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Trisectional fuzzy trapezoidal approach to optimize interval data based transportation problem

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ABSTRACT

This research article puts forward a combination of two new thoughts to solve interval data based transportation problems (IBTPs). Firstly IBTP is converted to fuzzy transportation problem using trisectional approach and secondly a newly proposed ranking technique based on in-centre concept is applied for conversion to crisp number. It is supported with numerical illustrations to test the relevance of the scheme. Comparison with other existing methods confirms its significance for transportation problems having interval data.

1. Introduction

In the current competitive environment, different transportation models exist in a variety of forms and everyone is keen to utilize best possible resources to optimize the profit and cost.

Optimization of transportation problems stands for optimizing cost of products to be transported from a number of sources to different destinations (Hitchcock, 1941). The classical transportation problems include fixed numbers of supply, demand and cost. But in real life parameters may be vague or uncertain with some limit (Zimmermann, 1978). Inclusion of uncertain environment raised a new model of transportation problem termed as fuzzy transportation problem (FTP). In FTPs the parameters costs, supply and demand are fuzzy quantities (ÓhÉigeartaigh, 1982); (Mathur et al., 2018). Various techniques have been applied to solve FTPs such as Chanas et al. (1984) used decision making technique developed by Bellman and Zadeh (1970) to solve FTPs. Chanas and Kuchta successfully worked out on fuzzy transportation with integer value problem (Chanas and Kuchta, 1998). Extension principle has also been established for solution of fuzzy transportation by Liu and Kao (2004). The separation method based on the zero point method provides an optimal value of the objective function for the fully interval transportation problem (Pandian and Natarajan, 2010; Natarajan, 2010). Kumar and Kaur represented all values as LR flat fuzzy numbers to optimize unbalanced FTPs (Kumar and Amarpreet, 2011). Ebrahimnejad proposed a two-step method to solve such problems (Ebrahimnejad, 2015). Jamrus et al. well thought-out of triangular fuzzy demands to optimize FTPs (Jamrus et al., 2015). Zheng and Ling introduced a cooperative optimization method to solve fuzzy emergency transportation problems (Zheng and Ling, 2013).

The solution of transportation problems using algorithms based on trapezoidal fuzzy numbers is wide spread. Gani and Razak minimized the transportation cost for demands and supplies using trapezoidal fuzzy numbers. Ritha and Vinotha (2009) used fuzzy geometric approach to find the solution of FTPs. These type of problems are also investigated by Stephen Dinagar and Palanivel, Kumar and Kaur, Ebrahimnejad using concept of trapezoidal fuzzy numbers (Dinagar and Palanivel, 2009; Kumar and Kumar, 2010; Kaur and Kumar, 2011; Ebrahimnejad, 2014). The recent works include newly developed ranking techniques based on trapezoidal fuzzy numbers show betterment over classical methods (Mathur et al., 2016; Bisht and Srivastava, 2017).

Till date all the FTPs are represented in doublet, triplet or 4-tuple forms but in some cases data may be available in interval or in a range due to flexible inputs and requirements. These transportation problems with flexible demand, supply and cost are termed as interval data based transportation problem (IBTP). Such type of problems cannot be solved directly using available methods

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for transportation problem. This motivates authors to produce a robust method which can easily implemented on IBTPs.

In this paper a trisectional fuzzy trapezoidal algorithm is proposed to solve IBTPs. To illustrate the proposed method two numerical examples of IBTPs are solved and the obtained results are compared with the results of existing methods.

2. Interval data based transportation problem

At present time the organizations need to announce their production or supply requirements very much in advance for transportation, so that all arrangements can be done well in time. Initially only a flexible data of demand or supply can be provided, but in the end a huge difference between the expected and actual quantity may occur. This problem can be treated by changing the problem representation in which data is provided in an interval form: minimum amount to a maximum amount. Such type of problems with flexible demand, supply and cost are termed as IBTPs.

3. Prefatory of trapezoidal fuzzy number

Fuzzy numbers have become a useful tool to optimize real life problems (Kauffman and Gupta, 1991; Kaufmann and Gupta, 1988). This section describes briefly trapezoidal fuzzy number and arithmetic operations used in present paper.

3.1. Definition

Trapezoidal fuzzy number $K^* = (k_1,\ k_2,\ k_3,\ k_4)$ is defined as:

$$\mu_{K^*}(Z) = \begin{cases} \frac{z-k_1}{k_2-k_1}, & k_1 \leqslant z < k_2 \\ a, & k_2 \leqslant z \leqslant k_3 \\ \frac{z-k_4}{k_3-k_4}, & \frac{z-k_4}{k_3-k_4}, \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

Here a is any real number satisfying $0 < a \leqslant 1$. If $a = 1$ then the trapezoidal fuzzy number is said to be normal.

3.2. Arithmetic operations

Let the two fuzzy numbers $P = (p_1,\ p_2,\ p_3,\ p_4)$ and $Q = (q_1,\ q_2,\ q_3,\ q_4)$ are trapezoidal, then

a. Addition $P + Q = (p_1 + q_1, p_2 + q_2, p_3 + q_3, p_4 + q_4)$
b. Subtraction $P - Q = (p_1 - q_4, p_2 - q_3, p_3 - q_2, p_4 - q_1)$
c. Multiplication $P \times Q = (min\{p_1q_1, p_1q_4, p_4q_1, p_4q_4\},$ $min\{p_2q_2, p_2q_3, p_3q_2, p_3q_3\}$, $max\{p_2q_2, p_2q_3, p_3q_2, p_3q_3\}$, $max\{p_1q_1, p_1q_4, p_4q_1, p_4q_4\})$.

4. Proposed trisectional fuzzification approach

In IBTP the decision maker does not possess exact information about the demand, supply and cost, but are avialble in interval form. To apply fuzzy transportation approach this interval data needs to be fuzzified in fuzzy numbers. The proposed trisectional approach is used to fuzzify the given interval data to a trapezoidal fuzzy number.

Let us consider an interval data (L, H). The trisection of this interval is taken as

$$\mathrm{d} = (\mathrm{H} - \mathrm{L})/3 \tag{2}$$

hence the required trapezoidal fuzzy number will be

$(\mathrm{L}, \mathrm{L} + \mathrm{d}, \mathrm{L} + 2\mathrm{d}, \mathrm{H})$.

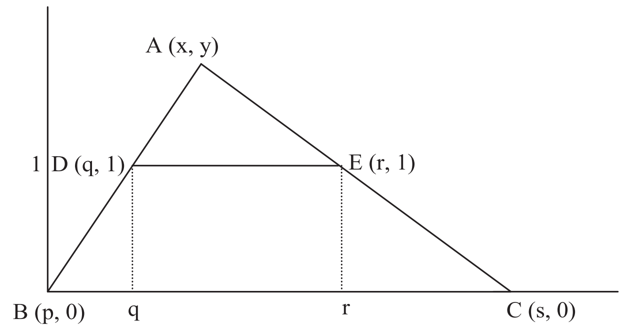

Fig. 1. Normal trapezoidal fuzzy number.

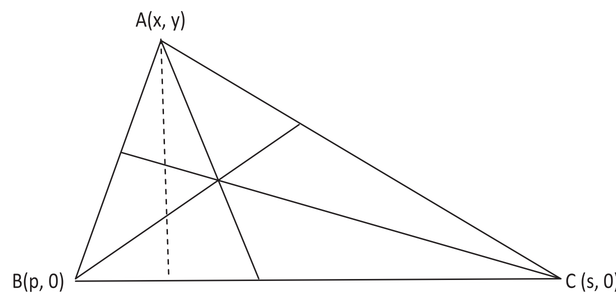

Fig. 2. In-center.

5. Proposed ranking technique

Consider a normal trapezoidal fuzzy number $P = (p, q, r, s)$ represented in Fig. 1. Extend the line joining $B(p, 0)$ and $D(q, 1)$ as BDA and the line joining $C(s, 0)$ and $E(r, 1)$ as CEA. The intersection of extended lines BD and CE is A. Coordinates for intersection point A is given by (x, y) where

$$x = \frac{pr - qs}{r - s - q + p},\ \ y = \frac{x - q}{q - p} + 1 \tag{3}$$

The proposed ranking technique is based on the concept of incentre (the centre of the incircle of a triangle) for triangle ABC Fig. 2) and is defined as

$$R(P) = \frac{ax + bp + cs}{a + b + c} \tag{4}$$

where $a = s - p,\ b = \sqrt{y^2 + (s - x)^2},\ c = \sqrt{y^2 + (x - p)^2}$

6. Mathematical formulation

The algorithm proposed in this paper involves following steps to solve IBTP:

Step 1. Conversion of the specified problem in tabular form.
Step 2. Fuzzification of the particular data using trisectional approach defined in Section 4.
Step 3. Application of ranking technique proposed in Section 5, formulation as LPP and check for balance.
Step 4. Application of classical least cost method for initial basic feasible solution and then modified distribution (MODI) technique (Mathur et al., 2018) to get optimal solution.

7. Numerical example

Example 1. A company has three sources or deliverers A,B,C with supply (1–9);(4–10);(4–11) respectively and it has three

Table 1
Transportation cost for example 1.

	R_1	R_2	R_3
A	[1,19]	[1,9]	[2,18]
B	[8,26]	[3,12]	[7,28]
C	[11,27]	[0,15]	[4,11]

Table 2
Tabular form of example 1.

	R_1	R_2	R_3	Supply
A	[1,19]	[1,9]	[2,18]	[1,9]
B	[8,26]	[3,12]	[7,28]	[4,10]
C	[11,27]	[0,15]	[4,11]	[4,11]
Demand	[3,12]	[4,10]	[2,8]	

receivers R_1, R_2, R_3 with demand values (3–12);(4–10);(2–8) respectively. The transport cost is given in following table (see Table 1).

Solution:

Step 1. Conversion in tabular form

Step 2. Fuzzification of interval data in Table 2 using equation (2)

Step 3. Ranking, Formulation as LPP and Check for balance

Applying ranking technique in equation (4), the Table 3 becomes

The problem obtained in Table 4 is not a balanced problem as the total demand and total supply are not equal. To make it balanced a dummy receiver is introduced with demand 0.30 units (see Table 5).

Step 4. Application of least cost method (Mathur et al., 2018) to solve LPP

The IBFS by least cost method is

$$x_{12} = 4.47;\ x_{14} = 0.30, x_{21} = 4.47,\ x_{22} = 2.11$$
$$x_{31} = 2.29,\ x_{33} = 4.58$$

The transportation cost is $Z = 173.20$.

Applying MODI method in the IBFS solution obtained by least cost method, the basic variables are

$$x_{11} = 4.47;\ x_{21} = 2.29,\ x_{22} = 4.29,\ x_{32} = 2.29,\ x_{33} = 4.58$$

The total transportation cost is $Z = 154.93$.

So the transportation cost for this IBTP will be optimum if the receivers R1 receives product from suppliers A and B, R2 receives product from suppliers B and C and R3 receives product from supplier C only Table 11).

Example 2. A company has factories at three different locations L_1, L_2 and L_3 with production quantities (1–12), (0–3) and (5–15.6) tons per month respectively and company has four warehouses H_1, H_2, H_3 and H_4 with demand (5–10), (1–10), (1–6) and (1–4). The transport cost is given in following table (see Table 6)

Step 1. Conversion in tabular form

Step 2. Fuzzification of interval data in Table 7 using equation (2)

Step 3. Ranking, Formulation as LPP and Check for balance

Table 4
Defuzzified data for example 1.

	R_1	R_2	R_3	Supply
A	9.75	4.77	9.48	4.77
B	14.72	6.76	15.48	6.58
C	17.61	7.50	6.87	6.87
Demand	6.76	6.58	4.58	

Table 5
Balanced table for example 1.

	R_1	R_2	R_3	Dummy	Supply
A	9.75	4.77	9.48	0	4.77
B	14.72	6.76	15.48	0	6.58
C	17.61	7.50	6.87	0	6.87
Demand	6.76	6.58	4.58	0.30	

Table 6
Transportation cost for example 2.

	H_1	H_2	H_3	H_4
L_1	[1,4]	[1,6]	[4,12]	[5,11]
L_2	[0,4]	[1,4]	[5,8]	[0,3]
L_3	[3,8]	[5,12]	[12,19]	[7,12]

Table 7
Tabular form of example 2.

	H_1	H_2	H_3	H_4	Supply
L_1	[1,4]	[1,6]	[4,12]	[5,11]	[1,12]
L_2	[0,4]	[1,4]	[5,8]	[0,3]	[0,3]
L_3	[3,8]	[5,12]	[12,19]	[7,12	[5,15.6]
Demand	[5,10]	[1,10]	[1,6]	[1,4]	

Applying ranking technique in equation (4), Table 8 becomes

The problem obtained in Table 9 is not a balanced problem as the total demand and total supply are not equal. To make it balanced a dummy supplier is introduced with supply 1.59 units (see Table 10)

Step 4. Application of Least Cost method (Mathur et al., 2018) to solve LPP

The IBFS using least cost method is

$$x_{11} = 5.91;\ x_{12} = 0.34,\ x_{24} = 1.50,\ x_{32} = 4.92,\ x_{33} = 3.30,$$
$$x_{34} = 0.88$$

The total transportation cost is $Z = 119.69$.

Applying MODI method in the IBFS solution obtained by least cost method, the basic variables will be

$$x_{12} = 2.95,\ x_{13} = 3.3,\ x_{24} = 1.50,\ x_{31} = 5.91,\ x_{32} = 2.31,$$
$$x_{34} = 0.88$$

The total transportation cost is $Z = 93.39$.

So the transportation cost for this IBTP will be optimum if the warehouses H1 receives product from suppliers L3, H2 receives product from suppliers L1 and L3, H3 receives product from

Table 3
Fuzzified interval data for example 1.

	R_1	R_2	R_3	Supply
A	(1,7,13,19)	(1,3.67,6.34,9)	(2,7.33,12.66,18)	(1,3.67,6.33, 9)
B	(8, 1.67, 13.33, 16)	(3, 6, 9, 12)	(7,14,21,28)	(4,6,8,10)
C	(11,16.33,21.67,27)	(0, 5, 10, 15)	(4,6.33,8.67,11)	(4,6.33,8.67,11)
Demand	(3, 6, 9, 12)	(4,6,8,10)	(2,4,6,8)	

Table 8
Fuzzified data for example 2.

	H_1	H_2	H_3	H_4	Supply
L_1	(1, 2, 3, 4)	(1, 2.67, 4.33, 6)	(4, 6.67, 9.33, 12)	(5, 7, 9, 11)	(1, 4.67, 8.33, 12)
L_2	(0, 1.33, 2.67, 4)	(1, 2, 3, 4)	(5, 6, 7, 8)	(0, 1, 2, 3)	(0, 1, 2, 3)
L_3	(3, 4.67, 6.33, 8)	(5, 7.33, 9.67, 12)	(12,14.33, 16.67, 19)	(7, 8.67, 10.33, 12)	(5, 8.53, 12.07, 15.6)
Demand	(5, 6.67, 8.33, 10)	(1, 4, 7, 10)	(1, 2.67, 4.33, 6)	(1, 2, 3, 4)	

Table 9
Defuzzified data for example 2.

	H_1	H_2	H_3	H_4	Supply
L_1	2.38	3.30	7.19	7.78	6.25
L_2	2.00	2.38	6.78	1.50	1.50
L_3	5.15	8.04	16.37	9.86	9.10
Demand	7.50	5.26	3.30	2.38	

Table 10
Balanced table for example 2.

	H_1	H_2	H_3	H_4	Supply
L_1	2.38	3.30	7.19	7.78	6.25
L_2	2.00	2.38	6.78	1.50	1.50
L_3	5.15	8.04	16.37	9.86	9.10
Dummy	0	0	0	0	1.59
Demand	7.50	5.26	3.30	2.38	

suppliers L1 and H3 receives product from suppliers L2 and L3 (Table 12).

8. Comparative study

The fuzzified interval data obtained in step 2 of both the examples under consideration are solved by proposed method and compared with Mathur et al. approach (Mathur et al., 2016). The proposed method either gives better alternate solutions to the problem or confirms that the solution obtained is the best possible optimal solution. The comparison of example 1 and 2 for both the methods is shown in Tables 11 and 12 respectively.

9. Merits of proposed scheme

There are several approaches to solve FTPs and considered trisectional trapezoidal approach appears better in comparison to other existing approaches. The proposed method has following merits:

i. Easily convert the interval based data to crisp form through proposed trisectional fuzzification and ranking technique approaches.
ii. Easily applicable to interval based transportation problems by converting it to fuzzy transportation problem.
iii. Comparison with available methods in literature is easily possible as in both the cases the optimal solution is in crisp number.
iv. Unique ranking approach is applied.

10. Conclusion

The proposed algorithm in this research article is the combination of two key ideas. First idea is to propose a unique trisectional fuzzification approach and second idea is to find optimal solution of IBTP through a new ranking technique. The algorithm is compared with the previous existing method. The presented method is found to be more effective to solve such transportation problem, which has a large difference in production and supply. This scheme suggests significant industrial and engineering applications in decision making.

Table 11
Comparative table of example 1.

	R_1		R_2		R_3	
	Mathur et al. approach (Mathur et al., 2016)	Proposed Trisectional Trapezoidal Ranking Approach	Mathur et al. approach (Mathur et al., 2016)	Proposed Trisectional Trapezoidal Ranking Approach	Mathur et al. approach (Mathur et al., 2016)	Proposed Trisectional Trapezoidal Ranking Approach
A	√	√	X	X	X	X
B	√	√	√	√	X	X
C	X	X	√	√	√	√

Table 12
Comparative table of example 2.

	H_1		H_2		H_3		H_4	
	Mathur et al. approach (Mathur et al., 2016)	Proposed Trisectional Trapezoidal Ranking Approach	Mathur et al. approach (Mathur et al., 2016)	Proposed Trisectional Trapezoidal Ranking Approach	Mathur et al. approach (Mathur et al., 2016)	Proposed Trisectional Trapezoidal Ranking Approach	Mathur et al. approach (Mathur et al., 2016)	Proposed Trisectional Trapezoidal Ranking Approach
L_1	X	X	√	√	√	√	X	X
L_2	X	X	X	X	√	X	X	√
L_3	√	√	X	√	√	X	√	√

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