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ORIGINAL ARTICLE

Ion-acoustic waves in an inhomogeneous plasma with negative ions

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Abstract The propagation of the ion-acoustic waves (IAWs) in a collisionless, unmagnetized inhomogeneous plasma composed of warm positive and negative ions with different masses and isothermal electrons is investigated. The reductive-perturbation method is employed to reduce the basic set of fluid equations to the variable coefficients Korteweg–de Varies (KdV) equation. It is found that both compressive and rarefactive solitons can propagate in the system. Below (above) certain values of positive ions densities the system supports rarefactive (compressive) solitons. The dependence of solitons amplitude and width on the system parameters is investigated.

1. Introduction

Negative ion plasma can be defined as plasma that contains both negative and positive ion species, as well as electrons. Negative ion plasma can appear as a result of elementary processes, such as dissociative or non-dissociative electron attachment to neutrals when an electronegative gas, such as halogens and hexafluorides, is introduced into an electrical gas discharge or injected from the external sources (Vladimirov et al., 2003; Mamun and Shukla, 2003; Djebli, 2003). Furthermore, negative ion plasma can be created in the plasma processing reactors (Gottscho and Gaebe, 1986), low-temperature laboratory experiments (Jacquinot et al., 1977; Nakamura et al., 1997; Weingarten et al., 2001; Ichiki et al., 2002), and neutral beam sources (Bacal and Hamilton, 1979). Also, negative ion plasma can be found in many space observations such as the *D*-region of the ionosphere (Massey, 1976; Portnyagin et al., 1991), cometary comae (Chaizy et al., 1991), etc.

The presence of the negative ion in a plasma modifies the charge neutrality condition i.e., $n_e = n_p - n_n$, where n_e, n_p, and n_n are the electron, positive ion, and negative ion number densities, respectively. It is clear that as the number density of the negative ions increases, the electrons' number density decreases. The result is a decrease of the shielding effect produced by the electrons, which is one of the main effects governing the behavior of plasmas. From this perspective, it seems to follow

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that the negative ions only influence plasmas secondarily. Although this is a fact, most of the phenomena are actually affected by the negative ions themselves, as well as by the lack of electrons (Ichiki et al., 2002).

In the absence of dissipation or if the dissipation is weak, the balance between the nonlinear and the dispersion effects can result in the formation of symmetrical solitary waves – soliton. The IAWs and their attributes have been the subject of many researches in plasma physics and complex (dusty) plasma as well. There has been considerable interest (see, for example, Das and Tagare, 1975; Nakamura, 1982; Watanabe, 1984; Tajiri and Tuda, 1985; Ludwig et al., 1984; Moslem, 1999; El-Labany et al., 2000, 2011a,b) in the investigation of nonlinear IAWs in plasmas with positive–negative ions and Boltzmann distributed electrons. Extensive work has been devoted to the study of IAWs in plasmas with positive–negative ions. For example, The propagation of IAWs in a multi-species plasma consisting of positive ions, electrons and negative ions have been investigated theoretically by Das and Tagare (1975), Watanabe (1984), Tagare (1986), and experimentally by Cooney et al. (1991), etc. and their studies showed the rarefactive solitary waves owing to the presence of negative ions. Chakraborty et al. (1992), Chakraborty et al. (1993), Chakraborty et al. (1994), Chattopadhyay et al. (2002), considered the effect of negative ions on the formation of ion acoustic solitons in relativistic plasmas. They have numerically estimated the width, amplitude and phase velocity of IAWs for a model plasma having the negative ions which have short lifetimes (or small probability of existence). Chattopadhyay et al. (2002), found that drift motion of ions has a significant contribution on the excitation of IAWs in the presence of negative ions in plasma. Chattopadhyay et al. (2009), studied warm positive and negative ions with two-temperature isothermal electrons using the pseudopotential method. They found that the concentration of negative ions, drift velocities, mass ratios, equal temperatures of ions (particular case) and presence of two groups of electrons and their ratios modify the profiles of the Sagdeev pseudopotential curves of the solitary waves in the plasma. Das and Nag (2010), used reductive-perturbation technique to study the propagation of IAWs in the vicinity of KdV equation in magnetized plasma with negative ions. They found that the presence of negative ions changes the nonlinearity as a result of which the formation of soliton exhibits different nature on soliton propagation. Furthermore, several authors (Das and Singh, 1992; Malik and Dahiya, 1994; Das and Devi, 2006; Malik and Stroth, 2008; Singh and Malik, 2008) have focused their attention on the investigation of IAWs in inhomogeneous plasmas with positive–negative ions. Since the propagation characteristics of IAWs may be modified by the effects of the presence of negative ion, the main concern of this paper is to study the IAWs in multicomponent inhomogeneous plasma consisting of warm positive- and negative ion species along with isothermal electrons. The salient feature is to demonstrate the existence of compressive and rarefactive IAWs in inhomogeneous plasma with two-ion species having different masses, concentrations, and temperatures. The two types of ions are assumed to have the same charge number = 1. Using the reductive-perturbation method, we have derived a KdV equation that admits a soliton solution. We have investigated the effect of mass, concentration, and temperature of different ion species on the characteristics of IAWs in detail. Furthermore, for numerical illustrations, we take a realistic example of plasma containing the ions (Ar^+, SF_6^-). The (Ar^+, SF_6^-) plasma composition has been used in the experimental investigation of strong double layers by Merlino and Loomis (1990).

The present paper is organized as follows. In Section 2, the basic set of fluid equations for the IAWs in positive–negative ion plasma with isothermal electrons is presented. The well known KdV equation is derived. In Section 3, the KdV equation is solved and analyzed to study the profiles of the nonlinear IAW. Section 4 is devoted to a concluding discussion.

2. Basic equations and derivation of the KdV equation

Let us consider a medium of an unmagnetized, collisionless, warm, adiabatic inhomogeneous plasma containing mixture of positive and negative ions and isothermal electrons. The fluid equations of motion governing the system can be expressed as follows (Kalita et al., 1996; Moslem et al. 2009):

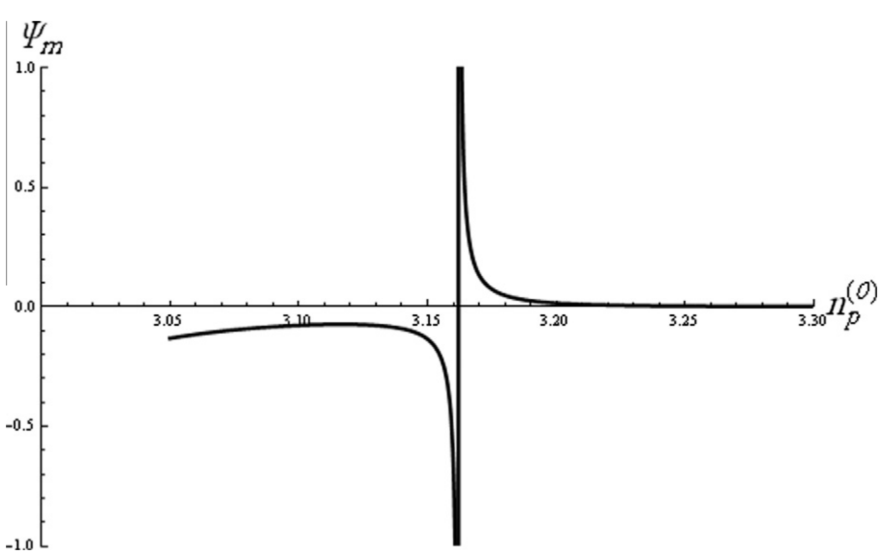

Figure 1 The variation of solitons amplitude (Ψ_m) against ion density ($n_p^{(0)}$) for $n_e^{(0)} = 1.8, q = 1.7, \sigma_p = 0.01$, and $\sigma_n = 0.001$.

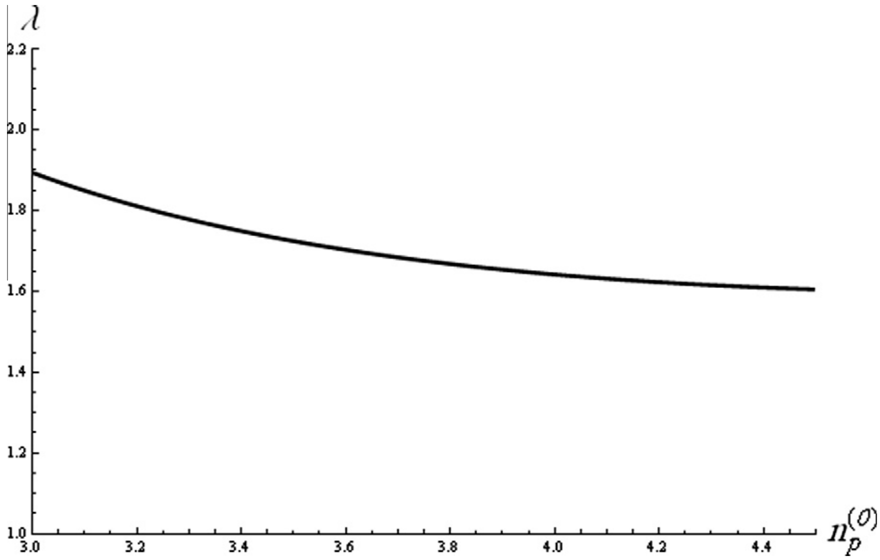

Figure 2 Graph of soliton velocity (λ) soliton against $n_p^{(0)}$ for $n_e^{(0)} = 1.8$ $q = 1.7$, $\sigma_p = 0.01$ and $\sigma_n = 0.001$.

$$\frac{\partial}{\partial t}n_p + \frac{\partial}{\partial x}(n_p u_p) = 0, \tag{1}$$

$$\left(\frac{\partial}{\partial t} + u_p\frac{\partial}{\partial x}\right)u_p + 3\sigma_p n_p\frac{\partial}{\partial x}n_p + \frac{\partial}{\partial x}\phi = 0, \tag{2}$$

$$\frac{\partial}{\partial t}n_n + \frac{\partial}{\partial x}(n_n u_n) = 0, \tag{3}$$

$$\left(\frac{\partial}{\partial t} + u_n\frac{\partial}{\partial x}\right)u_n + \frac{3\sigma_n}{q}n_n\frac{\partial}{\partial x}n_n - \frac{1}{q}\frac{\partial}{\partial x}\phi = 0, \tag{4}$$

$$n_e = e^{\phi}, \tag{5}$$

$$\frac{\partial^2}{\partial x^2}\phi = -n_p + n_n + n_e. \tag{6}$$

Here, u_p and u_n are the velocities of the positive and negative ions, respectively, ϕ the electrostatic potential, x the space coordinate, and t the time variable. Furthermore, $\sigma_P = T_P/T_e$ and $\sigma_n = T_n/T_e$ are the ratios of the temperatures of positive ions (T_p) and negative ions (T_n) to the electron temperature (T_e), and $q = m_n/m_p$ is the ratio of the negative ion mass (m_n) to the positive ion mass (m_p).

We normalized all the physical quantities as follows. The background electron density $n_e^{(0)}$ normalizes the densities, $u_{p,n}$ by the ion sound speed $(KT_e/m_p)^{1/2}$, ϕ by KT_e/e, t by the inverse of the

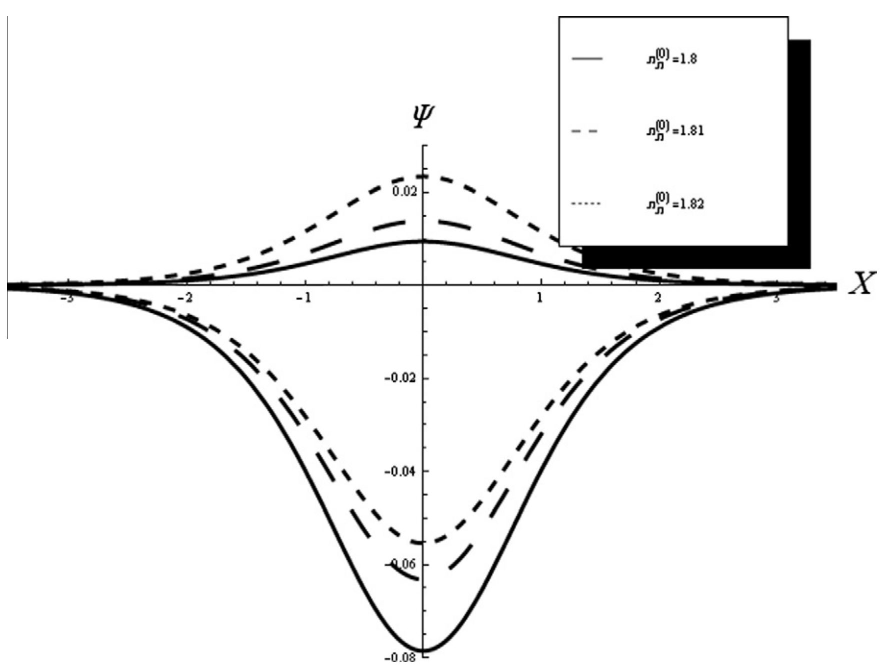

Figure 3 Graph of compressive (rarefactive) soliton at $n_p^{(0)} = 3.21(3.13)$ against x for $q = 1.7$, $\sigma_p = 0.01$ and $\sigma_n = 0.001$.

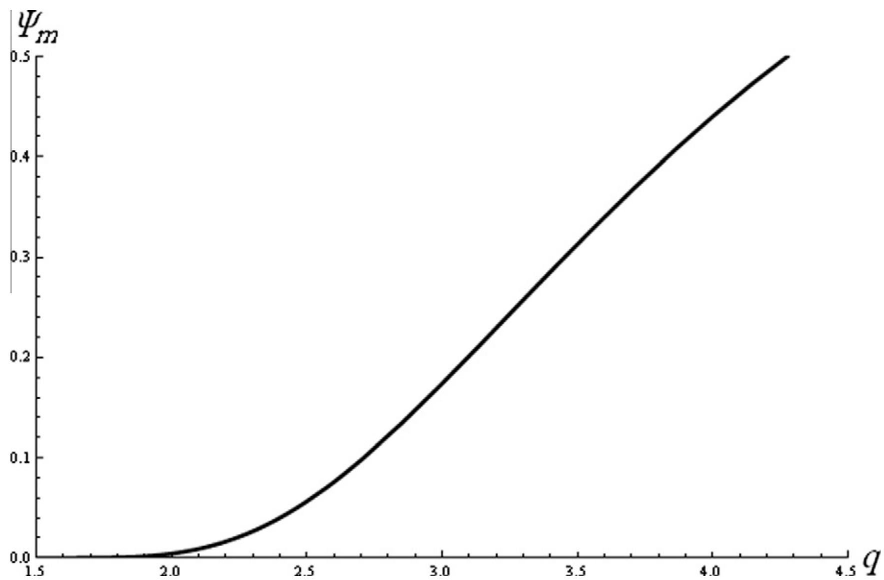

Figure 4 The compressive amplitude (Ψ_m) is plotted against q for ($n_p^{(0)}$) for $n_e^{(0)} = 3.35, n_n^{(0)} = 1.8, \sigma_p = 0.01$ and $\sigma_n = 0.001$.

plasma frequency $\omega_{pi}^{-1} = (m_p/4\pi e^2 n_e)^{1/2}$, x by the electron Debye length $\lambda_D = KT_e/4\pi e^2 n_e)^{1/2}$, where K is Boltzmann's constant. The independent variables can be stretched as (Asano, 1974)

$$\xi = \varepsilon^{1/2} - \left(\int \frac{dx}{\lambda(x)} - t\right), \quad s = \varepsilon^{3/2} x, \tag{7}$$

where ε is a small parameter less than one and λ is the wave propagation speed to be determined later. The dependent variables are expanded as

$$\begin{pmatrix} n_{n,p} \\ u_{n,p} \\ \phi \end{pmatrix} = \begin{pmatrix} n_{n,p}^{(0)} \\ u_{n,p}^{(0)} \\ \phi^{(0)} \end{pmatrix} + \begin{pmatrix} \varepsilon n_{n,p}^{(1)} + \varepsilon^2 n_{n,p}^{(2)} + \ldots \\ \varepsilon u_{n,p}^{(1)} + \varepsilon^2 u_{n,p}^{(2)} + \ldots \\ \varepsilon \phi^{(1)} + \varepsilon^2 \phi^{(2)} + \ldots \end{pmatrix}, \tag{8}$$

where $n_{n,p}^{(0)}$ are the unperturbed densities, which satisfy the neutrality condition

$$n_p^{(0)} = n_n^{(0)} + 1 + \phi^{(0)}, \tag{9}$$

for simplicity we shall consider the ionic charge number of positive and negative ions equal unity.

Using the stretching (7) and the expansion (8) into Eqs. (1)–(6), and assuming the gradients is in the spatial only, it follows that

$$\frac{\partial}{\partial \xi} n_{n,p}^{(0)} = \frac{\partial}{\partial \xi} u_{n,p}^{(0)} = \frac{\partial}{\partial \xi} \lambda. \tag{10}$$

In equilibrium, Eqs. (1)–(6) give the following relations:

$$\frac{\partial}{\partial s}(u_p^{(0)} u_p^{(0)}) = 0, \tag{11}$$

$$3\sigma_p n_p^{(0)} \frac{\partial}{\partial s} n_p^{(0)} + u_p^{(0)} \frac{\partial}{\partial s} u_p^{(0)} + \frac{\partial}{\partial s} \phi^{(0)} = 0, \tag{12}$$

$$\frac{\partial}{\partial s}(u_n^{(0)} u_n^{(0)}) = 0, \tag{13}$$

$$3\sigma_p n_p^{(0)} \frac{\partial}{\partial s} n_p^{(0)} + u_p^{(0)} \frac{\partial}{\partial s} u_p^{(0)} + \frac{\partial}{\partial s} \phi^{(0)} = 0, \tag{14}$$

$$\frac{3\sigma_n n_n^{(0)}}{q} \frac{\partial}{\partial s} n_n^{(0)} + u_n^{(0)} \frac{\partial}{\partial s} u_n^{(0)} - \frac{1}{q} \frac{\partial}{\partial s} \phi^{(0)} = 0. \tag{15}$$

The zero order of ε gives

$$n_p^{(0)} - n_n^{(0)} - 1 - \phi^{(0)} - \frac{\phi^{(0)^2}}{2} = 0, \tag{16}$$

The first order in ε yields the following relations:

$$n_p^{(1)} = -\frac{n_p^{(0)} \phi^{(1)}}{C_p}, \tag{17}$$

$$u_p^{(1)} = -\frac{\lambda \phi^{(1)}}{C_p}, \tag{18}$$

$$n_n^{(1)} = \frac{n_n^{(0)} \phi^{(1)}}{C_n}, \tag{19}$$

$$u_n^{(1)} = \frac{\lambda \phi^{(1)}}{C_n}, \tag{20}$$

$$1 + \frac{n_p^{(0)}}{C_p} + \frac{n_n^{(0)}}{C_n} + \phi^{(0)} = 0, \tag{21}$$

In deriving Eqs. (17)–(21), it has been assumed that the equilibrium flow velocity of positive and negative ions are neglected by assuming them to be much smaller than DIAW phase speed ($\lambda \gg u_{p,n}^{(0)}$). From Eq. (21), we obtain the linear phase velocity, which is equivalent to the linear wave dispersion relation

$$\lambda = \left(\frac{R_1 + \sqrt{(-R_1)^2 - 4(q + \phi^{(0)} q)(R_2)}}{2(q + \phi^{(0)} q)} \right)^{1/2}, \tag{22}$$

where

$$R_1 = n_n^{(0)} + n_p^{(0)} q + (3 n_n^{(0)^2} \sigma_n + 3 n_p^{(0)^2} q \sigma_p)(1 + \phi^{(0)}),$$

$$R_2 = 3 n_n^{(0)} n_p^{(0)} (n_n^{(0)} \sigma_n + n_p^{(0)} \sigma_p + 3 n_n^{(0)} n_p^{(0)} \sigma_n \sigma_p (1 + \phi^{(0)})),$$

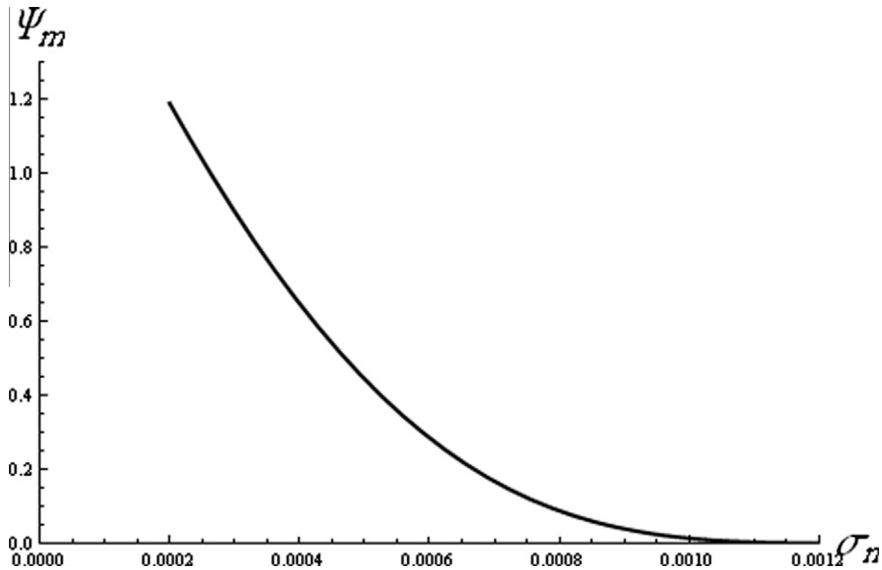

Figure 5 The variation of compressive amplitude (Ψ_m) against the ratio of the negative ions temperature to the free electron temperature (σ_n) for $n_p^{(0)} = 3.2, n_n^{(0)} = 1.8, p = 1.7$ and $\sigma_{p^-} = 0.01$.

$$C_p = 3\sigma_p n_p^{(0)^2} - \lambda^2, \quad C_n = 3\sigma_n n_n^{(0)^2} - q\lambda^2.$$

The second-order in ε yields

$$u_p^{(1)} \frac{\partial}{\partial s} n_p^{(0)} + n_p^{(1)} \frac{\partial}{\partial s} u_p^{(0)} + \frac{u_p^{(1)}}{\lambda} \frac{\partial}{\partial \xi} n_p^{(1)} - \frac{\partial}{\partial \xi} n_p^{(2)} + \frac{u_p^{(0)}}{\lambda} \frac{\partial}{\partial \xi} n_p^{(2)} + \frac{n_p^{(1)}}{\lambda} \frac{\partial}{\partial \xi} u_p^{(1)} + \frac{n_p^{(0)}}{\lambda} \frac{\partial}{\partial \xi} u_p^{(2)} + u_p^{(0)} \frac{\partial}{\partial s} n_p^{(1)} + n_p^{(0)} \frac{\partial}{\partial s} u_p^{(1)} = 0, \quad (23)$$

$$\sigma_p n_p^{(1)} \frac{\partial}{\partial s} n_p^{(0)} + u_p^{(1)} \frac{\partial}{\partial s} u_p^{(0)} + \frac{3\sigma_p n_p^{(1)}}{\lambda} \frac{\partial}{\partial \xi} n_p^{(1)} + \frac{3\sigma_p n_p^{(0)}}{\lambda} \frac{\partial}{\partial \xi} n_p^{(2)} + \frac{u_p^{(1)}}{\lambda} \frac{\partial}{\partial \xi} u_p^{(1)} - \frac{\partial}{\partial \xi} u_p^{(2)} + \frac{u_p^{(0)}}{\lambda} \frac{\partial}{\partial \xi} u_p^{(2)} + \frac{1}{\lambda} \frac{\partial}{\partial \xi} \phi^{(2)} + 3\sigma_p n_p^{(0)} \frac{\partial}{\partial s} n_p^{(1)} + u_p^{(0)} \frac{\partial}{\partial s} u_p^{(1)} + \frac{\partial}{\partial s} \phi^{(1)} = 0, \quad (24)$$

$$u_n^{(1)} \frac{\partial}{\partial s} n_n^{(0)} + n_n^{(1)} \frac{\partial}{\partial s} u_n^{(0)} + \frac{u_n^{(1)}}{\lambda} \frac{\partial}{\partial \xi} n_n^{(1)} - \frac{\partial}{\partial \xi} n_n^{(2)} + \frac{u_n^{(0)}}{\lambda} \frac{\partial}{\partial \xi} n_n^{(2)} + \frac{n_n^{(1)}}{\lambda} \frac{\partial}{\partial \xi} u_n^{(1)} + \frac{n_n^{(0)}}{\lambda} \frac{\partial}{\partial \xi} u_n^{(2)} + u_n^{(0)} \frac{\partial}{\partial s} n_n^{(1)} + n_n^{(0)} \frac{\partial}{\partial s} u_n^{(1)} = 0, \quad (25)$$

$$\frac{3\sigma_n n_n^{(1)}}{q} \frac{\partial}{\partial s} n_n^{(0)} + u_n^{(1)} \frac{\partial}{\partial s} u_n^{(0)} + \frac{3\sigma_n n_n^{(1)}}{q\lambda} \frac{\partial}{\partial \xi} n_n^{(1)} + \frac{3\sigma_n n_n^{(0)}}{q\lambda} \frac{\partial}{\partial \xi} n_n^{(2)} + \frac{u_n^{(1)}}{\lambda} \frac{\partial}{\partial \xi} u_n^{(1)} - \frac{\partial}{\partial \xi} u_n^{(2)} + \frac{u_n^{(0)}}{\lambda} \frac{\partial}{\partial \xi} u_n^{(2)} - \frac{1}{q\lambda} \frac{\partial}{\partial \xi} \phi^{(2)} + \frac{3\sigma_n n_n^{(0)}}{q} \frac{\partial}{\partial s} n_n^{(1)} + u_n^{(0)} \frac{\partial}{\partial s} u_n^{(1)} - \frac{1}{q} \frac{\partial}{\partial s} \phi^{(1)} = 0, \quad (26)$$

$$\frac{1}{\lambda^2} \frac{\partial^2}{\partial \xi^2} \phi^{(1)} + n_p^{(2)} - n_n^{(2)} - (1 + \phi^{(0)})\phi^{(2)} - \frac{1}{2} \phi^{(1)^2} = 0. \quad (27)$$

Eliminating the second-order perturbed quantities and making use of the first-order results, we finally get the following nonlinear partial differential equation with variable coefficients:

$$\frac{\partial}{\partial s} \phi^{(1)} + \frac{a_3}{a_2} \phi^{(1)} \frac{\partial}{\partial s} \phi^{(0)} + \frac{a_1}{a_2} \phi^{(1)} \frac{\partial}{\partial \xi} \phi^{(1)} + \frac{1}{a_2} \frac{\partial^3}{\partial \xi^3} \phi^{(1)} = 0, \quad (28)$$

where

$$a_1 = -1 + \frac{3n_n^{(0)} q\lambda^2}{C_n^3} - \frac{3n_p^{(0)} \lambda^2}{C_p^3} + \frac{3n_n^{(0)^3} \sigma_n}{C_n^3} - \frac{3n_p^{(0)^3} \sigma_p}{C_p^3},$$

$$a_2 = \frac{n_n^{(0)} u_n^{(0)} q\lambda^2}{C_n^2} + \frac{n_n^{(0)} q\lambda^3}{C_n^2} - \frac{n_n^{(0)} \lambda}{C_n} - \frac{n_p^{(0)} \lambda}{C_p} + \frac{3n_n^{(0)^3} \lambda\sigma_n}{C_n^2},$$

$$\begin{aligned} a_3 = {} & \frac{2n_n^{(0)} u_n^{(0)} q^2 \lambda^3}{C_1 C_n^3} + \frac{2n_n^{(0)} q^2 \lambda^4}{C_1 C_n^3} + \frac{n_n^{(0)} q\lambda^2}{C_1 C_n^2} - \frac{2n_n^{(0)^2} u_n^{(0)} q^3 \lambda^5}{C_1 C_n^5 v_n} \\ & - \frac{2n_n^{(0)^2} q^3 \lambda^6}{C_1 C_n^5 v_n} - \frac{n_n^{(0)^2} q^2 \lambda^4}{C_1 C_n^4 v_n} - \frac{n_n^{(0)} u_n^{(0)} q\lambda^2}{C_n^2 v_n} + \frac{n_n^{(0)} q\lambda^3}{C_n^2 v_n} \\ & + \frac{2n_p^{(0)} u_p^{(0)} \lambda^2}{C_p^2 v_p} - \frac{n_p^{(0)} \lambda^3}{C_p^2 v_p} + \frac{2n_n^{(0)} n_p^{(0)} u_n^{(0)} q^2 \lambda^5}{C_1 C_n^3 C_p^2 v_p} + \frac{2n_n^{(0)} n_p^{(0)} q^2 \lambda^6}{C_1 C_n^3 C_p^2 v_p} \\ & + \frac{n_n^{(0)} n_p^{(0)} q\lambda^4}{C_1 C_n^2 C_p^2 v_p} + \frac{6n_n^{(0)^3} q\lambda^2 \sigma_n}{C_1 C_n^3} - \frac{6n_n^{(0)^4} u_n^{(0)} q^2 \lambda^3 \sigma_n}{C_1 C_n^5 v_n} \\ & - \frac{12n_n^{(0)^4} q^2 \lambda^4 \sigma_n}{C_1 C_n^5 v_n} - \frac{3n_n^{(0)^4} q\lambda^2 \sigma_n}{C_1 C_n^4 v_n} - \frac{6n_n^{(0)^3} u_n^{(0)} q\lambda^2 \sigma_n}{C_n^3 v_n} \\ & - \frac{6n_n^{(0)^3} q\lambda^3 \sigma_n}{C_n^3 v_n} + \frac{6n_n^{(0)^3} \lambda\sigma_n}{C_n^2 v_n} + \frac{6n_n^{(0)^3} n_p^{(0)} q\lambda^4 \sigma_n}{C_1 C_n^3 C_p^2 v_p} \\ & - \frac{18n_n^{(0)^6} q\lambda^2 \sigma_n^2}{C_1 C_n^5 v_n} - \frac{18n_n^{(0)^5} \lambda\sigma_n^2}{C_n^3 v_n} - \frac{3n_p^{(0)^3} \lambda\sigma_p}{C_p^2 v_p} \\ & + \frac{6n_n^{(0)} n_p^{(0)^3} u_n^{(0)} q^2 \lambda^3 \sigma_p}{C_1 C_n^3 C_p^2 v_p} + \frac{6n_n^{(0)} n_p^{(0)^3} q^2 \lambda^4 \sigma_p}{C_1 C_n^3 C_p^2 v_p} \\ & + \frac{3n_n^{(0)} n_p^{(0)^3} q\lambda^2 \sigma_p}{C_1 C_n^2 C_p^2 v_p} + \frac{18n_n^{(0)^3} n_p^{(0)^3} q\lambda^2 \sigma_n \sigma_p}{C_1 C_n^3 C_p^2 v_p}, \end{aligned}$$

$$C_1 = -\frac{2qn_n^{(0)} \lambda}{C_n^2} - \frac{2n_p^{(0)} \lambda}{C_p^2}, \quad v_n = 3\sigma_n n_n^{(0)^2} - qu_n^{(0)^2},$$

$$v_p = 3\sigma_p n_p^{(0)^2} - u_p^{(0)^2}$$

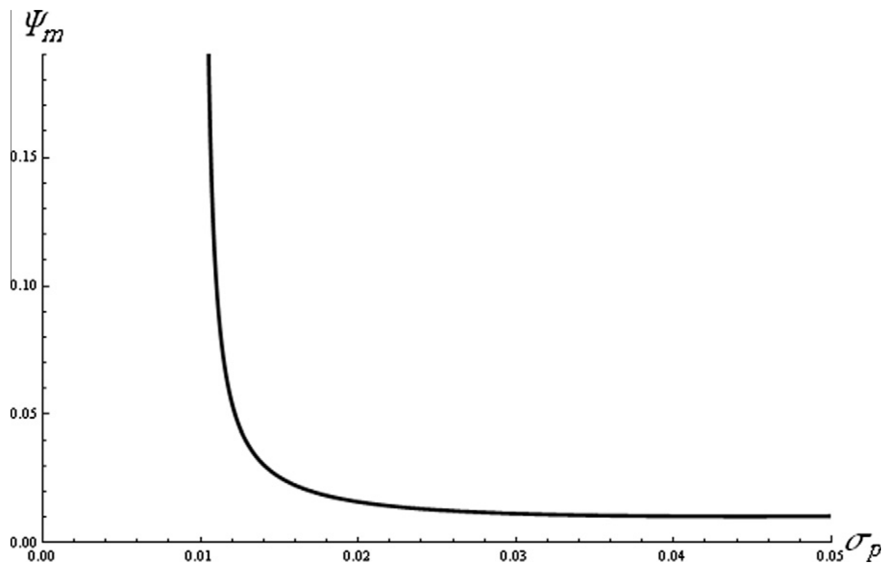

Figure 6 The compressive soliton amplitude (Ψ_m) is depicted against σ_p for $n_p^{(0)} = 1.7, n_n^{(0)} = 1.8, p = 1.7$ and $\sigma_n = 0.001$.

3. The stationary solutions

In order to obtain the solitary wave solutions, one can use the following transformation:

$$\phi^{(1)} = \Psi \exp\left(-\frac{a_3}{a_2}\phi^{(0)}\right). \quad (29)$$

In order to recast Eq. (28) to the standard form of Korteweg–de Vries (KdV) equation with variable coefficients,

$$\frac{\partial}{\partial s}\Psi + B\Psi\frac{\partial}{\partial \xi}\Psi + A\frac{\partial^3}{\partial \xi^3}\Psi = 0, \quad (30)$$

where

$$A = \frac{1}{a_2}, \quad B = \frac{a_1}{a_2}\exp\left(-\frac{a_3}{a_2}\phi^{(0)}\right).$$

However, the nonlinear coefficients functionally depend on the space of the plasma, for the sake of simplicity for mathematical development, the variations are assumed negligibly as compared to the scale length or it is assumed that all parameters are locally constant. This equation, as is well known, has the following solution given by Kodama and Taniuti (1978).

$$\Psi = \Psi_m \,\mathrm{Sech}^2\left(\frac{X}{W}\right), \quad (31)$$

where the amplitude $\Psi_m = 3U/B$, the width $W = (4A/U)^{1/2}$, and X is the transformed coordinate with respect to a frame moving with velocity U (i.e. $X = \xi - Us$).

4. Results and discussion

We have presented a study of the weakly nonlinear IAWs in plasma with two distinct ion species (i.e., positive and negative ions) and isothermal electrons. By employing the two-fluid equations for the ions and isothermal electron distribution, we have derived the KdV equation. The latter is used to examine the width and the amplitude of IAWs.

The dependence of the soliton amplitude Ψ_m on the ion density $n_p^{(0)}$ is depicted in Fig. 1. It is found that the soliton amplitude Ψ_m can be positive ($a_1/a_2 > 0$) or negative ($a_1/a_2 < 0$), indicating thereby that both rarefactive and compressive solitons can exist in the present system. It is seen that, the rarefactive amplitude increases as $n_p^{(0)}$ increases for its lower range (i.e. $n_p^{(0)} < 3.16$), on the other hand the compressive amplitude decreases with $n_p^{(0)}$ (i.e. $n_p^{(0)} > 3.16$). From Fig. 2, it is obvious that the soliton phase velocity gradually decreases with the increase of $n_p^{(0)}$. Fig. 3 shows that both the amplitude and width of the compressive (rarefactive) solitons increase (decrease) with the negative ion density$n_n^{(0)}$. The effect of the ratio of the negative ion mass to the positive ion mass (q) on the compressive soliton amplitude is depicted in Fig. 4. It is clear that the amplitude of the compressive soliton increases rapidly with q. Fig. 5 reveals how the compressive soliton amplitude decreases with the increase of the ratio of the negative ions temperature to the electron temperature σ_n. It is obvious from Fig. 6 that the amplitude of compressive soliton decreases sharply by increasing σ_p for $\sigma_p < 0.02$, but for $\sigma_p > 0.02$ the amplitude decreases gradually.

The present study is applied to examine the nonlinear IAWs excitations for the (Ar^+, SF_6^-) plasma, which has been used in the experimental investigation of strong double layers by Merlino and Loomis (1990). Finally, this investigation should be helpful in understanding the salient features of the nonlinear IAWs both in space and in laboratory experiments where two distinct groups of ions and isothermal electrons are present.

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