Journal of King Saud University – Science (2013) **25**, 129–133

King Saud University
Journal of King Saud University – Science
www.ksu.edu.sa
www.sciencedirect.com

ORIGINAL ARTICLE

Envelope solitons for generalized forms of the phi-four equation

Houria Triki [a], **Abdul-Majid Wazwaz** [b,*]

[a] Radiation Physics Laboratory, Department of Physics, Faculty of Sciences, Badji Mokhtar University, P.O. Box 12, 23000 Annaba, Algeria
[b] Department of Mathematics, Saint Xavier University, Chicago, IL 60655, United States

Received 20 June 2012; accepted 11 August 2012
Available online 22 August 2012

KEYWORDS
Phi-four equation;
Bright solitons;
Dark solitons;
Solitary wave ansatz

Abstract We consider two variants of the generalized phi-four equation with arbitrary constant coefficients and general values of the exponents in the dissipation and nonlinear terms. By using solitary wave ansatze in terms of $\text{sech}^p(x)$ and $\tanh^p(x)$ functions respectively, we find the non-topological (bright) as well as topological (dark) soliton solutions for the considered models. The physical parameters in the soliton solutions are obtained as a function of the dependent model coefficients. The conditions of existence of solitons are presented. Further, we show that the obtained soliton solutions depend on the exponent of the wave function $u(x,t)$, positive or negative, and on all the dependent model coefficients as well.

1. Introduction

In the theoretical investigation of the dynamics of nonlinear waves in physical systems several kinds of nonlinear partial differential equations (NLPDEs) take an important role. These equations appear in a great array of contexts such as, for example, in plasma physics, fluid mechanics, nonlinear optics, hydrodynamics, quantum mechanics and many other fields. It should be noted that the propagation behavior of nonlinear waves depends on the model coefficients which can be constant or variable parameters depending on the physical situation.

In the past decades, studies have been made on the aspect of integrability of NLPDEs. A given nonlinear evolution equation can be considered integrable when it is equivalent to the compatibility condition for the associated Lax pair (Ablowitz and Clarkson, 1992). Lax pair can be used not only to demonstrate the integrability but also to construct the soliton solutions via the Darboux transformation (Ablowitz and Clarkson, 1992). In many practical physics problems ((Triki and Wazwaz, 2010), the resulting nonlinear wave equations of interest are nonintegrable (Palacios, 2004). In some particular cases they may be close to an integrable one (Palacios, 2004). It is remarkable that non- integrability is not necessarily related to the nonlinear terms (Palacios and Fernandez-Diaz, 2000). Higher

* Corresponding author.
E-mail address: wazwaz@sxu.edu (A.-M. Wazwaz).

1018-3647 © 2012 King Saud University. Production and hosting by Elsevier B.V. All rights reserved.

Peer review under responsibility of King Saud University.
doi:10.1016/j.jksus.2012.08.001

Production and hosting by Elsevier

order dispersions, for example, also can make the system to be non- integrable (while it remains Hamiltonian) (Palacios and Fernandez-Diaz, 2000).

By means of different modern methods of integrability (Triki and Wazwaz, 2009a,b, 2010, 2011a,b; Triki and Ismail, 2010; Triki and Taha, 2012) such as the coupled amplitude-phase formalism (Du et al., 1995; Palacios et al., 1999), the hyperbolic tangent method (Malfliet, 1992), Hirota bilinear method (Nakkeeran, 2002; Wazwaz, 2005a,b, 2010), the sub-ODE method (Li and Wang, 2007; Triki and Wazwaz, 2009a), the solitary wave ansatz method (Biswas et al., 2011; Biswas, 2008a,b, 2009a,b; Saha et al., 2009) and other methods as well, a rich variety of exact solutions have been obtained. Based on these exact solutions directly, we can accurately analyze the properties of the propagating waves in dynamical systems. These methods work even though the Painleve test of integrability will fail (Biswas, 2009b).

The phi-four equation reads (Wazwaz, 2005b)

$$u_{tt} - au_{xx} - u + u^3 = 0, \quad a > 0, \tag{1}$$

arises in many branches of mathematical physics. Special solutions known as kink and anti-kink solitons,

$$u(x,t) = \pm\tanh\left(\frac{1}{\sqrt{2(1-c^2)}}(x-ct)\right), \tag{2}$$

have been found (Triki and Wazwaz, 2009b). Here c is the wave speed while the coefficient a satisfies the condition $a = 1$ for the solution (2) to exist. In the same context, they have investigated two generalized forms of the phi-four equation by using the sine-cosine ansatz. As an example of the considered generalized forms, one finds the following model equations:

$$(u^n)_{tt} - a(u^n)_{xx} - u^m + u^n = 0, \tag{3}$$
$$(u^{-n})_{tt} - a(u^{-n})_{xx} - u^m + u^{-n} = 0, \tag{4}$$

where the effect of the positive and negative exponents and the coefficient a of the second derivative u_{xx} on the obtained solutions have been studied.

For the purpose of better understanding the effect of the exponents and the dependent model coefficients on the properties of the resulting solitons, we consider the following two variants:

$$(u^l)_{tt} - a(u^n)_{xx} - bu^m + cu^n = 0, \tag{5}$$
$$(u^{-l})_{tt} - a(u^{-n})_{xx} - bu^m + cu^{-n} = 0, \tag{6}$$

where a, b and c are arbitrary nonzero constants and l, m and n are integers. If setting $l = n$ and $b = c = 1$, Eqs. (5) and (6) reduce to the model Eqs. (3) and (4).

Our interest in the present paper (Esfahani, 2011) is to search for the solitary wave solutions for Eqs. (5) and (6) as they appear, namely for three arbitrary coefficients a, b and c and general values of the integers l, m and n. The technique that will be used is the solitary ansatz method, which is one of the most effective direct methods to construct solitary wave solutions of nonlinear evolution equations, see for example (Biswas et al., 2011; Biswas, 2008a,b, 2009a,b; Saha et al., 2009) and references therein. In particular, we show that the existence of solitary wave solutions (Triki and Taha, 2012) depends essentially on the model coefficients a, b and c, and therefore on the specific nonlinear and dissipation features of the medium. Notably, solitary waves, which are localized traveling waves, asymptotically zero at large distances, are very interesting from the point of view of applications.

2. Soliton solutions via the solitary wave ansatz

2.1. Variant I

We first consider the variant I of the generalized phi-four equation with positive exponents (5):

$$(u^l)_{tt} - a(u^n)_{xx} - bu^m + cu^n = 0, \tag{7}$$

where $u(x, t)$ is the unknown function depending on the spatial variable x and the temporal variable t. The subscripts x and t denote partial derivatives with respect to these variables, and a, b and c are real constants. In (7) the first term is the evolution term, the second term represents the dissipation term, while the last two terms are the nonlinear terms.

2.1.1. Bright solitons

To find an exact bright soliton solution for (7), we use the following solitary wave ansatz (Biswas et al., 2011; Biswas, 2008a,b; Saha et al., 2009)

$$u(x,t) = \frac{A}{\cosh^p\tau}, \tag{8}$$

where

$$\tau = B(x - vt), \tag{9}$$

and

$$p > 0, \tag{10}$$

for solitons to exist. Here, in (8) and (9), A represents the amplitude of the soliton, while v is the velocity of the soliton and B represents the inverse width of the soliton. The exponent p will be determined as a function of l, m and n.

From the ansatz (8), one gets

$$(u^l)_{tt} = \frac{p^2l^2v^2A^lB^2}{\cosh^{pl}\tau} - \frac{pl(pl+1)v^2A^lB^2}{\cosh^{pl+2}\tau}, \tag{11}$$
$$(u^n)_{xx} = \frac{p^2n^2A^nB^2}{\cosh^{pn}\tau} - \frac{pn(pn+1)A^nB^2}{\cosh^{pn+2}\tau}, \tag{12}$$
$$u^n = \frac{A^n}{\cosh^{pn}\tau}, \tag{13}$$
$$u^m = \frac{A^m}{\cosh^{pm}\tau}. \tag{14}$$

Inserting the expressions (11)–(14) into (7) yields

$$\frac{p^2l^2v^2A^lB^2}{\cosh^{pl}\tau} - \frac{pl(pl+1)v^2A^lB^2}{\cosh^{pl+2}\tau} - \frac{ap^2n^2A^nB^2}{\cosh^{pn}\tau} + \frac{apn(pn+1)A^nB^2}{\cosh^{pn+2}\tau} - \frac{bA^m}{\cosh^{pm}\tau} + \frac{cA^n}{\cosh^{pn}\tau} = 0. \tag{15}$$

Now, from (15), matching the exponents of $1/\cosh^{pn+2}\tau$ and $1/\cosh^{pm}\tau$ functions gives

$$pn + 2 = pm, \tag{16}$$

so that

$$p = \frac{2}{m-n}. \quad (17)$$

Also, from (15), equating the exponents of $1/\cosh^{pl}\tau$ and $1/\cosh^{pn}\tau$ functions yields

$$pl = pn, \quad (18)$$

and therefore

$$l = n. \quad (19)$$

Now, the functions $1/\cosh^{pn+j}\tau$ with $n = l$, for $j = 0, 2$ in (15) are linearly independent. Thus, their respective coefficients must vanish. Setting their coefficients to zero gives the system of algebraic equations:

$$p^2n^2A^nB^2(v^2-a) + cA^n = 0,$$
$$-pn(pn+1)A^nB^2(v^2-a) - bA^m = 0. \quad (20)$$

Solving the above system gives

$$A = \left(\frac{(m+n)c}{2nb}\right)^{\frac{1}{m-n}}, \quad m > n > 1 \quad (21)$$

$$B = \frac{m-n}{2n}\sqrt{\frac{c}{a-v^2}}. \quad (22)$$

Thus, the 1-soliton solution of the generalized phi-four Eq. (7) with positive exponents is given by

$$u(x,t) = \left\{\frac{(m+n)c}{2nb\cosh^2[B(x-vt)]}\right\}^{\frac{1}{m-n}}, \quad (23)$$

where the width B is given by (22). In view of (22), we clearly see that this solution exists provided that $c(a - v^2) > 0$. Also (21) shows that it is necessary to have $bc > 0$ for the bright soliton to exist if $m - n$ is an even integer. However, if $m - n$ is an odd integer there is no such restriction but the soliton will be pointing downward. Finally, we would like to note that the solution (23) exists provided (Adem et al., 2011) that $m > n$ as seen from (10) and (17).

2.1.2. Dark solitons

In this subsection the search is going to be for shock wave solution or topological 1-soliton solution to the generalized phi-four equation given by (7). To start off, the hypothesis is given by (Saha et al., 2009; Triki and Wazwaz, 2009a,b)

$$u(x,t) = A\tanh^p\tau, \quad (24)$$

where

$$\tau = B(x - vt), \quad (25)$$

and

$$p > 0, \quad (26)$$

for solitons to exist. Here in (24) and (25), A and B are free parameters while v is the velocity of the wave. Also, the unknown exponent p will be determined during the course of the derivation of the soliton solution to (7).

From the ansatz (24), we get

$$(u^l)_{tt} = plA^lB^2v^2\{(pl-1)\tanh^{pl-2}\tau - 2pl\tanh^{pl}\tau$$
$$+ (pl+1)\tanh^{pl+2}\tau\}, \quad (27)$$

$$(u^n)_{xx} = pnA^nB^2\{(pn-1)\tanh^{pn-2} - 2pn\tanh^{pn}\tau$$
$$+ (pn+1)\tanh^{pn+2}\tau\}, \quad (28)$$

$$u^n = A^n\tanh^{pn}\tau, \quad (29)$$

$$u^m = A^m\tanh^{pm}\tau. \quad (30)$$

Substituting (27)–(30) into (7) yields

$$plA^lB^2v^2\{(pl-1)\tanh^{pl-2}\tau - 2pl\tanh^{pl}\tau$$
$$+ (pl+1)\tanh^{pl+2}\tau\} - apnA^nB^2\{(pn-1)\tanh^{pn-2}\tau$$
$$- 2pn\tanh^{pn}\tau + (pn+1)\tanh^{pn+2}\tau\} - bA^m\tanh^{pm}\tau$$
$$+ cA^n\tanh^{pn}\tau = 0. \quad (31)$$

From (31), equating the exponents $pn + 2$ and pm gives

$$pn + 2 = pm, \quad (32)$$

so that

$$p = \frac{2}{m-n}. \quad (33)$$

Next, equating the exponents pl and pn gives

$$pl = pn, \quad (34)$$

so that

$$l = n, \quad (35)$$

which also follows if the exponents pair $pl + 2$ and $pn + 2$, and $pl - 2$ and $pn - 2$, match up. Now, noting that the functions $\tanh^{pn+j}\tau$, with $j = -2,0,2$, are linearly independent, setting their coefficients to zero yields

$$-2p^2n^2A^nB^2(v^2-a) + cA^n = 0, \quad (36)$$

$$pnA^nB^2(pn+1)(v^2-a) - bA^m = 0, \quad (37)$$

$$pnA^nB^2(pn-1)(v^2-a) = 0. \quad (38)$$

To solve (38), we have considered the case $pn - 1 = 0$. This yields

$$p = \frac{1}{n}. \quad (39)$$

Substituting (39) into (36) and (37) gives

$$A = \left(\frac{c}{b}\right)^{\frac{1}{m-n}}, \quad m > n > 1 \quad (40)$$

$$B = \sqrt{\frac{c}{2(v^2-a)}}. \quad (41)$$

Hence, the topological 1-soliton solution to the generalized phi-four Eq. (7) is given by

$$u(x,t) = \left\{\frac{c}{b}\tanh^2\left[\sqrt{\frac{c}{2(v^2-a)}}(x-vt)\right]\right\}^{\frac{1}{m-n}}, \quad (42)$$

which exists provided that $m = 3n$ as seen from equating the two values of p from (33) and (39), and $c(v^2 - a) > 0$ following to (41).

It is of interest to note that when $m = 3$, $n = l = 1$ and $b = c = -1$, the generalized phi-four Eq. (7) will be reduced to (1) and the corresponding solution (42) will have a similar form as the solution (2).

2.2. Variant II

In this section, we consider the variant II of the generalized phi-four equation with negative exponents (6):

$$(u^{-l})_{tt} - a(u^{-n})_{xx} - bu^{m} + cu^{-n} = 0. \tag{43}$$

The focus will be on searching the bright and dark soliton solutions to (43).

2.2.1. Bright solitons

The starting hypothesis for the solution to (43) is the same as in the variant I that is given by (8) and (9). Thus from the ansatz (8), we obtain

$$(u^{-l})_{tt} = \frac{p^2l^2v^2A^{-l}B^2}{\cosh^{-pl}\tau} - \frac{pl(pl-1)v^2A^{-l}B^2}{\cosh^{-pl+2}\tau}, \tag{44}$$

$$(u^{-n})_{xx} = \frac{p^2n^2A^{-n}B^2}{\cosh^{-pn}\tau} - \frac{pn(pn-1)A^{-n}B^2}{\cosh^{-pn+2}\tau}, \tag{45}$$

$$u^{-n} = \frac{A^{-n}}{\cosh^{-pn}\tau}, \tag{46}$$

$$u^{m} = \frac{A^{m}}{\cosh^{pm}\tau}. \tag{47}$$

Inserting the expressions (44)–(47) into (43) yields

$$\frac{p^2l^2v^2A^{-l}B^2}{\cosh^{-pl}\tau} - \frac{pl(pl-1)v^2A^{-l}B^2}{\cosh^{-pl+2}\tau} - \frac{ap^2n^2A^{-n}B^2}{\cosh^{-pn}\tau} + \frac{apn(pn-1)A^{-n}B^2}{\cosh^{-pn+2}\tau} - \frac{bA^{m}}{\cosh^{pm}\tau} + \frac{cA^{-n}}{\cosh^{-pn}\tau} = 0. \tag{48}$$

Equating the exponents of $1/\cosh^{-pn+2}\tau$ and $1/\cosh^{pm}\tau$ functions, we get

$$-pn + 2 = pm, \tag{49}$$

so that

$$p = \frac{2}{n+m}. \tag{50}$$

Next, equating the exponents $-pl$ and $-pn$ gives

$$-pl = -pn. \tag{51}$$

so that

$$l = n, \tag{52}$$

which is also obtained by equating the exponents $-pl + 2$ and $-pn + 2$. Now, noting that the functions $1/\cosh^{-pn+j}\tau$, with $j = 0{,}2$, are linearly independent (Adem et al., 2011), setting their coefficients to zero yields

$$p^2n^2A^{-n}B^2(v^2 - a) + cA^{-n} = 0, \tag{53}$$

$$-pn(pn-1)A^{-n}B^2(v^2 - a) - bA^{m} = 0. \tag{54}$$

Solving the above equations gives

$$A = \left(\frac{c(n-m)}{2nb}\right)^{\frac{1}{n+m}}, \quad n > m, \tag{55}$$

$$B = \frac{n+m}{2n}\sqrt{\frac{c}{a-v^2}}. \tag{56}$$

Thus, the 1-soliton solution of the generalized phi-four equation with negative exponents (43) is given by

$$u(x,t) = \left\{\frac{c(n-m)}{2nb}\sec h^2\left[\frac{n+m}{2n}\sqrt{\frac{c}{a-v^2}}(x-vt)\right]\right\}^{\frac{1}{n+m}}. \tag{57}$$

In view of (56), we clearly see that this solution exists provided that $c(a - v^2) > 0$. Finally, we would like to note that the solution (57) exists under the conditions $n + m > 0$, $n > m$ and $l = n$.

2.2.2. Dark solitons

Now, we are interested in finding the dark soliton solution for the considered generalized phi-four Eq. (43). To do this, we use an ansatz solution of the form (24) and (25). Thus from assumption (24), we obtain

$$(u^{-l})_{tt} = plA^{-l}B^2v^2\{(pl-1)\tanh^{-pl+2}\tau, -2pl\tanh^{-pl}\tau + (pl+1)\tanh^{-pl-2}\tau\} \tag{58}$$

$$(u^{-n})_{xx} = pnA^{-n}B^2\{(pn-1)\tanh^{-pn+2}\tau, -2pn\tanh^{-pn}\tau + (pn+1)\tanh^{-pn-2}\tau\} \tag{59}$$

$$u^{-n} = A^{-n}\tanh^{-pn}\tau, \tag{60}$$

$$u^{m} = A^{m}\tanh^{pm}\tau. \tag{61}$$

Substituting (58)–(61), (61) into (43), we have

$$plA^{-l}B^2v^2\{(pl-1)\tanh^{-pl+2}\tau - 2pl\tanh^{-pl}\tau + (pl+1)\tanh^{-pl-2}\tau\} - apnA^{-n}B^2\{(pn-1)\tanh^{-pn+2}\tau - 2pn\tanh^{-pn}\tau + (pn+1)\tanh^{-pn-2}\tau\} - bA^{m}\tanh^{pm}\tau + cA^{-n}\tanh^{-pn}\tau = 0. \tag{62}$$

From (62), equating the exponents $-pn + 2$ and pm gives

$$-pn + 2 = pm, \tag{63}$$

so that

$$p = \frac{2}{n+m}. \tag{64}$$

Again from (62), equating the exponents $-pl$ and $-pn$ gives

$$-pl = -pn, \tag{65}$$

that yields

$$l = n, \tag{66}$$

which is also obtained by equating the exponents' pairs $-pl + 2$ and $-pn + 2$, $-pl - 2$ and $-pn - 2$.

Setting the coefficients of the linearly independent functions $\tanh^{-pn+j}\tau$, where $j = -2{,}0{,}2$ to zero yields

$$pn(pn-1)A^{-n}B^2(v^2 - a) - bA^{m} = 0, \tag{67}$$

$$-2p^2n^2A^{-n}B^2(v^2 - a) + cA^{-n} = 0, \tag{68}$$

$$pnA^{-n}B^2(pn+1)(v^2 - a) = 0. \tag{69}$$

To solve (69), we have considered the case $pn + 1 = 0$. This yields

$$p = -\frac{1}{n}. \tag{70}$$

Note that in the sense of constructing the dark soliton solutions for Eq. (43), one need to have $p > 0$ for solitons to exist. This condition implies that $n < 0$ in Eq. (70). Substituting (70) into (67) and (68) gives

$$A = \left(\frac{c}{b}\right)^{\frac{1}{n+m}}, \tag{71}$$

$$B = \sqrt{\frac{c}{2(v^2 - a)}}. \tag{72}$$

Also from equating the two values of p from (64) and (70), one gets the condition

$$m = -3n. \tag{73}$$

Further, the free parameter A in (71) becomes

$$A = \left(\frac{c}{b}\right)^{-\frac{1}{2n}}. \tag{74}$$

Lastly, we can determine the topological 1-soliton solution to the generalized phi-four Eq. (43) with negative exponents as

$$u(x,t) = \left\{\sqrt{\frac{c}{b}}\tanh\left[\sqrt{\frac{c}{2(v^2 - a)}}(x - vt)\right]\right\}^{-\frac{1}{n}}. \tag{75}$$

which exist provided that $c(v^2 - a) > 0$ as seen from (72) and $n < 0$.

3. Conclusion

In this work, making use of solitary wave ansatze in terms of sec$h$ and tanh functions, respectively, solitary wave solution or bell-shaped soliton solutions and shock wave solution or kink-shaped soliton solutions are obtained for two variants of the generalized phi-four equations, including general values of the exponents and arbitrary model coefficients. The physical parameters in the soliton solutions are obtained as a function of the dependent model coefficients. Parametric conditions for the existence of envelope solitons have also been reported. We have found that the key factors, which determine the closed form solutions are the dependent exponents which can be positive or negative, and the model coefficients a, b and c. It should be noted that the existence of the resulting solutions is related on whether $c(v^2 - a) > 0$ or $c(a - v^2) > 0$. In view of the analysis, we clearly see that the solitary wave ansatz method is very efficient for solving this very interesting nonlinear equation after proving its consistency to a wide range of NLPDEs with constant and time-dependent coefficients.

References

Ablowitz, M.J., Clarkson, P., 1992. Solitons, Nonlinear Evolution Equations and Inverse Scattering. Cambridge University Press, Cambridge, England.

Adem, A., Khalique, C., Biswas, A., 2011. Solutions of Kadomtsev–Petviashvili equation with power law nonlinearity in 1 + 3 dimensions. Math. Meth. Appl. Sci. 34 (5), 532–543.

Biswas, A., Triki, H., Hayat, T., Olaleru, J., 2011. 1-Soliton solution of the generalized Burgers equation with generalized evolution, Appl. Math. Comput. 217 (24), 10289–10294.

Biswas, A., 2008a. 1-Soliton solution of the $K(m, n)$ equation with generalized evolution. Phys. Lett. A 372, 4601–4602.

Biswas, A., 2008b. 1-soliton solution of (1 + 2) dimensional nonlinear Schrödinger's equation in dual-power law media. Phys. Lett. A 372, 5941–5943.

Biswas, A., 2009a. Solitary wave solution for the generalized Kawahara equation. Appl. Math. Lett. 22, 208–210.

Biswas, A., 2009b. 1-Soliton solution of the $B(m, n)$ equation with generalized evolution. Commun. Nonlinear Sci. Numer. Simul. 14, 3226–3229.

Du, M., Chan, A., Chui, C., 1995. A novel approach to solving the nonlinear Schrödinger equation by the coupled amplitude-phase formulation. IEEE J. Quant. Electron. 31 (1), 177–182.

Esfahani, A., 2011. Solitary waves for the perturbed nonlinear Klein–Gordon equation. Appl. Math. Lett. 24 (2), 204–209.

Li, X., Wang, M., 2007. A sub-ODE method for finding exact solutions of a generalized KdV–mKdV equation with high-order nonlinear terms. Phys. Lett. A 361, 115–118.

Malfliet, W., 1992. Solitary wave solutions of nonlinear wave equations. Am. J. Phys. 60 (7), 650–654.

Nakkeeran, K., 2002. Bright and dark optical solitons in fiber media with higher-order effects. Chaos Solitons Fract. 13, 673–679.

Palacios, S., 2004. Two simple ansätze for obtaining exact solutions of high dispersive nonlinear Schrödinger equations. Chaos Solitons Fract. 19, 203–207.

Palacios, S., Fernandez-Diaz, J., 2000. Black optical solitons for media with parabolic nonlinearity law in the presence of fourth order dispersion. Opt. Commun. 178, 457–460.

Palacios, S., Guinea, A., Fernandez-Diaz, J., Crespo, R., 1999. Dark solitary waves in the nonlinear Schrödinger equation with third order dispersion, self-steepening, and self-frequency shift. Phys. Rev. E 60 (1), R45–R47.

Saha, M., Sarma, A., Biswas, A., 2009. Dark optical solitons in power law media with time-dependent coefficients. Phys. Lett. A 373, 4438–4441, HUNA.

Triki, H., Wazwaz, A.M., 2009a. Sub-ODE method and soliton solutions for the variable-coefficient mKdV equation. Appl. Math. Comput. 214, 370–373.

Triki, H., Wazwaz, A.M., 2009b. Bright and dark soliton solutions for a $K(m, n)$ equation with t-dependent coefficients. Phys. Lett. A 373, 2162–2165.

Triki, H., Wazwaz, A.M., 2010. Soliton solutions for (2 + 1)-dimensional and (3 + 1)-dimensional $K(m, n)$ equations. Appl. Math. Comput. 217, 1733–1740.

Triki, H., Wazwaz, A.M., 2011a. Dark solitons for a combined potential KdV and Schwarzian KdV equations with t-dependent coefficients and forcing term. Appl. Math. Comput. 217, 8846–8851.

Triki, H., Wazwaz, A.M., 2011b. 1-soliton solution of the $ZK(m, n, k)$ equation with generalized evolution and time-dependent coefficients. Nonlinear Anal. Ser. B: Real World Appl. 12 (5), 2822–2825.

Triki, H., Ismail, M.S., 2010. Soliton solutions of a BBM(m, n) equation with generalized evolution. Appl. Math. Comput. 217 (1), 48–54.

Triki, H., Taha, T., 2012. Solitary wave solutions for a higher order nonlinear Schrodinger equation. Math. Comput. Simul. 82 (7), 1333–1340.

Wazwaz, A.M., 2010. Integrable (2 + 1)-dimensional and (3 + 1)-dimensional breaking soliton equations. Phys. Scr. 035005 (5pp).

Wazwaz, A.M., 2005a. Generalized forms of the phi-four equation with compactons, solitons and periodic solutions. Math. Comput. Simul. 69, 580–588.

Wazwaz, A.M., 2005b. Generalized forms of the Phi-Four equation with compactons, solitons and periodic solutions. Math. Comput. Simul. 69 (5-6), 580–588.