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ORIGINAL ARTICLE

An aircraft acquisition decision model under stochastic demand

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Abstract This study proposes the probabilistic dynamic programming model to address the stochastic demand issue in aircraft acquisition problem. A probable phenomena is defined to comprehend the uncertain state variables so that the targeted level of service could be achieved profitably by the airline company. The objective function and the constraints have a linear expression with respect to the decision variables, and hence the proposed model is then converted as a linear programming model. The proposed model and the solution method are then examined with an illustrative case study to determine the number and the types of new aircraft that should be purchased at every time period. The results show that the proposed methodology is viable in providing the optimal solution.

1. Introduction

The level of passenger demand varies from time to time due to unpredicted events, such as the outbreaks of flu diseases. Such uncertainty in travel demand could affect the operational profit and the level of services of airline companies. One major operational decision that requires careful planning is the acquisition of new aircraft. Based on the projected level of demand, airline companies have to decide on their aircraft's fleet size. In such a case, how to determine the number of aircraft and the types of the aircraft required in meeting the level of demand is utmost important especially when the demand level is uncertain about the point of planning. Past literature studies (New, 1975; Wei and Hansen, 2005) revealed that the level of travellers' demand needs to be considered in obtaining the optimal solution for the aircraft acquisition model. Listes and Dekker (2005) mentioned that if stochasticity is considered, the solution obtained is more robust and closer to the realistic implementation.

Considering stochastic demand for airline operational planning, Pitfield et al. (2009) adopted simultaneous equations approach to evaluate the real airline data. They found that the level of demand elasticity could affect the aircraft size as well

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as the service frequency. Apart from this, uncertainty of future demand was inspected by List et al. (2003) by using a partial moment measure of risk to solve the robust optimization problem. Listes and Dekker (2005) then investigated the best fleet composition (i.e. choice of aircraft for the most profitable operation subject to the airline's planned schedule) by using scenario aggregation-based approach. They looked into the robustness of an airline fleet configuration that accounts explicitly for short-term stochastic demand fluctuations. In the study, they found that the stochastic approach is pertinent and viable in capturing the robustness of a larger set of realistic data. However, the limitation of the study is that only short term planning horizon is considered. Besides, there are some studies in fleet assignment and scheduling problem that consider stochastic demand. Yan et al. (2008) developed a stochastic programming model for the flight scheduling model, and Feldman (2002) reported that stochastic demand should be considered when carrying out the aircraft assignment.

It could be seen that there is limited studies in the aircraft acquisition model that consider stochastic demand. Nevertheless, this is important as, in the reality, the airline company has to consider the demand stochasticity when they are planning for the aircraft acquisition. It would be more challenging when a long term (more than 1 year) planning is to be considered as the forecasting of the demand might be difficult. As such, the approaches and models developed for short term planning might not be useful in such circumstances. Therefore, in this study, a long term planning of the aircraft acquisition considering the stochastic demand is proposed. An optimization model that aims to maximize the operational profit of airline companies is developed by using the probabilistic dynamic programming model. This approach is selected as it is capable of decomposing the proposed model into a series of simpler single-period sub-problems during the planning horizon. More importantly, this approach considers states (i.e. decision variables) and the corresponding profits which are probabilistic (not deterministic) at each stage. The decision variables of the acquisition model are the number and types of aircraft that need to be purchased in order to achieve the objective. To capture the demand uncertainty, it is assumed that the travel demand could be described by some probabilistic distributions. Besides, the probable phenomena are defined according to the targeted confidence level as the occurrence of state variables is probabilistic due to uncertain demand. This is necessary in order to capture the uncertainty of state variables properly as well as to ensure that the airline company could achieve the targeted level of service profitably. It is then shown that the probabilistic dynamic programming model could be converted as a linear programming model if the objective function and the constraints are assumed to have a linear relationship with respect to the decision variables. An illustrative case study is developed to test the proposed model and methodology. For simplicity, only two types of aircraft are considered. Gu et al. (1994) mentioned that if more than two aircraft types are considered, the problem might become a NP-hard problem which can only be solved by meta-heuristic methods.

2. Nomenclature

Following are the notations used in this study (apply for n types of aircraft at age y for which t is the operating period):

Parameters	
T	Horizon length for the planning period
$MAX_{budget(t)}$	Budget constraint allocated for the acquisition of new aircraft
D_t^S	Random variable for stochastic demand (correspond to phenomenon S)
$ORDER_t$	Total number of aircraft that could be purchased in the market
$PARK_t$	Area of hangar (as geometry limitation)
r_t	Discount rate for which the discount factor is $(1 + r_t)^{-t}$
α	Significance level of demand constraint
β	Significance level of lead time constraint
γ	Significance level of selling time constraint
$E(fare_t)$	Expected value of flight fare per passenger
$E(\cos t_t)$	Expected value of flight cost per passenger
p_s	Probability to own I_t as the initial number of aircraft (at phenomenon S)
A_t^n	Total of aircraft owned
Functions	
$P(I_t)$	Function of discounted profit (with I_t as initial number of aircraft)
$f(D_t, A_t^n)$	Function of number of flights in terms of D_t and A_t^n
$hgf(D_t, A_t^n)$	Maintenance cost function in terms of the function of total mileage travelled, g, and $f(D_t, A_t^n)$
Sets	
$X_t = (x_{t1}, x_{t2}, \ldots, x_{tn})$	Number of aircraft to be purchased
$I_t = (In_{t1y}, In_{t2y}, \ldots, In_{tny})$	Initial number of aircraft
$O_t = (O_{t1}, O_{t2}, \ldots, O_{tn})$	Number of aircraft to be ordered
$R_t = (R_{t1}, R_{t2}, \ldots, R_{tn})$	Number of aircraft to be released for sales
$U_t = (u_{t1}, u_{t2}, \ldots, u_{tn})$	Setup cost for the acquisition of aircraft
$S = (s_1, s_2, \ldots, s_k)$	Phenomenon of owning I_t as the initial number of aircraft
$PURC_t = (purc_{t1}, purc_{t2}, \ldots, purc_{tn})$	Purchase cost of aircraft
$DP_t = (dp_{t1}, dp_{t2}, \ldots, dp_{tn})$	Payable deposit of aircraft
$SEAT_n = (seat_1, seat_2, \ldots, seat_n)$	Number of seats of aircraft owned
$SOLD_t = (sold_{t1y}, sold_{t2y}, \ldots, sold_{tny})$	Number of aircraft sold

$RESALE_t = (resale_{t1y}, \ldots, resale_{tny})$	Resale price of aircraft
$DEP_t = (dep_{t1y}, dep_{t2y}, \ldots, dep_{tny})$	Depreciation values of aircraft
$SIZE = (size_1, size_2, \ldots, size_n)$	Size of aircraft
$RLT_t = (RLT_{t1}, RLT_{t2}, \ldots, RLT_{tn})$	Real lead time of aircraft
$DLT_t = (DLT_{t1}, DLT_{t2}, \ldots, DLT_{tn})$	Desired lead time of aircraft
$RST_t = (RST_{t1}, RST_{t2}, \ldots, RST_{tn})$	Real selling time of aircraft
$DST_t = (DST_{t1}, DST_{t2}, \ldots, DST_{tn})$	Desired selling time of aircraft

3. Problem formulation

Assume that there is a choice of n types of aircraft that could be purchased and used for a given origin–destination (OD) pair. The objective of the study is to find the number and types of aircraft that should be purchased in order to maximize the operational profit of the airline company. The passenger demand for the mentioned OD pair is assumed to be stochastic and could be expressed by some random distributions. To deal with this stochastic element, the problem is formulated as a probabilistic dynamic programming problem. The objective function is to maximize the expected profit of the airline companies, by considering various practical constraints faced in the operational planning.

3.1. Probabilistic dynamic programming model

3.1.1. Stage, state variables and optimal decision

The stage of the model is the planning horizon of the aircraft acquisition period. In this study, the planning period, t, in terms of years is the stage variable of the model. The state variable at each stage t consisted of various inter-correlated variables, namely the number of aircraft to be purchased (i.e. main decision variable for this study), initial number of aircraft owned, number of aircraft to be sold, number of aircraft to be ordered, number of aircraft to be released for sales and stochastic demand. The optimal decision (i.e. the alternatives at each stage) for the study is the acquisition decision of new aircraft in order to meet stochastic demand while making decision to sell ageing aircraft with the aim to maximize the expected profit.

3.1.2. Constraints

There are some constraints that need to be considered for the efficiency of the operational planning of airline companies. They are explained as follows:

Budget constraint: This is the most practical constraint in order to ascertain that the solution obtained is financially feasible for the airline companies. Accordingly, the total purchase cost of the aircraft should be less than or equal to the allocated budget, expressed as follows:

$$\sum_{i=1}^{n} purc_{ti} x_{ti} \leqslant MAX_{budget(t)} \tag{1}$$

Demand constraint: The stochastic demand can be represented by some probability distributions. Let α indicates the significance level to meet stochastic demand; the following expression can be formulated to achieve the targeted level of service:

$$P\left(\sum_{i=1}^{n} (SEAT_i)\big(f\big(D_t, A_t^i\big)\big) \geqslant D_t^S\right) \geqslant 1-\alpha \tag{2}$$

where $1-\alpha$ is the confidence level (i.e. targeted level) while P is the probability of the occurrence of the desired level of service. If the demand is assumed to follow the normal distribution with mean μ and standard deviation σ, the demand constraint could be expressed by,

$$\sum_{i=1}^{n} (SEAT_i)\big(f\big(D_t, A_t^i\big)\big) \geqslant F^{-1}(1-\alpha)\sigma + \mu \tag{3}$$

where $F^{-1}(1-\alpha)$ is the inverse cumulative probability of $1-\alpha$.

Parking constraint: When the aircraft is "off-duty", it has to be parked at the hangar of the airport. In such a case, the choice of the aircraft would sometimes be constrained by the geometry layout of the airports. As such, parking constraint is ought to be considered feasibly. The constraint is shown as follows:

$$\sum_{i=1}^{n} \sum_{y=0}^{m} (In_{tiy} + x_{ti})(SIZE_i) \leqslant PARK_t \tag{4}$$

Sales of aircraft constraint: For some airlines, ageing aircraft which is less cost-effective might be sold at the beginning of a certain operating period t when the airlines make the decision to purchase new aircraft. However, to maintain a certain level of operational efficiency, the number of aircraft sold should not be more than what was owned by the aircraft' companies. It is expressed as follows:

$$sold_{tiy} \leqslant In_{(t-1)i(y-1)} \quad \text{for } t = 1, 2, \ldots, T,\ i = 1, 2, \ldots, n,\ y = 1, 2, \ldots, m \tag{5}$$

Order delivery constraint: The delivery of the new aircraft ordered is depended on the efficiency of the manufacturing company. Sometimes, there might be a delay in delivering the new aircraft. As such, the aircraft that one could purchase should not be more than the number of aircraft available in the market, which is expressed as follows:

$$\sum_{i=1}^{n} x_i \leqslant ORDER_t \tag{6}$$

Lead time constraint: It is important to note that in the real practice, the airline company would get an agreeable lead time (the period between placing and receiving an order) from the aircraft manufacturer when they order new aircraft that needs to be purchased. However, the real lead time would be longer than the agreeable lead time and this will result in the delay of aircraft' delivery. This signifies that lead time constraint is necessary as it is able to indicate when the airline company supposes to place an order for their new aircraft. This constraint can be expressed as follows:

$$P(RLT_{tn} \geqslant DLT_{tn}) \leqslant \beta \tag{7}$$

By assuming that lead time is normally distributed with mean μ_{LT} and standard deviation σ_{LT}, this constraint could be stated by,

$$DLT_{tn} \geqslant F^{-1}(1-\beta)\sigma_{LT} + \mu_{LT} \tag{8}$$

where $F^{-1}(1-\beta)$ is the inverse cumulative probability of $1-\beta$.

Selling time constraint: Sales of ageing aircraft generate income for the airline company. In such a case, the airline company needs to know the most suitable time to release their ageing aircraft for sales particularly to look for prospect buyers in advanced. In the real practice, the real selling time might be longer than the desired selling time. Therefore, this constraint is formed with the aim to reduce the possibility for this incident as least as possible. This constraint could be defined as follows:

$$P(RST_{tn} \geqslant DST_{tn}) \leqslant \gamma \tag{9}$$

Subsequently, this constraint could be stated as follows by assuming selling time is normally distributed with mean μ_{ST} and standard deviation σ_{ST}:

$$DST_{tn} \geqslant F^{-1}(1-\gamma)\sigma_{ST} + \mu_{ST} \tag{10}$$

where $F^{-1}(1-\gamma)$ is the inverse cumulative probability of $1-\gamma$.

3.1.3. Objective function

The objective of the study is to maximize the expected operational profit of the airline companies. The profit could be derived by subtraction of the total operating cost from the total revenue obtained. For an airline company, the total revenue comes from the operational income (i.e. the sales of the air ticket) and the sales of ageing aircraft. The total operating cost considers the total purchasing cost of new aircraft, the total operational cost of aircraft owned, the total maintenance cost of aircraft owned, the total depreciation expenses of aircraft owned and the payable deposit of new aircraft to be purchased.

The total revenue for the operating period t, $TR(I_t)$, is expressed as follows:

$$TR(I_t) = E(fare_t)D_t^S + \sum_{i=1}^{n}\sum_{y=1}^{m} sold_{tiy} resale_{tiy} \tag{11}$$

The first term of the right hand side of Eq. (11) indicates the expected income obtained from the sale of flight tickets by considering the stochastic demand D_t^S for which $D_t^S \geqslant F^{-1}(1-\alpha)\sigma + \mu$. The second term indicates the revenue obtained from selling the ageing aircraft.

The total operating cost for the operating period t, $TC(I_t)$ is expressed as follows:

$$\begin{aligned} TC(I_t) = {} & \sum_{i=1}^{n} u_{ti} + purc_{ti}(x_{ti}) + E(\cos t_t)D_t^S \\ & + \sum_{i=1}^{n} hgf(D_t, A_t^i) + \sum_{i=1}^{n}\sum_{y=1}^{m}(In_{tiy})(dep_{tiy}) \\ & + \sum_{i=1}^{n} dp_{ti}(x_{ti}) \end{aligned} \tag{12}$$

The first term of the right hand side of Eq. (12) indicates the setup cost for the acquisition of aircraft; the second term indicates the purchasing cost of the new aircraft; the third term indicates the expected operating cost; the forth term indicates the maintenance cost; the fifth term indicates the total depreciation expenses; and the last term indicates the total of payable deposit for n types of aircraft.

3.1.4. The probable phenomena, $s_1, \ldots, s_k$

Since the demand is stochastic, the probable phenomenon for which the likely state variables to be occurred should be defined accordingly in order to capture the uncertainty properly. To account for the possible phenomenon appropriately, the airline company needs to consider all possible levels of service (i.e. actual level of demand) in order to plan their profitable operations strategically. In general, let $s_1, \ldots, s_k$ be k possible phenomenon to meet the level of service at a targeted confidence level. Apparently, it is extremely significant as it turns out to be an essential indicator to imply the possession of aircraft in order to capture the actual occurrence in the real practice. Only with this indicator, the actual operation under uncertainties will then be monitored closely with the developed optimization model. Correspondingly, the possibility for the phenomenon $s_1, \ldots, s_k$ to be happened, i.e. $p_{s_1}, \ldots, p_{s_k}$ is included necessarily in the developed model.

For the real practice, the phenomenon $s_1, \ldots, s_k$ and the corresponding probability $p_{s_1}, \ldots, p_{s_k}$ ought to be treated tactically not only based on the company's decision policy, qualitative judgement from experts or consultants, but also based on the past operational performance (i.e. the real historical data) which includes the records of the number of passengers and the trend of travel, which associate closely to the number of aircraft owned by the airline companies. The real data from the past performance could be a useful indicator for the airline company to forecast the future trend of demand and hence constitutes to the probable phenomena. At a certain extent, opinions from air transportation users should be considered as well.

3.1.5. The optimization model

It is now ready to present the optimization model considering the stage and state variables. With the aim to maximize the expected profit earned by acquiring new aircraft to meet the travellers demand under uncertainty, the formulation of the optimization model can be phrased as follows:

For $t = 1, 2, \ldots, T$

$$\begin{aligned} P(I_t) = \max_X {} & \frac{1}{(1+r_t)^t} \\ & \times \Bigg\{ p_{s_1}\Bigg(E\big(fare_t^{s_1}\big)D_t^{s_1} + \sum_{i=1}^{n}\sum_{y=1}^{m} sold_{tiy} resale_{tiy} \\ & - \sum_{i=1}^{n} u_{ti} + purc_{ti}(x_{ti}) - E\big(\cos t_t^{s_1}\big)D_t^{s_1} - \sum_{i=1}^{n} hgf(D_t, A_t) \\ & - \sum_{i=1}^{n}\sum_{y=1}^{m}(In_{tiy})(dep_{tiy}) - \sum_{i=1}^{n} dp_{ti}(x_{ti})\Bigg) + \cdots + p_{s_k} \\ & \times \Bigg(E\big(fare_t^{s_k}\big)D_t^{s_k} + \sum_{i=1}^{n}\sum_{y=1}^{m} sold_{tiy} resale_{tiy} \\ & - \sum_{i=1}^{n} u_{ti} + purc_{ti}(x_{ti}) - E\big(\cos t_t^{s_k}\big)D_t^{s_k} - \sum_{i=1}^{n} hgf(D_t, A_t) \\ & - \sum_{i=1}^{n}\sum_{y=1}^{m}(In_{tiy})(dep_{tiy}) - \sum_{i=1}^{n} dp_{ti}(x_{ti})\Bigg) + P_{t+1}(I_t)\Bigg\} \end{aligned} \tag{13}$$

subject to (1), (3)–(6), (8) and (10) for which D_t^S, X_t, I_t, $SOLD_t$, O_t, $R_t \in Z^+ \cup \{0\}$. The term, $\frac{1}{(1+r_t)^t}$ is needed in order to obtain the discounted value across the period of time while k indicates the kth possible phenomenon for owning I_t as the initial number of aircraft. Only two phenomenon, namely s_1 and s_2 are considered in this study in order to reduce the complexity.

It is important to note that the model formulation is formed by assuming that the developed model drives operational decision of airline companies particularly from the aspect of flight's frequency and its scheduling to meet stochastic demand. In other words, the acquisition decision of new aircraft will subsequently lead to the optimal operational decision of a fleet routing at a desired targeted level of service.

4. Solution method

The proposed probabilistic dynamic programming can be solved by decomposing it into a chain of simpler sub-problems. With the working backward, the solution method commences by solving the sub-problem at the last period of the planning horizon, T. The current optimal solutions found for the states at current stage leads to the problem solving at the period of $T-1, T-2, \ldots, 1$. This procedure continues until all the sub-problems have been solved optimally so that the decision policy to acquire new aircraft can be determined eventually. For the developed optimization model (13), the type of solution method (i.e. linear programming problem or non-linear programming problem) can be identified clearly with a careful inspection upon the developed model particularly from the key components as follows:

- function of the number of flights, $f(D_t, A_t^n)$;
- function of the maintenance cost, $hgf(D_t, A_t^n)$;
- constraints (1), (3)–(6), (8) and (10).

In general, the developed probabilistic dynamic programming model could be equivalent to linear programming or nonlinear programming model based on the nature of linearity. For model (13), the interested parameters appear to be discrete or continuous variables while the function of the number of flights, $f(D_t, A_t^n)$ and maintenance cost, $hgf(D_t, A_t^n)$ could be a linear or nonlinear function. If they are in the form of linear function in terms of decision variables, then the model (13) will be solved as a linear programming model, or else it is converted as a nonlinear programming model. In reality, the linearity of these components is based on the data collected for the particular airline company. It shall then be validated by using the regression test with the aid of some mathematical software.

In the illustrative case study shown in the following section, linear relationship was adopted for the above-mentioned components. Nonetheless, due to the consideration of stochastic demand which contributed to the probabilistic dynamic programming, the linear programming model obtained from the conversion could not be solved directly using any conventional methods available for solving linear programming model. One has to write his or her own algorithm in solving the model. In this study, spreadsheet functionality of Excel 2007 was deployed to find the optimal solution.

Table 1 Expected value of flight fare and flight cost per passenger for the period of t.

	Period, t							
	1	2	3	4	5	6	7	8
$E(fare_t^{s_1})$, \$	235	243	254	263	273	284	294	304
$E(fare_t^{s_2})$, \$	205	216	228	237	246	256	265	274
$E(\cos t_t^{s_1})$, \$	152	158	162	167	171	176	181	186
$E(\cos t_t^{s_2})$, \$	135	140	146	150	154	158	163	167

5. An illustrative case study

An illustrative case study is shown to test the proposed model. Assume that there are two types of aircraft choice where $n = 1$ for A320-216 and $n = 2$ for A340-300. The task is to decide when and which type of aircraft should be purchased over the planning horizon, i.e. 8 years. To avoid choosing some unreality value for the parameters and functions, some information from the published reports and accessible websites of airline companies are gathered. Tables 1 and 2 show the data input of the model. From the Airbus published statement (Airbus, 2010a,b), it is obtained that the capacity of aircraft A320-216 and A340-300 is 180 (with total size 1300 m^2) and 295 (with total size 3900 m^2), respectively. The expected flight fare and cost shown in Table 1 is generated based on the available financial reports of Malaysia Airlines (MAS) (Malaysia Airlines, 2010). In addition, the purchase prices of aircraft as shown in Table 2 were obtained from the published data of Airbus (Airbus, 2010c). With the purchase prices of aircraft and the estimated useful life of aircraft, i.e. 5 years, the depreciation values of aircraft are calculated accordingly by using the sum of the years' digits approach. The resale prices and depreciation values as shown in Table 2 are obtained based on the assumed residual value, i.e. salvage cost of aircraft, which is 10% of the purchase cost.

There are many variables and parameters in the model. Since not all real data can be obtained, it is interesting to investigate how the results of the model changes if the values of the variables are changed. Six scenarios are created besides the

Table 2 Resale price, depreciation values and purchase prices of aircraft.

y	$resale_{51y}$ (\$ millions)	$resale_{52y}$ (\$ millions)	dep_{51y} (\$ millions)	dep_{52y} (\$ millions)	$purc_{51}$ (\$ millions)	$purc_{52}$ (\$ millions)
1	56	159.6	24	68.4	80	228
2	36.8	104.88	19.2	54.72		
3	22.4	63.84	14.4	41.04		
4	12.8	36.84	9.6	27.36		
5	8	22.78	4.8	13.7		
		Average	14.4	41.0		

benchmark scenarios to test the difference of the results obtained.

5.1. Benchmark scenario

The list shown as follows is used in benchmark scenario:

- Two possible phenomenon are considered, where $k = 2$ for the model (13).
- At $t = 1$, the initial number of A320-216 and A340-300 are $In_{11} = 50$ and $In_{12} = 50$, respectively.
- The probability of posses these aircraft at initial period is $p_{s_1} = 0.5$ and $p_{s_2} = 0.5$.
- The budget, $MAX_{budget(t)}$ = \$6,500,000,000.
- Hangar area, $PARK_t$ = 500,000 m^2.
- Order delivery, $ORDER_t = 25$.
- Discount rate is fixed, $r = 5\%$ per annum.
- Confidence level of demand constraint, $1 - \alpha = 95\%$.
- Significance level of lead time constraint, $\beta = 5\%$.
- Significance level of selling time constraint, $\gamma = 5\%$.
- Salvage cost of aircraft = 10% of purchase cost of aircraft.
- At $t = 1$, the initial number of A320-216 and A340-300 to be 2 years old is $In_{112} = In_{122} = 2$.
- $$D_t^{s_2} = 0.95 D_t^{s_1} \tag{14}$$
- Number of flights,
$$f(A_t^n) = 54379 + 483A_t^n \tag{15}$$
- Maintenance cost,
$$hgf(A_t^n) = 81031 + 705A_t^n \tag{16}$$
- Number of aircraft,
$$NA = 17.9 + 0.000002NP \tag{17}$$

where NP is the number of passengers

Eq. (15) indicates that 483 flights are operated practically for each additional aircraft. The constant in this equation has no practical interpretation. Eq. (16) denotes that \$705 is the estimated increase of maintenance cost for each additional aircraft and \$81,031 is the overall estimated maintenance cost without considering additional aircraft. These functions signify that the respective function is strongly affected by the number of aircraft owned. Eq. (17) implies that each additional 500,000 passengers require one additional aircraft (or one passenger requires 0.000002 aircraft).

With the backward working, model (13) is simplified to model (18)–(26) when $t = T = 8$:

$$\begin{aligned} P(I_8) = \max_X \frac{1}{(1.05)^8} \big[& p_{s_1} \big(118D_8^{s_1} + (8 \times 10^6 sold_{815} + 2.278 \times 10^7 sold_{825}) \\ & - (8 \times 10^7 x_{81} + 2.28 \times 10^8 x_{82}) - (81031 + 705A_8) \\ & - (1.44 \times 10^7 In_{81} + 4.1 \times 10^7 In_{82}) \\ & - \big(8 \times 10^6 x_{81} + 2.28 \times 10^7 x_{82}\big)\big) + p_{s_2}(96.3D_8^{s_2} \\ & + (8 \times 10^6 sold_{815} + 2.278 \times 10^7 sold_{825}) - (8 \times 10^7 x_{81} \\ & + 2.28 \times 10^8 x_{82}) - (81031 + 705A_8) - (1.44 \times 10^7 In_{81} \\ & + 4.1 \times 10^7 In_{82}) - (8 \times 10^6 x_{81} + 2.28 \times 10^7 x_{82}))\big] \end{aligned} \tag{18}$$

subject to

$$80x_{81} + 228x_{82} \leqslant 6500 \tag{19}$$
$$In_{81} + In_{82} + x_{81} + x_{82} \geqslant 93 \tag{20}$$
$$D_8^{s_1} \geqslant 10,645,000, \quad D_8^{s_2} \geqslant 10,645,000 \tag{21}$$
$$13In_{81} + 13x_{81} + 39In_{82} + 39x_{82} \leqslant 5000 \tag{22}$$
$$sold_{815} \leqslant In_{81}, \quad sold_{825} \leqslant In_{82} \tag{23}$$
$$x_{81} + x_{82} \leqslant 25 \tag{24}$$
$$DLT_{81} \geqslant 30, \quad DLT_{82} \geqslant 30 \tag{25}$$
$$DST_{81} \geqslant 30, \quad DST_{82} \geqslant 30 \tag{26}$$
$$D_t^S,\ X_t,\ I_t,\ SOLD_t,\ O_t,\ R_t \in Z^+ \cup \{0\}$$

Eq. (19) takes the budget constraint of \$6500 million. The total demand simulated for $t = 8$ is to follow the Normal distribution, i.e. $D_8 \sim N(9 \times 10^6, 1 \times 10^6)$. With a 95% confidence level, it is found that the total aircraft owned at this period must be greater than 93, i.e. $A_8 \geqslant 93$, which is indicated in Eq. (20). Eq. (21) indicates that with the verified normal distribution, the actual level of demand for $t = 8$ is predicted to be at least 10,645,000 at a confidence level of 95%, which is derived by (2) and (3). Eq. (22) is the parking constraint; Eq. (23) is the sales of aircraft constraint, which is derived with the assumption that aircraft at the age which is equal to or greater than 5 years old are considered to be sold, thus: $sold_{815} \leqslant In_{714}$, and $sold_{825} \leqslant In_{724}$. Eq. (24) indicates the order delivery constraint. With the assumed normal distribution of $RLT_{8n} \sim N(1.918, 0.3613)$ and $RST_{8n} \sim N(1.918, 0.3613)$, Eqs. (25) and (26) represent lead time and selling time constraints respectively for which the desired period to order new aircraft as well as to the period to release ageing aircraft for sales is at least 30 months (i.e. 2.5 years ≈ 3 years) in advanced.

The function of the number of flights, $f(A_t^n) = 54,379 + 483A_t^n$ and the maintenance cost, $hgf(A_t^n) = 81,031 + 705A_t^n$ are both linear functions in terms of the total of aircraft owned, A_t^n and hence the developed model (13) is solved as a linear programming model. The procedure can be repeated to formulate the optimization model for the operating period, $t = 7, 6, 5, 4, 3, 2, 1$.

5.1.1. Other scenarios

Another six scenarios with variations to some of the parameters used in the benchmark scenarios are developed to investigate the impact of the changes on the results obtained. The following lists the scenario developed and the value of parameters used.

- Scenarios A and B has a confidence level of 90% and 99%, respectively.
- Scenarios C and D has the probability of owning the initial aircraft of 0.6:0.4 and 0.4:0.6, respectively.
- Scenarios E and F has the order delivery constraint value, $ORDER_t \leqslant 20$ and $ORDER_t \leqslant 30$, respectively.

6. Results and discussion

The results obtained for benchmark scenario is shown in Table 3. It could be seen that the proposed model and solution method could produce the optimal solution for the new aircraft acquisition problem. Table 3 shows a consistent increasing trend of discounted annual profit earned except the period for which there's a decrease in stochastic demand or when a payment is charged for deposit and purchase cost of new air-

Table 3 Benchmark scenario.

t	Discounted annual profit of period t Future value	Number of aircraft to be ordered		Number of aircraft to be received		Initial number of aircraft		Number of aircraft to be released for sales		Number of aircraft to be sold		Total demand, $D_t^{s_1}$
		A320-216	A340-300	A320-216	A340-300	A320-216	A340-300	A320-216	A340-300	A320-216	A340-300	
1	$1,752,427,113	0	0	0	0	50	50	0	0	0	0	16,000,000
2	$1,316,665,278	7	7	0	0	50	50	2	2	0	0	15,000,000
3	$1,433,492,145	0	0	0	0	50	50	0	0	0	0	14,955,000
4	$1,067,558,039	12	12	0	0	50	50	0	0	0	0	15,000,000
5	$264,749,080	5	5	7	7	55	55	0	0	2	2	20,000,000
6	$1,773,187,933	0	0	0	0	55	55	0	0	0	0	18,000,000
7	$659,375,524	0	0	12	12	67	67	0	0	0	0	30,000,000
8	$2,860,787,049	0	0	5	5	72	72	0	0	0	0	35,000,000

craft. This happening to be created purposely in terms of demand fluctuation as it is able to capture the uncertainty of demand in the real practice in a fairly better manner. In addition, the obtained result is capable of demonstrating a better view for airlines in making decision for aircraft acquisition to account for the inconsistency of demand.

The results of Scenarios A and B (for simplicity, the results of Scenarios A–F are not shown in this paper) display that when the confidence level changes, it has an impact on the value of the total demand. The confidence level indicates the targeted level of a service by an airline company and hence the level of profit for the airline company is affected if the targeted level of service changes. The results of Scenarios A and B established the fact a higher profit is gained (at higher value of confidence level) when the value of confidence level is on the rise. Apart from this, the comparison of results shows that there is a tendency for the airline company to acquire more aircraft to meet a higher increase of demand but yet subject to the constraints as elaborated earlier. In overall, the sensitivity results show that the airline company has to set their target properly in order to maximize their profit.

From the generated results of Scenarios C and D, it could be observed that the profit level of the airline company has a smaller effect when the setting of the probability of owning an initial number of aircraft changes. Contrary to Scenario D, expected profit generated by Scenario C is higher as it is outlined at a higher probability of s_1, i.e. $p_{s_1} = 0.6$ which is 20% higher than p_{s_1} for Scenario D. Similarly, the profit gained by Scenario C is higher than benchmark scenario during the planning horizon. This shows that the higher value of p_{s_1} which correspond to be higher level of demand subsequently results in a higher return. Therefore, the proposed model is sensitive to the setting of the initial number of aircraft owned by the airline company.

The results for Scenarios E and F show that the order delivery constraint could affect the optimal decision but the level of profit of the airline company is not much affected. This happens mainly due to the consideration (or decision) of the airline company in purchasing the least number of aircraft as long as the total number of aircraft owned is sufficient to provide the service. Hence, it is important to note that it's not certainly profitable to acquire more aircraft as higher purchasing cost and maintenance cost will occur. In other words, purchase lesser aircraft probably contributes higher expected profit due to the less charged costs.

In a nutshell, it could be seen that the parameters setting in the model could affect the results, to some extent. The results are more sensitive to the confidence level compared to other parameters. This means that there is no ideal means to obtain a supreme profit as the optimal acquisition decision is decidedly dependent on several factors, i.e. current management policy in practice for airline companies, the desired scenarios, to be optimized and also unpredictable uncertainties. Therefore, in order to improve the decision making for air transportation system, those aspects as mentioned and illustrated earlier should be taken into consideration considerably.

7. Conclusions

This study formulated an aircraft acquisition decision model with the aim to maximize the airline companies' profit. In doing this, an optimization model is developed by using probabilistic dynamic programming approach in order to capture the stochastic demand which is assumed to be normally distributed. The proposed model and solution method is tested with an illustrative case study, in which most of the input data and functions is either obtained or simulated using the real data. The model is solved in determining the optimal decision for the number and the types of new aircraft that should be purchased during the planning horizon. It is observed that the outputs are sensitive to the values of parameters setting, to some extent, and the results obtained indicated that the proposed methodology is viable.

With reasonable assumptions that pertain closely to realistic practice, the results reveal that aircraft acquisition decision is strongly influenced by stochastic demand as well as the policy of airlines, for instance, the pre-determined age of aircraft to be sold in this study. Generally, the profit earned is increasing when the level of demand is on the rise except for unexpected drop of demand, which could be taken place in the real practice or when the deposit and purchase cost are charged for new aircraft. In addition, six scenarios are created to test the sensitivity of the parameters setting to the outcome. Remarkably, order delivery constraint has a little impact for aircraft acquisition decision as compared to the benchmark problem. Nonetheless, the acquisition decision is comparatively influenced by the confidence level and the probability of owning the initial aircraft. It is shown that the significant findings in this study are able to steer the relevant authorities at the management level as well as the decision makers in making a wise profitable operational decision to perform better in such a competitive airline industry. For the future work, the proposed model will be tested with a set of real data collected from the airline company. In addition, the service frequency assignment will be considered as well.

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