



REVIEW ARTICLE

# Analytical study of the non orthogonal stagnation point flow of a micro polar fluid



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Received 24 March 2015; accepted 15 May 2015

Available online 22 May 2015

**KEYWORDS**

Micro polar fluid;  
 Hiemenz flow;  
 Homotopy perturbation  
 method (HPM)

**Abstract** In this paper we consider the steady two dimensional flow of micro polar fluids on a flat plate. The flow under discussion is the modified Hiemenz flow for a micro polar fluid which occurs in the  $hjkns + skms$  boundary layer near an orthogonal stagnation point. The full governing equation reduced to a modified Hiemenz flow. The solution to the boundary value problem is governed by two non dimensional parameters, the material parameter  $K$  and the ratio of the micro rotation to skin friction parameter  $n$ . The obtained nonlinear coupled ordinary differential equations are solved by using the Homotopy perturbation method. Comparison between numerical and analytical solutions of the problem is shown in tables form for different values of the governing parameters  $K$  and  $n$ . Effects of the material parameter  $K$  on the velocity profile and microrotation profiles for different cases of  $n$  are discussed graphically as well as numerically. Velocity profile decreases as the material parameter  $K$  increases and the microrotation profile increases as the material parameter  $K$  increases for different cases of  $n$ .

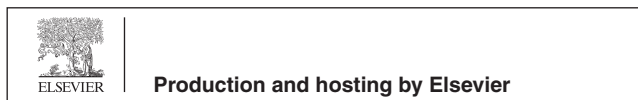
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## 1. Introduction

The behavior of the flow near a stagnation-point is a fundamental topic in fluid dynamics, and it has attracted the attention of many researchers during the past several decades because of its wide industrial and technical applications, such as heat exchangers placed in a low-velocity environment, cooling of nuclear reactors during emergency shutdown, solar central receivers exposed to wind currents, cooling of electronic devices by fans, and many hydrodynamic processes. [Hiemenz \(1911\)](#) and [Homann \(1936\)](#) initiated the study of two dimensional and axisymmetric three dimensional stagnation point flows, respectively. [Eckert \(1942\)](#) extended Hiemenz's work by including the energy equation and obtained an exact similarity solution for the thermal field. Later, the problem of stagnation point flow was extended numerically by [Schlichting and Busmann \(1943\)](#) and analytically by [Ariel \(1994\)](#) to include the effect of suction. Further, [Mabood et al. \(2015\)](#) worked on heat and mass transfer by Magnetohydrodynamics (MHD) stagnation point flow toward a permeable stretching surface. [Rosali et al. \(2014\)](#) analyzed the unsteady boundary layer flow toward a mixed convection stagnation point flow on a heated vertical surface. [Borrelli et al. \(2015\)](#) has discussed the influence of a non uniform external magnetic field on the steady three dimensional stagnation point flow of a micropolar fluid.

In fluid mechanics we study the behavior of particles at every point within the domain under various physical conditions. To describe the physical phenomena in fluid mechanics we use mathematical models for different types of fluids such as Newtonian fluids ([Picchi et al., 2014](#); [Fzal and Kim, 2014](#); [Villone et al., 2014](#); [Hatami and Domairry, 2014](#)) and non Newtonian fluids ([Hatami and Ganji, 2014a,b](#); [Liu et al., 2013](#)). There are many special cases of non-Newtonian fluids such as nano fluids ([Domairry and Hatami, 2014](#); [Hatami et al., 2014](#); [Ahmadi et al., 2014](#); [Shaikholeslam et al., 2014](#)), micropolar fluid ([Srinivasacharya and Upendar, 2014](#); [Hatami and Ganji, 2014c](#); [Pažanin, 2013](#)).

Models of micropolar fluids were first pioneered by [Eringen \(1966\)](#) and explained the characters of certain real fluid flows. The attractiveness and power of the model of micropolar fluids come from the fact that it is both a significant and a simple generalization of the classical Navier Stokes model. Recently many researchers worked on different models of micropolar fluids. [Faltas and Saad \(2014\)](#) obtained a solution of Stokes axisymmetrical flow problems of viscous fluid moving perpendicular to an impermeable bounding surface for cylindrical and spherical cases. [Borrelli et al. \(2015\)](#) examined the effects of the magnetic field and the temperature on the steady mixed convection of a micropolar fluid. Similarly, [Mohanty et al. \(2015\)](#) investigated heat and mass transfer effects of micropolar fluid over stretching sheet numerically. [Shaikholeslam et al. \(2014\)](#) has studied the flow of micropolar fluid and heat transfer in a permeable channel.

Micropolar fluid problems can be studied in different applications like Liquid crystals, animal blood, colloidal fluids, flow

of low concentration suspensions etc. Some researchers worked on the two main applications. [Oahimire and Olajuwa \(2014\)](#) solved heat and mass transfer effects on an unsteady flow of a chemically reacting micropolar fluid through porous medium. [Abd Alla et al. \(2013\)](#) studied effects of rotation and magnetic field of a micropolar fluid through a porous medium. Abdalla et al. obtained closed form solution under the consideration of long wavelength and low reold number. Another important application is Lubrication theory. [Prakash and Sinha \(1975\)](#) considered a steady laminar flow of an incompressible micropolar fluid of Lubrication theory. [Bayada and kaszewic \(1996\)](#) derived an analog of the classical reold equation of the theory of lubrication and discussed its particular forms depending on the assumption imposed on the viscosities and the data.

[Lok and Pop \(2007\)](#) analyzes the steady two dimensional stagnation-point flow of a micropolar fluid impinging on a flat rigid wall obliquely. This flow appears when a jet of viscous fluids impinge on a rigid wall obliquely. In many cases, the jet may be oblique to the impinged surface due to surface contouring or physical constraints on the nozzle ([Wang, 1985](#)). In particular, we investigate the behavior of the micropolar fluid and velocity profile of the fluid particle near the wall for various values of the micropolar parameter and rate of particle rotation to the skin friction at the plate.

To deal with such kind of problems, we needed a strong analytical tool. Interest in analytical techniques for studying nonlinear problems increased dramatically over the past two decades. Analytical methods have significant advantages over numerical methods in providing analytic, verifiable, rapidly convergent approximations. Therefore, many methods have been used to deal with highly nonlinear problems such as the homotopy perturbation transform method ([Khan and Muhammad, 2014](#); [Khan and Wu, 2011](#)), differential transform method ([Masayebidorchen et al., 2014](#); [Hatami et al., 2014](#); [Shaikholeslam et al., 2015](#); [Domairry and Shaikholeslam, 2012](#)), optimal homotopy analysis method ([Shaikholeslam and Ganji, 2014](#); [Shaikholeslam et al., 2012](#)), least square method ([Sheikholeslami et al., 2013](#); [Hatami et al., 2014](#)), Hamiltonian approach ([Nianga and Recho, 2014](#); [Reinhardt and Heffner, 2012](#); [Lan, 2011](#)), homotopy analysis method ([Sheikholeslami et al., 2012, 2014](#)), variational iteration method ([Biazar et al., 2010](#); [Faraz, 2011](#); [Faraz et al., 2011](#)), and decomposition method ([Sheikholeslami et al., 2013](#); [Sheikholeslami et al., 2012](#); [Shakeri Aski et al., 2014](#); [Guo-cheng, 2011](#)). One of the semi-exact methods is the homotopy perturbation method (HPM) ([Sheikholeslami and Ganji, 2013](#); [Shaikholeslami et al., 2011](#); [Sheikholeslami et al., 2012, 2013](#); [Jun Yang et al., 2014](#)). [He \(2007, 2003, 2004a,b\)](#) developed and formulated HPM by merging the standard homotopy and perturbation. He's HPM is proved to be compatible with the versatile nature of physical problems and has been applied to a wide class of functional equations ([Brzdęk et al., 2014](#); [Galleas and Lamers, 2014](#)). In general the solutions produced by the HPM are as accurate as the solutions given by the other

methods like the tanh method (Shukri and Al-Khaled, 2010; Bekir and Cevikel, 2009), the transformed rational function method (Nasiri Soloklo and Maghfoori Farsangi, 2013) and linear superposition principle (Stiver and Mackay, 1995).

The aim of present study is to improve the results given by Lok for non orthogonal stagnation point flow of a micro polar fluid. Particularly, we investigated and analyzed the velocity profile and microrotation profile of fluid particles. Microrotation of the particles is represented as a vector quantity that is transported and dispersed inside the fluid. It also has influence over the fluid velocity presenting a two way coupled non linear system.

**2. Formulation of the problem**

We consider two dimensional steady flow of a micropolar fluid near a non-orthogonal stagnation point at a fixed flat plate coinciding with the plane  $y > 0$ , the flow being confined to  $y > 0$ . Cartesian coordinates  $(x, y)$  fixed in space are taken, the  $x$ -axis being along the plate and the  $y$ -axis normal to it, respectively. Under these assumptions the steady two dimensional flow of a micropolar fluid is described by the following equations:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{1}$$

$$\bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial \bar{x}} + \left(\frac{\mu + \kappa}{\rho}\right) \nabla^2 \bar{u} + \frac{\kappa}{\rho} \frac{\partial \bar{N}}{\partial \bar{y}} \tag{2}$$

$$\bar{u} \frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{v}}{\partial \bar{y}} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial \bar{y}} + \left(\frac{\mu + \kappa}{\rho}\right) \nabla^2 \bar{v} - \frac{\kappa}{\rho} \frac{\partial \bar{N}}{\partial \bar{x}} \tag{3}$$

$$\rho \zeta \left( \bar{u} \frac{\partial \bar{N}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{N}}{\partial \bar{y}} \right) = \gamma \nabla^2 N - \kappa \left( 2N + \frac{\partial \bar{u}}{\partial \bar{y}} - \frac{\partial \bar{v}}{\partial \bar{x}} \right) \tag{4}$$

where  $\bar{u}$  and  $\bar{v}$  are the velocity components along the  $\bar{x}$  and  $\bar{y}$ -axes, respectively.  $\bar{N}$  is the component of the microrotation vector normal to the  $\bar{x}\bar{y}$  plane,  $\rho$  is the density,  $\mu$  is the absolute viscosity,  $\kappa$  is the vortex viscosity,  $\gamma$  is the spin-gradient viscosity,  $\zeta$  is the micro inertia density and  $\nabla^2$  is the Laplacian in Cartesian coordinates  $(\bar{x}, \bar{y})$ . It is assumed that all physical quantities are constant. We eliminate the pressure from Eqs. (2) and (3) and introducing the non dimensional variables.

$$x = \frac{\bar{x}}{\lambda}, \quad y = \frac{\bar{y}}{\lambda}, \quad u = \frac{\lambda}{\nu} \bar{u}, \quad v = \frac{\lambda}{\nu} \bar{v}$$

$$N = \frac{\bar{N}}{a}, \quad p = \frac{\bar{p}}{\rho a^2 \lambda^2}$$

By eliminating pressure, Eqs. (2)–(4) can be written in the non dimensional form as,

$$\frac{\partial \Phi}{\partial x} \frac{\partial}{\partial y} (\nabla^2 \Phi) - \frac{\partial \Phi}{\partial y} \frac{\partial}{\partial x} (\nabla^2 \Phi) + (1 + K) \nabla^4 \Phi + K \nabla^2 N = 0 \tag{5}$$

$$\frac{\partial \Phi}{\partial y} \frac{\partial N}{\partial x} - \frac{\partial \Phi}{\partial x} \frac{\partial N}{\partial y} = + \left( 1 + \frac{K}{2} \right) \nabla^2 N - K(2N + \nabla^2 \Phi) = 0 \tag{6}$$

where  $\Phi$  is the non dimensional stream function which is defined by the usual way as  $u = \frac{\partial \Phi}{\partial y}$  and  $v = -\frac{\partial \Phi}{\partial x}$  with  $u$  and  $v$  being the non dimensional velocity components along  $x$ -axis and  $y$ -axis. Using non dimensional variables the skin friction can be shown as,

$$C_f = \left[ (1 + K) \left( \frac{\partial^2 \Phi}{\partial y^2} - \frac{\partial^2 \Phi}{\partial x^2} \right) + KN \right] \tag{7}$$

where  $K$  is the material parameter defined as  $K = \frac{\kappa}{\mu}$ .

In the case of orthogonal flow the boundary condition of Eqs. (5) and (6) can be written as

$$\begin{aligned} \Phi = \frac{\partial \Phi}{\partial y} = 0, N = -n \nabla^2 \Phi \quad \text{on} \quad y = 0 \\ \Phi \approx xy \quad N \rightarrow 0 \quad \text{as} \quad y \rightarrow \infty \end{aligned} \tag{8}$$

where  $n$  is the constant such that  $0 \leq n \leq 1$ . It should be mentioned that the case  $n = 0$ , called strong concentration by Guram and Smith (1980), indicating  $N = 0$  near the wall, represents concentrated particle flows in which the microelements close to the wall surface are unable to rotate. The case  $n = 1/2$  indicates the vanishing of anti-symmetrical part of the stress tensor and denotes weak concentration. The  $n = 0$  as suggested by Peddieson (1972), is used for the modeling of turbulent boundary layer flows. The solution of Eqs. (5) and (6) subject to the boundary condition (8) can be sought in the form

$$\Phi(x, y) = xf(y) \quad N(x, y) = xg(y), \tag{9}$$

By using Eqs. (9), (5) and (6), we get the following set of ordinary differential equations:

For case  $n = 0$ :

$$\begin{aligned} (1 + K)f''' + ff'' + 1 - f'^2 + Kg' = 0 \\ (1 + \frac{K}{2})g'' + fg' - f'g - K(2g + f'') = 0 \\ f(0) = f'(0) = 0, \quad g(0) = 0 \\ f'(\infty) = 1, \quad g(\infty) = 0 \end{aligned} \tag{10}$$

For case  $n = 1/2$ :

$$\begin{aligned} (1 + \frac{K}{2})f''' + ff'' + 1 - f'^2 = 0 \\ f(0) = f'(0) = 0, f'(\infty) = 1 \end{aligned} \tag{11}$$

where prime denotes differentiation with respect to  $y$ . The skin friction becomes:

$$C_f = [(1 + (1 - n)K)]xf''(0) \tag{12}$$

In the case of the Oblique flow, it can be assumed that the stream function from the wall has the form

**Table 1** Comparison of the velocity profile between HPM and KB (Lok and Pop, 2007) for different values of  $K$  and  $n$ .

$K$	HPM	HPM	KB (Lok and Pop, 2007)	KB Lok and Pop, 2007
	$f''(0)$	$f''(0)$	$f''(0)$	$f''(0)$
	$n = 0$	$n = 1/2$	$n = 0$	$n = 1/2$
0.0	1.221927	1.223627	1.232627	1.232627
0.5	0.932657	1.082486	0.992657	1.102486
1.0	0.821085	1.016423	0.841085	1.006423
1.5	0.706835000	0.922764	0.736835	0.931764
2.0	0.620349	0.869584	0.660479	0.871584
2.5	0.84321	0.869584	0.601906	0.821736
3.0	0.504374	0.735566	0.555374	0.779566

**Table 2** Comparison of the velocity profile between HPM and KB for different values of  $K$  and  $n$ .

$K$	HPM	HPM	KB	KB
	$h'(0)$ $n = 0$	$h'(0)$ $n = 1/2$	$h'(0)$ $n = 0$	$h'(0)$ $n = 1/2$
0.0	1.406592	1.386592	1.406592	1.406592
0.5	1.305981	1.376579	1.305981	1.406579
1.0	1.230837	1.366572	1.230837	1.406572
1.5	1.174002	1.386569	1.174002	1.406569
2.0	1.129859	1.396566	1.129859	1.406566
2.5	1.0948	1.4165	1.094646	1.406563
3.0	1.065887	1.376561	1.065887	1.406561

$$\Phi(x, y) = xy + \alpha y^2, \quad y \rightarrow \infty \tag{13}$$

where  $\alpha$  is a constant and the microrotation  $N \rightarrow$  constant as  $y \rightarrow 1$ . Expression (14) suggests that Eqs. (5) and (6) have the solution of the form

$$\Phi(x, y) = xF(x) + G(y), N(x, y) = xH(y) + T(y) \tag{14}$$

The boundary conditions take the form:

$$\begin{aligned} F(0) = F'(0) = 0, \quad G(0) = G'(0) = 0 \\ H(0) = -nF''(0), \quad T(0) = -nG''(0) \end{aligned} \tag{15}$$

substituting (14) in (5) and (6) and we get the following equations:

$$(1 + K)F''' + FF'' + 1 - F'^2 + KH' = 0 \tag{16}$$

$$(1 + K)G''' + FG'' - F'G' + KT' = 2\alpha A \tag{17}$$

$$\left(1 + \frac{K}{2}\right)H'' + FH' - F'H - K(2H + F'') = 0 \tag{18}$$

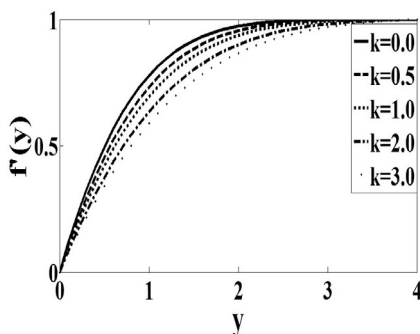
$$\left(1 + \frac{K}{2}\right)T'' + FT' - G'H - K(2T + G'') = 0 \tag{19}$$

subject to the boundary conditions

$$\begin{aligned} F(0) = F'(0) = 0, \quad G(0) = G'(0) = 0 \\ H(0) = -nF''(0), \quad T(0) = -nG''(0) \end{aligned} \tag{20}$$

$$F'(\infty) = 1, \quad G''(\infty) = 2\alpha, \quad H(\infty) = 0, \quad T(\infty) = -\alpha$$

We can see that Eqs. (16) and (18) subject to the boundary condition are identical with those of Eq. (10), by introducing new variables as:



**Figure 1** Velocity profile for the case  $n = 0.5$ .

$$G(y) = 2\alpha h(y), \quad T(y) = 2\alpha t(y)$$

Eqs. (17) and (19) can be written as,

$$(1 + K)h'' + Fh' - F'h + Kt' = A$$

$$\left(1 + \frac{K}{2}\right)t'' + Ft' - Hh - K(2t + h') = 0$$

$$h(0) = 0, \quad t(0) = -nh'(0) \tag{21}$$

$$h'(\infty) = 1, \quad t(\infty) = -\frac{1}{2}$$

where  $A$  can be calculated from the following equation:

$$f(y) \approx y + A \tag{22}$$

Values for  $F''(0)$  and  $h'(0)$  can be calculated for some values of parameters  $K$  and  $n$ . It is worth to mention that  $F''(0) = f''(0)$  because solution of Eqs. (16) and (18) subject to the boundary condition (20) is identical with the solution of (10).

In the view of HPM (Sheikholeslami et al., 2012, 2013), Eq. (10) can be expressed as,

$$(1 - p)L_1(f - f_0) + p[(1 + K)f''' + ff'' + 1 - f'^2 + Kg] = 0$$

$$\begin{aligned} (1 - p)\left(1 + \frac{K}{2}\right)L_2(g - g_0) \\ + p\left[\left(1 + \frac{K}{2}\right)g'' + fg' - f'g - K(2g + f'')\right] = 0 \end{aligned} \tag{23}$$

Assuming  $L_1 f = 0$ , and  $L_2 g = 0$ , the following expansions for  $f$  and  $g$  can be introduced into the Eq. (23) as,

$$\begin{aligned} f = f_0 + pf_1 + p^2f_2 + \dots \\ g = g_0 + pg_1 + p^2g_2 + \dots \end{aligned} \tag{24}$$

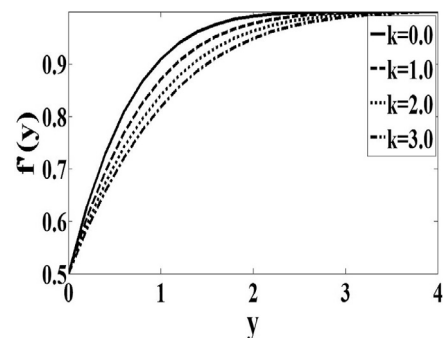
After some simplifications and rearrangements based on the powers of  $p$ -terms, the following equations can be obtained,

$$\begin{aligned} p^{(0)} : L_1f_0 = 0 \quad \text{and} \quad L_2g_0 = 0 \\ f_0(0) = f_0'(0) = 0, \quad g_0(0) = 0 \\ f_0'(\infty) = 1, \quad g_0(\infty) = 0 \end{aligned} \tag{25}$$

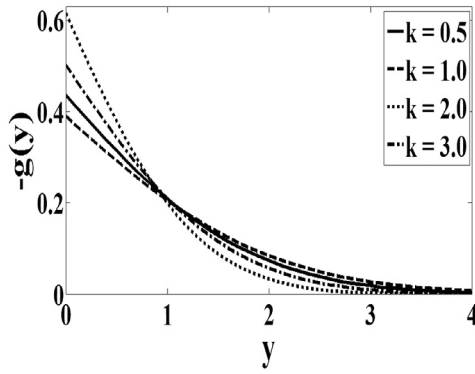
where  $L_1$  and  $L_2$  can be defined as,

$$L_1 = \frac{\partial^3}{\partial y^3} + \frac{\partial^2}{\partial y^2} \quad \text{and} \quad L_2 = \frac{\partial^2}{\partial y^2} - 1 \tag{26}$$

Initial guesses can be obtained by solving Eq. (25) as,



**Figure 2** Velocity profile for the case  $n = 0$ .



**Figure 3** Micro rotation profile for case  $n = 0.5$ .

$$f_0(y) = y + e^{-y} - 1, \quad g_0(y) = e^{-y} - e^{-2y} \quad (27)$$

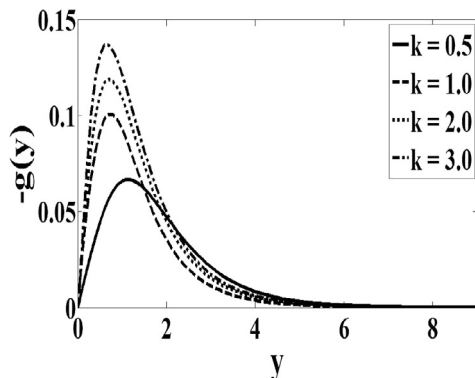
Higher order terms can be determined as,

$$\begin{aligned} (1-p)L_1(f-f_0) + p[(1+K)f''' + ff'' + 1 - f^2 + Kg] &= 0 \\ (1-p)(1 + \frac{K}{2})L_2(g-g_0) + p[(1 + \frac{K}{2})g'' + fg' - f'g - K(2g + f'')] &= 0 \\ p^{(1)} : L_1 f_1 + [(1+K)f_0''' + f_0 f_0' + 1 - f_0^2 + Kg_0] &= 0 \\ f(0) = f'(0) = 0, \quad f'(\infty) = 1 \end{aligned} \quad (28)$$

$$\begin{aligned} p^{(1)} : L_2 g_1 + \left[ (1 + \frac{K}{2})g_0'' + f_0 g_0' - f_0' g_0 - K(2g_0 + f_0'') \right] &= 0 \\ g(0) = 0, \quad g(\infty) = 0 \\ p^{(j)} : L_1 f_j - L_1 f_{j-1} \\ + \left[ (1+K)f_{j-1}''' + \sum_{k=0}^{j-1} f_k f_{j-1-k}' + 1 - \sum_{k=0}^{j-1} f_k' f_{j-1-k} + Kg_{j-1} \right] &= 0 \\ f(0) = f'(0) = 0, \quad f'(\infty) = 1 \end{aligned} \quad (29)$$

$$\begin{aligned} p^{(j)} : L_2 g_j - L_2 g_{j-1} \\ + \left[ (1 + \frac{K}{2})g_{j-1}'' + \sum_{k=0}^{j-1} f_k g_{j-1-k}' - \sum_{k=0}^{j-1} f_k' g_{j-1-k} - K(2g_{j-1} + f_{j-1}'') \right] \\ g(0) = 0, \quad g(\infty) = 0 \\ \vdots \end{aligned} \quad (30)$$

Similarly for Eq. (21) we define the initial guess as



**Figure 4** Micro rotation profile for case  $n = 0$ .

$$h_0(y) = y + e^{-y} - 1, \quad t_0(y) = -\frac{1}{2} + \frac{e^{-y}}{2} \quad (31)$$

We can obtain the higher order term for Eq. (21) as we obtained in Eq. (9)...

Graphical behavior of the physical parameters has been calculated for the 10th order approximation.

### 3. Results and discussion

In this study we have examined the two dimensional stagnation point flow of a micropolar fluid. The obtained nonlinear coupled ordinary differential equations are solved by using Homotopy perturbation method. Effects of the material parameter  $K$  on the velocity profile and microrotation profile for different cases of  $n$  are investigated and obtained results compared with the Kaller Box method (Lok and Pop, 2007).

Tables 1 and 2 shows the comparison of  $f''(0)$ ,  $h'(0)$  for some values of  $K$  and  $n$  between HPM and Kaller Box. Values for  $g(0)$  and  $t(0)$  are not calculated as we calculate them from  $f''(0)$ ,  $h'(0)$ . They result from the boundary condition (10) and (21) where  $g(0) = -nf''(0)$ ,  $t(0) = -nh'(0)$ . It is noticed from the table that the effect of increasing values of material parameter  $K$  results in a decrease in values of reduced skin friction  $f''(0)$  and also the values are greater for  $n = 1/2$  than for  $n = 0$ . This indicates that the micropolar fluid displays a reduction in drag as compared to Newtonian fluids.

In order to provide physical insights into the flow problems, Figs. 1-4 are plotted for the velocity profile and the microrotation profile. The material parameter  $K$  is the ratio of vortex viscosity to absolute viscosity. If this parameter is small then it can be said that the viscous effect is confined in a very thin layer near the wall and the thickness of this layer is proportional to  $\kappa = \frac{\mu}{\mu}$  and inversely proportional to  $f'(0)$ . Therefore it can be analyzed from Figs. 1 and 2 that as the values of  $K$  increase, values of  $f'(0)$  near the wall are decreasing. Consequently, the velocity gradient at the wall decreases as  $K$  increases. This describes very well the trend of the values of obtained in Table 1. For case  $n = 1/2$  (Fig. 3), the microrotation decreases continuously from its maximum value at the wall to zero far from the wall. On the other hand, the microrotation increases in function with parameter  $K$ . It can be seen from Fig. 4 that the microrotation increases as  $K$  increases for the case when  $n = 0$ . The peak value of microrotation occurs near the wall then decreases monotonically to zero as  $y$  increases. Finally, it is observed that Figs. 1-4 that, as expected, the velocity and microrotation boundary layer thicknesses increases as the material parameter  $K$  increases.

### 4. Conclusion

The series solution of steady two dimensional non orthogonal stagnation point flow of the micropolar fluid is studied and the main points of the study are listed below.

- The velocity profile strongly decreases as the material parameter increases at strong concentrations, which clearly shows that the viscous effects are confined in a very thin layer near the wall.



- For the weak concentration the velocity profile decreases as material parameter increases and remains uniform throughout the domain.
- The microrotation velocity decreases continuously from its maximum value at the wall to zero far from the wall for the weak concentration case.
- The microrotation velocity profile has a parabolic distribution in the case of strong concentration. It increases strongly near the wall then decreases continuously to zero as  $y$  increases.

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