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Enhanced estimation of population mean in the presence of auxiliary information

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ABSTRACT

In this article, we propose a general family of exponential-type estimators for enhanced estimation of population mean in simple random sampling. These estimators are based on the available parameters of the auxiliary variable such as coefficient of skewness, coefficient of kurtosis, standard deviation and coefficient of variation etc. Expressions for bias, mean squared error and minimum mean squared error of the proposed family are derived up to first degree of approximation. Five natural populations are considered to assess the performance of the proposed estimators. Numerical findings confirm that the proposed estimators dominate over the existing estimators such as sample mean, ratio, regression, Singh et al. (2008, 2009), Upadhyaya et al. (2011), Yadav and Kadilar (2013) and Kadilar (2016) in terms of mean squared error.

1. Introduction

Survey sampling is broadly used in agriculture, business management, demography, economics, education, engineering, industry, medical sciences, political science, social sciences and many others. The main objective of sample survey theory is to make inferences about the unknown population parameters like population total, population proportion, population mean or population variance etc. One of the hottest issues in survey sampling is to enhanced the efficiency of ratio, product and regression type estimators in the presence of known auxiliary information for estimating the unknown population parameters of the study variable under different sampling techniques. Ratio method of estimation proposed by Cochran (1940) is at its best than the usual estimators of mean/total when the correlation between study variable and auxiliary variable is positively high and regression line of study variable on auxiliary variable is linear and passes through or nearly through the origin. In many real situations, auxiliary variables are highly correlated with the study variable for instance, person's age and duration of sleep, income and expenditure, person's age and blood pressure etc. Product method of estimation was first suggested by Robson (1957) and rediscovered by Murthy (1964). It is preferably used when the correlation between study variable and auxiliary variable is negatively high. Regression method of estimation proposed by Watson (1937) is the appropriate choice when the regression line is linear and passes through a point away from the origin.

Many authors have suggested estimators for the estimation of population mean based on auxiliary information in simple random sampling without replacement (SRSWOR). Readers may refer to Singh and Tailor (2003), Kadilar and Cingi (2004, 2006a, 2006b, 2006c), Koyuncu and Kadilar (2009), Singh et al. (2009), Yan and Tian (2010), Upadhyaya et al. (2011), Yadav and Kadilar (2013), Khan et al. (2015), Kadilar (2016) and the references cited therein.

The enthusiasm behind this article is to propose an improved general family of exponential-type estimators for the estimation of finite population mean under SRSWOR. A brief introduction of some conventional and exponential-type estimators for population mean is provided in section 2. In section 3, we proposed a general exponential-type family of estimators for estimating finite population mean $\bar{Y}$ under SRSWOR scheme and derived their properties up to first order of approximation. An empirical study using five real data sets is performed to compare the proposed and existing estimators in terms of MSE in section 4. Finally, concluding remarks are addressed in the last section.

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2. Existing estimators

Consider a sample of size "n" is selected from a population of size "N" subject to the constraint $n < N$ under SRSWOR. Let n pair of observations $(y_i, x_i), i = 1, 2, 3, \cdots, n$ for the study (y) and auxiliary variable (x) respectively. Let $\bar{Y} = N^{-1}\sum_{i=1}^{N} y_i$ and $X- = N^{-1}\sum i = 1_N xi$ be the population means of the study and auxiliary variables respectively. Similarly, $\bar{y} = n^{-1}\sum_{i=1}^{n} y_i$ and $\bar{x} = n^{-1}\sum_{i=1}^{n} x_i$ be the respective sample means of the study and auxiliary variables.

To drive the expressions for bias and mean squared error (MSE) of the existing and proposed estimators, we consider $\zeta_0 = \frac{\bar{y}-\bar{Y}}{\bar{Y}}$ and $\zeta_1 = \frac{\bar{x}-\bar{X}}{\bar{X}}$ such that

$$E(\zeta_0) = E(\zeta_1) = 0$$

$$V(\zeta_0) = E(\zeta_0^2) = \frac{V(\bar{y})}{\bar{Y}^2} = \lambda C_y^2$$

$$V(\zeta_1) = E(\zeta_1^2) = \lambda C_x^2$$

$$Cov(\zeta_0, \zeta_1) = E(\zeta_0\zeta_1) = \frac{Cov(\bar{y}, \bar{x})}{\bar{Y}\bar{X}} = \lambda\rho_{yx}C_yC_x$$

where

$$\lambda = \left(\frac{1}{n} - \frac{1}{N}\right), C_y^2 = \frac{S_y^2}{\bar{Y}^2}, C_x^2 = \frac{S_x^2}{\bar{X}^2}, \rho_{yx} = \frac{S_{yx}}{S_yS_x},$$

$$S_y^2 = \frac{\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)^2}{N-1}, S_x^2 = \frac{\sum_{i=1}^{N}\left(x_i - \bar{X}\right)^2}{N-1} \text{ and}$$

$$S_{yx} = \frac{\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)\left(x_i - \bar{X}\right)}{N-1}.$$

The conventional unbiased estimator without utilizing auxiliary information is defined by

$$\eta_0 = \bar{y} \quad (1)$$

Variance/MSE of η_0 is given by

$$MSE(\eta_0) = Var(\eta_0) = \lambda\bar{Y}^2C_y^2 \quad (2)$$

The conventional ratio estimator proposed by Cochran (1940) is defined as

$$\hat{\bar{Y}}_R = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right), \quad \bar{x} \neq 0 \quad (3)$$

Expressions for the bias and MSE of the $\hat{\bar{Y}}_R$ estimator, up to first degree of approximation, are respectively given by

$$Bias\left(\hat{\bar{Y}}_R\right) \cong \lambda\bar{Y}\left[C_x^2 - \rho_{yx}C_yC_x\right] \quad (4)$$

and

$$MSE\left(\hat{\bar{Y}}_R\right) \cong \lambda\bar{Y}^2\left[C_y^2 + C_x^2(1 - 2A)\right] \quad (5)$$

where $A = \rho_{yx}\frac{C_y}{C_x}$

Exponential ratio type estimator initiated by Bahl and Tuteja (1991) of population mean is as below

$$\hat{\bar{Y}}_{BT} = \bar{y}\exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right) \quad (6)$$

Bias and MSE of the $\hat{\bar{Y}}_{BT}$ estimator, up to first order of approximation, are respectively given as

$$Bias\left(\hat{\bar{Y}}_{BT}\right) \cong \frac{\lambda}{2}\bar{Y}\left[\frac{3C_x^2}{4} - \rho_{yx}C_yC_x\right] \quad (7)$$

and

$$MSE\left(\hat{\bar{Y}}_{BT}\right) \cong \frac{\lambda}{4}\bar{Y}^2\left[4C_y^2 + C_x^2 - 4\rho_{yx}C_yC_x\right] \quad (8)$$

Singh et al. (2008) suggested a ratio-product exponential type estimator for population mean given as

$$\hat{\bar{Y}}_{SI} = \bar{y}\left[\mu\exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right) + (1 - \mu)\exp\left(\frac{\bar{x} - \bar{X}}{\bar{X} + \bar{x}}\right)\right] \quad (9)$$

where μ is a real constant.

The optimal value of $\mu = \frac{1}{2} + \frac{\rho_{yx}C_y}{C_x}$.

Minimum MSE of $\hat{\bar{Y}}_{SI}$ estimator, up to first degree of approximation, as follows

$$MSE_{min}\left(\hat{\bar{Y}}_{SI}\right) \cong \lambda\bar{Y}^2C_y^2\left(1 - \rho_{yx}^2\right) \cong MSE\left(\hat{\bar{Y}}_{Reg}\right) \quad (10)$$

Motivated by Khoshnevisan et al. (2007), Singh et al. (2009) proposed an exponential family of estimators for $\bar{Y}$ as

$$\eta = \bar{y}\exp\left[\frac{(\alpha\bar{X} + \beta) - (\alpha\bar{x} + \beta)}{(\alpha\bar{X} + \beta) + (\alpha\bar{x} + \beta)}\right] \quad (11)$$

where α and β are either real numbers or function of the known parameters of the auxiliary variable such as coefficient of skewness $\left(\beta_{1(x)}\right)$, coefficient of kurtosis $\left(\beta_{2(x)}\right)$, standard deviation (S_x), coefficient of variation (C_x) and coefficient of correlation $\left(\rho_{yx}\right)$ of the population.

Following are the bias and MSE up to first degree of approximation

$$Bias(\eta) \cong \lambda\bar{Y}\left[\frac{3}{2}\theta_i^2C_x - \theta_i\rho_{yx}C_y\right]C_x \quad (12)$$

and

$$MSE(\eta) \cong \lambda\bar{Y}^2\left[C_y^2 + \theta_i^2C_x^2 - 2\theta_i\rho_{yx}C_yC_x\right] \quad (13)$$

where

$$\theta_i = \frac{\alpha\bar{X}}{2(\alpha\bar{X} + \beta)}, i = 1, 2, 3, ..., 10$$

and

$$\theta_1 = 0.5; \theta_2 = \frac{\bar{X}}{2\left(\bar{X} + \beta_{2(x)}\right)}; \theta_3 = \frac{\bar{X}}{2(\bar{X} + C_x)}; \theta_4 = \frac{\bar{X}}{2\left(\bar{X} + \rho_{yx}\right)};$$

$$\theta_5 = \frac{\beta_{2(x)}\bar{X}}{2\left(\beta_{2(x)}\bar{X} + C_x\right)}; \theta_6 = \frac{C_x\bar{X}}{2\left(C_x\bar{X} + \beta_{2(x)}\right)}; \theta_7 = \frac{C_x\bar{X}}{2\left(C_x\bar{X} + \rho_{yx}\right)};$$

$$\theta_8 = \frac{\rho_{yx}\bar{X}}{2\left(\rho_{yx}\bar{X} + C_x\right)}; \theta_9 = \frac{\beta_{2(x)}\bar{X}}{2\left(\beta_{2(x)}\bar{X} + \rho_{yx}\right)}; \theta_{10} = \frac{\rho_{yx}\bar{X}}{2\left(\rho_{yx}\bar{X} + \beta_{2(x)}\right)}.$$

Some members of the η- family of estimators are given in Table 1. Furthermore, Singh et al. (2009) proposed a generalized estimator by combining the estimator η_1 and $\eta_i (i = 2, 3, 4, \cdots, 10)$ as follows

$$\eta^* = \gamma\eta_1 + (1 - \gamma)\eta_i, i = 2, 3, 4, \cdots, 10 \quad (14)$$

where γ is a suitable chosen weight.

The optimal value of $\gamma = \frac{2(A - \theta_i)}{(1 - 2\theta_i)}$.

Table 1
Members of η- family of estimators.

Estimators	α	β
Sample Mean $\eta_0 = \bar{y}$	0	0
Bahl and Tuteja (1991) $\eta_1 = \bar{y}\exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right]$	1	0
Singh et al. (2009) $\eta_2 = \bar{y}\exp\left[\frac{[\bar{X}-\bar{x}]}{[\bar{X}+\bar{x}]+2\beta_{2(x)}}\right]$	1	$\beta_{2(x)}$
$\eta_3 = \bar{y}\exp\left[\frac{[\bar{X}-\bar{x}]}{[\bar{X}+\bar{x}]+2C_x}\right]$	1	C_x
$\eta_4 = \bar{y}\exp\left[\frac{[\bar{X}-\bar{x}]}{[\bar{X}+\bar{x}]+2\rho_{yx}}\right]$	1	ρ_{yx}
$\eta_5 = \bar{y}\exp\left[\frac{\beta_{2(x)}[\bar{X}-\bar{x}]}{\beta_{2(x)}[\bar{X}+\bar{x}]+2C_x}\right]$	$\beta_{2(x)}$	C_x
$\eta_6 = \bar{y}\exp\left[\frac{C_x[\bar{X}-\bar{x}]}{C_x[\bar{X}+\bar{x}]+2\beta_{2(x)}}\right]$	C_x	$\beta_{2(x)}$
$\eta_7 = \bar{y}\exp\left[\frac{C_x[\bar{X}-\bar{x}]}{C_x[\bar{X}+\bar{x}]+2\rho_{yx}}\right]$	C_x	ρ_{yx}
$\eta_8 = \bar{y}\exp\left[\frac{\rho_{yx}[\bar{X}-\bar{x}]}{\rho_{yx}[\bar{X}+\bar{x}]+2C_x}\right]$	ρ_{yx}	C_x
$\eta_9 = \bar{y}\exp\left[\frac{\beta_{2(x)}[\bar{X}-\bar{x}]}{\beta_{2(x)}[\bar{X}+\bar{x}]+2\rho_{yx}}\right]$	$\beta_{2(x)}$	ρ_{yx}
$\eta_{10} = \bar{y}\exp\left[\frac{\rho_{yx}[\bar{X}-\bar{x}]}{\rho_{yx}[\bar{X}+\bar{x}]+2\beta_{2(x)}}\right]$	ρ_{yx}	$\beta_{2(x)}$

Table 2
Members of ω- family of estimators.

Estimators	α	β	k
Bahl and Tuteja (1991) $\omega_0 = \bar{y}\exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right]$	1	0	1
Yadav and Kadilar (2013) $\omega_1 = k\bar{y}\exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right]$	1	0	k
$\omega_2 = k\bar{y}\exp\left[\frac{[\bar{X}-\bar{x}]}{[\bar{X}+\bar{x}]+2\beta_{2(x)}}\right]$	1	$\beta_{2(x)}$	k
$\omega_3 = k\bar{y}\exp\left[\frac{[\bar{X}-\bar{x}]}{[\bar{X}+\bar{x}]+2C_x}\right]$	1	C_x	k
$\omega_4 = k\bar{y}\exp\left[\frac{[\bar{X}-\bar{x}]}{[\bar{X}+\bar{x}]+2\rho_{yx}}\right]$	1	ρ_{yx}	k
$\omega_5 = k\bar{y}\exp\left[\frac{\beta_{2(x)}[\bar{X}-\bar{x}]}{\beta_{2(x)}[\bar{X}+\bar{x}]+2C_x}\right]$	$\beta_{2(x)}$	C_x	k
$\omega_6 = k\bar{y}\exp\left[\frac{C_x[\bar{X}-\bar{x}]}{C_x[\bar{X}+\bar{x}]+2\beta_{2(x)}}\right]$	C_x	$\beta_{2(x)}$	k
$\omega_7 = k\bar{y}\exp\left[\frac{C_x[\bar{X}-\bar{x}]}{C_x[\bar{X}+\bar{x}]+2\rho_{yx}}\right]$	C_x	ρ_{yx}	k
$\omega_8 = k\bar{y}\exp\left[\frac{\rho_{yx}[\bar{X}-\bar{x}]}{\rho_{yx}[\bar{X}+\bar{x}]+2C_x}\right]$	ρ_{yx}	C_x	k
$\omega_9 = k\bar{y}\exp\left[\frac{\beta_{2(x)}[\bar{X}-\bar{x}]}{\beta_{2(x)}[\bar{X}+\bar{x}]+2\rho_{yx}}\right]$	$\beta_{2(x)}$	ρ_{yx}	k
$\omega_{10} = k\bar{y}\exp\left[\frac{\rho_{yx}[\bar{X}-\bar{x}]}{\rho_{yx}[\bar{X}+\bar{x}]+2\beta_{2(x)}}\right]$	ρ_{yx}	$\beta_{2(x)}$	k

Minimum MSE of η^* estimator is equal to the MSE of the regression estimator $\hat{\bar{Y}}_{Reg}$.

$$MSE_{min}(\eta^*) \cong \lambda\bar{Y}^2C_y^2\left(1-\rho_{yx}^2\right) \cong MSE\left(\hat{\bar{Y}}_{Reg}\right) \tag{15}$$

Upadhyaya et al. (2011) proposed ratio exponential type estimator as

$$\hat{\bar{Y}}_{UP} = \bar{y}\exp\left(1-\frac{2\bar{x}}{\bar{X}+\bar{x}}\right) \tag{16}$$

MSE of $\hat{\bar{Y}}_{UP}$ estimator, up to first degree of approximation, as follows

$$MSE_{min}\left(\hat{\bar{Y}}_{UP}\right) \cong \lambda\bar{Y}^2C_y^2\left(1-\rho_{yx}^2\right) \cong MSE\left(\hat{\bar{Y}}_{Reg}\right) \tag{17}$$

Yadav and Kadilar (2013) suggested an improved exponential family of estimators for population mean $\bar{Y}$ by utilizing Singh et al. (2009) as

$$\omega = k\bar{y}\exp\left[\frac{(\alpha\bar{X}+\beta)-(\alpha\bar{x}+\beta)}{(\alpha\bar{X}+\beta)+(\alpha\bar{x}+\beta)}\right] \tag{18}$$

where k is a suitable constant and α and β are defined earlier. Some members of the ω- family of estimators are given in Table 2.

Following are the expressions of bias and MSE of the ω- family of estimators

$$Bias(\omega) \cong \lambda k\bar{Y}\left[\frac{3}{2}\theta_i^2C_x-\theta_i\rho_{yx}C_y\right]C_x+\bar{Y}(k-1) \tag{19}$$

and

$$MSE(\omega) \cong \lambda\bar{Y}^2\left[k^2C_y^2+k\theta_i^2C_x^2(4k-3)-2k\theta_i\rho_{yx}C_yC_x(2k-1)\right] + \bar{Y}^2(k-1)^2 \tag{20}$$

where θ_i is defined earlier.

Partially differentiating Eq. (20) w.r.t. "k" and setting $\frac{\partial MSE(t)}{\partial k}=0$, the optimal value of "$k$" is given by

$$k = \frac{1+\lambda\left[\frac{3}{2}\theta_i^2C_x^2-\theta_i\rho_{yx}C_yC_x\right]}{1+\lambda\left[C_y^2+4\theta_i^2C_x^2-4\theta_i\rho_{yx}C_yC_x\right]} = \frac{C_{1i}}{C_{2i}} \tag{21}$$

where

$$C_{1i} = 1+\lambda\left[\frac{3}{2}\theta_i^2C_x^2-\theta_i\rho_{yx}C_yC_x\right]$$

and

$$C_{2i} = 1+\lambda\left[C_y^2+4\theta_i^2C_x^2-4\theta_i\rho_{yx}C_yC_x\right]$$

Substituting Eq. (21) in Eq. (20), we get minimum MSE of the ω-family of estimators as

$$MSE_{min}(\omega) \cong \bar{Y}^2\left[1-\frac{C_{1i}^2}{C_{2i}}\right] \tag{22}$$

Kadilar (2016) suggested a modified exponential type estimator for population mean as

$$\hat{\bar{Y}}_K = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{\delta}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) \tag{23}$$

where δ is a real constant.

The optimal value of $\delta = \frac{(C_x-2\rho_{yx}C_y)}{2C_x}$.

Minimum MSE of $\hat{\bar{Y}}_K$ estimator, up to first degree of approximation, as follows

$$MSE_{min}\left(\hat{\bar{Y}}_K\right) \cong \lambda\bar{Y}^2C_y^2\left(1-\rho_{yx}^2\right) \cong MSE\left(\hat{\bar{Y}}_{Reg}\right) \tag{24}$$

Remark 2.1. Minimum MSE of $\widehat{\bar{Y}}_{SI}$, η^*, $\widehat{\bar{Y}}_{UP}$ and $\widehat{\bar{Y}}_K$ estimators, up to first order of approximation is exactly equal to the variance of the usual regression estimator $\widehat{\bar{Y}}_{Reg}$. Regression estimator suggested by Watson (1937) is given by

$$\widehat{\bar{Y}}_{Reg} = \bar{y} + b_{y.x}(\bar{X} - \bar{x}) \quad (25)$$

where $b_{y.x} = \frac{s_{yx}}{s_x^2}$ is the sample regression coefficient.

3. The suggested family of estimators

In this section, general exponential-type estimators for estimating finite population mean $\bar{Y}$ under SRSWOR is proposed. Some members of proposed family are given in Table 3. Expressions for the bias, MSE and minimum MSE are obtained to the first degree of approximation.

$$\xi = t_1\bar{y}\left(\frac{\bar{X}^*}{\bar{x}^*}\right) + t_2(\bar{X} - \bar{x})\exp\left(\frac{\bar{X}^* - \bar{x}^*}{\bar{X}^* + \bar{x}^*}\right) \quad (26)$$

where $\bar{X}^* = \alpha\bar{X} + \beta$ and $\bar{x}^* = \alpha\bar{x} + \beta$

where t_1 and t_2 are the appropriate constants to be determined such the MSE of ξ is minimum, $\alpha(\neq 0)$ and β are either real numbers or functions of the known parameters such as coefficient of variation (C_x), standard deviation (S_x), coefficient of skewness $\left(\beta_{1(x)}\right)$,coefficient of kurtosis $\left(\beta_{2(x)}\right)$ and coefficient of correlation $\left(\rho_{yx}\right)$ of the population.

3.1. Bias, MSE and minimum MSE of ξ

Eq. (26) can be transformed in terms of $\zeta_i's$ as

$$\xi = t_1\bar{Y}(1+\zeta_0)(1+\phi_i\zeta_1)^{-1} - t_2\bar{X}\zeta_1\exp\left(\frac{-\phi_i\zeta_1}{2}\left(1+\frac{\phi_i\zeta_1}{2}\right)^{-1}\right),$$
$$i = 1,2,3,\cdots,11 \quad (27)$$

Table 3
Some members of ξ- family of estimators.

Estimators	α	β
$\xi_1 = t_1\bar{y}\left(\frac{\bar{X}}{\bar{x}}\right) + t_2(\bar{X}-\bar{x})\exp\left(\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})}\right)$	1	0
$\xi_2 = t_1\bar{y}\left(\frac{\beta_{2(x)}\bar{X}+\beta_{1(x)}}{\beta_{2(x)}\bar{x}+\beta_{1(x)}}\right) + t_2(\bar{X}-\bar{x})\exp\left(\frac{\beta_{2(x)}(\bar{X}-\bar{x})}{\beta_{2(x)}(\bar{X}+\bar{x})+2\beta_{1(x)}}\right)$	$\beta_{2(x)}$	$\beta_{1(x)}$
$\xi_3 = t_1\bar{y}\left(\frac{S_x\bar{X}+\beta_{2(x)}}{S_x\bar{x}+\beta_{2(x)}}\right) + t_2(\bar{X}-\bar{x})\exp\left(\frac{S_x(\bar{X}-\bar{x})}{S_x(\bar{X}+\bar{x})+2\beta_{2(x)}}\right)$	S_x	$\beta_{2(x)}$
$\xi_4 = t_1\bar{y}\left(\frac{C_x\bar{X}+\beta_{1(x)}}{C_x\bar{x}+\beta_{1(x)}}\right) + t_2(\bar{X}-\bar{x})\exp\left(\frac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2\beta_{1(x)}}\right)$	C_x	$\beta_{1(x)}$
$\xi_5 = t_1\bar{y}\left(\frac{\bar{X}+\beta_{1(x)}}{\bar{x}+\beta_{1(x)}}\right) + t_2(\bar{X}-\bar{x})\exp\left(\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2\beta_{1(x)}}\right)$	1	$\beta_{1(x)}$
$\xi_6 = t_1\bar{y}\left(\frac{\rho_{yx}\bar{X}+\beta_{2(x)}}{\rho_{yx}\bar{x}+\beta_{2(x)}}\right) + t_2(\bar{X}-\bar{x})\exp\left(\frac{\rho_{yx}(\bar{X}-\bar{x})}{\rho_{yx}(\bar{X}+\bar{x})+2\beta_{2(x)}}\right)$	ρ_{yx}	$\beta_{2(x)}$
$\xi_7 = t_1\bar{y}\left(\frac{\beta_{1(x)}\bar{X}+\beta_{2(x)}C_x}{\beta_{1(x)}\bar{x}+\beta_{2(x)}C_x}\right) + t_2(\bar{X}-\bar{x})\exp\left(\frac{\beta_{1(x)}(\bar{X}-\bar{x})}{\beta_{1(x)}(\bar{X}+\bar{x})+2\beta_{2(x)}C_x}\right)$	$\beta_{1(x)}$	$\beta_{2(x)}C_x$
$\xi_8 = t_1\bar{y}\left(\frac{S_x\bar{X}+\beta_{1(x)}}{S_x\bar{x}+\beta_{1(x)}}\right) + t_2(\bar{X}-\bar{x})\exp\left(\frac{S_x(\bar{X}-\bar{x})}{S_x(\bar{X}+\bar{x})+2\beta_{1(x)}}\right)$	S_x	$\beta_{1(x)}$
$\xi_9 = t_1\bar{y}\left(\frac{C_x\bar{X}+\rho_{yx}}{C_x\bar{x}+\rho_{yx}}\right) + t_2(\bar{X}-\bar{x})\exp\left(\frac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2\rho_{yx}}\right)$	C_x	ρ_{yx}
$\xi_{10} = t_1\bar{y}\left(\frac{\beta_{2(x)}\bar{X}+\rho_{yx}}{\beta_{2(x)}\bar{x}+\rho_{yx}}\right) + t_2(\bar{X}-\bar{x})\exp\left(\frac{\beta_{2(x)}(\bar{X}-\bar{x})}{\beta_{2(x)}(\bar{X}+\bar{x})+2\rho_{yx}}\right)$	$\beta_{2(x)}$	ρ_{yx}
$\xi_{11} = t_1\bar{y}\left(\frac{\bar{X}+\rho_{yx}}{\bar{x}+\rho_{yx}}\right) + t_2(\bar{X}-\bar{x})\exp\left(\frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2\rho_{yx}}\right)$	1	ρ_{yx}

where

$$\phi_i = \frac{\alpha\bar{X}}{\alpha\bar{X}+\beta}, i = 1,2,3,\ldots,11$$

and

$$\phi_1 = 1; \phi_2 = \frac{\bar{X}\beta_{2(x)}}{\bar{X}\beta_{2(x)}+\beta_{1(x)}}; \phi_3 = \frac{\bar{X}S_x}{\bar{X}S_x+\beta_{2(x)}}; \phi_4 = \frac{\bar{X}C_x}{\bar{X}C_x+\beta_{1(x)}}; \phi_5 = \frac{\bar{X}}{\bar{X}+\beta_{1(x)}};$$

$$\phi_6 = \frac{\bar{X}\rho_{yx}}{\bar{X}\rho_{yx}+\beta_{2(x)}}; \phi_7 = \frac{\bar{X}\beta_{1(x)}}{\bar{X}\beta_{1(x)}+\beta_{2(x)}C_x}; \phi_8 = \frac{\bar{X}S_x}{\bar{X}S_x+\beta_{1(x)}}; \phi_9 = \frac{\bar{X}C_x}{\bar{X}C_x+\rho_{yx}};$$

$$\phi_{10} = \frac{\bar{X}\beta_{2(x)}}{\bar{X}\beta_{2(x)}+\rho_{yx}} \text{ and } \phi_{11} = \frac{\bar{X}}{\bar{X}+\rho_{yx}}.$$

Subtracting $\bar{Y}$ on both sides of the Eq. (27) and expanding up to first degree of approximation, we have

$$\xi - \bar{Y} = t_1(\bar{Y}-1) + t_1\bar{Y}\zeta_0 - t_1\bar{Y}\phi_i\zeta_1 - t_2\bar{X}\zeta_1 + t_1\bar{Y}\phi_i^2\zeta_1^2 + \frac{t_2\bar{X}\phi_i\zeta_1^2}{2} - t_1\bar{Y}\phi_i\zeta_0\zeta_1 \quad (28)$$

Taking expectation on both sides of Eq. (28), we obtained bias of the ξ-family of estimators to the first degree of approximation, given as

$$Bias(\xi) = E(\xi - \bar{Y}) \cong t_1(\bar{Y}-1) + t_1\bar{Y}\phi_i^2\lambda C_x^2 + \frac{t_2\bar{X}\phi_i\lambda C_x^2}{2} - t_1\bar{Y}\phi_i\lambda\rho_{yx}C_yC_x \quad (29)$$

Squaring and taking expectation on both sides of Eq. (28), we get MSE of the proposed ξ-family of estimators to the first degree of approximation as:

$$MSE(\xi) = E(\xi - \bar{Y})^2 \cong \bar{Y}^2\left[1 + t_1^2A + t_2^2R^2\lambda C_x^2 - 2t_1t_2RB - 2t_1C - t_2R\phi_i\lambda C_x^2\right] \quad (30)$$

where $R = \frac{\bar{X}}{\bar{Y}}$

To obtained minimum MSE of the proposed estimators "ξ", Eq. (30) is differentiated with respect to the unknown parameters t_1 and t_2 and setting $\frac{\partial MSE(\xi)}{\partial t_1} = 0$ and $\frac{\partial MSE(\xi)}{\partial t_2} = 0$, we obtained the optimal values of t_1 and t_2 as given below

$$t_{1(opt)} = \frac{\lambda C_x^2(2C + B\phi_i)}{2\left(A\lambda C_x^2 - B^2\right)}$$

and

$$t_{2(opt)} = \frac{2BC + A\phi_i\lambda C_x^2}{2R\left(A\lambda C_x^2 - B^2\right)}$$

Minimum MSE of ξ-family up to first degree of approximation is obtained by substituting the optimal values of $t_1$andt_2 in Eq. (30) and simplifying as

$$MSE_{min}(\xi) \cong \frac{\bar{Y}^2\lambda C_x^2}{4D^2}\left[\frac{4D^2}{\lambda C_x^2} + E^2\left(D + B^2\right) + F^2 - 2BEF - 4CED - 2\phi_iFD\right] \quad (31)$$

where

$$A = 1 + \lambda C_y^2 + 3\phi_i^2\lambda C_x^2 - 4\phi_i\lambda\rho_{yx}C_yC_x, \quad B = \lambda\rho_{yx}C_yC_x - \frac{3\phi_i\lambda C_x^2}{2}$$

$$C = 1 + \phi_i^2\lambda C_x^2 - \phi_i\lambda\rho_{yx}C_yC_x, \quad D = A\lambda C_x^2 - B^2$$

$$E = 2C + B\phi_i, \quad F = 2BC + A\phi_i\lambda C_x^2$$

Remark 3.1. We can also obtain many more exponential-type estimators by putting different choices of α and β in the estimator given in Eq. (26).

4. Empirical study

In this section, we demonstrate the performance of the proposed estimators over simple mean, ratio, regression, Bahl and Tuteja (1991), Singh et al. (2008, 2009), Upadhyaya et al. (2011), Yadav and Kadilar (2013) and Kadilar (2016) estimators in terms of MSE. Five natural populations are considered to access the performance of the proposed estimators. Parameters of all the populations are detailed below in this section.

Population 1 (*Source: Koyuncu and Kadilar, 2009*). Let y be the number of teachers and x be the number of students in both primary and secondary schools for 923 districts of Turkey in 2007. The summary statistics are shown as follows.

$$N = 923, n = 180, \bar{Y} = 436.4345, \bar{X} = 11440.4984, \rho_{yx} = 0.9543, S_y = 749.9395,$$

$$C_y = 1.7183, S_x = 21331.1315, C_x = 1.8645, \beta_{1(x)} = 3.9365, \beta_{2(x)} = 18.7208$$

Population 2 (*Source: Srisodaphol et al., 2015*). Let y be the entire height in feet and x be the diameter in centimeters of a breast height of conifer (Pinus palustric) trees. The statistic values are shown as follows.

$$N = 396, n = 30, \bar{Y} = 20.9629, \bar{X} = 52.6742, \rho_{yx} = 0.9073, S_y = 17.6164, C_y = 0.8404,$$

$$S_x = 57.1132, C_x = 1.0843, \beta_{1(x)} = 1.6157, \beta_{2(x)} = 1.7785$$

Population 3 (*Source: Cochran, 1977*). Let y be the population size in 1930 and x be the population size in 1920. The statistic values are shown as follows.

$$N = 49, n = 20, \bar{Y} = 127.7959, \bar{X} = 103.1429, \rho_{yx} = 0.9817, S_y = 123.1212, C_y = 0.9634,$$

$$S_x = 104.4051, C_x = 1.0122, \beta_{1(x)} = 2.2553, \beta_{2(x)} = 5.1412$$

Population 4 (*Source: Kadilar and Cingi, 2004*). Let y be the number of teachers and x be the number of students in both primary and secondary schools of Turkey in 2007. The summary statistics are shown as follows.

$$N = 106, n = 20, \bar{Y} = 2212.59, \bar{X} = 27421.70, \rho_{yx} = 0.86, S_y = 11549.72, C_y = 5.22,$$

$$S_x = 57585.57, C_x = 2.10, \beta_{1(x)} = 5.1237, \beta_{2(x)} = 34.5723$$

Population 5 (*Source: Kadilar and Cingi, 2003*). Let y be the apple production quantity and x be the count of apple trees in 854 villages in Turkey. The summary statistics are shown as follows.

$$N = 94, n = 20, \bar{Y} = 9384, \bar{X} = 72410, \rho_{yx} = 0.901, S_y = 29906.810, C_y = 3.187,$$

$$S_x = 160750.20, C_x = 2.22, \beta_{1(x)} = 4.611, \beta_{2(x)} = 26.136$$

We computed MSE of all the proposed and competing estimators including simple mean $\bar{y}$, ratio $\hat{\bar{Y}}_R$, Bahl and Tuteja (1991) $\hat{\bar{Y}}_{BT}$, regression $\hat{\bar{Y}}_{Reg}$, Singh et al. (2008), Singh et al. (2009) Upadhyaya et al. (2011) , Yadav and Kadilar (2013) $\hat{\bar{Y}}_{SI}, \hat{\bar{Y}}_{UP}$, and Kadilar (2016) $\hat{\bar{Y}}_K$. Findings are presented in Tables 4–9. It is clearly observed from Tables 4–9 that

Table 4
Var/MSE of the estimators. $\bar{y}, \hat{\bar{Y}}_R, \hat{\bar{Y}}_{BT}, \hat{\bar{Y}}_{Reg}, \hat{\bar{Y}}_{SI}, \hat{\bar{Y}}_{UP}$ and $\hat{\bar{Y}}_K$

Population	$Var(\bar{y})$	$MSE(\hat{\bar{Y}}_R)$	$MSE(\hat{\bar{Y}}_{BT})$	$MSE(\hat{\bar{Y}}_{Reg}) = MSE_{min}(\hat{\bar{Y}}_i)$, $i = SI, UP, K$
1	2515.074	267.644	651.042	224.625
2	9.562	3.093	2.348	1.691
3	448.563	18.362	109.665	16.230
4	5,411,348	2,542,740	3,758,095	1,409,115
5	35,205,782	8,096,858	17,380,652	6,625,693

Table 5
MSE of existing and proposed estimators for Population 1.

η- family	MSE	t- family	MSE	ξ- family	MSE
η_0	2515.0740	t_0	651.0423	ξ_1	223.7713
η_1	651.0423	t_1	649.9361	ξ_2	223.7714
η_2	652.8800	t_2	651.7722	ξ_3	223.7713
η_3	651.2254	t_3	650.1190	ξ_4	223.7726
η_4	651.1360	t_4	650.0297	ξ_5	223.7737
η_5	651.0520	t_5	650.0342	ξ_6	223.7833
η_6	652.0282	t_6	650.9211	ξ_7	223.7767
η_7	651.0925	t_7	649.9863	ξ_8	223.7713
η_8	651.2341	t_8	650.1278	ξ_9	223.7716
η_9	651.0473	t_9	649.9411	ξ_{10}	223.7714
η_{10}	652.9680	t_{10}	651.8601	ξ_{11}	223.7719

Table 6
MSE of existing and proposed estimators for Population 2.

η- family	MSE	t- family	MSE	ξ- family	MSE
η_0	9.5618	t_0	2.3479	ξ_1	1.6534
η_1	2.3479	t_1	2.3312	ξ_2	1.6565
η_2	2.4578	t_2	2.4422	ξ_3	1.6535
η_3	2.4148	t_3	2.3987	ξ_4	1.6583
η_4	2.4038	t_4	2.3877	ξ_5	1.6586
η_5	2.3854	t_5	2.3935	ξ_6	1.6597
η_6	2.4492	t_6	2.4335	ξ_7	1.6574
η_7	2.3995	t_7	2.3833	ξ_8	1.6535
η_8	2.4216	t_8	2.4057	ξ_9	1.6562
η_9	2.3793	t_9	2.3629	ξ_{10}	1.6551
η_{10}	2.4691	t_{10}	2.4535	ξ_{11}	1.6565

Table 7
MSE of existing and proposed estimators for Population 3.

η- family	MSE	t- family	MSE	ξ- family	MSE
η_0	448.5631	t_0	109.6657	ξ_1	16.1653
η_1	109.6657	t_1	109.4151	ξ_2	16.1696
η_2	120.1576	t_2	119.8725	ξ_3	16.1658
η_3	111.7679	t_3	111.5113	ξ_4	16.1853
η_4	111.7048	t_4	111.4484	ξ_5	16.1855
η_5	110.0760	t_5	111.4860	ξ_6	16.2048
η_6	120.0335	t_6	119.7489	ξ_7	16.1859
η_7	111.6803	t_7	111.4239	ξ_8	16.1655
η_8	111.8068	t_8	111.5501	ξ_9	16.1746
η_9	110.0636	t_9	109.8119	ξ_{10}	16.1672
η_{10}	120.3483	t_{10}	120.0624	ξ_{11}	16.1747

Table 8
MSE of existing and proposed estimators for Population 4.

η- family	MSE	t- family	MSE	ξ- family	MSE
η_0	5,411,348	t_0	3,758,095	ξ_1	1,246,075
η_1	3,758,095	t_1	2,423,766	ξ_2	1,246,075
η_2	3,759,902	t_2	2,424,194	ξ_3	1,246,075
η_3	3,758,205	t_3	2,423,792	ξ_4	1,246,075
η_4	3,758,140	t_4	2,423,777	ξ_5	1,246,075
η_5	3,758,098	t_5	2,423,779	ξ_6	1,246,070
η_6	3,758,956	t_6	2,423,970	ξ_7	1,246,073
η_7	3,758,117	t_7	2,423,771	ξ_8	1,246,075
η_8	3,758,223	t_8	2,423,797	ξ_9	1,246,075
η_9	3,758,097	t_9	2,423,767	ξ_{10}	1,246,075
η_{10}	3,760,195	t_{10}	2,424,264	ξ_{11}	1,246,075

Table 9
MSE of existing and proposed estimators for Population 5.

η- family	MSE	t- family	MSE	ξ- family	MSE
η_0	35,205,782	t_0	17,380,652	ξ_1	6,427,017
η_1	17,380,652	t_1	15,693,026	ξ_2	6,427,018
η_2	17,385,543	t_2	15,696,853	ξ_3	6,427,017
η_3	17,381,067	t_3	15,693,351	ξ_4	6,427,033
η_4	17,380,820	t_4	15,693,158	ξ_5	6,427,053
η_5	17,380,668	t_5	15,693,173	ξ_6	6,427,246
η_6	17,382,855	t_6	15,694,750	ξ_7	6,427,116
η_7	17,380,728	t_7	15,693,085	ξ_8	6,427,017
η_8	17,381,113	t_8	15,693,387	ξ_9	6,427,020
η_9	17,380,658	t_9	15,693,031	ξ_{10}	6,427,017
η_{10}	17,386,080	t_{10}	15,697,273	ξ_{11}	6,427,024

- our proposed estimators have minimum MSEs as compared to all other estimators for all data sets.
- in each data set, MSEs of all proposed estimators are almost equal which indicates that all proposed estimators are equally efficient. So, researcher may be used any one of them depending upon the availability of the values of α and β.

5. Concluding remarks

In this article, we proposed a general family of exponential-type estimators for finite population mean under simple random sampling. Known information of auxiliary variable including coefficient of skewness, coefficient of kurtosis, standard deviation and coefficient of variation etc. is utilized to enhance the estimation. We derived the expressions for bias, mean squared error (MSE) and minimum MSE of the proposed estimators up to first degree of approximation. A numerical study is conducted through five real data sets to highlight the applicability of proposed estimators. This study concludes that all the proposed estimators are i) more efficient over the competing estimators and ii) are equally efficient, so anyone of them can be used depending upon the availability of the auxiliary information (α and β). Therefore, the practitioners are suggested to use the proposed estimators to get more efficient results in future applications.

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