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Original article

Construction of traveling and solitary wave solutions for wave propagation in nonlinear low-pass electrical transmission lines

Aly R. Seadawy [a,*], Mujahid Iqbal [b], Dumitru Baleanu [c,d,e]

[a] Mathematics Department, Faculty of Science, Taibah University, Al-Madinah Al-Munawarah, Saudi Arabia
[b] Faculty of Science, Jiangsu University, Zhenjiang, Jiangsu 212013, PR China
[c] Department of Mathematics, Cankaya University, Ankara, Turkey
[d] Department of Medical Research, China Medical University Hospital, China Medical University, Taichung, Taiwan
[e] Institute of Space Sciences, 077125 Magurele, Romania

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ABSTRACT

In this study, our aim to constructed the traveling and solitary wave solutions for nonlinear evolution equation describe the wave propagation in nonlinear low-pass electrical transmission lines by implemented the modification of mathematical method. We obtained the new and more general solutions in rational, trigonometric, hyperbolic type which represent to kink and anti-kink wave solitons, bright-dark solitons and traveling waves. The physical interpretation of some results demonstrated by graphically with symbolic computation. We are hopefully determined results have numerous applications in optical fiber, geophysics, fluid dynamics, laser optics, engineering, and many other various kinds of applied sciences. The complete investigation prove that proposed technique is more reliable, efficient, straightforward, and powerful to investigate various kinds of nonlinear evolution equations involves in geophysics, fluid dynamics, nonlinear plasma, chemistry, biology, and field of engineering.
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1. Introduction

The investigations of nonlinear electrical transmission lines and its solitary wave solutions have marvelous applications in communication systems and electronic engineering such as distributing signal of cable television, computer network connection systems, data buses of high speed computer, mobile network systems, connection between transmitters and receivers of radio with their antennas, routing calls in trunk lines between telephone switching centers (Abdoulkary et al., 2013; Arshad et al., 2018; Seadawy, 2017a; Kumar et al., 2018; Aslan, 2016). Moreover, nonlinear electrical transmission lines are used as special medium to carrying the alternate current (AC) of frequency of radio (Abdoulkary et al., 2013). The NLETLs are also used to deliver the better way how to observe the nonlinear excitation be have under nonlinear media and modulate the features of exotic of new systems (Jaradat et al., 2018a,b,c,d; Alquran and Jaradat, 2018; Ali et al., 2019; Alquran et al., 2020; Pelap and Faye, 2005).

For demonstrated of solitary wave solutions for NLEEs describe the wave propagation in nonlinear low-pass electrical transmission lines (Abdoulkary et al., 2013; Arshad et al., 2018; Seadawy, 2017a; Kumar et al., 2018; Aslan, 2016), we consider as:

$$\frac{\delta^2 V(x,t)}{\delta t^2}+\alpha\frac{\delta^2 V^2(x,t)}{\delta t^2}-\gamma\frac{\delta^2 V^3(x,t)}{\delta t^2}+\delta^2\frac{\delta^2 V^2(x,t)}{\delta x^2}+\frac{\delta^4}{12}\frac{\delta^4 V^2(x,t)}{\delta x^4}=0. \qquad (1)$$

In Eq. (1), $\alpha,\gamma,\delta$are the real constants and $V(x,t)$represents to transmission line voltage. The variables x,trepresents to distance for propagation and slow time, respectively. The Eq. (1), is derived by applying the Kirchoff's laws are seen in Abdoulkary et al. (2013), in this current work, which are omitted due to shortness.

In previous years a huge research work have been done on nonlinear low-pass electrical transmission lines. Many researchers have been determined the various types of solutions for NLETLs by using the different techniques including jacobi elliptic technique, kudryashov technique, auxiliary equation technique, $(\frac{G'}{G})$-expansion method, extension technique of tanh function, the technique of racatti equation, modification of kudryashov technique, the technique of sine-Gorden and extension of sine-Gorden equation (Abdoulkary et al., 2013; Arshad et al., 2018; Seadawy,

* Corresponding author.
E-mail address: Aly742001@yahoo.com (A.R. Seadawy).

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2017a; Kumar et al., 2018; Aslan, 2016). The NLETLs are play important role in the investigation of propagation phenomena of electrical solitons, they are travel in nonlinear media of dispersion in the shape of voltage waves.

The nonlinear wave equations are describing the physical process in many areas of science and engineering include fiber optics, geophysics, chemistry, plasma physics, biology, fluid dynamics, laser physics and so on. The NLETLs are the nonlinear partial differential equations. Now a days it is a deals of huge interest to finding the soliton solutions for NLPDEs by implementing the various techniques. To determine the solutions for NLPDEs play key role and deliver the important information to know the applications and its mechanism. Further, this is very keen to know the concept of specific wave called soliton. John Scott Russel first time introduced a solitary wave in the past few decades (Russell, 1844). When solitary wave is discover after that, now its named solitons. Solitions have potential applications in various kinds of areas including soliton dynamics, fluid dynamic, fiber optics, adiabatic dynamic parameters, phenomena of industrial, biomedical problems, engineering and many various kinds of applied sciences to due to their important stability properties. The nonlinear electrical transmission lines are the example in physics. In previous few decades, a lot of researchers and mathematician introduced a various kinds of methods to determine the solitonic solutions for these NLEEs. The name of some important techniques as, Backlund transform method, Hirota bilinear transformation, the Darboux transform method, the technique of exp-function, the jacobian function expansion technique, the trial equation technique, Simple equation technique, modification of fan-Sub equation technique, extend mapping method, the technique of sinh-cosh, the extension of auxiliary equation technique, the extension of direct algebraic technique, modification of extended auxiliary equation technique, Racatti equation mapping method (Juan et al., 2007; Hirota, 1971; Zheng Yi et al., 2006; Naher et al., 2012; Seadawy and Manafian, 2018; Zhang and Xia, 2006; Seadawy and El-Rashidy, 2013; Seadawy and Lu, 2016; Seadawy, 2012; Iqbal et al., 2018a, b; Seadawy et al., 2019a; Iqbal et al., 2019a,b; Seadawy et al., 2020a; Seadawy, 2014a,b; Seadawy, 2015a,b; Seadawy, 2016a,b; Seadawy, 2017b; Seadawy and Wang, 2019; Seadawy and El-Rashidy, 2014; Lu et al., 2018a,b; Seadawy et al., 2020b; Seadawy et al., 2019d,e; Yaro et al., 2019). In this current work, our main purpose to find the solutions for NLETLs including bright-dark solitons, kink and anti-kink wave solitons and traveling wave by proposed technique (Seadawy et al., 2019f; Iqbal et al., 2019c; Seadawy et al., 2019b; Iqbal et al., 2020; Seadawy et al., 2019c; Seadawy et al., 2020c).

This work is organized as, introduction is explain in Section 1. The main feature of proposed method described in Section 2. In Section 3, determined the solutions for NLETLs with proposed technique. In Section 4, compare the obtained results with detail. In Section 5, explain the concluding summery of this article.

2. Algorithm of proposed method

The nonlinear PDEs in general form consider as:

$$E(V, V_t, V_x, V_{tt}, V_{xx}, V_{xt}, \ldots\ldots) = 0. \tag{2}$$

In Eq. (2), Erepresent to polynomial function in $V(x,t)$and its derivatives. The main features of proposed method explain as:

Step. 1 : We applying the linear transformations on Eq. (2) as:

$$V(x,t) = W(\zeta), \qquad \zeta = (\kappa x + \omega t). \tag{3}$$

The ODE for Eq. (2) obtain as:

$$H(W, W', W'', W''', \ldots\ldots) = 0. \tag{4}$$

In Eq. (4), Hrepresent to function of polynomial in $W(\zeta)$and its all derivative.

Step. 2 : The trial solution of Eq. (4), as:

$$W(\zeta) = \sum_{l=0}^{m} a_l \Psi(\zeta)^l + \sum_{l=-1}^{-m} b_{-l} \Psi(\zeta)^l + \sum_{l=2}^{m} c_2 \Psi(\zeta)^{l-2} \Psi\prime(\zeta) + \sum_{l=1}^{m} d_l \left(\frac{\Psi'(\zeta)}{\Psi(\zeta)}\right)^l. \tag{5}$$

Here a_l, b_l, c_l, d_l are constants which determined in later, $\Psi(\xi)$ and its derivatives satisfy the following auxiliary equation:

$$\left(\frac{d\Psi}{d\zeta}\right)^2 = \beta_1 \Psi^2(\zeta) + \beta_2 \Psi^3(\zeta) + \beta_3 \Psi^4(\zeta). \tag{6}$$

In Eq. (6), β_is are represents to real constants which determined to be later.

Step. 3 :We apply the homogeneous method on Eq. (4) to find the value of $m$in Eq. (5).

Step. 4 : Substitute Eq. (5) in Eq. (4), and combine each coefficients of $\Psi^l(\zeta)\Psi'^j(\zeta)(l = 1, 2, 3, \ldots.n : j = 0, 1)$, then each coefficient equate to zero and get a system of algebraic equations, after solving these system of equations using any symbolic computation and find the values of constant parameters.

Step 5 : Putting the values of obtain parameters and $\Psi(\zeta)$ into Eq. (5), then we find the required solutions for Eq. (2).

3. Application of proposed method

Here we apply proposed technique to determine the solutions for the NLETLs. We consider the traveling wave transformation as:

$$V(x,t) = W(\zeta), \qquad \zeta = (\kappa x + \omega t). \tag{7}$$

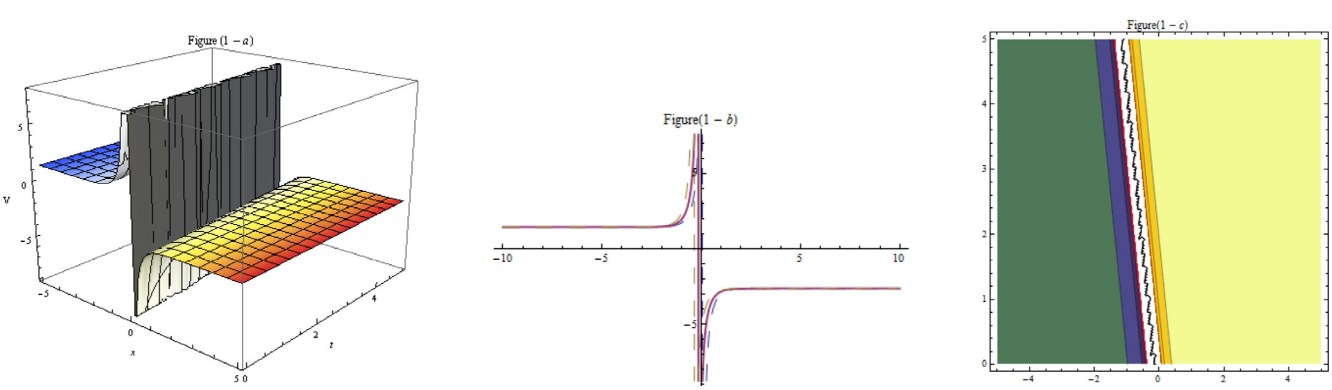

Fig. 1. (a) Three dimensional, (b) Two dimensional, (c) Contour plots for Eq. (11) represent to solitary wave while $\epsilon = 8, \zeta_0 = 0.3, \kappa = 2, \omega = 0.4, \alpha = 1, \gamma = -1, \delta = 1, \beta_1 = 2, \beta_2 = 4, \beta_3 = 2$.

Putting Eq. (7) in Eq. (1), the ODE obtain for Eq. (1), as:

$$12(\kappa^2\delta^2+\omega^2)W+12\alpha\omega^2W^2-12\gamma\omega^2W^3+\delta^4\kappa^4W''=0. \tag{8}$$

Balance the nonlinear term and derivative of highest order in Eq. (8), obtain $m=1$. General solution for Eq. (8), as:

$$W(\xi)=a_0+a_1\Psi(\zeta)+\frac{b_1}{\Psi(\zeta)}+d_1\frac{\Psi'(\zeta)}{\Psi(\zeta)}. \tag{9}$$

Putting Eq. (9) in Eq. (8), and combine each coefficients of $\Psi^l(\zeta)\Psi'^j(\zeta)(l=1,2,3,\dots n;j=0,1)$, and equate to each coefficient to zero. We get system of equations. System of equations are solve by using the Mathematica, the values of a_0, a_1, b_1, d_1 and frequency are obtain as:

Family-I

$$a_0=\frac{\alpha}{3\gamma},\ a_1=-\frac{\sqrt{\beta_3}\delta\kappa\sqrt{-(2\alpha^2+9\gamma)}}{6\sqrt{6}\gamma},\ b_1=0,\ d_1=-\frac{\delta\kappa\sqrt{-(2\alpha^2+9\gamma)}}{6\sqrt{6}\gamma},$$
$$\beta_1=-\frac{24\alpha^2}{\delta^2\kappa^2(2\alpha^2+9\gamma)},\ \omega=-\frac{3\sqrt{\gamma}\delta\kappa}{\sqrt{-2\alpha^2-9\gamma}}. \tag{10}$$

Putting the Eq. (10) in Eq. (9), then the solutions for Eq. (1), obtain as Figs. 1–14:

$$V_1(x,t)=\frac{24\alpha+\dfrac{\sqrt{6}\sqrt{\beta_1}\delta\kappa\sqrt{-2\alpha^2-9\gamma}\left(2\sqrt{\beta_1}\sqrt{\beta_3}\left(\epsilon\coth\left[\frac{1}{2}\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]+1\right)^2+\beta_2\epsilon\operatorname{csch}\left[\frac{1}{2}\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]^2\right)}{\beta_2\left(\epsilon\coth\left[\frac{1}{2}\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]+1\right)}}{72\gamma}, \tag{11}$$

$$V_2(x,t)=\frac{24\alpha+\dfrac{\sqrt{6}\sqrt{\beta_1}\delta\kappa\sqrt{-2\alpha^2-9\gamma}\left(\left(\eta+\cosh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]+\epsilon\sinh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]\right)^2-2\epsilon\left(\eta\cosh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]+1\right)\right)}{\left(\eta+\cosh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]\right)\left(\eta+\cosh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]+\epsilon\sinh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]\right)}}{72\gamma}, \tag{12}$$

$$\begin{aligned}V_3(x,t)=\ & -\tfrac{1}{36\gamma}(-12\alpha+\left(\sqrt{6}\sqrt{-2\alpha^2-9\gamma}\delta\epsilon\kappa\left(\eta\sqrt{p^2+1}\cosh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]+1\right.\right.\\ &+\sqrt{6}\sqrt{\beta_3}\delta\kappa\sqrt{-2\alpha^2-9\gamma}\left(-\frac{\epsilon\left(p+\sinh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]\right)}{\eta\sqrt{p^2+1}+\cosh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]}-1\right)\Bigg)\\ &-p\sinh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]\big)\sqrt{\beta_1}\big)/\left(\eta\sqrt{p^2+1}+\cosh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]\right.\\ &\left.\eta\sqrt{p^2+1}+\epsilon(p+\sinh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right])+\cosh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]\right).\end{aligned} \tag{13}$$

Family-II

$$a_0=\frac{\alpha}{3\gamma},\ a_1=\frac{\sqrt{\beta_3}\delta\kappa\sqrt{-(2\alpha^2+9\gamma)}}{6\sqrt{6}\gamma},\ b_1=0,\ d_1=\frac{\delta\kappa\sqrt{-(2\alpha^2+9\gamma)}}{6\sqrt{6}\gamma},$$
$$\beta_1=-\frac{24\alpha^2}{\delta^2\kappa^2(2\alpha^2+9\gamma)},\ \omega=\frac{3\sqrt{\gamma}\delta\kappa}{\sqrt{-2\alpha^2-9\gamma}}. \tag{14}$$

Putting Eq. (14) in Eq. (9), then solutions for Eq. (1), take as:

$$V_4(x,t)=-\frac{\dfrac{\sqrt{6}\sqrt{\beta_1}\delta\kappa\sqrt{-2\alpha^2-9\gamma}\left(2\sqrt{\beta_1}\sqrt{\beta_3}\left(\epsilon\coth\left[\frac{1}{2}\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]+1\right)^2+\beta_2\epsilon\operatorname{csch}\left[\frac{1}{2}\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]^2\right)}{\beta_2\left(\epsilon\coth\left[\frac{1}{2}\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]+1\right)}-24\alpha}{72\gamma}, \tag{15}$$

$$V_5(x,t)=\frac{24\alpha-\frac{\sqrt{6}\sqrt{\beta_1}\delta\kappa\sqrt{-2\alpha^2-9\gamma}\left(\left(\eta+\cosh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]+\epsilon\sinh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]\right)^2-2\epsilon\left(\eta\cosh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]+1\right)\right)}{\left(\eta+\cosh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]\right)\left(\eta+\cosh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]+\epsilon\sinh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]\right)}}{72\gamma}, \tag{16}$$

$$\begin{aligned}V_6(x,t)=\ &\tfrac{1}{36\gamma}(12\alpha+\left(\sqrt{6}\sqrt{-2\alpha^2-9\gamma}\delta\epsilon\kappa\left(\eta\sqrt{p^2+1}\cosh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]+1\right.\right.\\&-p\sinh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]\left.\right)\sqrt{\beta_1}\left.\right)/\left(\eta\sqrt{p^2+1}+\cosh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]\right.\\&\eta\sqrt{p^2+1}+\epsilon(p+\sinh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right])+\cosh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]\left.\right)+\\&\sqrt{6}\sqrt{\beta_3}\delta\kappa\sqrt{-2\alpha^2-9\gamma}\left(-\frac{\epsilon\left(p+\sinh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]\right)}{\eta\sqrt{p^2+1}+\cosh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]}-1\right)\Bigg).\end{aligned} \tag{17}$$

Family-III

$$a_0=0,\ a_1=-\frac{4\alpha\beta_3}{3\beta_2\gamma},\ b_1=d_1=0,\ \omega=\frac{\sqrt{\frac{3}{2}}\beta_2\sqrt{\gamma}\delta^2\kappa^2}{4\alpha\sqrt{\beta_3}},$$
$$\beta_1=-\frac{9\beta_2^2\gamma}{8\alpha^2\beta_3}-\frac{12}{\delta^2\kappa^2}. \tag{18}$$

Putting the Eq. (18) in Eq. (9), take solutions for Eq. (1), as:

$$V_7(x,t)=\frac{4\alpha\beta_1\beta_3\left(\epsilon\coth\left[\frac{1}{2}\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]+1\right)}{3\beta_2^2\gamma}, \tag{19}$$

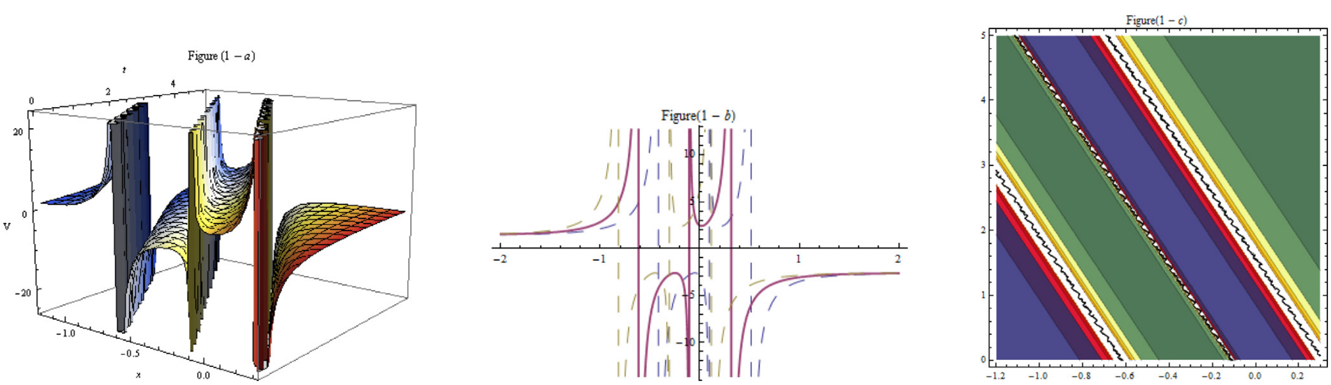

Fig. 2. (a) Three dimensional, (b) Two dimensional, (c) Contour plots for Eq. (12) represent to traveling wave while $\epsilon=8,\eta=-2,\zeta_0=0.3,\ \kappa=2,\omega=0.4,\alpha=1,\gamma=-1,\delta=1,\beta_1=2,\beta_2=4,\beta_3=2$.

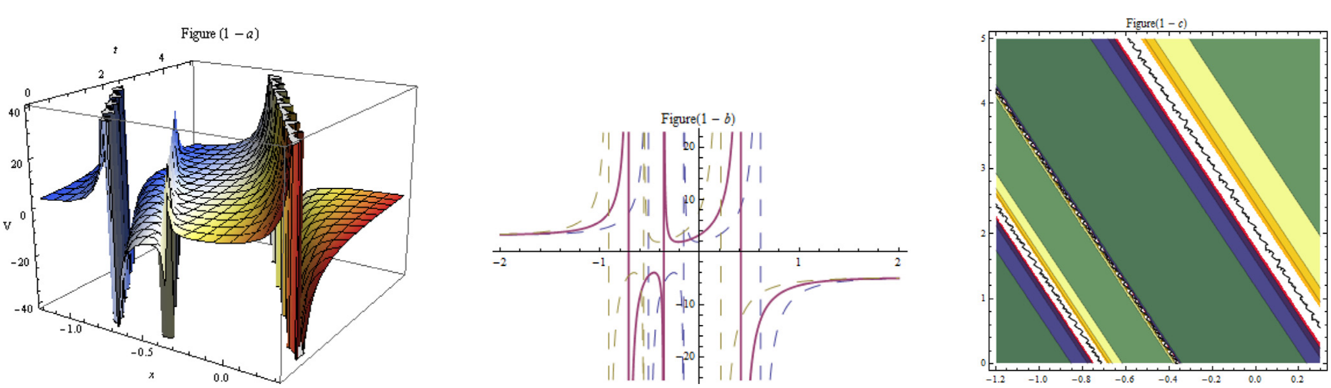

Fig. 3. (a) Three dimensional, (b) Two dimensional, (c) Contour plots for Eq. (13) represent to traveling wave while $\epsilon=8,\eta=-2,\ \zeta_0=0.3,\kappa=2,\omega=0.4,\alpha=1,\gamma=-1,P=0.8,\delta=1,\beta_1=2,\beta_2=4,\beta_3=2$.

$$V_8(x,t)=\frac{2\alpha\sqrt{\frac{\beta_1}{\beta_3}}\beta_3\left(\frac{\epsilon\sinh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]}{\eta+\cosh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]}+1\right)}{3\beta_2\gamma}, \tag{20}$$

$$V_9(x,t)=-\frac{4\alpha\beta_3\left(-\frac{\epsilon\left(p+\sinh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]\right)}{\eta\sqrt{p^2+1}+\cosh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]}-1\right)}{3\beta_2\gamma}. \tag{21}$$

Family-IV

$$a_0=\frac{3\alpha^2\beta_3\gamma\delta^2\kappa^2\left(\beta_2^2\delta^2\kappa^2-64\beta_3\right)+A}{6\alpha\beta_3\gamma^2\delta^2\kappa^2\left(\beta_2^2\delta^2\kappa^2-96\beta_3\right)},\ a_1=\frac{2\left(\alpha^2\beta_2^2\beta_3\gamma\delta^4\kappa^4+A\right)}{3\alpha\beta_2\gamma^2\delta^2\kappa^2\left(\beta_2^2\delta^2\kappa^2-96\beta_3\right)},\ b_1=d_1=0,$$
$$\omega=-\frac{1}{8}\sqrt{\frac{3\beta_3\gamma\delta^2\kappa^2\left(\beta_2^2\delta^2\kappa^2\left(5\alpha^2+18\gamma\right)-192\beta_3\left(2\alpha^2+9\gamma\right)\right)-3A}{\beta_3^2\left(2\alpha^2+9\gamma\right)^2}},$$
$$\beta_1=\frac{3\left(3\beta_2^2\beta_3\gamma\delta^4\kappa^4\left(\alpha^2+4\gamma\right)\left(\alpha^2+9\gamma\right)+64\alpha^2\beta_3^2\gamma\delta^2\kappa^2\left(2\alpha^2+9\gamma\right)+A\left(\alpha^2+3\gamma\right)\right)}{16\beta_3^2\gamma\delta^4\kappa^4\left(2\alpha^2+9\gamma\right)^2}, \tag{22}$$

where

$$A=\sqrt{3}\sqrt{\alpha^2\beta_2^2\beta_3^2\gamma^2\delta^6\kappa^6\left(3\beta_2^2\delta^2\kappa^2(\alpha^2+4\gamma)-128\beta_3(2\alpha^2+9\gamma)\right)}.$$

Putting the Eq. (22) in Eq. (9), then solutions for Eq. (1), as:

$$V_{11}(x,t)=\frac{\frac{3\alpha^2\beta_3\gamma\delta^2\kappa^2\left(\beta_2^2\delta^2\kappa^2-64\beta_3\right)+A}{\beta_3}-\frac{2\sqrt{\frac{\beta_1}{\beta_3}}\left(\alpha^2\beta_2^2\beta_3\gamma\delta^4\kappa^4+A\right)\left(\frac{\epsilon\sinh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]}{\eta+\cosh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]}+1\right)}{\beta_2}}{6\alpha\gamma^2\delta^2\kappa^2\left(\beta_2^2\delta^2\kappa^2-96\beta_3\right)}, \tag{24}$$

$$V_{12}(x,t)=\frac{\frac{3\alpha^2\beta_3\gamma\delta^2\kappa^2\left(\beta_2^2\delta^2\kappa^2-64\beta_3\right)+A}{\beta_3}+\frac{4\left(\alpha^2\beta_2^2\beta_3\gamma\delta^4\kappa^4+A\right)\left(-\frac{\epsilon\left(p+\sinh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]\right)}{\eta\sqrt{p^2+1}+\cosh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]}-1\right)}{\beta_2}}{6\alpha\gamma^2\delta^2\kappa^2\left(\beta_2^2\delta^2\kappa^2-96\beta_3\right)}. \tag{25}$$

Family-V

$$a_0=\frac{\alpha\omega^2-\sqrt{\omega^4\left(\alpha^2+4\gamma\right)+4\gamma\delta^2\kappa^2\omega^2}}{2\gamma\omega^2},\ a_1=\frac{\sqrt{\beta_3}\delta^2\kappa^2}{\sqrt{6}\sqrt{\gamma}\omega},\ b_1=d_1=0,$$
$$\beta_1=\frac{6\left(-\alpha\sqrt{\omega^4\left(\alpha^2+4\gamma\right)+4\gamma\delta^2\kappa^2\omega^2}+\alpha^2\omega^2+4\gamma\left(\delta^2\kappa^2+\omega^2\right)\right)}{\gamma\delta^4\kappa^4}, \tag{26}$$
$$\beta_2=\frac{2\sqrt{\frac{2}{3}}\sqrt{\beta_3}\left(\alpha\omega^2-3\sqrt{\omega^4\left(\alpha^2+4\gamma\right)+4\gamma\delta^2\kappa^2\omega^2}\right)}{\sqrt{\gamma}\delta^2\kappa^2\omega}.$$

$$V_{10}(x,t)=\left(-4A\beta_1\beta_3\left(\epsilon\coth\left[\tfrac{1}{2}\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]+1\right)+3\alpha^2\beta_2^4\beta_3\gamma\delta^4\kappa^4+\beta_2^2\left(A-4\alpha^2\beta_3^2\gamma\delta^2\kappa^2\left(\beta_1\delta^2\kappa^2\left(\epsilon\coth\left[\tfrac{1}{2}\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]+1\right)+48\right)\right)\right)\Big/ 6\alpha\beta_2^2\beta_3\gamma^2\delta^2\kappa^2\left(\beta_2^2\delta^2\kappa^2-96\beta_3\right), \tag{23}$$

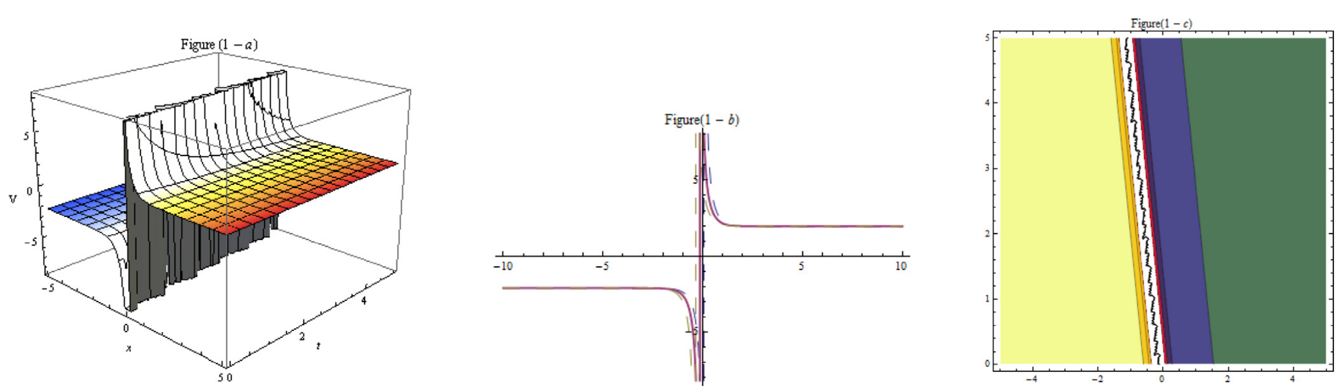

Fig. 4. (a) Three dimensional, (b) Two dimensional, (c) Contour plots for Eq. (15) represent to solitary wave while $\epsilon=8, \zeta_0=0.3, \kappa=2, \omega=0.4, \alpha=1, \gamma=-1, \delta=1, \beta_1=2, \beta_2=4, \beta_3=2$.

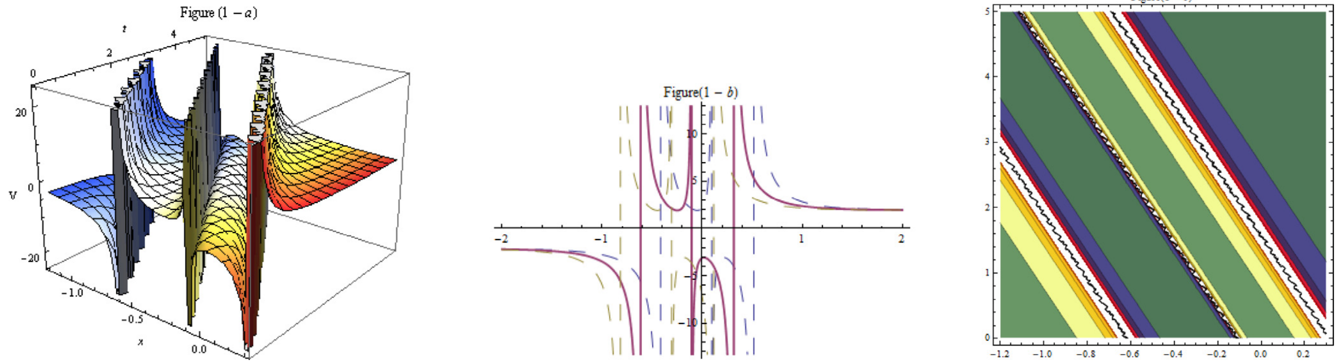

Fig. 5. (a) Three dimensional, (b) Two dimensional, (c) Contour plots for Eq. (16) represent to traveling wave while $\epsilon=8, \eta=-2, \zeta_0=0.3, \kappa=2, \omega=0.4, \alpha=1, \gamma=-1, \delta=1, \beta_1=2, \beta_2=4, \beta_3=2$.

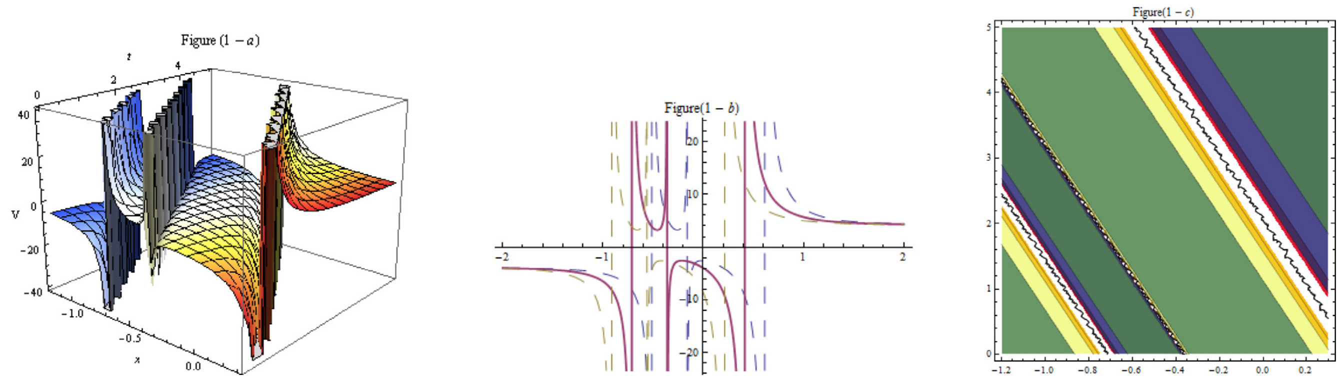

Fig. 6. (a) Three dimensional, (b) Two dimensional, (c) Contour plots for Eq. (17) represent to traveling wave while $\epsilon = 8, \eta = -2, \zeta_0 = 0.3, \kappa = 2, \omega = 0.4, \alpha = 1, \gamma = -1, P = 0.8, \delta = 1, \beta_1 = 2, \beta_2 = 4, \beta_3 = 2$.

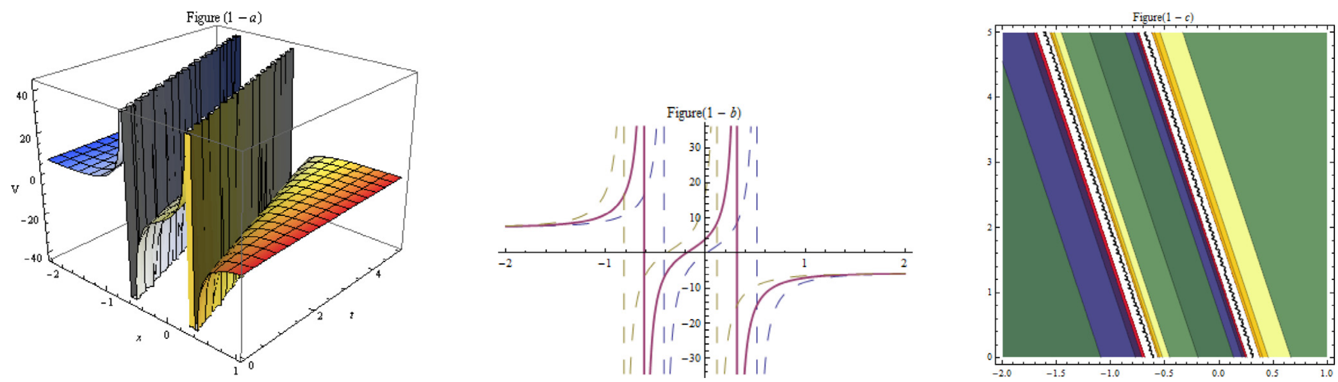

Fig. 7. (a) Three dimensional, (b) Two dimensional, (c) Contour plots for Eq. (20) represent to traveling wave while $\epsilon = -8, \eta = -2, \zeta_0 = 0.3, \kappa = 2, \omega = 0.4, \alpha = 0.5, \gamma = 0.2, \beta_1 = 2, \beta_2 = 4, \beta_3 = 2$.

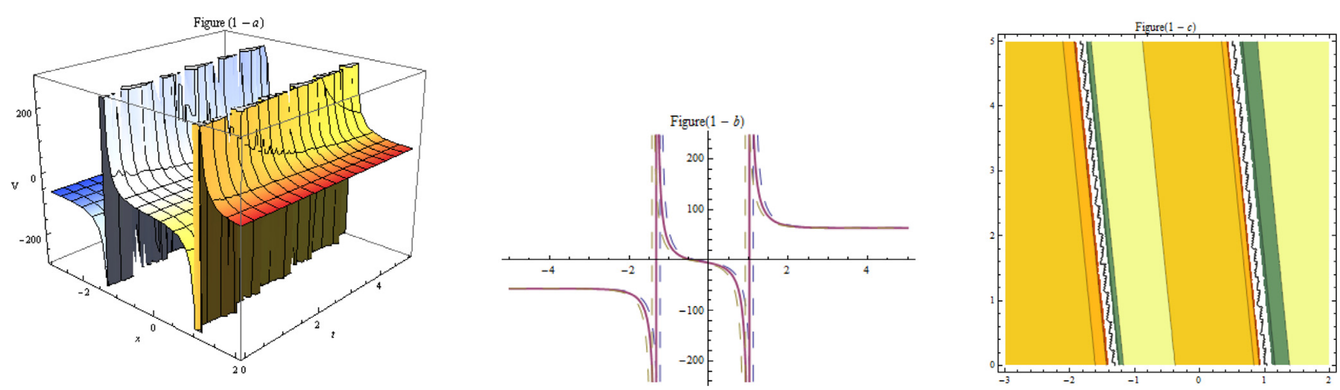

Fig. 8. (a) Three dimensional, (b) Two dimensional, (c) Contour plots for Eq. (21) represent to traveling wave while $\epsilon = 18, \eta = -8, \zeta_0 = 0.3, \kappa = 2, \omega = 0.2, \alpha = 0.5, \gamma = 0.2, p = 1.4, \beta_1 = 2, \beta_2 = 4, \beta_3 = 2$.

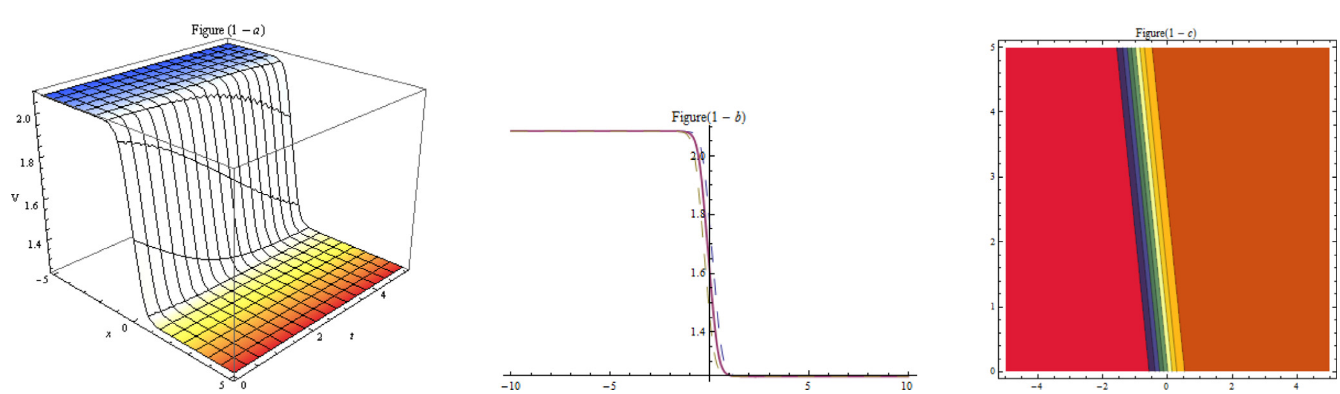

Fig. 9. (a) Three dimensional, (b) Two dimensional, (c) Contour plots for Eq. (23) represent to kink wave soliton while $\epsilon = -8, \eta = 2, \zeta_0 = 0.3, \kappa = 4, \omega = .8, \alpha = 1.5, \gamma = 0.3, \delta = 0.2, A = 1, \beta_1 = 2, \beta_2 = 4, \beta_3 = 2$.

Putting the Eq. (25) in Eq. (9), then solutions for Eq. (1), as:

$$V_{13}(x,t) = \frac{\alpha\omega^2 - \sqrt{\omega^4(\alpha^2+4\gamma)+4\gamma\delta^2\kappa^2\omega^2}}{2\gamma\omega^2} - \frac{\beta_1\sqrt{\beta_3}\delta^2\kappa^2\left(\epsilon\coth\left[\frac{1}{2}\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]+1\right)}{\sqrt{6}\beta_2\sqrt{\gamma}\omega}, \tag{27}$$

Family-VI

$$a_0 = 0,\ a_1 = \frac{\sqrt{\beta_3}\delta^2\kappa^2}{\sqrt{6}\sqrt{\gamma}\omega},\ b_1 = d_1 = 0,\ \beta_1 = -\frac{12\left(\delta^2\kappa^2+\omega^2\right)}{\delta^4\kappa^4},\ \beta_2 = -\frac{4\sqrt{\frac{2}{3}}\alpha\sqrt{\beta_3}\omega}{\sqrt{\gamma}\delta^2\kappa^2}. \tag{30}$$

Putting the Eq. (29) in Eq. (9), then solutions for Eq. (1), as:

$$V_{14}(x,t) = -\frac{6\sqrt{\omega^4(\alpha^2+4\gamma)+4\gamma\delta^2\kappa^2\omega^2} - 6\alpha\omega^2 + \sqrt{6}\sqrt{\beta_1}\sqrt{\gamma}\delta^2\kappa^2\omega\left(\frac{\epsilon\sinh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]}{\eta+\cosh\left(\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right)}+1\right)}{12\gamma\omega^2}, \tag{28}$$

$$V_{15}(x,t) = \frac{-3\sqrt{\omega^4(\alpha^2+4\gamma)+4\gamma\delta^2\kappa^2\omega^2} + 3\alpha\omega^2 + \sqrt{6}\sqrt{\beta_3}\sqrt{\gamma}\delta^2\kappa^2\omega\left(-\frac{\epsilon\left(p+\sinh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]\right)}{\eta\sqrt{p^2+1}+\cosh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]}-1\right)}{6\gamma\omega^2}. \tag{29}$$

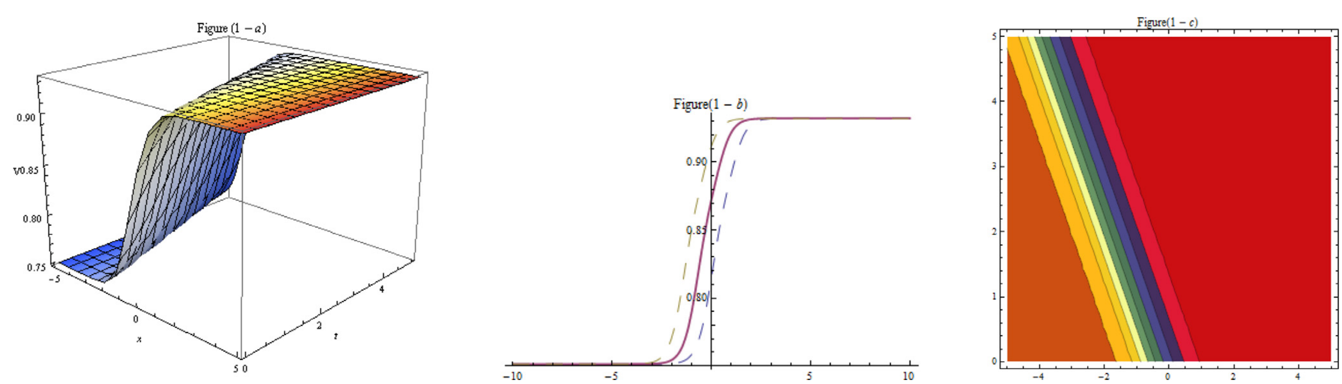

Fig. 10. (a) Three dimensional, (b) Two dimensional, (c) Contour plots for Eq. (24) represent to anti-kink wave soliton while $\epsilon = -8, \eta = 2, \zeta_0 = 0.3, \kappa = 2, \omega = 1.4, \alpha = 5, \gamma = 2, \delta = 0.2, p = 0.8, A = 1, \beta_1 = 2, \beta_2 = 4, \beta_3 = 2$,.

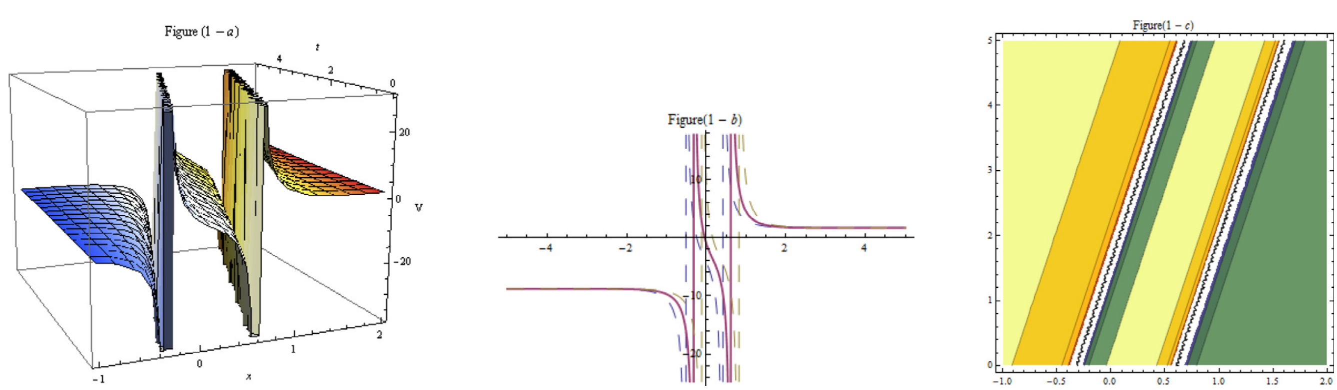

Fig. 11. (a) Three dimensional, (b) Two dimensional, (c) Contour plots for Eq. (27) represent to traveling wave while $\epsilon = 18, \eta = -8, \zeta_0 = 0.3, \kappa = 0.2, \omega = 4, \alpha = 0.3, \gamma = 0.5, \delta = 0.4, \beta_1 = 2$.

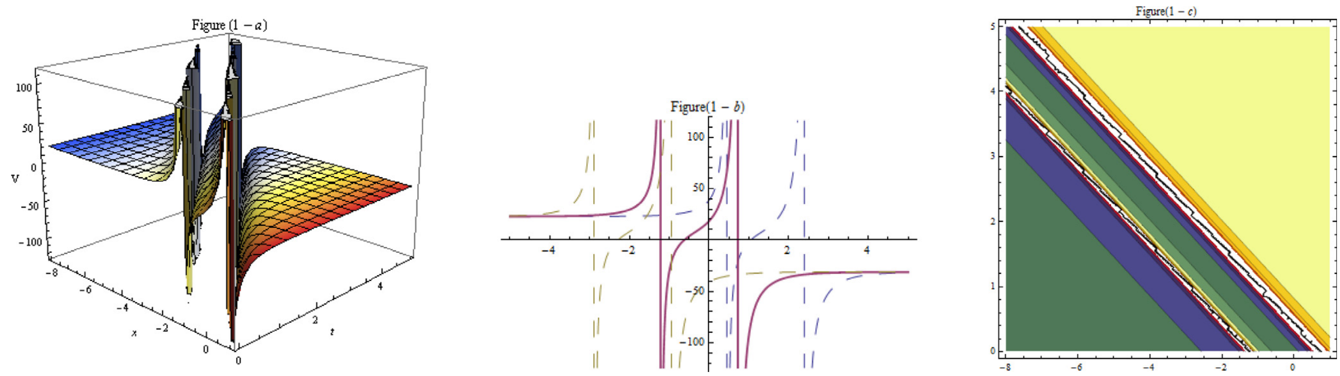

Fig. 12. (a) Three dimensional, (b) Two dimensional, (c) Contour plots for Eq. (28) represent to traveling wave while $\beta_1 = 2, \beta_3 = 4, \epsilon = -8, \eta = -2, \zeta_0 = 0.3, \kappa = 1.2, \omega = 2, \alpha = 3, \gamma = 1, \delta = 2.4, p = 0.9$.

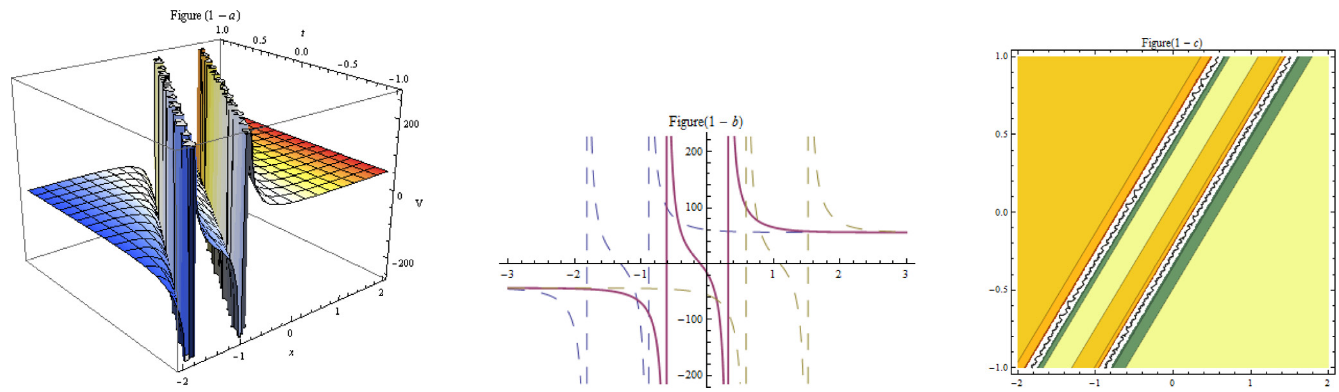

Fig. 13. (a) Three dimensional, (b) Two dimensional, (c) Contour plots for Eq. (31) represent to traveling wave while $\beta_1 = 2, \epsilon = 8, \eta = -2, \zeta_0 = 0.3, \kappa = 2, \omega = -2.4, \gamma = 0.2, \delta = 2.4$.

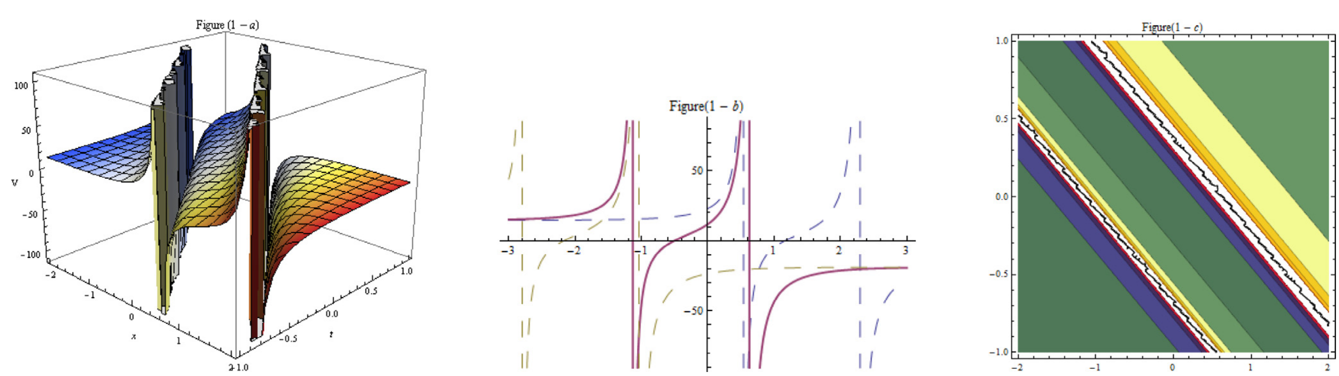

Fig. 14. (a) Three dimensional, (b) Two dimensional, (c) Contour plots for Eq. (28) represent to traveling wave while $\beta_1 = 2, \beta_3 = 4, \epsilon = 8, \eta = -2, \zeta_0 = 0.3, \kappa = 1.2, \omega = 2, \gamma = 0.4, \delta = 1.5, p = 0.6$.

$$V_{16}(x,t) = -\frac{\beta_1\sqrt{\beta_3}\delta^2\kappa^2\left(\epsilon\coth\left[\frac{1}{2}\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]+1\right)}{\sqrt{6}\beta_2\sqrt{\gamma}\omega}, \quad (31)$$

$$V_{17}(x,t) = -\frac{\sqrt{\beta_1}\delta^2\kappa^2\left(\frac{\epsilon\sinh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]}{\eta+\cosh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]}+1\right)}{2\sqrt{6}\sqrt{\gamma}\omega}, \quad (32)$$

$$V_{18}(x,t) = \frac{\sqrt{\beta_3}\delta^2\kappa^2\left(-\frac{\epsilon\left(p+\sinh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]\right)}{\eta\sqrt{p^2+1}+\cosh\left[\sqrt{\beta_1}(\kappa x+\omega t+\zeta_0)\right]}-1\right)}{\sqrt{6}\sqrt{\gamma}\omega}. \quad (33)$$

4. Results and discussion

Here we explain and compare our obtained results with other which already have been determined before this work in previous literature by other various methods. In the current study, we have constructed new and more general results. The main features of this work is the general solution Eq. (5) with range of four parameters and having various structure. The constant parameters values $a_l, b_l, c_l, d_l$are combine by any symbolic computation then Eq. (6) has various kinds of solutions. We obtained new and more general results by the proposed method.

Now we make compare the obtained results with other methods. In past research many scholars have been found different kinds of solutions for NLETLs equation e.g., hyperbolic, elliptic, trigonometric, rational type including kink and anti-kink wave solitons, singular and combined bright-dark solitions, bell shaped, periodic wave by implemented the jacobi elliptic technique, kudryashov technique, auxiliary equation technique, $\left(\frac{G'}{G}\right)$-expansion method, extension technique of tanh function, the technique of racatti equation, modification of kudryashov technique, the technique of sine-Gorden and extension of sine-Gorden equation (Abdoulkary et al., 2013; Arshad et al., 2018; Seadawy, 2017a; Kumar et al., 2018; Aslan, 2016).

From the above detail discussion and comparison conclude that our constructed results are new and more general which have been not determined in the past literature. The complete investigation prove that proposed technique is more reliable, efficient, straightforward, and powerful to investigate various kinds of nonlinear evolution equations.

5. Conclusion

In this study, we proposed successfully modified mathematical method on NLETLs equation and demonstrated new results. The obtained results are new and more general like rational, trigonometric, hyperbolic type including kink and anti-kink wave, bright-dark solitons and traveling waves. The physical interpretation of some obtained results demonstrated by graphically with symbolic computation. The complete investigation prove that proposed technique is more reliable, efficient, straightforward, and powerful to investigate various kinds of nonlinear evolution equations involves in geophysics, fluid dynamics, nonlinear plasma, chemistry, biology, and field of engineering.We are hopefully determined results have numerous applications in optical fiber, geophysics, fluid dynamics, laser optics, engineering, and many other various kinds of applied sciences.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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