Journal of King Saud University – Science 35 (2023) 102748

Contents lists available at ScienceDirect

Journal of King Saud University – Science

journal homepage: www.sciencedirect.com

Original article

Some new constructions of minimal efficient circular nearly strongly balanced neighbor designs

Khadija Noreen [a], Talha Omer [b,*], Jamshaid ul Hassan [a], H.M. Kashif Rasheed [a], Rashid Ahmed [a]

[a] Department of Statistics, The Islamia University of Bahawalpur, Pakistan
[b] Department of Economics, Finance and Statistics, Jönköping International Business School, Jönköping University, Sweden

ARTICLE INFO

Article history:
Received 8 August 2022
Revised 15 May 2023
Accepted 2 June 2023
Available online 9 June 2023

Mathematics Subject Classification (2010):
05B05
62K10
62K05

Keywords:
Rule I
Rule II
Neighbor effects
CNSBNDs
CSBNDs
CBNDs

ABSTRACT

Neighbor designs are popular to control neighbor effects. Among neighbor designs, strongly balanced neighbor designs are important to estimate treatment effects and neighbor effects independently. Minimal circular strongly balanced neighbor designs (MCSBNDs) can be obtained only for odd v (number of treatments). For v even, minimal circular nearly strongly balanced neighbor designs are used which satisfied all conditions of MCSBNDs except that the treatment labeled as $(v-1)$ does not appear as its own neighbor. These designs can be converted directly in some other useful classes of neighbor designs. These designs are efficient to minimize the bias due to the neighbor effects.

1. Introduction

If response of a treatment (treatment effect) is affected by the treatment(s) applied in neighboring units then such neighbor effects become major source of bias, in estimating the treatment effects. This bias can be minimized with the use of neighbor balanced designs, see Azais (1987), Azais et al. (1993), Kunert (2000) and Tomar et al. (2005).

- A circular design in which every treatment appears once as neighbors with all others (excluding it) is called a minimal circular balanced neighbor design (MCBND). If it also appears as its own neighbor then it is called MCSBND. MCBNDs and MCSBNDs can only be obtained for v odd.
- A circular design is called minimal circular nearly SBND (MCNSBND) if each treatment appears once as neighbor with other (v-2) treatments exactly once and (i) appear twice with only one treatment, labeled as (v-1), (ii) appear once as neighbor with itself except the treatment labeled as (v-1) which does not appear as its own neighbor. For v even, MCNSBNDs should be used as the best alternate of the MCSBNDs.

Rees (1967) introduced MCBNDs in serology for v odd. Azais et al. (1993) constructed some CBNDs using border plots. Jaggi et al. (2006) constructed some partially BNDs. Nutan (2007), Kedia & Misra (2008), Ahmed et al. (2009) constructed generalized neighbor designs (GNDs). Iqbal et al. (2009) constructed some classes of CBNDs using cyclic shifts. Akhtar et al. (2010) constructed CBNDs for k = 5. Meitei (2010) constructed new series of (i) CBNDs and (ii) one-sided CBNDs. Ahmed and Akhtar (2011) constructed CBNDs for k = 6. Shehzad et al. (2011) constructed some CBNDs. Jaggi et al. (2018) described some methods to construct CBNDs and circular partially BNDs. Singh (2019) developed new series of universally optimal one-sided CBNDs. Meitei (2020) presented a new series of universally optimal one-sided CBND for k = 5. Salam et al. (2022) introduced MCNSBNDs for (i) $v = 8i + 4$,

* Corresponding author.
E-mail address: talha.omer@ju.se (T. Omer).

Peer review under responsibility of King Saud University.

Production and hosting by Elsevier

https://doi.org/10.1016/j.jksus.2023.102748

k = 4, (ii) $v = 10i + 6$, k = 5, (iii) $v = 12i + 8$, k = 6, (iv) $v = 2ik_1 + 2$, $k_1 = 4j$, $k_2 = 3$, (v) $v = 2ik_1 + 4$, $k_1 = 4j$, $k_2 = 4$, (vi) $v = 2ik_1 + 2$, $k_1 > 3$ and $k_2 = 3$, (vii) $v = 2ik_1 + 4$, $k_1 > 4$ and $k_2 = 4$, and (viii) $v = 2ik_1 + 6$, $k_1 > 5$ and $k_2 = 5$.

In this article, (i) a generator is developed which produces the MCNSBNDs in equal as well as in unequal block sizes, with smallest of size at least three, (ii) some generators are developed which produce the MCNSBNDs which can directly be converted into MCSBNDs and MCBNDs, in blocks of equal as well as in unequal sizes, where smallest block size should be at least six.

2. Method of construction

Iqbal (1991) introduced method of cyclic shifts (Rule I & II) to construct experimental designs of several types. Its construction procedures are described here for MCNSBNDs, MCSBNDs and MCBNDs.

2.1. Rule II to obtain MCNSBNDs

Let $S_j = [q_{j1}, q_{j2}, \ldots, q_{j(k-1)}]$ and $S_i = [q_{i1}, q_{i2}, \ldots, q_{i(k-2)}]$t be the sets, where $0 \le q_{ji} \le v$-2. If each of 0, 1, 2, …, v-2 appears once in S*, where $S^* = [q_{j1}, q_{j2}, \ldots, q_{j(k-1)}, (q_{j1} + q_{j2} + \ldots + q_{j(k-1)}) \bmod (v-1), (v-1)-q_{j1}, (v-1)-q_{j2}, \ldots, (v-1)-q_{j(k-1)}, (v-1)-[(q_{j1} + q_{j2} + \ldots + q_{j(k-1)}) \bmod (v-1)], q_{i1}, q_{i2}, \ldots, q_{i(k-2)}, (v-1)-q_{i1}, (v-1)-q_{i2}, \ldots, (v-1)-q_{i(k-2)}]$ then it is MCNSBND. In Rule II, at least one set will contain k-2 elements which will be expressed as $[q_1, q_2, \ldots, q_{(k-2)}]$t. Here 't' is just to specify the set containing k-2 elements.

Example 2.1. Following MCNSBND is constructed from S_1 = [4,5,6,7,9,10,11], S_2 = [0,1,3,8,13]t for v = 26, k_1 = 8 & k_2 = 7.

Take (v-1) blocks for every set of shifts to get the complete design through Rule II. Consider 0, 1, …, v-2 as 1st unit of each block. Obtain 2nd unit elements by adding 4 (mod (v-1)) to 1st unit elements, where 4 is the 1st element of S_1. Obtain 3rd unit elements by adding 5 (mod 25) to 2nd unit elements, where 5 is the 2nd element of S_1. Similarly add 6, 7, 9, 10 and 11, see Table 1.

For S_2, take (v-1) more blocks. Obtain the blocks as are taken from S_1 except one extra row containing (v-1) in its each cell, see Table 2.

2.2. Rule I to obtain MCSBNDs and MCBNDs

Let $S_j = [q_{j1}, q_{j2}, \ldots, q_{j(k-1)}]$ be i sets, where j = 1, 2,…, i and u = 1, 2,…, k-1. If S* contains each of:

- 1, 2, …, v-1 once and $1 \le q_{ju} \le v$-1 then design will be MCBND.
- 0, 1, 2, …, v-1 once and $0 \le q_{ju} \le v$-1 then design will be MCSBND.

Here S* contains:

i. All elements of S_j.
ii. Sum of all elements (mod v) in each of S_j.
iii. Complements of all elements in (i) and (ii). In Rule I, the complement of 'a' is 'v-a'.

Example 2.2. Following MCBND is constructed from S_1 = [4,5,6,7,9,10,11] and S_2 = [1,3,8] for v = 25, k_1 = 8 & k_2 = 4 using Rule I.

Take v blocks for every set of shifts to get the complete design through Rule I. Consider 0, 1, …, v-1 as 1st unit of each block. Obtain 2nd unit elements by adding 4 (mod 25) to 1st unit elements. Similarly add 5, 6, 7, 9, 10 and 11, see Table 3.

Take more 25 blocks for S_2 and obtain blocks as taken from S_1, see Table 4.

2.3. Efficiency of Separability

Divecha and Gondaliya (2014) derived following expression for the efficiency of Separability (Es) which is also applicable for MCNSBNDs.

$ES = \left[\frac{v\sqrt{v-1}-1}{v\sqrt{v-1}}\right] \times 100\%$, where v is the number of treatments.

MCNSBND possessing Es at least 70% is considered efficient to reduce bias due to neighbor effects.

3. Construction of MCNSBNDs and their Conversion into MCSBNDs and MCBNDs

Here, the procedure to obtain the sets of shifts from generators developed in Section 4 is described. Non-zero elements of generator 'A' are divided into the required number of groups such that sum of elements in each group is divisible by (v-1). Sets to generate

Table 1
Blocks obtained from S_1 = [4,5,6,7,9,10,11].

Blocks												
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
0	1	2	3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13	14	15	16
9	10	11	12	13	14	15	16	17	18	19	20	21
15	16	17	18	19	20	21	22	23	24	0	1	2
22	23	24	0	1	2	3	4	5	6	7	8	9
6	7	8	9	10	11	12	13	14	15	16	17	18
16	17	18	19	20	21	22	23	24	0	1	2	3
2	3	4	5	6	7	8	9	10	11	12	13	14

Blocks											
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
13	14	15	16	17	18	19	20	21	22	23	24
17	18	19	20	21	22	23	24	0	1	2	3
22	23	24	0	1	2	3	4	5	6	7	8
3	4	5	6	7	8	9	10	11	12	13	14
10	11	12	13	14	15	16	17	18	19	20	21
19	20	21	22	23	24	0	1	2	3	4	5
4	5	6	7	8	9	10	11	12	13	14	15
15	16	17	18	19	20	21	22	23	24	0	1

Table 2
Blocks obtained from S_2 = [0,1,3,8,13]t.

Blocks												
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
0	1	2	3	4	5	6	7	8	9	10	11	12
0	1	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12	13
4	5	6	7	8	9	10	11	12	13	14	15	16
12	13	14	15	16	17	18	19	20	21	22	23	24
0	1	2	3	4	5	6	7	8	9	10	11	12
25	25	25	25	25	25	25	25	25	25	25	25	25

Blocks											
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
13	14	15	16	17	18	19	20	21	22	23	24
13	14	15	16	17	18	19	20	21	22	23	24
14	15	16	17	18	19	20	21	22	23	24	0
17	18	19	20	21	22	23	24	0	1	2	3
0	1	2	3	4	5	6	7	8	9	10	11
13	14	15	16	17	18	19	20	21	22	23	24
25	25	25	25	25	25	25	25	25	25	25	25

Table 1 & 2 jointly present MCNSBND for v = 26, k_1 = 8 and k_2 = 7, using 50 blocks.

Table 3
Blocks obtained from S_1 = [4,5,6,7,9,10,11].

Blocks												
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
0	1	2	3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13	14	15	16
9	10	11	12	13	14	15	16	17	18	19	20	21
15	16	17	18	19	20	21	22	23	24	0	1	2
22	23	24	0	1	2	3	4	5	6	7	8	9
6	7	8	9	10	11	12	13	14	15	16	17	18
16	17	18	19	20	21	22	23	24	0	1	2	3
2	3	4	5	6	7	8	9	10	11	12	13	14

Blocks											
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
13	14	15	16	17	18	19	20	21	22	23	24
17	18	19	20	21	22	23	24	0	1	2	3
22	23	24	0	1	2	3	4	5	6	7	8
3	4	5	6	7	8	9	10	11	12	13	14
10	11	12	13	14	15	16	17	18	19	20	21
19	20	21	22	23	24	0	1	2	3	4	5
4	5	6	7	8	9	10	11	12	13	14	15
15	16	17	18	19	20	21	22	23	24	0	1

Table 4
Blocks obtained from S_2 = [1,3,8].

Blocks												
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
0	1	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12	13
4	5	6	7	8	9	10	11	12	13	14	15	16
12	13	14	15	16	17	18	19	20	21	22	23	24

Blocks											
14	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
13	14	15	16	17	18	19	20	21	22	23	24
14	15	16	17	18	19	20	21	22	23	24	0
17	18	19	20	21	22	23	24	0	1	2	3
0	1	2	3	4	5	6	7	8	9	10	11

Table 3 and Table 4 jointly present MCBND for v = 25, k_1 = 8 & k_2 = 4.

MCNSBNDs are obtained by deleting one value (any) from each group containing non-zero values. The group containing ‘0’ will remain unchanged.

MCNSBNDs which can directly be converted into MCSBNDs and MCBNDs are constructed for following cases. Here i (integer) > 0 and A will be selected from Section 4.

- **For equal block sizes**
  - (i) $v = 2(i + 1)k-4$, $k > 5$. Divide the non-zero values of selected A into i groups each of k elements. Last will contain the remaining k-2 values.
- **For two different block sizes**
  - (i) $v = 2ik_1 + 2k_2-4$, $k_1 > k_2 > 5$. Divide the non-zero values of selected A into i groups each of k_1 elements. Last will contain the remaining k_2-2 values.
  - (ii) $v = 2ik_1 + 4k_2-4$, $k_1 > k_2 > 5$. Divide the non-zero values of selected A into i groups each of k_1 elements and one group of k_2 elements. Last will contain the remaining k_2-2 values.
- **For three different block sizes**
  - (i) $v = 2ik_1 + 2k_2 + 2k_3-4$, $k_1 > k_2 > k_3 > 5$. Divide the non-zero values of selected A into i groups each of k_1 elements and one group of k_2 elements. Last will contain the remaining k_3-2 values.
  - (ii) $v = 2ik_1 + 4k_2 + 2k_3-4$, $k_1 > k_2 > k_3 > 5$. Divide the non-zero values of selected A into i groups each of k_1 elements and two groups of k_2 elements. Last will contain the remaining k_3-2 values.
  - (iii) $v = 2ik_1 + 2k_2 + 4k_3-4$, $k_1 > k_2 > k_3 > 5$. Divide the non-zero values of selected A into i groups each of k_1 elements, one group of k_2 elements and one of k_3 elements. Last will contain the remaining k_3-2 values.
  - (iv) $v = 2ik_1 + 4k_2 + 4k_3-4$, $k_1 > k_2 > k_3 > 5$. Divide the non-zero values of selected A into i groups each of k_1 elements, two groups of k_2 elements and one of k_3 elements. Last will contain the remaining k_3-2 values.

4. Generator to generate MCNSBNDs which cannot be converted directly into MCSBNDs and MCBNDs

Generator 4.1: A = [0, 1, 2, ..., m] produces sets of shifts to obtain MCNSBNDs for every block sizes with smallest of size at least three, where $m = (v-2)/2$. The designs obtained from generator 4.1 cannot be converted directly into MCSBNDs and MCBNDs.

Example 4.1.1. S_1 = [3,5,6,7,8] and S_2 = [0,1,2,4]t produce MCNSBND for v = 20 & k = 6 with Es = 0.7415.

Example 4.1.2. S_1 = [2,3,5,6,7,9] and S_2 = [0,1,4,8]t produce MCNSBND for v = 22, k_1 = 7 & k_2 = 6 with Es = 0.7837.

5. Generators to generate MCNSBNDs which can directly be converted into MCSBNDs and MCBNDs

According to the value of m, generators 'A' are developed here using the logic behind Rule II, where $m = (v-2)/2$. These generators produce the sets of shifts to obtain MCNSBNDs which can directly be converted into MCSBNDs and MCBNDs.

Generator 5.1: A = [0, 1, 2, ..., j-1, j + 1, j + 2, ..., m, v-j] produces sets of shifts to obtain MCNSBNDs for $m \equiv 0 (\text{mod } 8)$, $j = m/8$, $j \geq 1$.

Example 5.1. S_1 = [5,6,7,8,10,18,23,24], S_2 = [4,9,11,12,13,14,16,17], S_3 = [0,1,15,19,20,21,22]t obtained from A = [0,1,2,**46**,4,...,24] produce MCNSBND for v = 50 & k = 9 with Es = 0.8574.

Generator 5.2: A = [0, 1, 2, ..., 3j, 3j + 2, 3j + 3, ..., m-1, m + 1, v-(3j + 1)] produces sets of shifts to obtain MCNSBNDs for $m \equiv 1 (\text{mod } 8)$, $j = (m-1)/8$, $j \geq 1$.

Example 5.2. S_1 = [1,2,3,4,5,6,7,8,11,14], S_2 = [9,12,13,15,17,18,19,24], S_3 = [0,16,20,21,22, 23]t obtained from A = [0,1,2,...,9,**41**,11,12,...,24,**26**] produce MCNSBND for v = 52, k_1 = 11, k_2 = 9 & k_3 = 8 with Es = 0.8439.

Generator 5.3: A = [0, 1, 2, ..., 5j + 1, 5j + 3, 5j + 4, ..., m-1, m + 1, v-(5j + 2)] produces sets of shifts to obtain MCNSBNDs for $m \equiv 2 (\text{mod } 8)$, $j = (m-2)/8$, $j \geq 1$.

Example 5.3. S_1 = [2,3,4,5,6,7], S_2 = [1,8,9,11,13,15] and S_3 = [0,14,16,19,25]t obtained from A = [0,1,2,...,11,**25**,13,14,15,16,17,**19**] produce MCNSBND for v = 38 & k = 7 with Es = 0.8513.

Generator 5.4: A = [0, 1, 2, ..., m-1-j, m + 1-j, m + 2-j, ..., m, v-(m-j)] produces sets of shifts to obtain MCNSBNDs for $m \equiv 3 (\text{mod } 8)$, $j = (m-3)/8$, $j \geq 0$.

Example 5.4. S_1 = [1,3,4,5,6,7,9], S_2 = [0,2,8,13]t obtained from A = [0,1,2,...,9,**13**,11] produce MCNSBND for v = 24, k_1 = 8 & k_2 = 6 with Es = 0.7680.

Generator 5.5: A = [0, 1, 2, ..., j, j + 2, j + 3, ..., m-1, m + 1, v-(j + 1)] produces sets of shifts to obtain MCNSBNDs for $m \equiv 4 (\text{mod } 8)$, $j = (m-4)/8$, $j \geq 0$.

Example 5.5. S_1 = [1,3,4,5,6,7,11], S_2 = [0,8,9,10,23]t obtained from A = [0,1,**23**,3,4,5,6,7,8,9, 10,11,**13**] produce MCNSBND for v = 26, k_1 = 8 & k_2 = 7 with Es = 0.7581.

Generator 5.6: A = [0, 1, 2, ..., 3j + 1, 3j + 3, 3j + 4, ..., m, v-(3j + 2)] produces sets of shifts to obtain MCNSBNDs for $m \equiv 5 (\text{mod } 8)$, $j = (m-5)/8$, $j \geq 0$.

Example 5.6. S_1 = [3,4,6,10,11,12,13], S_2 = [0,1,2,7,8,9]t obtained from A = [0,1,2,3,4,**22**,6,7, ...,13] produce MCNSBND for v = 28 & k = 8 with Es = 0.8318.

Generator 5.7: A = [0, 1, 2, ..., 5j + 3, 5j + 5, 5j + 6, m, v-(5j + 4)] produces sets of shifts to obtain MCNSBNDs for $m \equiv 6 (\text{mod } 8)$, $j = (m-6)/8$, $j \geq 0$.

Example 5.7. S_1 = [1,2,3,4,6,7,8,13], S_2 = [0,5,10,11,12,20]t obtained from A = [0,1,2,...,8,**20**, 10,11,...,14] produce MCNSBND for v = 30, k_1 = 9 & k_2 = 8 with Es = 0.7963.

Generator 5.8: A = [0, 1, 2, ..., m-1-j, m + 1-j, m + 2-j, ..., m-1, m + 1, v-(m-j)] produces sets of shifts to obtain MCNSBNDs for $m \equiv 7 (\text{mod } 8)$, $j = (m-7)/8$, $j \geq 1$.

Example 5.8.1. S_1 = [1,2,3,4,5], S_2 = [7,8,9,10,11], S_3 = [0,6,12,13] obtained from A = [0,1,2,..., 13,**17**,**16**] produce MCNSBND for v = 32 & k = 6 with Es = 0.8404.

Catalogues are also presented in Appendices A–C.

6. Conversion of proposed MCNSBNDs into MCSBNDs and MCBNDs

Conversion 6.1: Considering the Rule II as Rule I, MCNSBNDs constructed in Section 5 for $v = 2ik-4$, $i > 1$, $k > 5$ can be converted into:

1. MCSBNDs for $v = 2ik-5$, $k_1 = k$, $k_2 = k-2$. For it, delete '0' from the set of shifts.
2. MCBNDs for $v = 2ik-5$, $k_1 = k$, $k_2 = k-3$. For it, delete '0' and one more value (any) from the set containing '0'.

Example 6.1. MCNSBND constructed in example 5.2.1 for v = 20 and k = 6 through S_1 = [1,2,3,7,10], S_2 = [0,5,6,8]t will be converted into:

(a) MCSBND for v = 19, k_1 = 6 & k_2 = 4, with S_1 = [1,2,3,7,10], S_2 = [5,6,8].
(b) MCSBND for v = 19, k_1 = 6 & k_2 = 3, with S_1 = [1,2,3,7,10], S_2 = [5,6].

7. Remarks

Salam et al. (2022) introduced MCNSBNDs for some specific cases of $3 \le k_2 \le 5$. In this article, generator is developed for MCNSBNDs in equal as well as in unequal block sizes, with smallest block size at least three. Some generators are developed MCNSBNDs for v even with smallest block size at least six and these designs can directly be converted into MCSBNDs and MCBNDs for v odd.

MCSBNDs require at least $v(v$-1) experimental units for v even while our proposed MCNSBNDs require $v(v$-1)/2 units. Our proposed designs lose $\frac{100}{v(v)}$% neighbor balance and save at least 50 % experimental material. Our designs possess Es at least 70% therefore, these are efficient to minimize bias due to neighbor effects.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgement

Authors are thankful to anonymous Reviewer for the valuable corrections.

Appendix A

Catalogue of MCNSBNDs for $6 \le k \le 8$ and $v \le 60$.

v	k	Sets of Shifts	Es
20	6	[5,3,15,7,3] + [1,8,10,0]t	0.7415
32	6	[1,2,3,4,5] + [7,8,9,10,11] + [13,12,6,0]t	0.8404
44	6	[21,2,3,4,7] + [5,9,10,35,11] + [1,15,19,13,20] + [14,17,12,0]t	0.8363
56	6	[12,4,25,1,10] + [14,26,21,16,11] + [17,20,15,27,8] + [9,7,19,13,5] + [6,31,18,0]t	0.8653
24	7	[6,2,3,4,13,11] + [8,9,5,1,0]t	0.8215
38	7	[1,2,3,4,5,6] + [19,17,10,25,11,14] + [7,13,9,8,0]t	0.8513
52	7	[24,2,3,4,5,6] + [26,9,23,11,12,13] + [22,16,17,19,18,20] + [15,21,1,14,0]t	0.8510
28	8	[1,2,3,4,22,7,6] + [8,10,11,12,13,0]t	0.8318
44	8	[22,2,3,4,5,6,10] + [15,7,11,1,13,14,9] + [17,18,19,20,12,0]t	0.8474
60	8	[1,25,3,4,5,6,7] + [9,29,48,27,13,15,20] + [24,26,21,14,19,22,23]+[17,18,12,10,2,0]t	0.8699

Appendix B

Catalogue of MCNSBNDs in two different block sizes.

v	k1	k2	Sets of Shifts	Es
22	7	6	[1,9,3,6,5,4] + [8,2,11,0]t	0.7837
36	7	6	[18,2,3,4,5,10] + [1,9,6,15,12,13] + [11,16,8,0]t	0.8027
50	7	6	[24,2,46,4,5,6] + [23,9,20,7,12,13] + [1,16,15,10,17,18] + [22,8,19,0]t	0.8140
24	8	6	[11,2,3,4,5,6,7] + [9,13,1,0]t	0.7680
40	8	6	[1,2,3,4,5,9,7] + [19.18,10,12,13,15,14] + [22,11,6,0]t	0.8996
56	8	6	[20,2,3,4,5,7,6] + [9,10,11,12,13,14,25] + [17,26,19,1,21,27,23]+[15,18,22,0]t	0.8418
26	8	7	[13,23,4,3,11,6,7] + [9,10,5,1,0]t	0.7581
42	8	7	[1,2,38,5,4,6,7] + [15,9,16,21,13,18,14] + [12,10,11,8,0]t	0.8320
58	8	7	[29,2,3,5,6,53,7] + [8,10,26,12,13,14,15]+[17,18,19,27,21,22,23] + [25,11,20,1,0]t	0.8650

Appendix C

Catalogue of MCNSBNDs in three different block sizes.

v	k1	k2	k3	Sets of shifts	Es
38	8	7	6	[1,2,3,4,5,6,7] + [17,10,25,11,19,14] + [8,16,13,0]t	0.8514
54	8	7	6	[18,2,3,4,5,6,7] + [9,10,11,12,13,20,15]+[36,24,19,14,22,21] + [1,25,27,0]t	0.8722
40	9	7	6	[19,2,18,4,5,6,8,7] + [10,11,12,13,14,15] + [22,16,1,0]t	0.8545
58	9	7	6	[1,22,3,53,6,5,7,8] + [1,29,27,25,14,15,16,17]+['9,12,21,2,24,23] + [26,20,11,0]t	0.8453
42	9	8	6	[1,2,38,5,4,6,7,8] + [18,9,21,13,14,16,15] + [10,19,12,0]t	0.8575
60	9	8	6	[25,16,3,22,5,23,7,8] + [10,11,12,13,14,21,2,17]+[28,20,15,29,27,24,1] + [6,19,4,30]t	0.8779
44	9	8	7	[7,2,3,17,4,6,1,5] + [19,10,11,21,14,16,20] + [3,12,13,15,0]t	0.8877

Appendix D. Supplementary material

Supplementary material to this article can be found online at https://doi.org/10.1016/j.jksus.2023.102748.

References

Ahmed, R., Akhtar, M., Tahir, M.H., 2009. Economical generalized neighbor designs of use in Serology. Comput. Stat. Data Anal. 53, 4584–4589.
Ahmed, R., Akhtar, M., 2011. Designs balanced for neighbor effects in circular blocks of size six. J. Statist. Plann. Inference 141, 687–691.
Akhtar, M., Ahmed, R., Yasmin, F., 2010. A catalogue of nearest neighbor balanced-designs in circular blocks of size five. Pakistan J. Statist. 26 (2), 397–405.
Azais, J.M., 1987. Design of experiments for studying intergenotypic competition. J. R. Stat. Soc. Ser. B 49, 334–345.
Azais, J.M., Bailey, R.A., Monod, H., 1993. A catalogue of efficient neighbor designs with border plots. Biometrics 49 (4), 1252–1261.
Divecha, J., Gondaliya, J., 2014. Construction of minimal balanced crossover designs having good efficiency of separability. Electron. J. Statist. 8, 2923–2936.
Iqbal, I., 1991. Construction of experimental design using cyclic shifts Ph.D. Thesis. University of Kent at Canterbury, U.K.
Iqbal, I., Tahir, M.H., Ghazali, S.S.A., 2009. Circular neighbor-balanced designs using cyclic shifts. Sci. China Ser. A Math 52 (10), 2243–2256.
Jaggi, S., Gupta, V.K., Ashraf, J., 2006. On block designs partially balanced for neighbouring competition effects. J. Indian Statist. Assoc. 44 (1), 27–41.
Jaggi, S., Pateria, D.K., Varghese, C., Varghese, E., Bhowmik, A., 2018. A note on circular neighbour balanced designs. Commun. Statist.-Simulat. Comput. 47, 2896–2905.
Kedia, R.G., Misra, B.L., 2008. On construction of generalized neighbor design of use in serology. Statist. Probab. Lett. 18, 254–256.
Kunert, J., 2000. Randomization of neighbour balanced designs. Biom. J. 42 (1), 111–118.
Meitei, K.K.S., 2010. Circular neighbour-balanced designs and one-sided circular neighbour-balanced designs. Sankhyā: Indian J. Statist. Ser. B 72, 175–188.
Meitei, K.K.S., 2020. Universally Optimal One-sided Circular Neighbour-balanced Designs with k = 5. Calcutta Statist. Assoc. Bull. 72 (2), 84–88.
Nutan, S.M., 2007. Families of proper generalized neighbor designs. J. Statist. Plann. Inference 137, 1681–1686.
Rees, D.H., 1967. Some designs of use in serology. Biometrics 23, 779–791.
Salam, A., Ahmed, R., Daniyal, M., Berihan, R.A., 2022. Minimal circular nearly strongly-balanced neighbor designs when left and right neighbor effects are equal. J. Statist. Appl. Prob. 11 (3), 883–891.
Shehzad, F., Zafaryab, M., Ahmed, R., 2011. Minimal neighbor designs in circular blocks of unequal sizes. J. Statist. Plann. Inference 141, 3681–3685.
Singh, K.P., 2019. A neighbor balanced design and its optimal certainty contribution. Bull. Pure Appl. Sci. Sect. - E – Math. Statist. 38E (2), 636–640.
Tomar, J.S., Jaggi, S., Varghese, C., 2005. On totally balanced block designs for competition effects. J. Appl. Statist. 32 (1), 87–97.