Rule I: Let $\underline{S}_j = [q_{j1}, q_{j2}, \ldots, q_{j(k-1)}]$ be a set of shifts where $1 \le q_{ji} \le v-1$. A design is first order neighbor balanced designs in linear blocks if each element of $\underline{S}_j$ along with its complement contains all elements 1, 2, ..., $v-1$ equally often, say, λ_1 times. In Rule I, complement of q_i is $v - q_i$.

Rule II: Let $\underline{S}_j = [q_{j1}, q_{j2}, \ldots, q_{j(k-2)}]$t be a set of shifts where $1 \le q_{ji} \le v-2$. A design is first order neighbor balanced designs in linear blocks if each element of $\underline{S}_j$ along with its complement contains all elements 1, 2, ..., $v-2$ equally often, say, λ_1 times. In Rule II, complement of q_i is $v - 1 - q_i$.

3.2. Efficiency factor (E_f)

In literature, neighbor designs in incomplete block designs are compared with (i) complete block designs and (ii) complete block design balanced for neighbor effects. Hinkelmann and Kempthorne (2005) suggested former approach to calculate relative efficiency, Efficiency Factor E_f. The approach allows comparing an incomplete block design (IBD) with CRD or RCBD and two competing IBDs with each other. The residual variances (σ_c^2) is assumed same for both digns to be compared. For equal blocks of size k the information matrix of a model without and interference effect can be defined as:

$$\mathbf{C} = \mathbf{R} - \frac{1}{k}\mathbf{NN}'$$

where, $\mathbf{R}$ is a diagonal matrix of order $(v \times v)$ with diagonal elements (*r-number of replications*). The matrix $\mathbf{C}$ has one Eigen value (root) $d_v = 0$ with normalized associated Eigenvector $\mathbf{D_v} = (1/v)\mathbf{J}$ and the other non-zero roots will be $d_1, d_2, \cdots, d_{v-1}$ with orthonormal associated Eigenvectors $\mathbf{D_1}, \mathbf{D_2}, \cdot\cdot, \mathbf{D_{v-1}}$. For a BIBD, $d_1 = d_2 = \cdot\cdot = d_{v-1}$ whereas in PBIBD, number of different values of $d_1, d_2, \cdot\cdot, d_{v-1}$ relate with association scheme. Consider same residual variances σ_e^2 for CRD and IBD;

Then

$$\mathrm{E} = \frac{\underset{i \neq i'}{av}.\,\mathrm{var}(\hat{\tau}_i - \hat{\tau}_{i'})_{CRD/RCBD}}{\underset{i \neq i'}{av}.\,\mathrm{var}(\hat{\tau}_i - \hat{\tau}_{i'})_{IBD}} = \frac{2/r}{2/c} = \frac{2/r}{2/(rE_f)} \text{ where } \sigma^2_{e(CRD)} = \sigma^2_{e(IBD)}$$

The quantity $c = rE_f$ is harmonic mean of non-zero Eigen values of matrix $\mathbf{C}$. While comparing two competing IBDs having same number of observations (n), each IBD will have associated with an efficiency factor E_f, the IBD having higher E_f is considered as better design.

3.3. Upper bound (UB) for efficiency factor

In mathematics, an upper bound of a subset S of some partially ordered set P is an element which is greater than or equal to every element of S. The upper bound given by Hinkelmann and Kempthorne (2005) for an incomplete block designs is

$$E_f = \frac{(k-1)v}{(v-1)k}S.$$

A balanced incomplete block design always achieve its upper bound equal to efficiency factor while partially balanced incomplete block design attain smaller E_f value than the upper bound.

4. First order neighbor balanced designs in linear blocks of size three

In this section, some infinite series to generate the first order neighbor balanced designs are developed in linear blocks of size three.

Series 4.1. Minimal first order NBDs can be constructed for $v = 4i$, i integer, $k = 3$ and $\lambda_1 = 1$ in $i(v-1)$ linear blocks from the following i sets of shifts.

$S_j = [p, p+1]$; $j = 1, \ldots, i-1$ and $p = 2j - 1$.
$S_i = [(v-2)/2]t$

Example 4.1. Following is minimal first order NBD for $v = 8$ and $k = 3$ with $\lambda_1 = 1$ generated through the sets of shifts [1, 2] and [3]t mod 7.

B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	B11	B12	B13	B14
0	1	2	3	4	5	6	0	1	2	3	4	5	6
1	2	3	4	5	6	0	3	4	5	6	0	1	2
3	4	5	6	0	1	2	7	7	7	7	7	7	7

Series 4.2. First order NBDs can be constructed for $v = 2i$, i integer greater than 1 and $k = 3$ with $\lambda_1 = 2$ through the following sets of shifts mod $v - 1$.

$\mathbf{S_j} = [j, j]$; $j = 1, 2, \ldots, i-2$.
$\mathbf{S_{i-1}} = [(v-2)/2]t$, $\mathbf{S_i} = [v/2]t$

Example 4.2. First order NBD for $v = 10$ and $k = 3$ with $\lambda_1 = 2$ can be generated through the sets of shifts [1, 1], [2, 2],[3, 3], [4]t and [5]t mod 9.

5. First order neighbor balanced designs in linear blocks of size four

In this section, some infinite series to generate the first order neighbor balanced designs are developed in linear blocks of size four.

Series 5.1. Minimal first order NBDs can be constructed for $v = 6i + 1$,*i*integer and $k = 4$ with $\lambda_1 = 1$ through the following *i sets* of shifts.

$S_j = [3j-2, 3j-1, 3j]$; $j = 1, \ldots, i$.

Example 5.1. Minimal first order NBD for $v = 13$ and $k = 4$ with $\lambda_1 = 1$ can be generated through the sets of shifts [1, 2, 3] and [4, 5, 6] mod 13.

Series 5.2. First order NBDs can be constructed for $v = 3i + 1$ and $k = 4$ with $\lambda_1 = 2$ through the following sets of shifts mod v.

$\mathbf{S_{j+1}} = [(3j)/2 + 1, (3j)/2 + 2, v - ((3j)/2 + 3)]$; $j = 0, 2, \ldots, i$.
$\mathbf{S_{j+1}} = [3(j-1)/2 + 3, 3(j-1)/2 + 2, v - (3(j-1)/2 + 3)]$; $j = 1, 3, \ldots, i-1$.

Example 5.2. First order NBD for $v = 7$ and $k = 4$ with $\lambda_1 = 2$ can be generated through the sets of shifts [1, 2, 6] and [3, 2, 4] mod 7.

Series 5.3. Minimal first order NBDs can be constructed for $v = 6i + 3$ and $k = 4$ with $\lambda_1 = 1$ through the following sets of shifts mod v.

$\mathbf{S_{j+1}} = [3j + 1, v - (3j + 2), 3j + 3]$; $j = 0, 1, \ldots, i-1$.
$\mathbf{S_{i+1}} = [(v-1)/2, (v-1)/2, (v-1)/2](1/3)$ (Every third block be taken from).

Example 5.3. Minimal first order NBD for $v = 15$ and, $k = 4$ with $\lambda_1 = 1$ can be generated through the sets of shifts [1, 13, 3] and [4, 10, 6] and [7, 7, 7](1/3) (Consider every third block) mod 15.

Series 5.4. Minimal first order NBDs can be constructed for $v = 6i + 4$ and $k = 4$ with $\lambda_1 = 1$ through the following sets of shifts mod $v - 1$.

$\mathbf{S_{j+1}} = [3j + 1, v - (3j + 2), 3j + 3]$; $j = 0, 1, \ldots, i-1$.
$\mathbf{S_{i+1}} = [(v-2)/2, v/2, (v+2)/2](1/2)$

Example 5.4. Minimal first order NBD for $v = 16$ and $k = 4$ with $\lambda_1 = 1$ can be generated through the sets of shifts [1, 14, 3], [4, 11, 6] and [7, 8, 9](1/2) mod 16.

Series 5.5. Minimal first order NBDs can be constructed for $v = 3i$, i even and $k = 4$ with $\lambda_1 = 1$ through the following sets of shifts mod $v - 1$.

$S_j = [p, p+1, p+2]$; $j = 1, \ldots, c-1$ and $p = 3j - 2$.
$S_c = [(v-4)/2, (v-2)/2]t$, where $c = i/2$

Example 5.5. Minimal first order NBD for $v = 12$ and $k = 4$ with $\lambda_1 = 1$ can be generated through the sets of shifts [1, 2, 3] and [4, 5]t mod 11.

Series 5.6. First order NBDs can be constructed for $v = 6i + 2$ and $k = 4$ with $\lambda_1 = 3$ through the following sets of shifts mod $v - 1$.

$\mathbf{S_j} = [j, j, j]$; $j = 1, \ldots, 3i - 2$.
$\mathbf{S_{3i-1}} = [(v-4)/2, (v-4)/2]t$,
$\mathbf{S_{3i}} = [(v-4)/2, (v-2)/2]t$,
$\mathbf{S_{3i+1}} = [(v-2)/2, (v-2)/2]t$

Example 5.6. First order NBD for $v = 8$ and $k = 4$ with $\lambda_1 = 3$ can be generated through the sets of shifts [1, 1, 1] + [2, 2]t + [2, 3]t + [3, 3]t mod 7.

A catalogue of First order NBD is developed for linear blocks of size three using Series 3.1 & 3.2 and of size four using series 4.1, 4.2, …, 4.6 which is given as Supplementary Material.

6. Second order NBD in linear blocks of size three

In this Section, a method to construct the second order neighbor balanced designs through combining two minimal first order neighbor balanced designs in linear binary blocks of size three is developed.

A design is called second order neighbor balanced in linear blocks if each unordered pair of distinct treatments appears:

i) An equal number of times, say, λ_1 as first order/ adjacent neighbors, and
ii) An equal number of times, say, λ_2 as second order neighbors.

A minimal second order neighbor balanced designs will be with $\lambda_1 = 2$ and $\lambda_2 = 1$.

Theorem 6.1. If $v = 4i + 1$, i odd then second order neighbor balanced designs with $\lambda_2 = 1$ can be constructed in linear binary blocks of size three by combing two first order neighbor balanced minimal designs through the following sets of shifts.

Design I (first order neighbor balanced minimal design):

$\mathbf{S_{j+1}} = [2j+1, 2j+2]$; $j = 0, 1, \ldots, i-1$.

Design II (first order neighbor balanced minimal design):

$\mathbf{S_{j+1}} = [(2j+2), 2j+3]$; $j = 0, 1, \ldots, i-2$ where $j \neq (i-3)/2$.
$\mathbf{S_{i-1}} = [3i+2, i]$ for $i > 1$
$\mathbf{S_i} = [1, 2i+1]$

Required Design (first-order and second order neighbor balanced minimal design):

$\mathbf{S_{i+1}} = [2j+1, 2j+2]$; $j = 0, 1, \ldots, i-1$.
$\mathbf{S_{j+i+1}} = [2j+2, 2j+3]$; $j = 0, 1, \ldots, i-2$ where $j \neq (i-3)/2$.
$\mathbf{S_{2i-1}} = [3i+2, i]$
$\mathbf{S_{2i}} = [1, 2i+1]$ for $i > 1$

Example 6.1. For $v = 5$ the following are two different first order neighbor balanced designs in linear binary blocks of size three where each pair of distinct treatments appears twice as first order neighbors.

Design I					Design II				
B_1	B_2	B_3	B_4	B_5	B_1	B_2	B_3	B_4	B_5
0	1	2	3	4	0	1	2	3	4
1	2	3	4	0	1	2	3	4	0
3	4	0	1	2	4	0	1	2	3

Combing these two designs, we get required design which is also second order neighbor balanced design with $\lambda_1 = 2$ and $\lambda_2 = 1$

B1	B2	B3	B4	B5	B6	B7	B8	B9	B10
0	1	2	3	4	0	1	2	3	4
1	2	3	4	0	1	2	3	4	0
3	4	0	1	2	4	0	1	2	3

Theorem 6.2. If $v = 8i + 1$, i be an integer then second order neighbor balanced designs $\lambda_2 = 1$ can be constructed in linear binary blocks of size three by combing two minimal first order neighbor balanced designs through the following sets of shifts.

Design I (first order neighbor balanced minimal design):

$\mathbf{S_{j+1}} = [2j+1, 2j+2]$; $j = 0, 1, \ldots, 2i-1$.

Design II (first order neighbor balanced minimal design):

$\mathbf{S_{j+1}} = [2j+2, 2j+3]$; $j = 0, 1, \ldots, 2i-2$ where $j \neq i-1$.
$\mathbf{S_{2i-1}} = [6i+1, 2i+1]$ $\mathbf{S2}\boldsymbol{i} = [1, 4i]$.

Required Design (first-order and second order neighbor balanced minimal design):

$\mathbf{S_{j+1}} = [2j+1, 2j+2]$; $j = 0, 1, \ldots, 2i-1$.
$\mathbf{S_{j+2t+1}} = [2j+2, 2j+3]$; $j = 0, 1, \ldots, 2i-2$ where $j \neq i-1$.
$\mathbf{S_{4i-1}} = [6i+1, 2i+1]$ $\mathbf{S_{4i}} = [1, 4i]$.

Example 6.2. For $v = 9$ the following are two different first order neighbor balanced designs in linear binary blocks of size three where each pair of distinct treatments appears twice as first order neighbors.

Design I

B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	B11	B12	B13	B14	B15	B16	B17	B18
0	1	2	3	4	5	6	7	8	0	1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8	0	3	4	5	6	7	8	0	1	2
3	4	5	6	7	8	0	1	2	7	8	0	1	2	3	4	5	6

Design II

B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}	B_{11}	B_{12}	B_{13}	B_{14}	B_{15}	B_{16}	B_{17}	B_{18}
0	1	2	3	4	5	6	7	8	0	1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8	0	3	4	5	6	7	8	0	1	2
3	4	5	6	7	8	0	1	2	7	8	0	1	2	3	4	5	6

Combing these two designs, we get required design which is also minimal second order neighbor balanced design with $\lambda_1 = 2$ and $\lambda_2 = 1$.

B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	B11	B12
0	1	2	3	4	5	6	7	8	0	1	2
1	2	3	4	5	6	7	8	0	3	4	5
3	4	5	6	7	8	0	1	2	7	8	0

B13	B14	B15	B16	B17	B18	B19	B20	B21	B22	B23	B24
3	4	5	6	7	8	0	1	2	3	4	5
6	7	8	0	1	2	1	2	3	4	5	6
1	2	3	4	5	6	3	4	5	6	7	8

B25	B26	B27	B28	B29	B30	B31	B32	B33	B34	B35	B36
6	7	8	0	1	2	3	4	5	6	7	8
7	8	0	3	4	5	6	7	8	0	1	2
0	1	2	7	8	0	1	2	3	4	5	6

Acknowledgment

Authors are highly thankful to the Editor and Reviewers for their valuable suggestions and corrections.

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at https://doi.org/10.1016/j.jksus.2017.10.001.

References

Ahmed, R., 2010. Designs Robust to Neighbor Effects with Minimum Number of Blocks (Unpublished Ph.D. thesis). The Islamia University of Bahawalpur, Pakistan.

Ahmed, R., Akhtar, M., 2011. Construction of neighbor balanced designs in linear blocks. Commun. Stat. – Theory Methods 40, 3198–3205.

Ahmed, R., Shehzad, F., Akhtar, M., 2013. Minimal neighbor designs in linear Blocks. Commun. Stat. – Simul. Comput. 42, 2135–2139.

Azais, J.M., Bailey, R.A., Monod, H., 1993. A catalogue of efficient neighbor – designs with border plots. Biometrics 49 (4), 1252–1261.

Cheng, C.S., 1983. Construction of optimal balanced incomplete block designs for correlated observations. Ann. Stat. 11, 240–246.

Hinkelmann, K., Kempthorne, O., 2005. Design and Analysis of Experiments. John and Wiley & Sons Inc, Hoboken, New Jersey.

Iqbal, I., 1991. Construction of Experimental Design Using Cyclic Shifts (Unpublished Ph.D Thesis). University of Kent at Canterbury, U.K.

Iqbal, I., Aman Ullah, M., Nasir, J.A., 2006. The construction of second order neighbor designs. J. Res. (Sci.) 17 (3), 191–199.

Iqbal, I., Tahir, M.H., Ghazali, S.S.A., 2009. Circular neighbor-balanced designs using cyclic shifts. Sci. China Ser. A. 10, 2243–2256.

Jacroux, M., 1998. On the construction of efficient equineighbored incomplete block designs having block size 3. Sankhya, Ser. B 60 (3), 488–495.

Kiefer, J., Wynn, H., 1981. Optimum balanced block and Latin square designs for correlated observations. Ann. Stat. 9, 737–757.

Shahid, M.R., Zakria, M., Shehzad, F., Ahmed, R., 2017. Some important classes of generalized neighbor designs for linear blocks. Commun. Stat. – Simul. Comput. 46 (3), 1991–1997.

Tomar, J.S., Jaggi, S., Varghese, C., 2005. On totally balanced block designs for competition effects. J. Appl. Stat. 32 (1), 87–97.