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Study of shear Alfve'n waves with Landau quantization effect in degenerate relativistic plasma

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ABSTRACT

In a ground breaking endeavor, we investigate a fundamental quantum property of fermions, specifically electrons, within a non-uniform quantized magnetic field, unveiling novel insights into shear Alfve'n waves in the relativistic realm of quantum plasma, particularly within the context of a degenerate Fermi gas. Employing the quantum magnetohydrodynamic model, we delve into the profound implications of the Landau quantization effect. This quantum phenomenon holds paramount importance across diverse fields such as condensed matter physics, astrophysics, and quantum information. Our study focuses on a plasma composed of degenerate relativistic electrons coexisting with cold fluid ions, which are non-degenerate particles. The findings of this research reveal substantial modifications in the thermodynamic, kinetic, and dispersion relation characteristics of shear Alfve'n waves when subjected to a non-uniform quantized magnetic field. This exploration offers a pioneering perspective on the intricate interplay between quantum physics and plasma dynamics, with implications for a wide array of scientific disciplines.

1. Introduction

Quantum plasma physics has emerged as a captivating and highly relevant field, garnering significant attention from the scientific community. Its significance spans diverse realms, from super dense astrophysical plasmas, such as those found in neutron stars and white dwarfs, to the burgeoning applications in modern technology, including nanoplasmonic devices, quantum X-ray lasers, spintronics, nanotubes, quantum dots, and quantum wells (El-Taibany et al., 2012). A fundamental characteristic that distinguishes quantum plasmas is their high number density, which leads to degeneracy, a state where the average inter-electron distance $n_e^{-1/3}$ becomes comparable to or less than the De Broglie wavelength λ_B associated with electrons, as expressed by $n_e\lambda_B^3 \geq 1$. Here, λ_B is defined as $\lambda_B = \frac{\hbar}{(2\pi m_e k_B T)^{1/2}}$. Moreover, when the temperature of the system is comparable to or less than the electron's Fermi temperature $\chi = \frac{T_{Fe}}{T} = \frac{1}{2}\left(3\pi^2 n_e \lambda_B^3\right)^{2/3} \geq 1$, quantum effects become pronounced and cannot be disregarded(Malay Kumar Ghorui et al. 2013; El-Labany et al., 2020; Passot et al., 2005). This introduces intriguing quantum phenomena, with magnetic fields playing a crucial role in shaping the behavior of degenerate plasmas. While the significance of magnetic fields in degenerate plasmas has been somewhat overshadowed in the past, recent research has underscored their pivotal role across a spectrum of natural and astrophysical systems, spanning both microscopic and macroscopic scales. Magnetic fields, particularly strong or superstrong ones, influence a multitude of astrophysical environments. They are instrumental in processes ranging from the formation of stars to the generation of stellar winds, cosmic rays, accretion disks, and the production of jets in X-ray binaries and active galactic nuclei. Notably, the influence of strong magnetic fields extends beyond astrophysics; they also impact the properties of atoms, molecules, and condensed matter systems. To achieve these transformative effects, a condition must be met where $\hbar\omega_{ce} \gg k_B\, T$, ensuring that the electron cyclotron energy vastly exceeds the Coulomb energy.

The interaction of a strong magnetic field with a degenerate Fermi gas introduces fascinating quantum phenomena, such as Landau Quantization and Geometric Phase. Even in the presence of a weak magnetic field, the electron gas exhibits two distinct forms of magnetization: paramagnetic and diamagnetic. This arises from the fact that moving charges, whether through spin or orbital motion, generate a magnetic field and possess a magnetic moment along their axis of rotation. In degenerate plasma, the ambient magnetic field's strength influences

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electron motion in two primary ways. Firstly, it impacts the unpaired spin of electrons, giving rise to Pauli Paramagnetism. Secondly, it influences the orbital motion of paired electrons, leading to the formation of Landau Diamagnetism/Quantization (Shah et al., 2012).

The phenomenon of Landau quantization is inherently quantum in nature and lacks a classical counterpart. When charged particles move along magnetic field lines, they remain largely unaffected by external magnetic fields. However, the mean energy of the system, within the Landau levels, increases due to the application of an external magnetic field (Landau and Lifshitz, 2013; Sumera et al., 2017); (Shaukat, 2017). In this context, extensive research has been conducted on both linear and nonlinear electromagnetic waves in quantum plasmas. A recent study by (Rozina et al., 2020) delved into the characteristics of magnetized, weakly ionized linear and nonlinear fast magnetosonic waves within pulsar environments. This research illuminated how the orbital and spinning motion of electrons significantly influences the dispersion relations of electrostatic longitudinal waves in degenerate Fermi gases (Tsintsadze, 2010).

Furthermore, Tsintsadze and Tsintsadze (2012) explored the thermodynamic properties of magnetized quantum electron gases under the influence of strong magnetic fields. They successfully derived a new type of sound velocity, termed the quantum magnetosound velocity, which adds to the understanding of the dynamics of these complex systems. Shukla and Rahman (1996) delved into the profiles of both shear Alfve'n waves and magnetosonic waves in classical plasmas, employing the plasma fluid model to gain insights into these wave phenomena. Salimullah and Rosenberg (1999) were pioneers in analytically exploring low-frequency dust kinetic Alfve'n waves in dusty plasmas, marking a significant milestone in the study of these waves, especially in their applications to comets and planetary rings. Moreover, Reddy et al. (1996) contributed to the field by investigating low-frequency Alfve'n waves in multibeam dusty plasmas, uncovering their relevance in astrophysical contexts such as comets and planetary rings. Lastly, Masood et al. (2010) made substantial progress by studying both linear and nonlinear electromagnetic waves in astrophysical systems, establishing a robust theoretical framework for calculating the magnetoacoustic speed in white dwarfs. This work considered electrons in both relativistic and non-relativistic regimes, expanding our understanding of the intricate interplay between quantum effects, magnetic fields, and plasma dynamics in these celestial objects.

In this paper, we build upon these foundational studies to explore the behavior of shear Alfve'n waves in the context of quantum plasmas, considering the Landau quantization effect. We aim to deepen our comprehension of the interplay between quantum phenomena and magnetic fields in degenerate relativistic plasmas, shedding light on their implications for astrophysical and technological applications.

2. A realtivistic multifluid model

The work of Nodar L. Tsintsadze and Levan N. Tsintsadze has been instrumental in advancing our understanding of the relativistic thermodynamic properties of degenerate Fermi gases when subjected to super strong magnetic fields. In their research, they meticulously considered the quantized motion of particles, specifically their motion in planes perpendicular to the magnetic field. Their ground breaking contributions led to the development of a novel relativistic thermodynamic potential that sheds light on the intricate behavior of matter in these extreme conditions.

In the realm of astrophysical investigations, notable researchers such as Landstreet (1967), Shapiro and Teukolsky (2008), and Lipunov (1987) have meticulously examined various astrophysical environments, accumulating invaluable data sets replete with rich information. According to their meticulous calculations, the magnetic fields encountered on the surfaces of neutron stars typically range from 10^{11} to 10^{13} Gauss, with the potential for even more staggering intensities exceeding 10^{15}5Gauss within the interior regions of neutron stars or possibly even higher. Notably, Bisnovatyi-Kogan (1970) has expounded on the intriguing phenomenon whereby the rotation of stars can augment the strength of magnetic fields by factors ranging from 10^3 to 10^4. This underscores the profound influence of rotation on the magnetic characteristics of celestial bodies. It is imperative to recognize that within the confines of such super strong magnetic fields, the dynamics and thermodynamic properties of degenerate electron gases undergo profound transformations. The interplay between quantum effects and the overwhelming magnetic field strengths gives rise to novel and intriguing phenomena. These phenomena have far-reaching implications, not only in the context of astrophysical environments like the interiors of white dwarfs, neutron star magnetospheres, and magnetars but also in the domain of modern technology. The newly formulated relativistic thermodynamics model for magnetized electron gases has found widespread application in elucidating the behavior of matter within these extreme conditions. This model has become an indispensable tool for unraveling the enigmatic secrets of degenerate environments, offering insights into the exotic physics at play in neutron stars and white dwarfs. Furthermore, its relevance extends to modern technology, where the knowledge gleaned from astrophysical investigations can be harnessed for technological advancements. Consequently, the study of relativistic thermodynamics in the presence of super strong magnetic fields stands as a crossroads between fundamental astrophysical research and cutting-edge technological innovation. Nodar L. Tsintsadze and Levan N. Tsintsadze paved a way for researcher to crack the hidden secrets of plasma by calculating many phenomenon such as, they calculated the thermodynamic potential Ω_T for degenerate electron gas in the relativistic limit i.e. $\mu \sim \hbar\omega_c \gg k_B T$.

$$\Omega_T = -\frac{V m_e c^2 D}{12\lambda_c^3}\left(\frac{k_B T}{m_e c^2}\right)^2 \left\{ \frac{\bar{\mu}}{\sqrt{\bar{\mu}^2 - 1}} + \frac{2\bar{\mu}\sqrt{\bar{\mu}^2 - 1 - D}}{D} \right\} \tag{1}$$

Where $\bar{\mu} = \frac{\mu}{m_e c^2} = \sqrt{1 + \frac{2E_F}{m_e c^2}}, \qquad D = \frac{2\hbar\omega_c}{m_e c^2}$

μ is chemical potential, D is Landau quantization, while, $E_F, m_e, c, \hbar, \omega_c, \lambda_c, k_B, T$ is Fermi energy, mass of electrons, speed of light, Dirac constant, cyclotron frequency of electrons, Compton wavelength, Boltzmann constant and thermal temperature of plasma species respectively. The expression for relativistic pressure can be constructed by using the equation $P = -\frac{\Omega_T}{V}$. The most astonishing fact is that the Landau quantization D is appearing in the relativistic pressure and thus we do not need to write separate part of Landau quantization.

$$P = -\frac{\Omega_T}{V} = \frac{m_e c^2 D}{12\lambda_c^3}\left(\frac{k_B T}{m_e c^2}\right)^2 \left\{ \frac{\bar{\mu}}{\sqrt{\bar{\mu}^2 - 1}} + \frac{2\bar{\mu}\sqrt{\bar{\mu}^2 - 1 - D}}{D} \right\} \tag{2}$$

Simplification leads us to the following form,

$$P = \frac{k_B^2 T^2}{12\lambda_c^3}\frac{1}{m_e c^2}\left[2\beta n^{\frac{2}{3}} + 1\right]$$

Where $\beta = \frac{3^{\frac{2}{3}}\pi^{\frac{4}{3}}\hbar^2}{m_e^2 c^2}$ is a parameter.

$$\nabla P_e = N_T n_e^{-\frac{1}{3}} \nabla n_e \tag{3}$$

For our simplification, we have supposed $N_T = \frac{k_B^2 T^2}{9\lambda_c^3}\frac{1}{m_e c^2}\beta$

Now, keeping the higher order in the expansion of pressure term, the N_T will be replaced by N^L which has been defined separately from Eq. (3).

$$N^L = \frac{1}{12\lambda_c^3}\frac{(k_B T)^2}{m_e c^2 \beta}\left\{\frac{4}{3}\beta^2 + \frac{1}{6}n_{e0}^{-\frac{4}{3}}\left(1 + D^2\right)\right\}$$

In the present work, we aim to investigate the exotic physics of shear Alfve'n waves in an in homogeneous two component collisionless quantum electron ion plasma in the presence of strongly spatially variable quantized magnetic field via Landau quantization in the atmosphere of white dwarf. The orientation of ambient magnetic field is taken along z-axis i.e $\mathbf{B} = B_0\hat{z}$, B_0 is magnetic field strength, $\hat{z}$ is unit vector in the Cartesian coordinate system. The lighter particles, electrons, are treated as relativistic, quantized and magnetized particles. On the other hand, ion population is magnetized but do not behave as degenerate species due to their tenuous mass as compared to electrons. Brodin and Marklund were the first who developed spin-$\frac{1}{2}$ quantum magnetohydrodynamic model (QMHD) for description of magnetoplasma (hydrodynamic waves) and applied this model for various quantum plasma systems such as, astrophysical plasma, solid state plasma and dusty plasma (Brodin and Marklund, 2007; Marklund and Brodin, 2007). The quantum magnetohydrodynamic model (QMHD) or hydrodynamic model (MHD) is just an extension of classical fluid model with incorporated quantum effects and thus,describe the microscopic variables of plasma system (Manfredi, 2005; Sutar et al., 2022; Ahmad and Qamar, 2009). The trajectories of plasma particles can be find out by using the equation of motion. For our theoretical description, the governing equation of motion for degenerate relativistic electrons is given by

$$m_e n_e(\frac{\partial \gamma \mathbf{v}_e}{\partial t}) = q_e n_e(\mathbf{E} + \frac{1}{c}\gamma \mathbf{v}_e \times \mathbf{B}) - \nabla P_e \tag{4}$$

Where $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$ is relativistic effect introduced through velocity of wave. $\gamma_e = \gamma_{e1} = \left(1 - \frac{v_{e1}^2}{c^2}\right)^{-1/2} \simeq 1$ is linearized form of relativistic effect. The coordinate system for propagation of wave vector is chosen along xz-plane i.e $\mathbf{k} = (k_x, 0, k_z)$ such that $k_x = k\sin\theta$ and $k_z = k\cos\theta$ where θ represents the angle between wave vector and positive z-direction. The magnitude of wave vector is related with wavelength as $k = \frac{2\pi}{\lambda}$. The propagation vector $\mathbf{k}$ and electric field $\mathbf{E}$ of low frequency (electromagnetic) shear Alfve'n waves are assumed in same plane, thus, $E_y = 0$. The x and z components of velocity for electrons is determined from Eq. (4) Where $\omega_{ce} = \frac{e\mathbf{B}_0}{m_{e0}c}$ represents the cyclotron frequency of electrons. Finally, x and z component of velocity are following,

$$v_{e1x} = -i\frac{e}{\omega m_{e0}}\beta_1\left(\frac{1}{(\omega^2 - \xi k_z^2 - \omega_{ce}^2)}\right)E_x - i\frac{e}{m_{e0}\omega}\beta_2\xi k_x k_z E_z \tag{5}$$

$$v_{e1z} = -i\frac{e\xi k_x k_z}{\omega m_{e0}}\beta_2 E_x - i\frac{e}{\omega m_{e0}}\beta_3 E_z \tag{6}$$

Where variables are $\xi = \frac{N_T}{m_{e0}n_{e0}^{1/3}}$ for first order expansion of pressure term and $\xi = \frac{N_L}{m_{e0}n_{e0}^{1/3}}$ in case of higher order expansion in order to sustain Landau quantization term and

$$\beta_1 = \frac{\xi^2 k_x^2 k_z^2\left(\omega^2 - \xi k_x^2\right)}{(\omega^2 - \xi k_x^2 - \xi k_z^2)\left(\omega^2 - \xi k_x^2 - \omega_{ce}^2\right) - \frac{\omega_{ce}^2\xi^2 k_x^2 k_z^2}{\omega^2}} + \omega^2$$

$$\beta_2 = \frac{\left(\omega^2 - \xi k_x^2\right)}{(\omega^2 - \xi k_x^2 - \xi k_z^2)\left(\omega^2 - \xi k_x^2 - \omega_{ce}^2\right) - \frac{\omega_{ce}^2\xi^2 k_x^2 k_z^2}{\omega^2}}$$

$$\beta_3 = \frac{\left(\omega^2 - \xi k_x^2 - \omega_{ce}^2\right)\left(\omega^2 - \xi k_x^2\right)}{(\omega^2 - \xi k_x^2 - \xi k_z^2)\left(\omega^2 - \xi k_x^2 - \omega_{ce}^2\right) - \frac{\omega_{ce}^2\xi^2 k_x^2 k_z^2}{\omega^2}}$$

The governing equation of motion for non-quantum particles i.e for ions is given by

$$m_i n_i\left(\partial_t + \mathbf{v}_i.\nabla\right) = n_i e\left(E_1 + \frac{1}{c}\mathbf{v}_i \times \mathbf{B}\right) - \gamma k_B T \nabla n_i \tag{7}$$

Where γ is an adiabatic factor defined as $\gamma = \frac{N+2}{2}$, N represents the dimensions of the system. We use equation of continuity for determining the density.

$$\frac{\partial n_i}{\partial t} + \nabla . n_i \mathbf{v}_i = 0 \tag{8}$$

After linearization, we get

$$\frac{n_{i1}}{n_{i0}} = \frac{\mathbf{k}\cdot\mathbf{v}_{i1}}{\omega} \tag{9}$$

Substituting the value of $\frac{n_{i1}}{n_{i0}}$ in Eq. (8). Thus, required x and z-component of velocity of ions is

$$\begin{aligned} v_{i1x} = {} & \frac{i\omega e}{\omega^2 - v_{Ti}^2 k_x^2 . m_{i0}}\left\{1 + \frac{\Omega_{ci}^2}{(\omega^2 - v_{Ti}^2 k_x^2 - \Omega_{ci}^2)}\right. \\ & \left.\times\left(1 + \frac{(\frac{v_{Ti}^4}{\omega^2})\left(\omega^2 - v_{Ti}^2 k_x^2\right)k_x^2 k_z^2}{\left[\left(\omega^2 - v_{Ti}^2 k_x^2 - v_{Ti}^2 k_z^2\right)\left(\omega^2 - v_{Ti}^2 k_x^2 - \Omega_{ci}^2\right) - \frac{\Omega_{ci}^2 v_{Ti}^4 k_x^2 k_z^2}{\omega^2}\right]}\right)\right\}E_x \\ & + \frac{i\omega}{\omega^2 - v_{Ti}^2 k_x^2}\frac{e}{m_{i0}} \\ & \times\left\{\frac{v_{Ti}^4\left(\omega^2 - v_{Ti}^2 k_x^2\right)}{\omega^2\left[\left(\omega^2 - v_{Ti}^2 k_x^2 - v_{Ti}^2 k_z^2\right)\left(\omega^2 - v_{Ti}^2 k_x^2 - \Omega_{ci}^2\right) - \frac{\Omega_{ci}^2 v_{Ti}^4 k_x^2 k_z^2}{\omega^2}\right]}k_x^2 k_z^2\right\}E_x \\ & + i\frac{e v_{Ti}^2\left(\omega^2 - v_{Ti}^2 k_x^2\right)}{\omega m_{i0}\left[\left(\omega^2 - v_{Ti}^2 k_x^2 - v_{Ti}^2 k_z^2\right)\left(\omega^2 - v_{Ti}^2 k_x^2 - \Omega_{ci}^2\right) - \frac{\Omega_{ci}^2 v_{Ti}^4 k_x^2 k_z^2}{\omega^2}\right]}k_x k_z E_z \end{aligned} \tag{10}$$

$$\begin{aligned} v_{i1z} = {} & i\frac{e v_{Ti}^2\left(\omega^2 - v_{Ti}^2 k_x^2\right)}{\omega m_{i0}\left[\left(\omega^2 - v_{Ti}^2 k_x^2 - v_{Ti}^2 k_z^2\right)\left(\omega^2 - v_{Ti}^2 k_x^2 - \Omega_{ci}^2\right) - \frac{\Omega_{ci}^2 v_{Ti}^4 k_x^2 k_z^2}{\omega^2}\right]}k_x k_z E_x \\ & + i\frac{1}{\omega m_{i0}}\frac{e\left(\omega^2 - v_{Ti}^2 k_x^2\right)\left(\omega^2 - v_{Ti}^2 k_x^2 - \Omega_{ci}^2\right)}{\left[\left\{\omega^2 - v_{Ti}^2(k_x^2 + k_z^2)\right\}\left(\omega^2 - v_{Ti}^2 k_x^2 - \Omega_{ci}^2\right) - \frac{\Omega_{ci}^2 v_{Ti}^4 k_x^2 k_z^2}{\omega^2}\right]}E_z \end{aligned} \tag{11}$$

Where $v_{Ti} = \left(\frac{\gamma k_B T_i}{m_i}\right)^{\frac{1}{2}}$ is thermal velocity of ions. Since shear Alfve'n wave is low frequency (but longer wavelength) electromagnetic wave, thus, it is necessary to execute the conditions $\omega^2 \ll \Omega_{ci}^2$. The ion particles are treated as cold particles which implies that thermal velocity of ions will be zero i.e $v_{Ti} = 0$. Then above equation assume the following form.

$$v_{ixz} = v_{izx} = 0 \tag{12}$$

$$v_{izz} = \frac{ie}{\omega m_{i0}}E_z \tag{13}$$

$$\mathbf{J} = e n_{i0} v_{i1} - e n_{e0} v_{e1}$$

$$\mathbf{J} = \begin{pmatrix} J_{11} & J_{12} \\ J21 & J_{22} \end{pmatrix} . \begin{pmatrix} E_x \\ E_z \end{pmatrix} \tag{14}$$

where current components of Eq. (14) given in matrix are defined as

$$J_{11} = i\frac{e^2 n_{e0}}{\omega m_{e0}}\left[\frac{\xi^2 k_x^2 k_z^2\left(\omega^2 - \xi k_x^2\right)}{(\omega^2 - \xi k_x^2 - \xi k_z^2)\left(\omega^2 - \xi k_x^2 - \omega_{ce}^2\right) - \frac{\omega_{ce}^2\xi^2 k_x^2 k_z^2}{\omega^2}} + \omega^2\right] \times\left(\frac{1}{(\omega^2 - \xi k_x^2 - \omega_{ce}^2)}\right)$$

$$J_{12} = i\frac{e^2 n_{e0}}{\omega m_{e0}}\xi k_x k_z\left(\frac{\omega^2 - \xi k_x^2}{(\omega^2 - \xi k_x^2 - \xi k_z^2)\left(\omega^2 - \xi k_x^2 - \omega_{cez}^2\right) - \frac{\omega_{ce}^2\xi^2 k_x^2 k_z^2}{\omega^2}}\right)$$

$$J21 = i\frac{e^2 n_{e0}}{\omega m_{e0}}\xi k_x k_z\left(\frac{\omega^2 - \xi k_x^2}{(\omega^2 - \xi k_x^2 - \xi k_z^2)\left(\omega^2 - \xi k_x^2 - \omega_{ce}^2\right) - \frac{\omega_{ce}^2\xi^2 k_x^2 k_z^2}{\omega^2}}\right)$$

$$J_{22} = i\frac{e^2 n_{i0}}{\omega m_{i0}} + i\frac{e^2 n_{e0}}{\omega m_{e0}}\left(\frac{\left(\omega^2 - \xi k_x^2 - \omega_{ce}^2\right)\left(\omega^2 - \xi k_x^2\right)}{(\omega^2 - \xi k_x^2 - \xi k_z^2)\left(\omega^2 - \xi k_x^2 - \omega_{ce}^2\right) - \frac{\omega_{ce}^2\xi^2 k_x^2 k_z^2}{\omega^2}}\right)$$

The wave equation in tensor form is defined as following

$$-\mathbf{kk}.\mathbf{E} + k^2\mathbf{I}.\mathbf{E} - \frac{\omega^2}{c^2}\mathbf{E} = \frac{4\pi i\omega}{c^2}\mathbf{J} \tag{15}$$

By substituting the value of $\mathbf{J}$ in Eq. (15) leads the equation to following form

$$\left[-\mathbf{kk} + k^2\mathbf{I} - \frac{\omega^2}{c^2}\epsilon\right].\mathbf{E} = \mathbf{0}$$

$\mathbf{I}$ is identity tensor. Unit Dyadic product is defined as $\mathbf{kk} = \begin{pmatrix} k_{xx} & k_{xz} \\ k_{zx} & k_{zz} \end{pmatrix}$, and $\epsilon = \left(\mathbf{I} - \frac{4\pi i}{\omega}\right)$ is dielectric tensor. The $\epsilon(\omega)$ is known as dielectric function for any varying value of frequency and this function will be known as dielectric constant for specific value of frequency. It is worthy to note that $\epsilon(\omega)$ and $\boldsymbol{\sigma}(\omega)$ are simple scaler functions of wave frequency in unmagnetized plasma (isotropic media) but these quantities assume the form of tensors in magnetized plasma due to anisotropy (different motion along and across the magnetic field). Moreover, above equation can be written as

$$\mathbf{D}.\mathbf{E} = 0 \tag{16}$$

The non vanishing solution for electric field $\mathbf{E} \neq \mathbf{0}$ is only possible if determinant of matrix becomes zero. The dispersion relation $\mathbf{D}(\boldsymbol{\omega}, \mathbf{k})$ can be decoded from determinant condition which is the implicit function of both frequency and wave number.

$$Det(\mathbf{D}) = \begin{pmatrix} k_z^2 - \frac{\omega^2}{c^2}\epsilon_{xx} & -k_x k_z - \frac{\omega^2}{c^2}\epsilon_{xz} \\ -k_z k_x - \frac{\omega^2}{c^2}\epsilon_{zx} & k_x^2 - \frac{\omega^2}{c^2}\epsilon_{zz} \end{pmatrix} = 0 \tag{17}$$

The polarization current is produced by wiggling motion of electrons and ions that specifies the plasma as a dielectric medium. The specific properties of plasma are encoded in the elements of dielectric tensor. The distinct components of medium response function are

$$\epsilon_{xx} = 1 + \frac{\omega_{pe}^2}{\omega^2}\left[\frac{\xi^2 k_x^2 k_z^2\left(\omega^2 - \xi k_x^2\right)}{\left(\omega^2 - \xi k^2\right)\left(\omega^2 - \xi k_x^2 - \omega_{ce}^2\right) - \frac{\omega_{ce}^2 \xi^2 k_x^2 k_z^2}{\omega^2}} + \omega^2\right] \times \frac{1}{\left(\omega^2 - \xi k_x^2 - \omega_{ce}^2\right)}$$

$$\epsilon_{xz} = \epsilon_{zx} = \frac{\omega_{pe}^2}{\omega^2}\xi k_x k_z\left(\frac{\omega^2 - \xi k_x^2}{\left(\omega^2 - \xi k_x^2 - \xi k_z^2\right)\left(\omega^2 - \xi k_x^2 - \omega_{ce}^2\right) - \frac{\omega_{ce}^2 \xi^2 k_x^2 k_z^2}{\omega^2}}\right)$$

$$\epsilon_{zz} = 1 + \frac{\omega_{pi}^2}{\omega^2} + \frac{\omega_{pe}^2}{\omega^2}\left(\frac{\left(\omega^2 - \xi k_x^2 - \omega_{ce}^2\right)\left(\omega^2 - \xi k_x^2\right)}{\left(\omega^2 - \xi k^2\right)\left(\omega^2 - \xi k_x^2 - \omega_{ce}^2\right) - \frac{\omega_{ce}^2 \xi^2 k_x^2 k_z^2}{\omega^2}}\right)$$

The low frequency oblique shear Alfve'n wave has both $\mathbf{k}$ and $\mathbf{E}$ in same plane. By executing the condition $E_y = 0$ and further this low frequency wave demands the implications of $\omega^2 \ll \omega_{ce}^2$ and $\omega^2 \ll \xi k_x^2$. The modified form medium response functions after the applications of conditions,

$$\epsilon_{xx} = 1 - \frac{\omega_{pe}^2}{\omega^2}\left[\frac{-\xi^3 k_x^4 k_z^2}{\xi k^2\left(\xi k_x^2 + \omega_{ce}^2\right) - \frac{\omega_{ce}^2 \xi^2 k_x^2 k_z^2}{\omega^2}} + \omega^2\right]\frac{1}{\xi k_x^2 + \omega_{ce}^2}$$

$$\epsilon_{xz} = \epsilon_{zx} = -\frac{\omega_{pe}^2}{\omega^2}\frac{\xi^2 k_x^4 k_z}{\left(\omega^2 - \xi k_x^2 - \xi k_z^2\right)\left(\omega^2 - \xi k_x^2 - \omega_{ce}^2\right) - \frac{\omega_{ce}^2 \xi^2 k_x^2 k_z^2}{\omega^2}}$$

$$\epsilon_{zz} = 1 + \frac{\omega_{pi}^2}{\omega^2} + \frac{\omega_{pe}^2}{\omega^2}\left(\frac{\xi k_x^2\left(\xi k_x^2 + \omega_{ce}^2\right)}{\xi k^2\left(\xi k_x^2 + \omega_{ce}^2\right) - \frac{\omega_{ce}^2 \xi^2 k_x^2 k_z^2}{\omega^2}}\right)$$

The determinant of $\mathbf{D}$ is

$$\omega^2\left(\epsilon_{xx}\epsilon_{zz} - \epsilon_{xz}^2\right) - 2c^2 k_x k_z \epsilon_{xz} - c^2 k_x^2 \epsilon_{xx} - c^2 k_z^2 \epsilon_{zz} = 0 \tag{18}$$

After substituting the values of $\epsilon_{xx}, \epsilon_{xz}$ and ϵ_{zz} in above equation, we obtain a biquadratic equation at $\frac{\omega_{pi}^2}{\omega_{ci}^2} = \frac{c^2}{v_{Ai}^2}$,

$$A\omega^4 + B\omega^2 + C = 0 \tag{19}$$

where

$$A = \xi k_x^2 k_z^2 \frac{\omega_{pe}^2}{\xi k_x^2 + \omega_{ce}^2}\left(\omega_{ce}^2 + \xi k_x^2\right) + \omega_{pe}^2 k_x^2\left(\xi k_x^2 + \omega_{ce}^2\right) - \omega_{pe}^4 k_x^2 - \omega_{ce}^2 \xi k_x^2 k_z^2$$

$$B = c^2\omega_{ce}^2 \xi k_x^4 k_z^2 + c^2\omega_{ce}^2 \xi k_x^2 k_z^4 + \frac{\omega_{pe}^2}{\xi k_x^2 + \omega_{ce}^2}\xi k_x^2 k_z^2 \times \left(\omega_{pi}^2\omega_{ce}^2 - \omega_{ce}^2 c^2 k_x^2 - c^2\xi k_x^4\right) - \omega_{ci}^2\frac{c^2}{v_{Ai}^2}\xi k_x^2 k_z^2 \omega_{ce}^2 - \frac{\omega_{pi}^2\omega_{pe}^2}{\xi k_x^2 + \omega_{ce}^2}\xi^2 k_x^4 k_z^2 + \omega_{pe}^2 c^2 k_x^2 k_z^2\left(\xi k_x^2 - \omega_{ce}^2\right)$$

$$C = c^2\omega_{ci}^2\frac{c^2}{v_{Ai}^2}\omega_{ce}^2 \xi k_x^2 k_z^4$$

The solution of Eq. (20) becomes as in Box I.

3. Discussion

$$\omega^2 = -\left[\frac{v_{Ae}^2\left(k^2 - \frac{\omega_{ci}^2}{v_{Ai}^2}\right)}{\left(2 + \eta - \frac{v_{Ae}^2}{c^2}\right)} + \frac{\left(\omega_{ci}^2\frac{v_{Ae}^2 c^2}{v_{Ai}^2}k_z^2\left\{2 + \eta - \frac{v_{Ae}^2}{c^2}\right\} + v_{Ae}^2\left(k^2 - \frac{\omega_{ci}^2}{v_{Ai}^2}\right)\left\{c^2 k_x^2 + \delta\left(\omega_{pi}^2 - c^2 k_x^2 - \xi k_x^2\sigma\right)\right\}\right)}{\left(2 + \eta - \frac{v_{Ae}^2}{c^2}\right)\left[-v_{Ae}^2\left(k^2 - \frac{\omega_{ci}^2}{v_{Ai}^2}\right) + \frac{\omega_{ce}^2}{\xi k_x^2 + \omega_{ce}^2}\left(\omega_{pi}^2 - c^2 k_x^2 - \xi k_x^2\sigma\right) + c^2 k_x^2\right]}\right] \tag{21}$$

Where

$$\frac{\omega_{ce}^2}{\xi k_x^2 + \omega_{ce}^2} = \delta, \quad \frac{\omega_{ce}^2 - \omega_{pe}^2}{\xi k_z^2} = \eta, \quad \frac{c^2 k_x^2 + \omega_{pi}^2}{\omega_{ce}^2} = \sigma$$

This long manifestation of dispersion relation Eq. (21) was obtained by using the thermodynamic potential Ω_T coined by Nodar L. Tsintsadze and Levan N. Tsintsadze. Then thermodynamic potential played vital role for determining the relativistic pressure in degenerate relativistic plasma which purely depends upon number density and thermal temperature of electrons. The relation between ω and $\mathbf{k}$ illustrates the possible wavelengths and frequencies of shear Alfve'n waves in degenerate relativistic plasma.

After dong some straightforward calculations, we obtain a dispersion relation which depicts the modified relation with quantum effect i.e. Landau quantization in degenerate relativistic plasma, which is geometry dependent and plays no role for parallel propagation. However, it is worthy to note that only electrons are contributing in the quantum effects but ion species exhibit classical behavior due to their ignorable De Broglie wavelength but these ions are treated as magnetized and cold particles. Eq. (21) shows that frequency of shear Alfve'n waves depends upon plasma and cyclotron frequencies of electrons and ions, Alfve'n speed of electrons and ions and angle between magnetic field and propagation vector.

4. Numericalanalysis

In a low thermal temperature and high density plasma systems microscopic quantum features modify the macroscopic properties significantly. The number densities in certain dense astrophysical plasmas are about $10^{30} - 10^{32}$ cm^{-3} and magnetic field about $10^9 - 10^{11}G$ (Shah et al., 2011; Deka and Dev, 2020). Quantum effects are significant in these severe astrophysical conditions. We select some common parameters seen in the interiors of neutron stars, magnet stars, and white dwarfs,

$n_{e0} = n_{i0} = (2.5 \mapsto 8.5)\times10^{31}cm^{-3},B_0$ = $(4.2 \mapsto 6.2)\times10^{11}$G,T =$(2.1 \mapsto 4.1)\times10^7$k, Ti = $(2.1 \mapsto 4.1)\times10^7$k, x=$12^0$ (Deka and Dev, 2020).

This research present a numerical investigation of the normalized dispersion relation for Shear Alfven waves within a degenerate

$$\omega^2 = \frac{-v_{Ae}^2\left(k^2-\frac{\omega_{ci}^2}{v_{Ai}^2}\right)-\frac{\left(\omega_{ci}^2\frac{v_{Ae}^2c^2}{v_{Ai}^2}k_z^2\left\{2+\frac{\omega_{ce}^2-\omega_{pe}^2}{\xi k_z^2}-\frac{v_{Ae}^2}{c^2}\right\}+v_{Ae}^2\left(k^2-\frac{\omega_{ci}^2}{v_{Ai}^2}\right)\left\{c^2k_x^2+\frac{\omega_{ce}^2}{\xi k_x^2+\omega_{ce}^2}\left(\omega_{pi}^2-c^2k_x^2-\xi k_x^2\left(\frac{c^2k_x^2+\omega_{pi}^2}{\omega_{ce}^2}\right)\right)\right\}\right)}{\left[-v_{Ae}^2\left(k^2-\frac{\omega_{ci}^2}{v_{Ai}^2}\right)+\frac{\omega_{ce}^2}{\xi k_x^2+\omega_{ce}^2}\left(\omega_{pi}^2-c^2k_x^2-\xi k_x^2\left(\frac{c^2k_x^2+\omega_{pi}^2}{\omega_{ce}^2}\right)\right)+c^2k_x^2\right]}}{\left(2+\frac{\omega_{ce}^2-\omega_{pe}^2}{\xi k_z^2}-\frac{v_{Ae}^2}{c^2}\right)} \quad (20)$$

Box I.

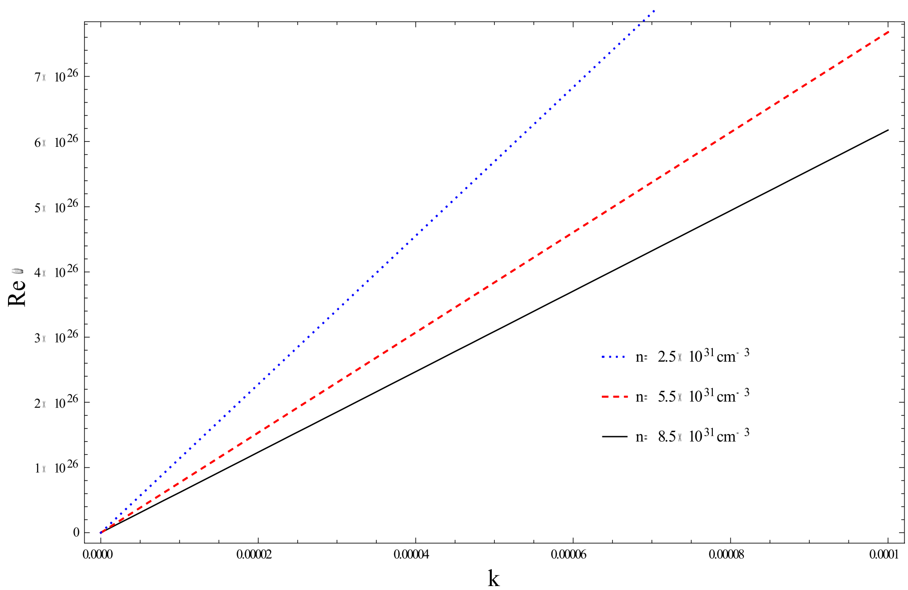

Fig. 1. Relationship of (ω, k) at $n = (2.5to8.5)\times10^{31}\,\mathrm{cm}^{-3}$, $B_0 = 5.2\times10^{11}G$, $T = 2.1\times10^7k$, $X = 12^o$.

relativistic quantum plasma, incorporating the effects of Landau quantization. Our study, depicted through Figs. 1 to 5, unveils a rich spectrum of phenomena, elucidating how these waves behave under a range of distinct conditions and parameter configurations.

Fig. 1 illustrates the propagation characteristics of Shear Alfve'n waves within a degenerate relativistic quantum plasma, with a focus on varying number density while keeping other parameters constant. The blue dotted line depicts the (ω, k) relationship at a reference number density of $n = 2.5\times10^{31}\mathrm{cm}^{-3}$, where we observe that as wavevector k increases, ω correspondingly increases while the phase speed remains constant. In contrast, the dashed red line $n = 5.5\times10^{31}\mathrm{cm}^{-3}$ and solid black line $n = 5.5\times10^{31}\mathrm{cm}^{-3}$ demonstrate a noteworthy trend: an increase in number density leads to a decrease in the phase speed of shear Alfve'n waves within the degenerate relativistic quantum plasma. Notably, at a specific number density, the phase speed exhibits consistent behavior. In the context of electromagnetic Shear Alfve'n wave propagation, the behavior of charged particles within the plasma is crucial. When the gyro scales (Larmor Radius) are shorter than the anticipated dimension of interplanetary magnetic fields, these particles become bound to the magnetic field lines (Fitzpatrick, 2022; Casse et al., 2001). This entrapment results in a combined motion of both particles and field lines, preventing their crossing and giving rise to the phenomenon known as "frozen-in flow". This phenomenon is responsible for magnetic compression and rarefaction (Matteucci, 2020). However, as particle density increases in the relativistic plasma regime while keeping other parameters constant, particles may diffuse across the magnetic field lines threading through the plasma. Consequently, magnetic field lines weaken, making it challenging to generate effective magnetic compressions and rare factions within the Landau-quantized relativistic density regime, thus lowering the phase speed. In contrast, in non-relativistic plasmas, phase speed reduction usually occurs due to the breaking of magnetic field lines through collisional processes (Huber et al., 2013; McCubbin et al., 2022).

Fig. 2 offers insights into the behavior of phase velocity in Shear Alfve'n waves within a degenerate relativistic quantum plasma, with a focus on varying the strength of the DC magnetic field while holding other parameters constant. The blue dotted line corresponds to a magnetic field strength of $B_0 = 4.2\times10^{11}$G, the dashed red line to $B_0 = 5.2\times10^{11}$ G, and the solid black line to $B_0 = 6.2\times10^{11}$G. Alongside these lines, one can observe a uniform phase speed at the corresponding magnetic field values. However, an intriguing trend emerges: as the magnetic field strength increases, the phase speed steadily decreases. This behavior can be understood from a physical perspective. In the context of relativistic quantized plasmas, a stronger magnetic field leads to a higher normalized Alfve'n speed. Importantly, the phase speed exhibits an inverse relationship with this normalized Alfve'n speed (Safdar et al., 2020). As a result, an increase in magnetic field strength corresponds to a decrease in phase speed, as indicated in the presented results. This observation underscores the intricate interplay between magnetic field strength and phase velocity in the context of magnetosonic waves within degenerate relativistic quantum plasmas.

Fig. 3 delves into the influence of electron and ion thermal temperatures on the propagation of Shear Alfve'n waves within the degenerate relativistic quantum plasma. The range considered spans from $T = 2.1\times10^7k$ to 4.1×10^7k, with all other parameters held constant. Notably, at a typical thermal temperature for electrons and ions, we observe a uniform slope, signifying a constant phase speed. Interestingly, it becomes evident that increasing the thermal temperature exerts no discernible impact on the propagation characteristics. This phenomenon can be elucidated by the nature of the system under study. In this scenario,

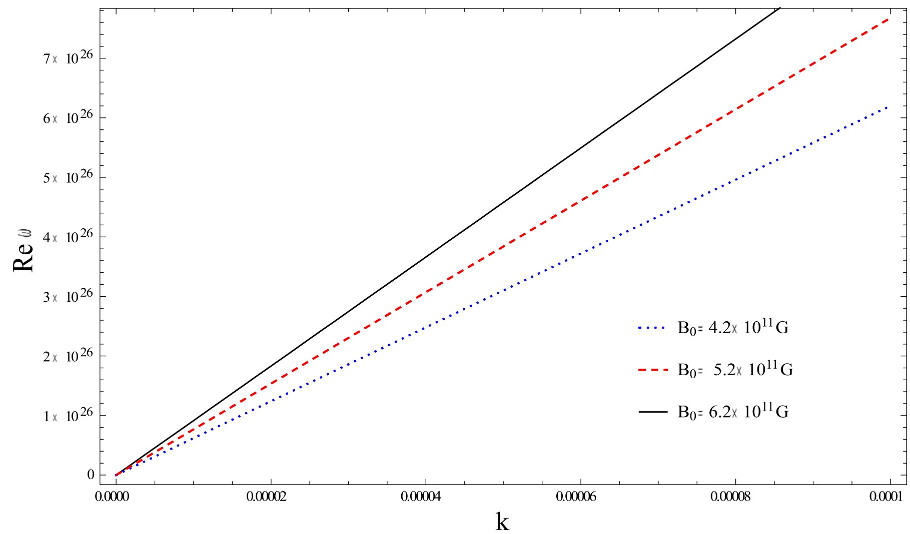

Fig. 2. Relationship of (ω, k) at $n = 5.5\times10^{31}$ cm^{-3}, $B_0 = (4.2to6.2)\times10^{11}G, T = 2.1\times10^7k, X = 12^o$.

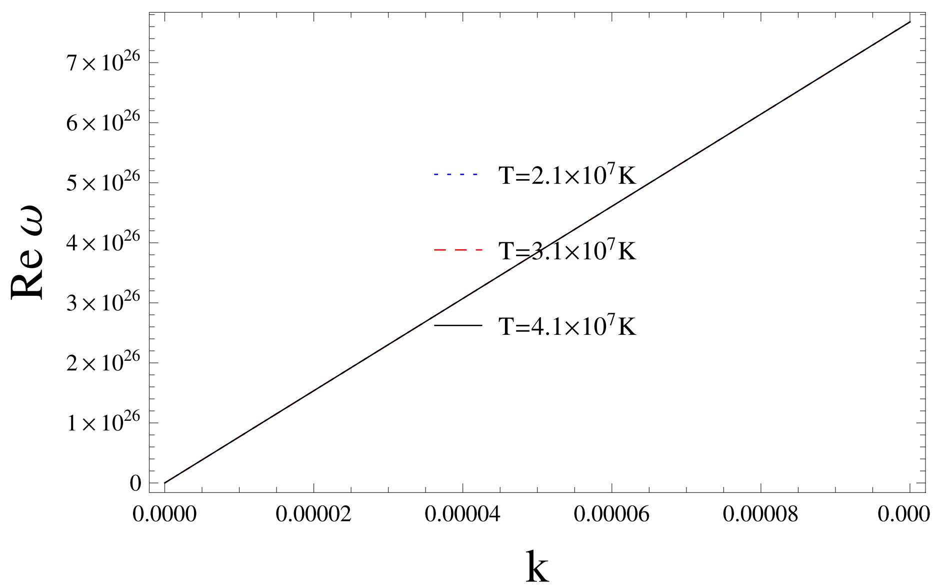

Fig. 3. Relationship of (ω, k) at $n = 5.5\times10^{31}$ cm^{-3}, $B_0 = 5.2\times10^{11}G, T = (2.1to4.1)\times10^7k, X = 12^o$.

electrons and ions are treated as degenerate and relativistic, rendering thermal effects negligible within the quantum plasma. As a result, alterations in the thermal temperatures of electrons and ions do not induce significant changes in the behavior of Shear Alfve'n waves (Mazzi et al., 2020). This observation underscores the unique characteristics of wave propagation in the context of a degenerate relativistic quantum plasma, where quantum effects dominate and thermal considerations take a backseat.

Fig. 4 provides a comprehensive view of the impact of varying the angle of Shear Alfve'n waves on their phase velocity within the degenerate relativistic quantum plasma. The angle of interest ranges from $X = 12°$ to $X = 18°$ with all other parameters held constant. A clear trend emerges as the angle between the x and z axes increases: the phase velocity of Shear Alfven waves steadily decreases. This intriguing behavior can be attributed to several factors. Firstly, the thermal temperature influences ion movement, prompting ions to move more rapidly, thereby enhancing the phase speed. However, the critical aspect to note is that Shear Alfven waves propagate more slowly when they are perpendicular to the magnetic field compared to when they align parallel to it. This anisotropy in phase velocity is a fundamental characteristic of magnetohydrodynamic (MHD) waves in plasma physics and is rooted in the intricate interplay between the wave and the magnetic field (Mazzi et al., 2020). The observations presented in Fig. 4 underscore the significance of this phenomenon within the context of degenerate relativistic quantum plasmas and its relevance in understanding wave behavior in extreme plasma environments.

The Fig. 5 depicts the behavior of phase speed of shear Alfvén waves when Landau quantization, in incorporated through higher order term defined in the expression of N^L. We know Landau quantization is based upon the external magnetic field. Physically on increasing the magnetic field, the phase speed is suppressed significantly as shown in figure from Dashed Red, Dotted Blue and Black. It is because the gyro frequency contributes inversely for the landau quantization which reduces the phase speed

In this comprehensive study, we investigate the intricate influence of Landau quantization on Shear Alfve'n waves within a degenerate relativistic plasma environment characterized by an ambient magnetic field aligned parallel to the z-axis. Within this framework, both electrons and ions are treated as relativistic entities, while the wavevector

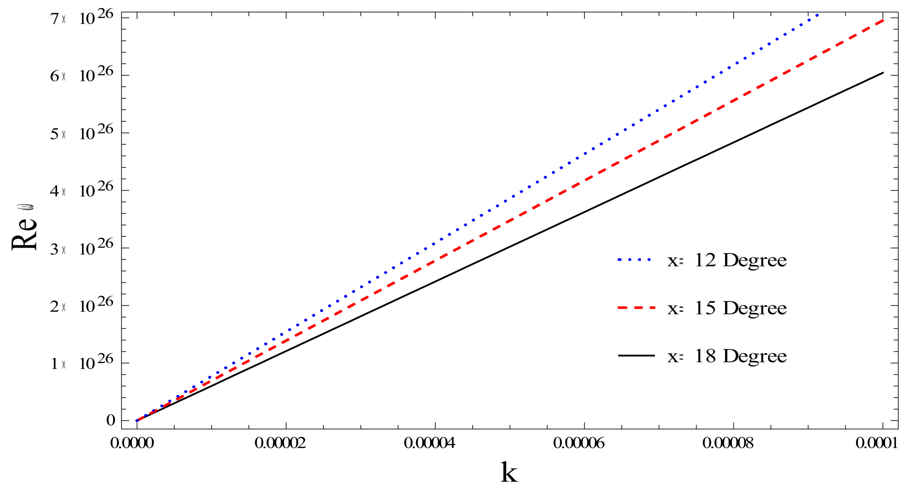

Fig. 4. Relationship of (ω, k) at $n = 5.5\times 10^{31}\,cm^{-3}$, $B_0 = 5.2\times10^{11}G$, $T = 2.1\times10^{7}k$, $X = (12-18^o)$.

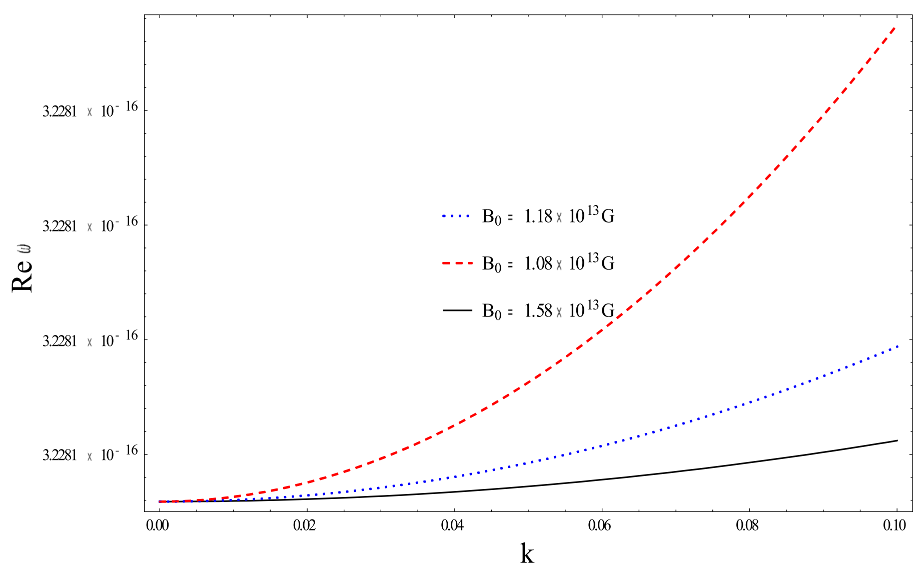

Fig. 5. Relationship of (ω, k) at $n = 1.0516\times10^{31}cm^{-3}$, $B_0 = (1.08-1.58)\times10^{13}G$, $T = 2.1\times10^{7}k$, $X = 10^o$.

is oriented perpendicularly to the applied magnetic field, along the y-axis, which is orthogonal to the uniform magnetic field. Employing the Quantum hydrodynamic model, we consider the interplay of fermi gas pressure, incorporating certain thermal effects and the Landau quantization phenomenon, within the context of relativistic plasma. Our analytical investigation unravels compelling insights into the modified dispersion relation arising from the coexistence of relativistic quantum electrons and classical thermal ions. The graphical representations of our findings emphasize the substantial impact of several key parameters, including the number density of plasma species, the strength of the ambient magnetic field, and the thermal temperatures of both ions and electrons. Notably, we observe that while the thermal temperature of relativistic electrons exerts minimal influence on the phase speed of magnetosonic waves, the angle between the wave vector (k) and the magnetic field emerges as a significant factor shaping wave behavior. Our research contributes to a deeper understanding of wave dynamics in extreme plasma conditions, shedding light on the complex interplay between quantum and thermal effects in the presence of Landau quantization.

Shear Alfve'n waves play a crucial role in various astrophysical and laboratory plasma environments. In space physics, they contribute to the transport of energy and momentum in magnetized plasmas, such as the solar wind and Earth's magnetosphere, influencing space weather and geomagnetic disturbances (Robertson et al., 2020). In controlled fusion experiments, Alfve'n waves are used for heating and stabilizing high-temperature plasmas in devices like tokamaks and stellarators (Kryzhanovskyy et al., 2022). Furthermore, they find applications in diagnostics, helping scientists probe and study plasma properties. Additionally, Alfve'n waves are relevant in understanding astrophysical phenomena, like the behavior of magnetic fields in stars and the interstellar medium (Melnikov, 2019). Their versatility in interacting with magnetized plasmas makes shear Alfve'n waves a valuable tool for both fundamental plasma physics research and practical applications in energy generation and space exploration.

CRediT authorship contribution statement

M. Rizwan: Writing – original draft. **A. Rasheed:** Conceptualization. **M. Jamil:** Investigation. **F. Areeb:** Formal analysis. **M. Fakhar-e-Alam:** Investigation. **M. Atif:** Writing – review & editing, Funding acquisition. **P. Sumera:** Conceptualization. **I. Ahsan:** Formal analysis. **Zulfiqar Ali:** Formal analysis, Investigation.

Declaration of competing interest

Authors have no conflict of interest.

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References

Ahmad, M., Qamar, A., 2009. Parametric studies of nonlinear magnetosonic waves in two-dimensional quantum magnetoplasmas. Phys. Plasmas 16 (2), http://dx.doi.org/10.1063/1.3073669.

Bisnovatyi-Kogan, G.S., 1970. The explosion of a rotating star as a supernova mechanism. Astron. Zh. 47 (813), https://ui.adsabs.harvard.edu/abs/1970AZh.47.813B/abstract.

Brodin, G., Marklund, M., 2007. Spin magnetohydrodynamics. New J. Phys. 9 (8), 277, https://iopscience.iop.org/article/10.1088/1367-2630/9/8/277/meta.

Casse, F., Lemoine, M., Pelletier, G., 2001. Transport of cosmic rays in chaotic magnetic fields. Phys. Rev. D 65 (2), http://dx.doi.org/10.1103/physrevd.65.023002.

Deka, M.K., Dev, A.N., 2020. Chin. Phys. Lett. 37, 016101, https://iopscience.iop.org/article/10.1088/0256-307X/37/1/016101/meta.

El-Labany, S.K., El-Taibany, W.F., Behery, E.E., Abd-Elbaki, R., 2020. Oblique collision of ion acoustic solitons in a relativistic degenerate plasma. Sci. Rep. 10 (1), http://dx.doi.org/10.1038/s41598-020-72449-x.

El-Taibany, W.F., Mamun, A.A., El-Shorbagy, Kh H., 2012. Nonlinear electromagnetic perturbations in a degenerate electron–positron plasma. Adv. Space Res. 50 (1), 101–107. http://dx.doi.org/10.1016/j.asr.2012.03.022.

Fitzpatrick, R., 2022. Plasma Physics: An Introduction. Crc Press, https://books.google.com/books?hl=en&lr=&id=8RmbEAAAQBAJ&oi=fnd&pg=PP1&dq=Fitzpatrick,R.(2022).Plasma+physics:+an+introduction.+Crc+Press.&ots=0ohHeCxiAh&sig=RdLmihoNR-DsDuBMylthHUxX7Ww.

Huber, M.C.E., Pauluhn, A., Culhane, J.L., Timothy, J.G., Wilhelm, K., Zehnder, A., 2013. Observing photons in space: A guide to experimental space astronomy. In: Google Books. Springer Science & Business Media, https://books.google.com/books?hl=en&lr=&id=c0-6BAAAQBAJ&oi=fnd&pg=PR5&dq=Matin%2BHarwit.

Kryzhanovskyy, A., Bonfiglio, D., Cappello, S., Veranda, M., Zuin, M., 2022. Alfvén waves in reversed-field pinch and tokamak ohmic plasmas: nonlinear 3D MHD modeling and comparison with RFX-mod. Nucl. Fusion 62 (8), 086019, https://iopscience.iop.org/article/10.1088/1741-4326/ac6ad3/meta.

Landau, L.D., Lifshitz, E.M., 2013. Statistical physics: volume 5. In: Google Books. Elsevier, https://books.google.com/books?hl=en&lr=&id=VzgJN-XPTRsC&oi=fnd&pg=PP1&dq=Landau.

Landstreet, J.D., 1967. Synchrotron radiation of neutrinos and its astrophysical significance. Phys. Rev. 153 (5), 1372–1377. http://dx.doi.org/10.1103/physrev.153.1372.

Lipunov, V.M., 1987. Neutron Star Astrophysics. Nauka, Moscow, https://scholar.google.com/scholar?cites=1394596258366388741&as_sdt=2005&sciodt=0,5&hl=en.

Manfredi, G., 2005. How to model quantum plasmas. Fields Inst. Commun. 46, 263–287, https://books.google.com/books?hl=en&lr=&id=HoG1hT8cpE0C&oi=fnd&pg=PA263&dq=Manfredi,+G.+(2005).+How+to+model+quantum+plasmas.+Fields+Inst.Commun,+46,+263-287.&ots=P-d_8IXfx9&sig=AtBwXXGEc5QafioEy_X9M78f1Jg.

Marklund, M., Brodin, G., 2007. Dynamics of spin-1 2 quantum plasmas. Phys. Rev. Lett. 98 (2), 025001, https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.98.025001.

Masood, W., Eliasson, B., Shukla, P.K., 2010. Electromagnetic wave equations for relativistically degenerate quantum magnetoplasmas. Phys. Rev. E 81 (6), http://dx.doi.org/10.1103/physreve.81.066401.

Matteucci, J.V.H., 2020. Magnetic Field Generation and Reconnection in High Energy Density Plasmas (Doctoral dissertation). Princeton University, https://search.proquest.com/openview/73e6fcb45db8429aef85737672325b72/1?pq-origsite=gscholar&cbl=18750&diss=y.

Mazzi, S., Zarzoso, D., Garcia, J., Görler, T., Di Siena, A., Camenen, Y., Shinohara, K., et al., 2020. Impact of fast ions on a trapped-electron-mode dominated plasma in a JT-60U hybrid scenario. Nucl. Fusion 60 (4), 046026, https://iopscience.iop.org/article/10.1088/1741-4326/ab74a1/meta.

McCubbin, A.J., Howes, G.G., TenBarge, J., 2022. Characterizing velocity–space signatures of electron energization in large-guide-field collisionless magnetic reconnection. Phys. Plasmas 29 (5), http://dx.doi.org/10.1063/5.0082213.

Melnikov, A.V., 2019. Electric Potential in Toroidal Plasmas. Springer International Publishing., Cham, Switzerland, https://link.springer.com/content/pdf/10.1007/978-3-030-03481-8.pdf.

Passot, T., Sulem, C., Sulem, P.L., 2005. Topics in kinetic theory. In: Google Books. American Mathematical Soc, https://books.google.com/books?hl=en&lr=&id=HoG1hT8cpE0C&oi=fnd&pg=PA263&dq=Manfredi.

Reddy, R.V., Lakhina, G.S., Verheest, F., Meuris, P., 1996. Alfvén modes in dusty cometary and planetary plasmas. Planet. Space Sci. 44 (2), 129–135. http://dx.doi.org/10.1016/0032-0633(95)00083-6.

Robertson, J., Carter, T.A., Vincena, S., 2020. Propagation of shear alfvén waves in a two-ion plasma and application as a diagnostic for the ion density ratio. J. Plasma Phys. 86 (6), 905860613, https://www.cambridge.org/core/journals/journal-of-plasma-physics/article/propagation-of-shear-alfven-waves-in-a-twoion-plasma-and-application-as-a-diagnostic-for-the-ion-density-ratio/88BE63344BA9E27E1908B6D98EE0D8ED.

Rozina, Ch., Tsintsadze, N.L., Tsintsadze, L.N., 2020. Magnetic field quantization in pulsars. J. Plasma Phys. 86 (2), http://dx.doi.org/10.1017/s0022377820000252.

Safdar, A., Mushtaq, A., Esmaeili, A., Ikram, M., Sadiq, N., 2020. MHD waves with Landau diamagnetic pressure and Pauli paramagnetizim in degenerate plasmas. Phys. Scr. 96 (1), 015603, https://iopscience.iop.org/article/10.1088/1402-4896/abc648/meta.

Salimullah, M., Rosenberg, M., 1999. On kinetic alfvén waves in a dusty plasma. Phys. Lett. A 254 (6), 347–350. http://dx.doi.org/10.1016/S0375-9601(99)00138-3.

Shah, H.A., Iqbal, M.J., Tsintsadze, N., Masood, W., Qureshi, M.N.S., 2012. Effect of trapping in a degenerate plasma in the presence of a quantizing magnetic field. Phys. Plasmas 19 (9), 092304, https://pubs.aip.org/aip/pop/article/19/9/092304/909143.

Shah, H.A., Masood, W., Qureshi, M.N., Tsintsadze, N.L., 2011. Phys. Plasmas 18, 102306, https://pubs.aip.org/aip/pop/article/18/10/102306/317202.

Shapiro, S.L., Teukolsky, S.A., 2008. Black holes, white dwarfs, and neutron stars: The physics of compact objects. In: Google Books. John Wiley & Sons, https://books.google.com/books?hl=en&lr=&id=d1CRQIcP1zoC&oi=fnd&pg=PP2&dq=Shapiro.

Shaukat, M.I., 2017. Nonlinear drift solitary structures in degenerate electron-positron-ion plasma with Landau quantization. 24(6). http://dx.doi.org/10.1063/1.4985681.

Shukla, P.K., Rahman, H.U., 1996. Low-frequency electromagnetic waves in nonuniform gravitating dusty magnetoplasmas. Planet. Space Sci. 44 (5), 469–472. http://dx.doi.org/10.1016/0032-0633(95)00132-8.

Sumera, P., Rasheed, A., Jamil, M., Siddique, Muhammad Saboor, Areeb, F., 2017. Landau quantization effects on hole-acoustic instability in semiconductor plasmas. Phys. Plasmas 24 (12), 122107. http://dx.doi.org/10.1063/1.5002675.

Sutar, D.L., Sharma, S., Saxena, A., Pathan, T.A., Pensia, R.K., 2022. A study of neutral collisions and viscous force on the formation of astronomical objects including with QMHD fluid model. Radiat. Eff. Defects Solids 177 (7–8), 727–742. http://dx.doi.org/10.1080/10420150.2022.2073884.

Tsintsadze, L.N., 2010. Quantization and excitation of longitudinal electrostatic waves in magnetized quantum plasmas. Nucleation Atmosph. Aerosols http://dx.doi.org/10.1063/1.3533197.

Tsintsadze, N.L., Tsintsadze, L.N., 2012. Relativistic thermodynamics of magnetized Fermi electron gas. http://dx.doi.org/10.48550/arXiv.1212.2830, ArXiv.org.