Journal of King Saud University – Science (2012) **24**, 295–299

King Saud University
Journal of King Saud University – Science
www.ksu.edu.sa
www.sciencedirect.com

ORIGINAL ARTICLE

Bright and dark soliton solutions for a new fifth-order nonlinear integrable equation with perturbation terms

Houria Triki [a], **Abdul-Majid Wazwaz** [b,*]

[a] Radiation Physics Laboratory, Department of Physics, Faculty of Sciences, Badji Mokhtar University, P.O. Box 12, Annaba 23000, Algeria
[b] Department of Mathematics, Saint Xavier University, Chicago, IL 60655, USA

Received 20 July 2011; accepted 14 August 2011
Available online 22 August 2011

KEYWORDS
Bright and dark solitons;
$K(m,n)$ equation;
Solitary wave ansatz

Abstract In this paper, we use the solitary wave ansatz method to carry out the integration of a new fifth-order nonlinear equation having perturbation terms. The perturbation terms that are considered are the first-order dispersion term, the power law nonlinearity term, and the fifth- order dispersion term. Both bright and dark soliton solutions are obtained. The physical parameters in the soliton solutions: amplitude, inverse width, and velocity are obtained as functions of the dependent model coefficients. The conditions of the existence of the derived solitons are presented.

1. Introduction

Recent advances in the study of the wave dynamics in nonlinear systems have led to the construction of new integrable models exhibiting a rich variety of exact solutions with interesting properties. A new class of nonlinear partial differential equations (NLPDEs) with constant and variable coefficients that model a lot of physical phenomena has been formulated depending on the physical situation. In parallel, some other NLPDEs are generalized which offer a rich knowledge on the propagation behavior of waves in nonlinear systems of all kinds. Examples of many equations that have been generalized are the $K(m,n)$ equation (Rosenau and Hyman, 1993), the $B(m,n)$ equation (Zhang, 2006; Biswas, 2009b), the $ZK(m,n,k)$ equation (Inc, 2007), the $CKG(m,n,k)$ (Yan, 2007) and so on.

The problem of finding exact solutions for nonlinear models is of great importance, both in mathematical and physical point of view. Based on the exact solutions, we can accurately analyze the properties of the propagating waves in a given nonlinear system. Several kinds of closed form solutions, including compactons, peakons, solitons, cuspons, similaritons, etc., have been obtained under specific conditions. In particular, exact solutions of soliton-type are of general physical interest, because the soliton approach is universal in different fields of modern physics. Notice that the soliton pulses are observed in many physics areas, such as fluid mechanics, plasma physics, optical fibers, hydrodynamics, biology, solid state physics, etc. It is worth noting that, the existence of such shape-preserving

* Corresponding author. Tel.: +1 773 298 3397.
E-mail addresses: houriatriki@yahoo.fr (H. Triki), wazwaz@sxu.edu (A.-M. Wazwaz).
Peer review under responsibility of King Saud University.

ELSEVIER Production and hosting by Elsevier

1018-3647 © 2011 King Saud University. Production and hosting by Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.jksus.2011.08.004

waves in the nonlinear systems is the result of exact balance among dispersion (or diffraction) and the nonlinearity effects.

Recently, in the mathematical physics literature has appeared a large amount of new powerful methods to calculate exact solutions to NLPDEs. Among these modern methods of integrability, we can cite the coupled amplitude-phase formalism (Palacios et al., 1999; Du et al., 1995), the hyperbolic tangent method (Malfliet, 1992), Hirota bilinear method (Nakkeeran, 2002; Wazwaz, 2010), the sub-ODE method (Li and Wang, 2007; Triki and Wazwaz, 2009), the solitary wave ansatz method (Biswas and Milovic, 2010; Saha et al., 2009; Triki and Wazwaz, 2009; Biswas, 2008a,b, 2009a) and other methods as well. The solitary wave ansatz has recently been applied successfully to a wide range of NLPDEs (Biswas and Milovic, 2010; Saha et al., 2009; Triki and Wazwaz, 2009; Biswas, 2008a,b, 2009a).

Very recently, Wazwaz (2011a,b) introduced a new fifth-order nonlinear integrable evolution equation in the form

$$u_{ttt} - u_{txxxx} - (u_x u_t)_{xx} - 4(u_x u_{xt})_x = 0, \tag{1.a}$$

and a new generalized fifth-order nonlinear integrable equation

$$u_{ttt} - u_{txxxx} - \alpha(u_x u_t)_{xx} - \beta(u_x u_{xt})_x = 0, \tag{1.b}$$

and derived the multiple soliton solutions for each equation by using the Hereman–Nuseri method.

In this work we prove that a family of this new evolution equation having perturbation terms in the form

$$u_{ttt} - a u_{txxxx} - b(u_x u_t)_{xx} - c(u_x u_{xt})_x = \alpha u_x + \gamma u^m u_x + \delta u_{xxxxx} \tag{1}$$

possesses exact bright and dark soliton solutions under certain parametric conditions. It should be noted that the finding of a new model supporting soliton-type solutions is a very interesting result and is also helpful for future research.

Here, in (2), a, b, c, α, γ and δ are all constants. The three perturbation terms on the right-hand side of Eq. (2) represent the first-order dispersion term, the power law nonlinearity term, and the fifth-order dispersion term, respectively. Also, noting that the parameter m indicates the power law nonlinearity. On setting $a = 1$, $b = c = 4$, and $\alpha = \gamma = \delta = 0$, Eq. (2) reduces to the model Eq. (1.a).

2. Bright solitons

In order to obtain the bright soliton solution to (2), the solitary wave ansatz is assumed as (Triki and Wazwaz, 2009; Biswas, 2008a; Biswas, 2009a)

$$u(x,t) = \frac{A}{\cosh^p \tau} \tag{2}$$

where

$$\tau = B(x - vt) \tag{3}$$

Here A is the soliton amplitude, B is the inverse width of the soliton and v is the soliton velocity. The unknown index p will be determined during the course of derivation of the solution of this Eq. (2).

From (3) and (4), we find

$$u_{ttt} = p^3 A v^3 B^3 \frac{\tanh\tau}{\cosh^p\tau} - p(p+1)(p+2)Av^3B^3\frac{\tanh\tau}{\cosh^{p+2}\tau} \tag{4}$$

$$u_{txxxx} = p^5 vAB^5 \frac{\tanh\tau}{\cosh^p\tau} - 2p(p+1)(p+2)(p^2+2p+2) \times vAB^5\frac{\tanh\tau}{\cosh^{p+2}\tau} + p(p+1)(p+2)(p+3)(p+4) \times vAB^5\frac{\tanh\tau}{\cosh^{p+4}\tau}, \tag{5}$$

$$(u_x u_t)_{xx} = -4p^4 vA^2B^4\frac{1}{\cosh^{2p}\tau} + 2p^2(4p^2+5p+2) \times vA^2B^4\frac{1}{\cosh^{2p+2}\tau} - 2p^2(p+1)(2p+3)vA^2B^4\frac{1}{\cosh^{2p+4}\tau}, \tag{6}$$

$$(u_x u_{xt})_x = -2p^4 vA^2B^4\frac{1}{\cosh^{2p}\tau} + p^2(4p^2+5p+2) \times vA^2B^4\frac{1}{\cosh^{2p+2}\tau} - p^2(p+1)(2p+3)vA^2B^4\frac{1}{\cosh^{2p+4}\tau}, \tag{7}$$

$$u_x = -pAB\frac{\tanh\tau}{\cosh^p\tau}, \tag{8}$$

$$u^m u_x = -pA^{m+1}B\frac{\tanh\tau}{\cosh^{p(m+1)}\tau}, \tag{9}$$

and

$$u_{xxxxx} = -p^5AB^5\frac{\tanh\tau}{\cosh^p\tau} + 2p(p+1)(p+2)(p^2+2p+2) \times AB^5\frac{\tanh\tau}{\cosh^{p+2}\tau} - p(p+1)(p+2)(p+3)(p+4)AB^5\frac{\tanh\tau}{\cosh^{p+4}\tau}. \tag{10}$$

Substituting (5)–(11) into (2) gives

$$\begin{aligned}&(p^3v^3AB^3 - ap^5vAB^5 + \delta p^5AB^5 + \alpha pAB)\frac{\tanh\tau}{\cosh^p\tau}\\
&+ \{-p(p+1)(p+2)Av^3B^3 + 2(av-\delta)p(p+1)(p+2)\\
&\times (p^2+2p+2)AB^5\}\frac{\tanh\tau}{\cosh^{p+2}\tau} - (av-\delta)p(p+1)(p+2)\\
&\times (p+3)(p+4)AB^5\frac{\tanh\tau}{\cosh^{p+4}\tau} + 2(2b+c)p^4vA^2B^4\frac{1}{\cosh^{2p}\tau}\\
&- (2b+c)p^2(4p^2+5p+2)vA^2B^4\frac{1}{\cosh^{2p+2}\tau}\\
&+ (2b+c)p^2(p+1)(2p+3)vA^2B^4\frac{1}{\cosh^{2p+4}\tau}\\
&+ \gamma pA^{m+1}B\frac{1}{\cosh^{(m+1)p}\tau} = 0.\end{aligned} \tag{11}$$

Now, from (12), equating the exponents $(m+1)p$ and $p+2$ leads to

$$(m+1)p = p+2, \tag{12}$$

which gives

$$p = \frac{2}{m}. \tag{13}$$

From (12) setting the coefficients of $\tanh\tau/\cosh^{p+j}\tau$ and $1/\cosh^{2p+j}\tau$ functions to zero where $j = 0, 2, 4$, since these are linearly independent functions, gives the following algebraic equations:

$$p^3v^3AB^3 - ap^5vAB^5 + \delta p^5AB^5 + \alpha pAB = 0, \tag{14}$$

$$-p(p+1)(p+2)Av^3B^3 + 2(av-\delta)p(p+1)(p+2) \times (p^2+2p+2)AB^5 + \gamma pA^{m+1}B = 0, \tag{15}$$

$$-(av-\delta)p(p+1)(p+2)(p+3)(p+4)AB^5 = 0, \tag{16}$$

$$2(2b+c)p^4vA^2B^4 = 0, \tag{17}$$

$$-(2b+c)p^2(4p^2+5p+2)vA^2B^4 = 0, \tag{18}$$

$$(2b+c)p^2(p+1)(2p+3)vA^2B^4 = 0. \tag{19}$$

Solving the above system yields

$$A = \left[-\frac{(m+1)(m+2)\alpha}{2\gamma}\right]^{\frac{1}{m}}, \quad \gamma \neq 0, \tag{20}$$

$$v = \frac{\delta}{a}, \quad a \neq 0, \tag{21}$$

$$B = \frac{ma}{2\delta}\sqrt{-\frac{\alpha a}{\delta}}, \tag{22}$$

and

$$c = -2b. \tag{23}$$

Eq. (21) shows that solitons will exist for

$$\alpha\gamma < 0, \tag{24}$$

if m is an even integer. However, if m is an odd integer there is no such restriction but the soliton will be pointing downwards.

From (23) we clearly see that the solitons will exist for

$$\delta\alpha a < 0. \tag{25}$$

Thus, finally, the bright soliton solution to the new fifth-order nonlinear evolution equation with perturbation terms (2) is given by

$$u(x,t) = \frac{A}{\cosh^{\frac{2}{m}}[B(x-vt)]}, \tag{26}$$

where the amplitude A and the width B are given by (21) and (23) respectively while the velocity of the soliton is given by (22). Finally the constraint relation between the nonlinear dispersion parameters c and b is displayed in (24).

3. Dark solitons

In this section the search is going to be for the topological 1-soliton solution to the new fifth-order nonlinear evolution equation given by (2). To start off, the hypothesis is given by (Triki and Wazwaz, 2009)

$$u(x,t) = A\tanh^p\tau, \tag{27}$$

where

$$\tau = B(x-vt), \tag{28}$$

where in (28) and (29), A and B are free parameters and v is the velocity of the wave. Also, the unknown exponent p will be determined during the course of the derivation of the soliton solution to (2). Therefore from (28), we get

$$\begin{aligned} u_{ttt} = -pAB^3v^3\big[&(p-1)(p-2)\tanh^{p-3}\tau \\ &-\{2p^2+(p-1)(p-2)\}\tanh^{p-1}\tau \\ &+\{2p^2+(p+1)(p+2)\}\tanh^{p+1}\tau \\ &-(p+1)(p+2)\tanh^{p+3}\tau\big], \end{aligned} \tag{29}$$

$$\begin{aligned} (u_xu_t)_{xx} = -2p^2A^2B^4v\big[&(p+1)(2p+3)\tanh^{2p+4}\tau \\ &+(p-1)(2p-3)\tanh^{2p-4}\tau \\ &-2(4p^2+5p+2)\tanh^{2p+2}\tau \\ &-2(4p^2-5p+2)\tanh^{2p-2}\tau + 2(6p^2+1)\tanh^{2p}\tau\big], \end{aligned} \tag{30}$$

$$\begin{aligned} (u_xu_{xt})_x = -p^2A^2B^4v\big[&(p+1)(2p+3)\tanh^{2p+4}\tau \\ &+(p-1)(2p-3)\tanh^{2p-4}\tau \\ &-2(4p^2+5p+2)\tanh^{2p+2}\tau \\ &-2(4p^2-5p+2)\tanh^{2p-2}\tau + 2(6p^2+1)\tanh^{2p}\tau\big], \end{aligned} \tag{31}$$

$$u_x = pAB(\tanh^{p-1}\tau - \tanh^{p+1}\tau), \tag{32}$$

$$u^mu_x = pA^{m+1}B(\tanh^{p(m+1)-1}\tau - \tanh^{p(m+1)+1}\tau), \tag{33}$$

$$\begin{aligned} u_{txxxx} = -pAB^5v\big\{&(p-1)(p-2)(p-3)(p-4)\tanh^{p-5}\tau \\ &-(p+1)(p+2)(p+3)(p+4)\tanh^{p+5}\tau \\ &-(p-1)(p-2)\{2p^2+2(p-2)^2+(p-3)(p-4)\}\tanh^{p-3}\tau \\ &+(p+1)(p+2)\{2p^2+2(p+2)^2+(p+3)(p+4)\}\tanh^{p+3}\tau \\ &+[2(p-1)(p-2)\{p^2+(p-2)^2\}+4p^4 \\ &+p(p-1)^2(p-2)+p(p+1)^2(p+2)]\tanh^{p-1}\tau \\ &-[2(p+1)(p+2)\{p^2+(p+2)^2\}+4p^4 \\ &+p(p-1)^2(p-2)+p(p+1)^2(p+2)]\tanh^{p+1}\tau\big\}, \end{aligned} \tag{34}$$

and

$$\begin{aligned} u_{xxxxx} = pAB^5\big\{&(p-1)(p-2)(p-3)(p-4)\tanh^{p-5}\tau \\ &-(p+1)(p+2)(p+3)(p+4)\tanh^{p+5}\tau \\ &-(p-1)(p-2)\{2p^2+2(p-2)^2 \\ &+(p-3)(p-4)\}\tanh^{p-3}\tau + (p+1)(p+2)\{2p^2 \\ &+2(p+2)^2+(p+3)(p+4)\}\tanh^{p+3}\tau \\ &+[2(p-1)(p-2)\{p^2+(p-2)^2\}+4p^4 \\ &+p(p-1)^2(p-2)+p(p+1)^2(p+2)]\tanh^{p-1}\tau \\ &-[2(p+1)(p+2)\{p^2+(p+2)^2\}+4p^4 \\ &+p(p-1)^2(p-2)+p(p+1)^2(p+2)]\tanh^{p+1}\tau\big\}. \end{aligned} \tag{35}$$

Now substituting (30)–(36) into (2) gives

$$\begin{aligned} &-pAB^3v^3[(p-1)(p-2)\tanh^{p-3}\tau - \{2p^2 \\ &+(p-1)(p-2)\}\tanh^{p-1}\tau + \{2p^2+(p+1)(p+2)\}\tanh^{p+1}\tau \\ &-(p+1)(p+2)\tanh^{p+3}\tau] + (av-\delta)pAB^5\{(p-1)(p-2) \\ &\times(p-3)(p-4)\tanh^{p-5}\tau - (p+1)(p+2)(p+3) \\ &\times(p+4)\tanh^{p+5}\tau - (p-1)(p-2)\{2p^2+2(p-2)^2 \\ &+(p-3)(p-4)\}\tanh^{p-3}\tau + (p+1)(p+2)\{2p^2+2(p+2)^2 \\ &+(p+3)(p+4)\}\tanh^{p+3}\tau + [2(p-1)(p-2)\{p^2+(p-2)^2\} \\ &+4p^4+p(p-1)^2(p-2)+p(p+1)^2(p+2)]\tanh^{p-1}\tau \\ &-[2(p+1)(p+2)\{p^2+(p+2)^2\}+4p^4+p(p-1)^2(p-2) \\ &+p(p+1)^2(p+2)]\tanh^{p+1}\tau\} + (2b+c)p^2A^2B^4v[(p+1) \\ &\times(2p+3)\tanh^{2p+4}\tau + (p-1)(2p-3)\tanh^{2p-4}\tau \\ &-2(4p^2+5p+2)\tanh^{2p+2}\tau - 2(4p^2-5p+2)\tanh^{2p-2}\tau \\ &+2(6p^2+1)\tanh^{2p}\tau] - \alpha pAB(\tanh^{p-1}\tau - \tanh^{p+1}\tau) \\ &-\gamma pA^{m+1}B(\tanh^{p(m+1)-1}\tau - \tanh^{p(m+1)+1}\tau) = 0. \end{aligned} \tag{36}$$

From (37), equating the exponents $p(m+1)+1$ and $p+3$ gives

$$p(m+1)+1 = p+3, \tag{37}$$

so that

$$p = \frac{2}{m}. \tag{38}$$

It needs to be noted that the same value of p is yielded when the exponents $p(m+1)-1$ and $p+1$ are equated with each other.

Now from (37) the linearly independent functions are $\tanh^{p+j}\tau$ for $j = \pm 1, \pm 3, \pm 5$ and $\tanh^{2p+k}\tau$ for $k = 0, \pm 2, \pm 4$. Hence setting their respective coefficients to zero yields the following set of equations:

$$\begin{aligned} &-pAB^3v^3(p-1)(p-2) \\ &-(av-\delta)pAB^5(p-1)(p-2)\{2p^2+2(p-2)^2 \\ &+(p-3)(p-4)\} = 0, \end{aligned} \tag{39}$$

$$\begin{aligned} &+pAB^3v^3\{2p^2+(p-1)(p-2)\} - \alpha pAB \\ &\quad +(av-\delta)pAB^5[2(p-1)(p-2)\{p^2+(p-2)^2\} \\ &\quad +4p^4+p(p-1)^2(p-2)+p(p+1)^2(p+2)] = 0, \end{aligned} \tag{40}$$

$$\begin{aligned} &-pAB^3v^3\{2p^2+(p+1)(p+2)\} + \alpha pAB - \gamma pA^{m+1}B \\ &\quad -(av-\delta)pAB^5[2(p+1)(p+2)\{p^2+(p+2)^2\}+4p^4 \\ &\quad +p(p-1)^2(p-2)+p(p+1)^2(p+2)] = 0, \end{aligned} \tag{41}$$

$$\begin{aligned} &pAB^3v^3(p+1)(p+2)+(av-\delta)pAB^5(p+1)(p+2) \\ &\qquad \times\{2p^2+2(p+2)^2+(p+3)(p+4)\} \\ &\quad +\gamma pA^{m+1}B = 0, \end{aligned} \tag{42}$$

$$(av-\delta)pAB^5(p-1)(p-2)(p-3)(p-4) = 0 \tag{43}$$

$$-(av-\delta)pAB^5(p+1)(p+2)(p+3)(p+4) = 0, \tag{44}$$

$$(2b+c)p^2A^2B^4v(p+1)(2p+3) = 0, \tag{45}$$

$$(2b+c)p^2A^2B^4v(p-1)(2p-3) = 0, \tag{46}$$

$$-2(2b+c)p^2A^2B^4v(4p^2+5p+2) = 0, \tag{47}$$

$$-2(2b+c)p^2A^2B^4v(4p^2-5p+2) = 0, \tag{48}$$

$$2(2b+c)p^2A^2B^4v(6p^2+1) = 0. \tag{49}$$

By solving (44)–(50), we get

$$v = \frac{\delta}{a} \tag{50}$$

and

$$c = -2b. \tag{51}$$

By substituting (51) into (40), we obtain

$$-pAB^3v^3(p-1)(p-2) = 0. \tag{52}$$

To solve (53), we have considered the following two cases:

3.1. Case 1: $p = 1$

From (39), this yields

$$m = 2. \tag{53}$$

Further substitution of $p = 1$ into (41)–(43), respectively, gives

$$A = \left(\frac{-3\alpha}{\gamma}\right)^{\frac{1}{m}}, \tag{54}$$

and

$$B = \frac{a}{\delta}\sqrt{\frac{\alpha a}{2\delta}}, \tag{55}$$

which shows that solitons will exist for

$$a\alpha\delta > 0. \tag{56}$$

3.2. Case 2: $p = 2$

From (39), this yields

$$m = 1. \tag{57}$$

By substituting $p = 2$ into (41)–(43), respectively, we obtain

$$A = \left(-\frac{3\alpha}{2\gamma}\right)^{\frac{1}{m}}, \tag{58}$$

and

$$B = \frac{a}{2\delta}\sqrt{\frac{\alpha a}{2\delta}}, \tag{59}$$

which shows that solitons will exist for

$$a\alpha\delta > 0. \tag{60}$$

Lastly, we can determine the dark soliton solution for the new evolution equation with perturbation terms (2) when we substitute (51), (55) and (56) in (28) with the respective constraint (54) for the first case of solution or we substitute (51), (59) and (60) in (28) with the respective constraint (58) for the second case of solution as

$$u(x,t) = A\tanh^{\frac{2}{m}}[B(x-vt)], \tag{61}$$

which exists provided that $a\alpha\delta > 0$ and $c = -2b$.

4. Conclusion

This paper obtains the bright and dark soliton solutions of a new fifth-order nonlinear equation in presence of perturbation terms including the power law nonlinearity. The solitary wave ansatz method is employed to integrate the considered equation. Parametric conditions for the existence of the soliton solutions are found. In view of the analysis, we see that the examined equation is an interesting model for soliton-type solutions. In addition, we note that the solitary wave ansatz method is an efficient method for constructing exact soliton solutions for such nonlinear evolution equation that includes perturbation terms.

References

Biswas, A., 2008a. 1-soliton solution of the K(m, n) equation with generalized evolution. Phys. Lett. A 372, 4601–4602.
Biswas, A., 2008b. 1-Soliton solution of (1 + 2) dimensional nonlinear Schrödinger's equation in dual-power law media. Phys. Lett. A 372, 5941–5943.
Biswas, A., 2009a. Solitary wave solution for the generalized Kawahara equation. Appl. Math. Lett. 22, 208–210.

Biswas, A., 2009b. 1-soliton solution of the B(m, n) equation with generalized evolution. Commun. Nonlinear Sci. Numer. Simul. 14, 3226–3229.

Biswas, A., Milovic, D., 2010. Bright and dark solitons of the generalized nonlinear Schrödinger's equation. Commun. Nonlinear Sci. Numer. Simulat. 15, 1473–1484.

Du, M., Chan, A.K., Chui, C.K., 1995. A novel approach to solving the nonlinear Schrödinger equation by the coupled amplitude-phase formulation. IEEE J. Quant. Electron. 31 (1), 177.

Inc, M., 2007. Compacton and periodic wave solutions of the nonlinear dispersive Zakharov–Kuznetsov equation. Centr. Eur. J. Phys. 5 (3), 351–366.

Li, X., Wang, M., 2007. A sub-ODE method for finding exact solutions of a generalized KdV–mKdV equation with high-order nonlinear terms. Phys. Lett. A 361, 115.

Malfliet, W., 1992. Solitary wave solutions of nonlinear wave equations. Am. J. Phys., 650.

Nakkeeran, K., 2002. Bright and dark optical solitons in fiber media with higher-order effects. Chaos, Solitons Fractals 13, 673.

Palacios, S.L., Guinea, A., Fernandez-Diaz, J.M., Crespo, R.D., 1999. Dark solitary waves in the nonlinear Schrödinger equation with third order dispersion, self-steepening, and self-frequency shift. Phys. Rev. E 60 (1), R45.

Rosenau, P., Hyman, J.M., 1993. Compactons: Solitons with finite wavelength. Phys. Rev. Lett. 70 (5), 564–567.

Saha, M., Sarma, A.K., Biswas, A., 2009. Dark optical solitons in power law media with time-dependent coefficients. Phys. Lett. A 373, 4438–4441.

Triki, H., Wazwaz, A.M., 2009. Sub-ODE method and soliton solutions for the variable-coefficient mKdV equation. Appl. Math. Comput. 214, 370.

Triki, H., Wazwaz, A.M., 2009. Bright and dark soliton solutions for a K(m,n) equation with t-dependent coefficients. Phys. Lett. A 373, 2162–2165.

Wazwaz, A.M., 2010. Integrable (2 + 1)-dimensional and (3 + 1)-dimensional breaking soliton equations. Phys. Scr. 81, 035005, 5p.

Wazwaz, A.M., 2011a. A new fifth-order nonlinear integrable equation: multiple soliton solutions. Phys. Scr. 83, 015012, 3p.

Wazwaz, A.M., 2011b. A new generalized fifth-order nonlinear integrable equation. Phys. Scr. 83, 035003, 4p.

Yan, Z., 2007. New exact solution structures and nonlinear dispersion in the coupled nonlinear wave systems. Phys. Lett. A 361, 194–200.

Zhang, Y., 2006. New exact special solutions with solitary patterns for Boussinesq-like B(m,n) equations with fully nonlinear dispersion. Appl. Math. Comput. 173, 1137–1148.