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Estimation of general parameters using auxiliary information in simple random sampling without replacement

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ABSTRACT

Estimation of population parameters plays a vital role in the area of sampling. Many authors have proposed several estimators for estimating population parameter(s) using auxiliary information. This paper has attempted to suggest a new estimator for estimating the general parameter $t_{(a,b)}$ using auxiliary information in simple random sampling without replacement (*SRSWOR*). A conventional estimator $t_{(a,b)}$ is used to define the population constants: coefficient of variation, population mean, standard deviation, and population mean square. The expression for the minimum mean squared error has been derived. The efficiency of the suggested estimator and the existing estimators has been analyzed using a simulation study. Theoretical and empirical studies reveal the effectiveness of the proposed estimator over other existing estimators.

1. Introduction

Auxiliary information, when suitably used, improves the efficiency of the population parameters estimators. There are various techniques for applying auxiliary information, which helps improve the estimator's performance. These techniques include product methods, ratio, and regression, among others. Auxiliary information can be available in various forms, such as in the form of variables or attributes. Many researchers have widely used it to formulate various estimators for estimating population parameters in different sampling schemes. Srivastava (1971), Kadilar and Cingi (2004) and Gupta and Shabbir (2008) have proposed estimators for estimating unknown population values. Another estimator's class that uses correlation coefficient value and minimum MSE have been used in population mean estimation as documented in Srivastava and Jhajj (1980), Srivastava and Jhajj (1981), Srivastava and Jhajj (1986). Shabbir and Gupta (2007) proposed an estimator using the information in the form of attributes. Tracy et al. (1996) and Gupta and Shabbir (2007) used information on two auxiliary variables and proposed estimators to estimate the population constants. Some other related work can be found in Singh and Singh (2001), Tripathi et al. (2002), Khoshnevisan et al. (2007), Singh et al. (2008a), Singh et al. (2008b), Singh and Kumar (2011), Singh and Solanki (2011), Singh and Malik (2014), Sharma et al. (2017), Adichwal et al. (2016), Adichwal et al. (2019), Adichwal et al. (2017), Singh et al. (2018) and Mishra and Singh (2017).

Motivated by the work of these authors, we have proposed an improved estimator in the SRSWOR scheme for the general parameter of the population.

2. Notations

Let us consider a sample of size n is drawn from a population $W = (W_1, W_2, \ldots, W_N)$ using SRSWOR scheme. Let Y_i and X_i be the study and auxiliary population variables for the i^{th} units (i = 1,2,3 ,. . ... ., N), and let y_i and x_i be i^{th} units, respectively, in the sample for i = 1,2,3,....,n.

In general, consider the following population parameters

$$\mu_{rs} = \frac{1}{N}\sum_{i=1}^{N}\left(y_i - \bar{y}\right)^r\left(x_i - \bar{X}\right)^s$$

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$\delta_{rs} = \mu_{rs}/\left(\mu_{20}^{r/2}\mu_{02}^{s/2}\right)$ and (r, s) is non-negative integers.

Note that

$\mu_{20} = S_Y^2,\ \mu_{02} = S_X^2$ and $\mu_{11} = S_{XY}$, So that $C_Y^2 = S_Y^2/\bar{Y}^2 = \mu_{20}/\bar{Y}^2$, $C_X^2 = S_X^2/\bar{X}^2 = \mu_{02}/\bar{X}^2$ and $\rho_{XY} = S_{XY}/(S_X S_Y) = \mu_{11}/(\sqrt{\mu_{20}}\sqrt{\mu_{02}})$

Let us define,

$\varepsilon_0 = \frac{\bar{y}}{\bar{Y}} - 1,\ \varepsilon_1 = \frac{s_y^2}{S_Y^2} - 1,\ \varepsilon_2 = \frac{\bar{x}}{\bar{X}} - 1$ and $\varepsilon_3 = \frac{s_x^2}{S_X^2} - 1$

Subject to the condition

$E(\varepsilon_i) = 0, i = 0, 1, 2, 3.$

Ignoring fpc, we have

$$E(\varepsilon_0^2) = n^{-1}C_Y^2 E(\varepsilon_1^2) = n^{-1}(\delta_{40} - 1)E(\varepsilon_2^2) = n^{-1}C_X^2 E(\varepsilon_3^2) = n^{-1}(\delta_{04} - 1)$$

$$E(\varepsilon_0\varepsilon_1) = n^{-1}\delta_{30}C_Y E(\varepsilon_0\varepsilon_2) = n^{-1}\rho_{yx}C_X C_Y E(\varepsilon_0\varepsilon_3) = n^{-1}\delta_{12}C_Y$$

$$E(\varepsilon_1\varepsilon_2) = n^{-1}\delta_{21}C_X E(\varepsilon_1\varepsilon_3) = n^{-1}(\delta_{22} - 1)E(\varepsilon_2\varepsilon_3) = n^{-1}\delta_{03}C_X$$

3. Conventional estimator

The general form of the parameter under consideration can be stated as

$$t_{(a,b)} = \bar{Y}^a S_y^b \tag{3.1}$$

In Eq. (3.1), a and b are scalars, suitably chosen and $\bar{y} = \frac{1}{n}\sum_{i=1}^n y_i$ and $s_y^2 = \frac{1}{n-1}\sum_{i=1}^n \left(y_i - \bar{y}\right)^2$ are unbiased estimators of $\bar{Y}$ and S_y^2, respectively.

For choices of a, b the general parameter $t_{(a,b)}$ will take form of

(i) If $(a = 1,\ b = 0)$, $t_{(a,\ b)}$ reduces to $t_{(1,\ 0)} = \bar{Y}$
(ii) If $(a = 0, b = 2)$, $t_{(a,b)}$ reduces to $t_{(0,2)} = S_Y^2$
(iii) If $(a = -1,\ 1)$, $t_{(a,b)}$ reduces to $t_{(-1,1)} = C_Y$
(iv) If $(a = 0, b = 1)$, $t_{(a,b)}$ reduces to $t_{(0,1)} = S_Y$

The general parameter $t_{(a,b)}$ conventional estimator is defined as-

$$\hat{t}_{(a,b)} = \bar{y}^a s_y^b \tag{3.2}$$

where $\bar{y} = \frac{1}{n}\sum_{i=1}^n y_i$ and $s_y^2 = \frac{1}{n-1}\sum_{i=1}^n \left(y_i - \bar{y}\right)^2$.

Expressing Eq. (3.2) in terms of ε's, we have

$$\hat{t}_{(a,b)} = \bar{Y}^a S_y^b (1 + \varepsilon_0)^a (1 + \varepsilon_1)^{\frac{b}{2}} \tag{3.3}$$

Eq. (3.3) can be written as

$$\hat{t}_{(a,b)} = t_{(a,b)}\left(1 + a\varepsilon_0 + \frac{b}{2}\varepsilon_1 + \frac{a(a-1)}{2}\varepsilon_0^2 + \frac{ab}{2}\varepsilon_0\varepsilon_1 + \frac{b(b-2)}{8}\varepsilon_1^2\right)$$

or equivalently,

$$\left(\hat{t}_{(a,b)} - t_{(a,b)}\right) = t_{(a,b)}\left(a\varepsilon_0 + \frac{b}{2}\varepsilon_1 + \frac{a(a-1)}{2}\varepsilon_0^2 + \frac{ab}{2}\varepsilon_0\varepsilon_1 + \frac{b(b-2)}{8}\varepsilon_1^2\right) \tag{3.4}$$

Squaring Eq. (3.4) and neglecting the terms of ε with power three or more, we have

$$\left(\hat{t}_{(a,b)} - t_{(a,b)}\right)^2 = t_{(a,b)}^2\left(a^2\varepsilon_0^2 + ab\varepsilon_0\varepsilon_1 + \frac{b^2}{4}\varepsilon_1^2\right) \tag{3.5}$$

Take the expectations on both sides of Eq. (3.5), we have MSE's estimators of $\hat{t}_{(a,b)}$ to as given by

$$MSE\left(\hat{t}_{(a,b)}\right) = \left(\frac{t_{(a,b)}^2}{n}\right)\left(a^2C_Y^2 + ab\delta_{30}C_Y + \frac{b^2}{4}(\delta_{40} - 1)\right) \tag{3.6}$$

or

$$MSE\left(\hat{t}_{(a,b)}\right) = \left(\frac{t_{(a,b)}^2}{n}\right)f_1(a,b) \tag{3.7}$$

where,

$$f_1(a,b) = \left(a^2C_Y^2 + ab\delta_{30}C_Y + \frac{b^2}{4}(\delta_{40} - 1)\right).$$

4. Proposed estimator

We propose a class of difference-cum exponential ratio type estimators to estimate the parameter of the population $t_{(a,b)}$ as

$$t = \left[\hat{t}_{(a,b)} + k\left(\bar{X} - \bar{x}\right)\right]\exp\left[\frac{w_1(\bar{X} - \bar{x})}{\bar{X} + (\alpha - 1)\bar{x}}\right]\exp\left[\frac{w_2(S_X^2 - s_x^2)}{S_X^2 + (\beta - 1)s_x^2}\right] \tag{4.1}$$

where w_1, w_2 being scalar having the values (0,-1,1), and k, α and β are constants and can be defined suitably.

We express Eq. (4.1) in terms of ε's to obtain

$$t = \left[t_{(a,b)}(1 + \varepsilon_0)^a(1 + \varepsilon_1)^{\frac{b}{2}} - k\bar{X}\varepsilon_2\right]\exp\left[\frac{-w_1\varepsilon_2}{\alpha}\left\{1 + \left(\frac{\alpha - 1}{\alpha}\right)\varepsilon_2\right\}^{-1}\right]\exp\left[\frac{-w_2\varepsilon_3}{\beta}\left\{1 + \left(\frac{\beta - 1}{\beta}\right)\varepsilon_3\right\}^{-1}\right]t$$
$$= t_{(a,b)}\left(1 + a\varepsilon_0 + \frac{b}{2}\varepsilon_1 + \frac{a(a-1)}{2}\varepsilon_0^2 + \frac{ab}{2}\varepsilon_0\varepsilon_1 + \frac{b(b-2)}{8}\varepsilon_1^2 + \dots\right)$$
$$\times\left[1 - \frac{w_2}{\beta}\varepsilon_3 - \frac{w_1}{\alpha}\varepsilon_2 + \frac{w_2\{2(\beta - 1) + w_2\}}{2\beta^2}\varepsilon_3^2 + \frac{w_1\{2(\alpha - 1) + w_1\}}{2\alpha^2}\varepsilon_2^2 + \frac{w_1}{\alpha}\frac{w_2}{\beta}\varepsilon_2\varepsilon_3 + \dots\right]$$
$$- k\bar{X}\varepsilon_2\left[1 - \frac{w_2}{\beta}\varepsilon_3 - \frac{w_1}{\alpha}\varepsilon_2 + \frac{w_2\{2(\beta - 1) + w_2\}}{2\beta^2}\varepsilon_3^2 + \frac{w_1\{2(\alpha - 1) + w_1\}}{2\alpha^2}\varepsilon_2^2 + \frac{w_1}{\alpha}\frac{w_2}{\beta}\varepsilon_2\varepsilon_3 + \dots\right] \tag{4.2}$$

Multiplying out and neglecting the higher-order terms of ε's which are greater than two powers in Eq. (4.2), we have

$$t = t_{(a,b)}\left[1 + a\varepsilon_0 + \frac{b}{2}\varepsilon_1 - \frac{w_1}{\alpha}\varepsilon_2 - \frac{w_2}{\beta}\varepsilon_3 + \frac{a(a-1)}{2}\varepsilon_0^2 + \frac{ab}{2}\varepsilon_0\varepsilon_1 + \frac{b(b-2)}{8}\varepsilon_1^2 - \frac{w_1}{\alpha}\left\{a\varepsilon_0\varepsilon_2 + \frac{b}{2}\varepsilon_1\varepsilon_2\right\}\right.$$

$$-\frac{w_2}{\beta}\left\{a\,\varepsilon_0\varepsilon_3+\frac{b}{2}\varepsilon_1\varepsilon_3\right\}+\frac{w_1}{\alpha}\frac{w_2}{\beta}\varepsilon_2\varepsilon_3+\frac{w_1\{2(\alpha-1)+w_1\}}{2\alpha^2}\varepsilon_2^2+\frac{w_2\{2(\beta-1)+w_2\}}{2\beta^2}\varepsilon_3^2\Bigg]-k\,\bar{X}\,\varepsilon_2+\frac{w_2}{\beta}k\,\bar{X}\,\varepsilon_2\,\varepsilon_3+\frac{w_1}{\alpha}k\,\bar{X}\,\varepsilon_2^2 \tag{4.3}$$

or

$$\left(t-t_{(a,b)}\right)=t_{(a,b)}\Bigg[a\varepsilon_0+\frac{b}{2}\varepsilon_1-\frac{w_1}{\alpha}\varepsilon_2-\frac{w_2}{\beta}\varepsilon_3+\frac{a(a-1)}{2}\varepsilon_0^2+\frac{ab}{2}\varepsilon_0\varepsilon_1+\frac{b(b-2)}{8}\varepsilon_1^2-\frac{w_1}{\alpha}\left\{a\varepsilon_0\varepsilon_2+\frac{b}{2}\varepsilon_1\varepsilon_2\right\}-\frac{w_2}{\beta}\left\{a\varepsilon_0\varepsilon_3+\frac{b}{2}\varepsilon_1\varepsilon_3\right\}+\frac{w_1}{\alpha}\frac{w_2}{\beta}\varepsilon_2\varepsilon_3+\frac{w_1\{2(\alpha-1)+w_1\}}{2\alpha^2}\varepsilon_2^2+\frac{w_2\{2(\beta-1)+w_2\}}{2\beta^2}\varepsilon_3^2-k\bar{X}\,\varepsilon_2+\frac{w_2}{\beta}k\,\bar{X}\,\varepsilon_2\,\varepsilon_3+\frac{w_1}{\alpha}k\,\bar{X}\,\varepsilon_2\,\varepsilon_3 \tag{4.4}$$

Squaring Eq. (4.4) both sides and neglect the higher order of ε's which are greater than two powers, the result is as follows

$$\left(t-t_{(a,b)}\right)^2=\left[t_{(a,b)}a\varepsilon_0+t_{(a,b)}\left(\frac{b}{2}\right)\varepsilon_1-\left\{t_{(a,b)}\left(\frac{w_1}{\alpha}\right)+k\bar{X}\right\}\varepsilon_2-t_{(a,b)}\left(\frac{w_2}{\beta}\right)\varepsilon_3\right]^2$$

or

$$\left(t-t_{(a,b)}\right)^2=\left[t_{(a,b)}\left\{a\varepsilon_0+\left(\frac{b}{2}\right)\varepsilon_1-\frac{w_1}{\alpha}\varepsilon_2-\frac{w_2}{\beta}\varepsilon_2\right\}-k\bar{X}\,\varepsilon_2\right]^2 \tag{4.5}$$

From Eq. (4.5), we have

$$\left(t-t_{(a,b)}\right)^2=t_{(a,b)}^2\Bigg[a^2\varepsilon_0^2+\left(\frac{b^2}{4}\right)\varepsilon_1^2+\left(\frac{w_1}{\alpha}\right)^2\varepsilon_2^2+\left(\frac{w_2}{\beta}\right)^2\varepsilon_3^2+2\left(\frac{ab}{2}\right)\varepsilon_0\varepsilon_1-2a\left(\frac{w_1}{\alpha}\right)\varepsilon_0\varepsilon_2-2a\left(\frac{w_2}{\beta}\right)\varepsilon_0\varepsilon_3-2\left(\frac{b}{2}\right)\left(\frac{w_1}{\alpha}\right)\varepsilon_1\varepsilon_2-2\left(\frac{b}{2}\right)\left(\frac{w_2}{\beta}\right)\varepsilon_1\varepsilon_3+2\left(\frac{w_1}{\alpha}\right)\left(\frac{w_2}{\beta}\right)\varepsilon_2\varepsilon_3\Bigg]-2t_{(a,b)}ak\bar{X}\,\varepsilon_0\varepsilon_2-2t_{(a,b)}\left(\frac{b}{2}\right)k\bar{X}\,\varepsilon_1\varepsilon_2+2t_{(a,b)}\left(\frac{w_1}{\alpha}\right)k\bar{X}\,\varepsilon_2^2+2t_{(a,b)}\left(\frac{w_2}{\beta}\right)k\bar{X}\,\varepsilon_2\varepsilon_3+k^2\bar{X}^2\varepsilon_2^2 \tag{4.6}$$

or

$$\left(t-t_{(a,b)}\right)^2=t_{(a,b)}^2\left\{a^2\varepsilon_0^2+\left(\frac{b^2}{4}\right)\varepsilon_1^2+2\left(\frac{ab}{2}\right)\varepsilon_0\varepsilon_1\right\}+t_{(a,b)}^2\Bigg[\left(\frac{w_1}{\alpha}\right)^2\varepsilon_2^2+\left(\frac{w_2}{\beta}\right)^2\varepsilon_3^2-2\left(\frac{w_1}{\alpha}\right)\left\{a\varepsilon_0\varepsilon_2+\left(\frac{b}{2}\right)\varepsilon_1\varepsilon_2\right\}-2\left(\frac{w_2}{\beta}\right)\left\{a\varepsilon_0\varepsilon_3+\left(\frac{b}{2}\right)\varepsilon_1\varepsilon_3\right\}+2\left(\frac{w_1}{\beta}\right)\left(\frac{w_2}{\beta}\right)\varepsilon_2\varepsilon_3\Bigg]+2t_{(a,b)}k\times\bar{X}\left\{\left(\frac{w_1}{\alpha}\right)\varepsilon_2^2+\left(\frac{w_2}{\beta}\right)\varepsilon_2\varepsilon_3\right\}-2t_{(a,b)}k\bar{X}\left\{a\varepsilon_0\varepsilon_2+\left(\frac{b}{2}\right)\varepsilon_1\varepsilon_2\right\}+k^2\bar{X}^2\varepsilon_2^2 \tag{4.7}$$

Take the expectation of Eq. (4.7), we have

$$MSE(t)=MSE\left(\hat{t}_{(a,b)}\right)+\frac{t_{(a,b)}^2}{n}\Bigg[\left(\frac{w_1}{\alpha}\right)^2C_X^2+\left(\frac{w_2}{\beta}\right)^2(\delta_{04}-1)-2\left(\frac{w_1}{\alpha}\right)\left\{a\rho_{XY}C_Y+\left(\frac{b}{2}\right)\delta_{21}\right\}C_X-2\left(\frac{w_2}{\beta}\right)\left\{a\delta_{12}C_Y+\left(\frac{b}{2}\right)(\delta_{22}-1)\right\}+2\left(\frac{w_1}{\beta}\right)\left(\frac{w_2}{\beta}\right)\delta_{03}C_X\Bigg]+2\times\frac{t_{(a,b)}}{n}k\bar{X}\left\{\left(\frac{w_1}{\alpha}\right)C_X^2+\left(\frac{w_2}{\beta}\right)\delta_{03}C_X\right\}-2\frac{t_{(a,b)}}{n}k\bar{X}\left\{a\rho_{XY}C_Y+\left(\frac{b}{2}\right)\delta_{21}\right\}C_X+\frac{k^2\bar{X}^2}{n}C_X^2 \tag{4.8}$$

or

$$MSE(t)=MSE\left(\hat{t}_{(a,b)}\right)+\frac{t_{(a,b)}^2}{n}\left[A_1^2C_X^2+A_2^2(\delta_{04}-1)-2A_1f_2(a,b)C_X-2A_2f_3(a,b)+2A_1A_2\delta_{03}C_X\right]+2\frac{t_{(a,b)}}{n}k\bar{X}\left\{A_1C_X^2+A_2\delta_{03}C_X\right\}-2\frac{t_{(a,b)}}{n}k\bar{X}f_2(a,b)C_X+\frac{k^2\bar{X}^2}{n}C_X^2 \tag{4.9}$$

where,

$$A_1=\frac{w_1}{\alpha}$$

$$A_2=\frac{w_2}{\beta}$$

$$f_2(a,b)=\left\{a\rho_{XY}C_Y+\left(\frac{b}{2}\right)\delta_{21}\right\}$$

$$f_3(a,b)=\left\{a\delta_{12}C_Y+\left(\frac{b}{2}\right)(\delta_{22}-1)\right\}.$$

Differentiate partially Eq. (4.9) *w.r.to* A_1 and A_2 and equate to zero, we have

$$\begin{bmatrix} t_{(a,b)}C_X & t_{(a,b)}\delta_{03} \\ t_{(a,b)}\delta_{03}C_X & t_{(a,b)}4(\delta_{03}-1)\end{bmatrix}[A_1A_2]=\begin{bmatrix} t_{(a,b)}\{a\rho_{XY}C_Y+(\frac{b}{2})\delta_{21}\}-k\bar{X}\,C_X \\ t_{(a,b)}\{a\delta_{12}C_Y+(\frac{b}{2})(\delta_{22}-1)\}-k\bar{X}\,C_X\delta_{03}\end{bmatrix}$$

After simplifying, we obtain the optimum value of A_1 and A_2, that is,

$$\left.\begin{aligned}(A_1)_{opt}&=\frac{(\delta_{04}-1)f_2(a,b)-\delta_{03}f_3(a,b)}{\left(\delta_{04}-\delta_{03}^2-1\right)C_X}-\frac{K\bar{X}}{t_{(a,b)}}\\ (A_2)_{opt}&=\frac{f_3(a,b)-\delta_{03}f_2(a,b)}{\left(\delta_{04}-\delta_{03}^2-1\right)}\end{aligned}\right\} \tag{4.10}$$

Substituting $(A_1)_{opt}$ and $(A_2)_{opt}$ from Eq. (4.10) in Eq. (4.9), we have

$$MSE(t)_{\min}=MSE\left(\hat{t}_{(a,b)}\right)-\frac{t_{(a,b)}^2}{n}\frac{\left[\{f_3(a,b)\}^2-2f_2(a,b)f_3(a,b)\delta_{03}+(\delta_{04}-1)\{f_2(a,b)\}^2\right]}{\left(\delta_{04}-\delta_{03}^2-1\right)} \tag{4.11}$$

$$=MSE\left(\hat{t}_{(a,b)}\right)-\left[\frac{t_{(a,b)}^2}{n}\{f_2(a,b)\}^2\right]-\left(\frac{t_{(a,b)}^2}{n}\right)\frac{\{f_2(a,b)\delta_{03}-f_3(a,b)\}^2}{\left(\delta_{04}-\delta_{03}^2-1\right)} \tag{4.12}$$

$$= MSE(\hat{t}_{(a,b)}) - \left(\frac{t^2_{(a,b)}}{n}\frac{\{f_3(a,b)\}^2}{(\delta_{04}-1)}\right) - \left(\frac{t^2_{(a,b)}}{n}\right)\frac{\{(\delta_{04}-1)f_2(a,b)-\delta_{03}f_3(a,b)\}^2}{(\delta_{04}-\delta^2_{03}-1)(\delta_{04}-1)} \quad (4.13)$$

Remark 4.1: The optimum values of constants A_1 and A_2 at (4.10) involve unknown population parameters. The values of these quantities can be guessed accurately through a pilot sample survey or sample data at hand or experience gathered in due course of time, see Srivastava and Jhajj (1980), Singh et al. (2003) and Singh and Solanki (2013).

When $\bar{X}$ is known, the following Difference-cum exponential ratio type estimator (DCERTE) to obtain population parameter $t_{(a,b)}$ on putting $w_2 = 0$ in Eq. (4.1) is defined as:

$$t_1 = \left[\hat{t}_{(a,b)} + k\left(\bar{X}-\bar{x}\right)\right]\exp\left[\frac{w_1(\bar{X}-\bar{x})}{\bar{X}+(\alpha-1)\bar{x}}\right] \quad (4.14)$$

where w_1 being scalar has real values (0,-1,1) and k and α is arbitrary constants.

MSE of the estimator t_1 is given by

$$MSE(t_1) = MSE(\hat{t}_{(a,b)}) + \frac{t^2_{(a,b)}}{n}\left[\left(\frac{w_1}{\alpha}\right)^2 C^2_X - 2\left(\frac{w_1}{\alpha}\right)f_2(a,b)C_X\right] \quad (4.15)$$

$MSE(t_1)$ defined in Eq. (4.15) is minimized for $\left(\frac{w_1}{\alpha}\right)_{opt} = \frac{f_2(a,b)}{C_X}$

$$MSE(t_1)_{\min} = MSE(\hat{t}_{(a,b)}) - \frac{t^2_{(a,b)}}{n}\{f_2(a,b)\}^2 \quad (4.16)$$

Table 4.1 presents the existing estimators obtained from Eq. (4.14) on taking appropriate values of a, b, k, w_1 and α accordingly.

Using known S^2_x, following exponential ratio type estimator to estimate the parameter $t_{(a,b)}$ of the population by putting k = 0 and $w_1 = 0$ in Eq. (4.1) is defined as

$$t_2 = \hat{t}_{(a,b)}\exp\left[\frac{w_2(S^2_X - s^2_x)}{S^2_X + (\beta-1)s^2_x}\right] \quad (4.17)$$

where w_2 being scalar has real values (0,-1,1) and β is suitably chosen constants, MSE of the estimator t_2 is defined as:

$$MSE(t_2) = MSE(\hat{t}_{(a,b)}) + \frac{t^2_{(a,b)}}{n}\left[\left(\frac{w_2}{\beta}\right)^2(\delta_{04}-1) - 2\left(\frac{w_2}{\beta}\right)f_3(a,b)\right] \quad (4.18)$$

$MSE(t_2)$ in (4.18) is minimized form for $\left(\frac{w_2}{\beta}\right)_{opt} = \frac{f_3(a,b)}{(\delta_{04}-1)}$

$$MSE(t_2)_{\min} = MSE(\hat{t}_{(a,b)}) - \frac{t^2_{(a,b)}}{n}\frac{\{f_3(a,b)\}^2}{(\delta_{04}-1)} \quad (4.19)$$

Table 4.2 presents a set of existing estimators obtained from (4.17) by suitable a, b, w_2 and β.

5. Efficiency comparison

$$MSE(\hat{t}_{(a,b)}) - MSE(t)_{\min} = \frac{t^2_{(a,b)}}{n}\frac{\left[\{f_3(a,b)\}^2 - 2f_2(a,b)f_3(a,b)\delta_{03} + (\delta_{04}-1)\{f_2(a,b)\}^2\right]}{(\delta_{04}-\delta^2_{03}-1)} \geqslant 0 \quad (5.1)$$

$$MSE(t_1)_{\min} - MSE(t)_{\min} = \left(\frac{t^2_{(a,b)}}{n}\right)\frac{\{f_2(a,b)\delta_{03} - f_3(a,b)\}^2}{(\delta_{04}-\delta^2_{03}-1)} \geqslant 0 \quad (5.2)$$

$$MSE(t_2)_{\min} - MSE(t)_{\min} = \left(\frac{t^2_{(a,b)}}{n}\right)\frac{\{(\delta_{04}-1)f_2(a,b)-\delta_{03}f_3(a,b)\}^2}{(\delta_{04}-\delta^2_{03}-1)(\delta_{04}-1)} \geqslant 0 \quad (5.3)$$

We note that $(\delta_{04}-\delta^2_{03}-1) \geqslant 0$ always.

It can be observed from (5.1), (5.2) and (5.3), the proposed difference-cum exponential ratio type conventional estimator t performs efficiently than the estimators $\hat{t}_{(a,b)}$, t_1 and t_2.

6. Estimation of the population mean of the study variable Y

For $(a,b,k,w_1,w_2,\alpha,\beta) = (1,0,k,w_1,w_2,\alpha,\beta)$, class of estimator 't' written in Eq. (4.1), will take the following form

$$t' = \left[\bar{y} + k\left(\bar{X}-\bar{x}\right)\right]\exp\left[\frac{w_1(\bar{X}-\bar{x})}{\bar{X}+(\alpha-1)\bar{x}}\right]\exp\left[\frac{w_2(S^2_X - s^2_x)}{S^2_X+(\beta-1)s^2_x}\right] \quad (6.1)$$

MSE's expressions of the estimator t' up to O(n^{-1}) is given by

$$MSE(t') = MSE\left(\bar{y}\right) + \frac{\bar{Y}^2}{n}\left[A^2_1C^2_x + A^2_2(\delta_{04}-1)\right] -2A_1\rho_{XY}C_XC_Y - 2A_2\delta_{12}C_Y + 2A_1A_2\delta_{03}C_X$$

$$+\frac{2}{n}k\bar{X}\,\bar{Y}\left\{A_1C^2_X + A_2\delta_{03}C_X\right\} - \frac{2}{n}k\bar{X}\,\bar{Y}\rho_{XY}C_XC_Y + \frac{k^2\bar{X}^2}{n}C^2_X \quad (6.2)$$

where, $A_1 = \frac{w_1}{\alpha}$ and $A_2 = \frac{w_2}{\beta}$

MSE of t' at (6.2) is minimized for values

$$\left.\begin{aligned}(A_1)_{opt} &= \frac{\{(\delta_{04}-1)\rho_{XY} - \delta_{03}\delta_{12}\}C_Y}{(\delta_{04}-\delta^2_{03}-1)C_X} - \frac{K\bar{X}}{\bar{Y}}\\ (A_2)_{opt} &= \frac{\{\delta_{12}-\delta_{03}\rho_{XY}\}C_Y}{(\delta_{04}-\delta^2_{03}-1)}\end{aligned}\right\} \quad (6.3)$$

Putting the value of $(A_1)_{opt}$ and $(A_2)_{opt}$ from Eq. (6.3) in Eq. (6.2) to get the minimum MSE of the estimator t' as (Table 6.1)

Table 4.1
Particular cases of the estimator t_1.

Subset of the proposed estimator	a	b	K	w_1	α
$t_{1(1)} = \hat{t}_{(a,b)}\exp\left[\frac{(\bar{X}-\bar{x})}{\bar{X}+(\alpha-1)\bar{x}}\right]$ (Singh and Pal, 2017)	a	b	0	1	α
$t_{1(2)} = \bar{y}\exp\left[\frac{(\bar{X}-\bar{x})}{\bar{X}+(\alpha-1)\bar{x}}\right]$ (Upadhyaya et al., 2011)	1	0	0	1	α
$t^*_{1(2)} = \bar{y}\exp\left[\frac{(\bar{X}-\bar{x})}{\bar{X}+\bar{x}}\right]$ (Bahl and Tuteja, 1991)	1	0	0	1	2
$t_{1(3)} = \left[\bar{y}+k\left(\bar{X}-\bar{x}\right)\right]$ (Difference Estimator)	1	0	0	0	α

Table 4.2
Particular cases of the estimator t_2.

Subset of the proposed estimator	A	b	w_2	β
$t_{2(1)} = \hat{t}_{(a,b)}\exp\left[\frac{(S_X^2 - s_x^2)}{S_X^2+(\beta-1)s_x^2}\right]$ (Singh and Pal, 2017)	a	b	1	β
$t^*_{2(1)} = s_y^2\exp\left[\frac{(S_X^2 - s_x^2)}{S_X^2+(\beta-1)s_x^2}\right]$ (Yadav and Kadilar, 2013)	0	2	1	β
$t^{**}_{2(1)} = s_y^2\exp\left[\frac{(S_X^2 - s_x^2)}{S_X^2+s_x^2}\right]$ (Singh et al., 2011)	0	2	1	2

Table 6.1
Particular cases of the estimator t'.

Subset of the proposed estimator	k	w_1	w_2	α
$t_{1(2)} = \bar{y}\exp\left[\frac{(\bar{X}-\bar{x})}{\bar{X}+(\alpha-1)\bar{x}}\right]$ (Upadhyaya et al., 2011)	0	1	0	α
$t^*_{1(2)} = \bar{y}\exp\left[\frac{(\bar{X}-\bar{x})}{\bar{X}+\bar{x}}\right]$ (Bahl and Tuteja, 1991)	0	1	0	2
$t_{1(3)} = \left[\bar{y} + k\left(\bar{X}-\bar{x}\right)\right]$ (Difference Estimator)	0	0	0	α

$$MSE(t')_{\min} = V\left(\bar{y}\right) - \frac{\bar{Y}^2}{n} \times \frac{\{\delta_{12}^2 - 2\rho_{XY}\delta_{03}\delta_{12} + (\delta_{04}-1)\rho_{XY}^2\}C_Y^2}{(\delta_{04} - \delta_{03}^2 - 1)} \quad (6.4)$$

The minimum MSE's of the estimator $t_{1(2)}$, $t^*_{1(2)}$ and $t_{1(3)}$ is given by

$$MSE(t_{1(2)})_{\min} = V\left(\bar{y}\right) - \frac{\bar{Y}^2}{n}\rho_{XY}^2 C_Y^2 \quad (6.5)$$

$$MSE\left(t^*_{1(2)}\right)_{\min} = V\left(\bar{y}\right) - \frac{\bar{Y}^2}{n}\left[\frac{1}{4}C_X^2 - \rho_{XY}C_X C_Y\right] \quad (6.6)$$

$$MSE(t_{1(3)})_{\min} = V\left(\bar{y}\right) - \frac{\bar{Y}^2}{n}\rho_{XY}^2 C_Y^2 \quad (6.7)$$

7. Efficiency comparison

$$V\left(\bar{y}\right) - MSE(t')_{\min} = \frac{\bar{Y}^2}{n}\frac{\{\delta_{12}^2 - 2\rho_{XY}\delta_{03}\delta_{12} + (\delta_{04}-1)\rho_{XY}^2\}C_Y^2}{(\delta_{04} - \delta_{03}^2 - 1)} \geqslant 0 \quad (7.1)$$

$$MSE(t_{1(2)}) - MSE(t')_{\min} = \frac{\bar{Y}^2}{n}\frac{\{\delta_{12}^2 - 2\rho_{XY}\delta_{03}\delta_{12} + (\delta_{04}-1)\rho_{XY}^2\}C_Y^2}{(\delta_{04} - \delta_{03}^2 - 1)} - \frac{\bar{Y}^2}{n}\rho_{XY}^2 C_Y^2 \geqslant 0 \quad (7.2)$$

$$MSE\left(t^*_{1(2)}\right)_{\min} - MSE(t')_{\min} = \frac{\bar{Y}^2}{n}\frac{\{\delta_{12}^2 - 2\rho_{XY}\delta_{03}\delta_{12} + (\delta_{04}-1)\rho_{XY}^2\}C_Y^2}{(\delta_{04} - \delta_{03}^2 - 1)} - \frac{\bar{Y}^2}{n}\left[\frac{1}{4}C_X^2 - \rho_{XY}C_X C_Y\right] \geqslant 0 \quad (7.3)$$

$$MSE(t_{1(3)})_{\min} - MSE(t')_{\min} = \frac{\bar{Y}^2}{n}\frac{\{\delta_{12}^2 - 2\rho_{XY}\delta_{03}\delta_{12} + (\delta_{04}-1)\rho_{XY}^2\}C_Y^2}{(\delta_{04} - \delta_{03}^2 - 1)} - \frac{\bar{Y}^2}{n}\left[\frac{1}{4}C_X^2 - \rho_{XY}C_X C_Y\right] \geqslant 0 \quad (7.4)$$

8. Empirical study

In this section, we compare the performance of the proposed estimators using a known population data set.

The description of population data sets is as follows.

Population

Y = Number of person per block

X = Number of rooms per block

The values of the parameters are

n = 10, $\bar{X} = 58.8$, $\bar{Y} = 101.1$, $C_X = 0.1281$, $C_Y = 0.1450$, $\rho_{XY} = 0.6500$, $\delta_{12} = 0.5714$, $\delta_{21} = 0.4537$, $\delta_{03} = 0.4861$, $\delta_{30} = 0.3248$, $\delta_{04} = 2.2387$, $\delta_{40} = 2.3523$, $\delta_{13} = 1.5041$, $\delta_{31} = 1.6923$, $\delta_{22} = 1.5432$.

Tables 8.1 is concluded that the performance of the proposed estimator t' is more efficient in comparison to the usual mean estimator and other existing estimators $t_{1(2)}$, $t^*_{1(2)}$ and $t_{1(3)}$ as the PRE of the proposed estimator t' is greater than the existing estimators.

9. Simulation study

This section shows the procedure for comparing estimators $t_{1(2)}$, $t^*_{1(2)}$ and $t_{1(3)}$ with the estimator t' computationally based on the Reddy et al. (2010) algorithm. The following stepwise simulation algorithm is used to evaluate the efficiency of $\bar{Y}$:

Step 1: Simulate $X\tilde{N}(\mu, \sigma^2)$ and $X_1\tilde{N}(\mu_1, \sigma_1^2)$ independently and randomly using the method of box-Muller.

Step 2: Let $Y = \rho X + \sqrt{1-\rho^2}X_1$ such that $0 < \rho = 0.4, 0.6, 0.8 < 1$.

Step 3: Gives the pair (Y, X).

Table 8.1
PRE's of the proposed and existing estimators.

Estimators	PRE
$\bar{y}$	100.0000
t'	195.1564
$t_{1(2)}$	173.1602
$t^*_{1(2)}$	72.5100
$t_{1(3)}$	173.1602

Step-4: Let define the parameters $\mu = 5, \sigma = 3$, $\mu_1 = 5$ and $\sigma_1 = 3$ for population-I in step 1, repeat steps 1 to 3 for 1000 times. Variable Y and X will have the same variances in the population.

Step-5: Similarly, use parameters $\mu = 3, \sigma = 2$, $\mu_1 = 5$ and $\sigma_1 = 3$ in step-1 to generate the population-II, and then repeat the process from steps 1 to 3 for 1000 times. Variable Y and X will have different variances in the population.

Step-6: Use SRS to draw 500 samples (y_i, x_i), for i = 1, 2,. . .,n from the population of size N = 1000, WOR of size n = 40, 50 and 60.

Step-7: Calculate $AMSE(t) = \frac{1}{500}\sum_{k=1}^{500} E\left(t_k - \bar{y}\right)^2$

and PRE of an estimator t with respect to the usual estimator $\bar{y}$ is $PRE(t) = \frac{Var(\bar{y}) \times 100}{MSE(t)}$

Tables 9.1 and 9.2 shows simulation results that have been obtained following the stepwise procedures.

From Tables 9.1 and 9.2, we observe that from the various values of correlation coefficient $\rho = 0.4, 0.6, 0.8$ and the sample size n = 40,50,60, it can be concluded that the performance of the proposed estimator t' is more efficient in comparison to the usual mean estimator and other existing estimators $t_{1(2)}$, $t^*_{1(2)}$ and $t_{1(3)}$. Table 9.1 and 9.2 indicates that the average PRE of the proposed t' estimator is higher in all the cases (for variations in ρ and n) and for both the simulated data sets discussed above.

10. Conclusion

The present study proposed an improved estimator for a general parameter estimation using the information on "additional value X". Differential cum exponential ratio type estimator 't' of a general parameter is suggested in SRSWOR. The estimator 't' proposed in Eq. (4.1) can be used to estimate different population parameters. We have verified the performance of the proposed estimator using simulated data set for the case of estimation of population mean only. The result of this study shows that the usual mean estimator, Singh and Pal (2017), Upadhyaya et al. (2011), Bahl and Tuteja (1991), Yadav and Kadilar (2013) and Singh et al. (2011) can be shown as the members of the proposed class. An empirical study has been carried out using real data set (presented in Table 8.1) and simulated data sets (presented in Table 9.1 and Table 9.2 by taking two different simulated population data sets) to illustrate the efficiency and effectiveness of the proposed estimator. The result of simulation studies shows that the average PRE of the suggested estimator t' is higher as compared to the existing estimators Upadhyaya et al. (2011) t_{1\left(2 \right)}, Tuteja and Bahl (1991)$t_{1(2)}$ $t^*_{1(2)}$ and difference estimator $t_{1(3)}$ for different choices of correlation coefficient ρ and sample size n in all the cases and for both the simulated data sets. The same pattern can also be obtained in case of real data set i.e. the proposed estimator t' performing better as comparison to the usual mean estimator and other existing estimators $t_{1(2)}$, $t^*_{1(2)}$ and $t_{1(3)}$ as the PRE of the proposed estimator t' is greater than the existing estimators. Through a literature survey, it can be found that the best estimator is one having the minimum mean square error. Hence, by the result of the simulation study, we can conclude that the proposed estimator t' performs better than the existing estimators $t_{1(2)}$, $t^*_{1(2)}$

Table 9.1
Average PRE of the estimators for Population-I.

ρ	N	Average PRE's				
		$\bar{y}$	t'	$t_{1(2)}$	$t^*_{1(2)}$	$t_{1(3)}$
0.5	40	100.0000	140.2242	138.7793	80.36167	138.7793
	50	100.0000	139.6806	138.6224	80.1948	138.6224
	60	100.0000	139.5516	138.5826	80.29106	138.5826
0.6	40	100.0000	165.8203	164.1184	72.5535	164.1184
	50	100.0000	165.1557	163.9098	72.47781	163.9098
	60	100.0000	164.9832	163.841	72.51843	163.841
0.8	40	100.0000	299.7129	296.6709	60.68814	296.6709
	50	100.0000	298.4635	296.2357	60.68819	296.2357
	60	100.0000	298.0521	296.0032	60.66814	296.0032

Table 9.2
Average PRE of the estimators for Population-II.

ρ	N	Average PRE's				
		$\bar{y}$	t'	$t_{1(2)}$	$t^*_{1(2)}$	$t_{1(3)}$
0.5	40	100.0000	119.2466	118.0128	97.4543	118.0128
	50	100.0000	118.8099	117.9053	96.43723	117.9053
	60	100.0000	118.7146	117.8876	96.7558	117.8876
0.6	40	100.0000	130.8356	129.4851	87.2013	129.4851
	50	100.0000	130.3386	129.3491	86.7238	129.3491
	60	100.0000	130.2249	129.3196	86.9226	129.3196
0.8	40	100.0000	191.0631	189.1082	68.4768	189.1082
	50	100.0000	190.2839	188.8529	68.3965	188.8529
	60	100.0000	190.0674	188.7543	68.4277	188.7543

and $t_{1(3)}$. Hence, this method is recommended for practical application.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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