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Original article

Dynamical behaviour of shallow water waves and solitary wave solutions of the Dullin-Gottwald-Holm dynamical system

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ABSTRACT

In this article, we recover a variety of new families of shallow water wave and solitary wave solutions to the (1+1)-dimensional Dullin "Gottwald" Holm (DGH) system by employing new extended direct algebraic method (NEDAM). The derived results are obtained in diverse hyperbolic and periodic function forms. The attained solutions are new addition in the study of solitary wave and shallow water wave theory. In addition, 3-dimensional, 2-dimensional, and contour graphs of secured results are plotted in order to observe their dynamics with the choices of involved parameters. On the basis of achieved outcomes, we may claim that the proposed computational method is direct, dynamics, well organized, and will be useful for solving the more complicated nonlinear problems in diverse areas together with symbolic computations.

1. Introduction

Due to the swift development of symbolic computation software systems, the construction of analytic (exact) solutions of nonlinear partial differential equations (NLPDEs) has a very prominent place in the research of some nonlinear intricate phenomena comprehensively (Bhatti et al., 2020; Cao et al., 2021; Khan et al., 2019; Marin et al., 2020; Seadawy et al., 2020; Seadawy et al., 2020). The forms of solutions of NLPDEs, that are combined employing several mathematical norms, are very substantial different sciences such as plasma physics, ocean engineering, fluid dynamics, biophysics, mathematical physics, chemistry, optical fiber,telecommunication, quantum field theory, and many others (Chen, 2020; Iqbal et al., 2018, 2020; Yang et al., 2020; Younis et al., 2020). In the recent past, many mathematician and researchers have established several efficient and powerful methodologies to retrieve exact solutions in the forms of traveling wave or solitary wave such as, Lie symmetry analysis, extended rational sine–cosine/ sinh-cosh schemes (Rehman and Ahmad, 2020), extended auxiliary equation method (Rezazadeh et al., 2019), $(\frac{G'}{G^2})$-expansion function method (Bilal et al., 2020), the extended fan sub-equation technique (Younis et al., 2020), F-expansion function method (Seadawy et al., 2020), Bernoulli sub-equation function method (Syam, 2019), $\exp(-\phi(\xi))$-expansion function method (Abdou, 2019), the new generalized rational function method (GERFM) (Younas et al., 2021), ansatz approach, new Φ^6-model expansion scheme (Rehman et al., 2020), Lie symmetries (Olver, 2000; Bluman et al., 2009), extended direct algebraic method and its modified form (Çelik et al., 2021; Seadawy et al., 2018), extended mapping method and Seadawy techniques (Khater et al., 2000; Seadawy, 2017; Seadawy et al., 2018) and several others (Ancol et al., 2015; da Silva and Freire, 2020; da Silva and Freire, 2019; Younas et al., 2020) were established for nonlinear physical models (El-Hameed, 2020). In this studies, (1+1)-dimensional Dullin-Gottwald-Holm (DGH) system is considered and analyzed analytically by deploying new extended direct algebraic method. It shows the unidirectional propagation of surface waves in a shallow water regime. Therefore, it is imperative to examine this considered model analytically and derive the solutions. The DGH system is read as (Dullin et al., 2001),

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$$\psi_t + c_0\psi_x - \alpha^2(\psi_{xxt} + \psi\psi_{xxx} + 2\psi_x\psi_{xx}) + 3\psi\psi_x + \gamma\psi_{xxx} = 0, \quad t > 0, \tag{1}$$

where fluid velocity is denoted by $\psi(x,t)$ in spatial direction x. $\alpha^2(\alpha > 0)$ and $\frac{c_0}{\gamma}$ represent squares of length scales, whereas $c_0 = \sqrt{gh}$ (where $c_0 = 2\omega$) denotes the linear wave speed for disturbed water at rest at spatial infinity. In this work, we will implement a mathematical technique NEDAM that will reveal a bunch of exact solutions (Mirhosseni-Alizamini et al., 2020). We note that the studied model has not yet been examined utilizing the NEDAM. NEDAM is one of the robust method to look for the exact solutions of NLPDEs. The layout of the article is arranged in the following order: Analysis of NEDAM is presented in Section 2. Exact solutions are revealed in Section 3. Results and discussion are displayed in Section 4. The concluding remarks are given in last Section 5.

2. Analysis of NEDAM

A detail description is given below.

Step 1: NLPDE is defined in general form as follows:

$$\Delta(\varphi, \varphi_t, \varphi_{tt}, \varphi_x, \varphi_{xx}, \ldots) = 0, \tag{2}$$

where Δ and φ represent a polynomial and unknown function respectively.

Step 2:

The variable η is defined to convert two variables x and t in a single form using the following compound transformation.

$$\varphi(x,t) = H(\eta), \qquad \eta = x - ct, \tag{3}$$

On solving Eqs. (3) and (2), following ODE is obtained

$$\Pi(H, H', H'', H''', \cdots) = 0, \tag{4}$$

where Π is a polynomial in H and its derivatives.

Step 3:

Consider the solutions of Eq. (4) in form of $\Upsilon(\eta)$ is written as

$$H(\eta) = \sum_{i=0}^{n} b_i \Upsilon^i(\eta), \quad b_n \neq 0, \tag{5}$$

where b_i $(0 \leqslant i \leqslant n)$ are unknown and $\Upsilon(\eta)$ satisfies the following nonlinear ODE,

$$\Upsilon'(\eta) = \ln(B)(\mu + \lambda\Upsilon(\eta) + \nu\Upsilon^2(\eta)), \quad B \neq 0, 1. \tag{6}$$

Further, Eq. (6) has the general solution.

(1) For $\lambda^2 - 4\mu\nu < 0$ and $\nu \neq 0$, there are five solutions,

$$\Upsilon_1(\eta) = -\frac{\lambda}{2\nu} + \frac{\sqrt{-(\lambda^2 - 4\mu\nu)}}{2\nu}\tan_B\left(\frac{\sqrt{-(\lambda^2 - 4\mu\nu)}}{2}\eta\right), \tag{7}$$

$$\Upsilon_2(\eta) = -\frac{\lambda}{2\nu} - \frac{\sqrt{-(\lambda^2 - 4\mu\nu)}}{2\nu}\cot_B\left(\frac{\sqrt{-(\lambda^2 - 4\mu\nu)}}{2}\eta\right), \tag{8}$$

$$\Upsilon_3(\eta) = -\frac{\lambda}{2\nu} + \frac{\sqrt{-(\lambda^2 - 4\mu\nu)}}{2\nu}\left(\tan_B\left(\sqrt{-(\lambda^2 - 4\mu\nu)}\eta\right) \pm \sqrt{pq}\sec_B\left(\sqrt{-(\lambda^2 - 4\mu\nu)}\eta\right)\right), \tag{9}$$

$$\Upsilon_4(\eta) = -\frac{\lambda}{2\nu} - \frac{\sqrt{-(\lambda^2 - 4\mu\nu)}}{2\nu}\left(\cot_B\left(\sqrt{-(\lambda^2 - 4\mu\nu)}\eta\right) \pm \sqrt{pq}\csc_B\left(\sqrt{-(\lambda^2 - 4\mu\nu)}\eta\right)\right), \tag{10}$$

$$\Upsilon_5(\eta) = -\frac{\lambda}{2\nu} + \frac{\sqrt{-(\lambda^2 - 4\mu\nu)}}{4\nu}\left(\tan_B\left(\frac{\sqrt{-(\lambda^2 - 4\mu\nu)}}{4}\eta\right) - \cot_B\left(\frac{\sqrt{-(\lambda^2 - 4\mu\nu)}}{4}\eta\right)\right). \tag{11}$$

(2) For $\lambda^2 - 4\mu\nu > 0$ and $\nu \neq 0$, there are five solutions,

$$\Upsilon_6(\eta) = -\frac{\lambda}{2\nu} - \frac{\sqrt{\lambda^2 - 4\mu\nu}}{2\nu}\tanh_B\left(\frac{\sqrt{\lambda^2 - 4\mu\nu}}{2}\eta\right), \tag{12}$$

$$\Upsilon_7(\eta) = -\frac{\lambda}{2\nu} - \frac{\sqrt{\lambda^2 - 4\mu\nu}}{2\nu}\coth_B\left(\frac{\sqrt{\lambda^2 - 4\mu\nu}}{2}\eta\right), \tag{13}$$

$$\Upsilon_8(\eta) = -\frac{\lambda}{2\nu} - \frac{\sqrt{\lambda^2 - 4\mu\nu}}{2\nu}\left(\tanh_B\left(\sqrt{\lambda^2 - 4\mu\nu}\eta\right) \pm i\sqrt{pq}\,\mathrm{sech}_B\left(\sqrt{\lambda^2 - 4\mu\nu}\eta\right)\right), \tag{14}$$

$$\Upsilon_9(\eta) = -\frac{\lambda}{2\nu} - \frac{\sqrt{-(\lambda^2 - 4\mu\nu)}}{2\nu}\left(\coth_B\left(\sqrt{\lambda^2 - 4\mu\nu}\eta\right) \pm \sqrt{pq}\,\mathrm{csch}_B\left(\sqrt{\lambda^2 - 4\mu\nu}\eta\right)\right), \tag{15}$$

$$\Upsilon_{10}(\eta) = -\frac{\lambda}{2\nu} + \frac{\sqrt{-(\lambda^2 - 4\mu\nu)}}{4\nu}\left(\tanh_B\left(\frac{\sqrt{\lambda^2 - 4\mu\nu}}{4}\eta\right) + \coth_B\left(\frac{\sqrt{\lambda^2 - 4\mu\nu}}{4}\eta\right)\right). \tag{16}$$

(3) For $\mu\nu > 0$ and $\lambda = 0$, there are five solutions,

$$\Upsilon_{11}(\eta) = \sqrt{\frac{\mu}{\nu}}\tan_B(\sqrt{\mu\nu}\eta), \tag{17}$$

$$\Upsilon_{12}(\eta) = -\sqrt{\frac{\mu}{\nu}}\cot_B(\sqrt{\mu\nu}\eta), \tag{18}$$

$$\Upsilon_{13}(\eta) = \sqrt{\frac{\mu}{\nu}}(\tan_B(2\sqrt{\mu\nu}\eta) \pm \sqrt{pq}\sec_B(2\sqrt{\mu\nu}\eta)), \tag{19}$$

$$\Upsilon_{14}(\eta) = -\sqrt{\frac{\mu}{\nu}}(\cot_B(2\sqrt{\mu\nu}\eta) \pm \sqrt{pq}\csc_B(2\sqrt{\mu\nu}\eta)), \tag{20}$$

$$\Upsilon_{15}(\eta) = \frac{1}{2}\sqrt{\frac{\mu}{\nu}}\left(\tan_B(\frac{\sqrt{\mu\nu}}{2}\eta) - \cot_B(\frac{\sqrt{\mu\nu}}{2}\eta)\right). \tag{21}$$

(4) For $\mu\nu < 0$ and $\lambda = 0$, there are five solutions,

$$\Upsilon_{16}(\eta) = -\sqrt{-\frac{\mu}{\nu}}\tanh_B(\sqrt{-\mu\nu}\eta), \tag{22}$$

$$\Upsilon_{17}(\eta) = -\sqrt{-\frac{\mu}{\nu}}\coth_B(\sqrt{-\mu\nu}\eta), \tag{23}$$

$$\Upsilon_{18}(\eta) = -\sqrt{-\frac{\mu}{\nu}}(\tanh_B(2\sqrt{-\mu\nu}\eta) \pm i\sqrt{pq}\,\mathrm{sech}_B(2\sqrt{-\mu\nu}\eta)), \tag{24}$$

$$\Upsilon_{19}(\eta) = -\sqrt{-\frac{\mu}{\nu}}(\coth_B(2\sqrt{-\mu\nu}\eta) \pm \sqrt{pq}\,\mathrm{csch}_B(2\sqrt{-\mu\nu}\eta)), \tag{25}$$

$$\Upsilon_{20}(\eta) = -\frac{1}{2}\sqrt{-\frac{\mu}{\nu}}\left(\tanh_B(\frac{\sqrt{-\mu\nu}}{2}\eta) - \coth_B(\frac{\sqrt{-\mu\nu}}{2}\eta)\right). \tag{26}$$

(5) For $\lambda = 0$ and $\nu = \mu$, there are five solutions,

$$\Upsilon_{21}(\eta) = \tan_B(\mu\eta), \tag{27}$$

$$\Upsilon_{22}(\eta) = -\cot_B(\mu\eta), \tag{28}$$

$$\Upsilon_{23}(\eta)=\tan_B(2\mu\eta)\pm\sqrt{pq}\sec_B(2\mu\eta), \tag{29}$$

$$\Upsilon_{24}(\eta)=-\cot_B(2\mu\eta)\pm\sqrt{pq}\csc_B(2\mu\eta), \tag{30}$$

$$\Upsilon_{25}(\eta)=\frac{1}{2}\left(\tan_B(\frac{\mu}{2}\eta)-\cot_B(\frac{\mu}{2}\eta)\right). \tag{31}$$

(6) For $\lambda=0$ and $\nu=-\mu$, there are five solutions,

$$\Upsilon_{26}(\eta)=-\tanh_B(\mu\eta), \tag{32}$$

$$\Upsilon_{27}(\eta)=-\coth_B(\mu\eta), \tag{33}$$

$$\Upsilon_{28}(\eta)=-\tanh_B(2\mu\eta)\pm i\sqrt{pq}\,\mathrm{sech}_B(2\mu\eta), \tag{34}$$

$$\Upsilon_{29}(\eta)=-\coth_B(2\mu\eta)\pm\sqrt{pq}\,\mathrm{csch}_B(2\mu\eta), \tag{35}$$

$$\Upsilon_{30}(\eta)=-\frac{1}{2}\left(\tanh_B(\frac{\mu}{2}\eta)+\coth_B(\frac{\mu}{2}\eta)\right). \tag{36}$$

(7) For $\lambda^2=4\mu\nu$, there is one solution,

$$\Upsilon_{31}(\eta)=\frac{-2\mu(\lambda\eta\ln B+2)}{\lambda^2\eta\ln B}. \tag{37}$$

(8) For $\lambda=\chi,\mu=r\chi(r\neq 0)$ and $\nu=0$, there is one solution,

$$\Upsilon_{32}(\eta)=B^{\chi\eta}-r. \tag{38}$$

(9) For $\lambda=\nu=0$, there is one solution,

$$\Upsilon_{33}(\eta)=\mu\eta\ln B. \tag{39}$$

(10) For $\lambda=\mu=0$, there is one solution,

$$\Upsilon_{34}(\eta)=\frac{-1}{\nu\eta\ln B}. \tag{40}$$

(11) For $\mu=0$ and $\lambda\neq 0$, there are two solutions,

$$\Upsilon_{35}(\eta)=-\frac{p\lambda}{\nu(\cosh_B(\lambda\eta)-\sinh_B(\lambda\eta)+p)}, \tag{41}$$

$$\Upsilon_{36}(\eta)=-\frac{\lambda(\sinh_B(\lambda\eta)+\cosh_B(\lambda\eta))}{\nu(\sinh_B(\lambda\eta)+\cosh_B(\lambda\eta)+q)}, \tag{42}$$

(12) For $\lambda=\chi,\nu=r\chi(r\neq 0)$ and $\mu=0$, there is one solution,

$$\Upsilon_{37}(\eta)=\frac{pB^{\chi\eta}}{p-rqB^{\chi\eta}}. \tag{43}$$

In the above solutions, the generalized hyperbolic and trigonometric functions are defined as the following citegroup cite36,

$$\sinh_B(\eta)=\frac{pB^{\eta}-qB^{-\eta}}{2},\quad \cosh_B(\eta)=\frac{pB^{\eta}+qB^{-\eta}}{2},$$

$$\tanh_B(\eta)=\frac{pB^{\eta}-qB^{-\eta}}{pB^{\eta}+qB^{-\eta}},\quad \coth_B(\eta)=\frac{pB^{\eta}+qB^{-\eta}}{pB^{\eta}-qB^{-\eta}},$$

$$\mathrm{sech}_B(\eta)=\frac{2}{pB^{\eta}+qB^{-\eta}},\quad \mathrm{csch}_B(\eta)=\frac{2}{pB^{\eta}-qB^{-\eta}},$$

$$\sin_B(\eta)=\frac{pB^{i\eta}-qB^{-i\eta}}{2},\quad \cos_B(\eta)=\frac{pB^{i\eta}+qB^{-i\eta}}{2},$$

$$\tan_B(\eta)=-i\frac{pB^{i\eta}-qB^{-i\eta}}{pB^{i\eta}+qB^{-i\eta}},\quad \cot_B(\eta)=i\frac{pB^{i\eta}+qB^{-i\eta}}{pB^{i\eta}-qB^{-i\eta}},$$

$$\sec_B(\eta)=\frac{2}{pB^{i\eta}+qB^{-i\eta}},\quad \csc_B(\eta)=\frac{2i}{pB^{i\eta}-qB^{-i\eta}},$$

where η is an independent variable and $p,q>0$.

Step 4:

The N can be calculated using Eq. (4). For this reason, the homogeneous balancing principle is useful in equating the nonlinear terms in Eq. (4) with higher derivatives.

Step 5:

The unknown constants can be obtained by substituting Eq. (5) into Eq. (4) and equating the coefficients of $\Upsilon(\eta)$ of similar power to zero, we achieve a set of algebraic expressions. On solving these equations through symbolic computation, we obtain sets of solution.

3. Exact solutions

In order to resolve the governing model by employing NEDAM, we operate the wave transformation $\varphi(x,t)$=$H(\eta)$, where$\eta=x-ct$ and $c\neq 0$. Substituting the given transformation into Eq. (1),

$$\alpha^2cH'''-cH'+c_0H'-\alpha^2HH'''+\gamma H'''+3HH'-2\alpha^2H'H\prime\prime=0, \tag{44}$$

where ′ represent derivative w.r.t η.

Integrating Eq. (44) one time w.r.t η and let the constant of integration equal to 0, turns into following

$$H''(\alpha^2(-c)-\gamma+\alpha^2H)-(c_0-c)H-\frac{3H^2}{2}+\frac{1}{2}\alpha^2(H')^2=0. \tag{45}$$

By making balance between in Eq. (45), yields, $n=2$. So, the non-trivial Eq. (45) transforms into following

$$H(\eta)=b_0+b_1\Upsilon(\eta)+b_2\Upsilon^2(\eta). \tag{46}$$

Putting Eq. (46) along its derivatives in Eq. (45) making the coefficients of similar powers of $\Upsilon(\eta)$ to 0, we achieve a system of algebraic equations. On solving these equations through symbolic computation, we seek clusters of solutions sets as follows

Family-1.

$$\left\{b_0=-4\gamma\mu\nu\ln^2(B),b_1=-4\gamma\lambda\nu\ln^2(B),b_2=-4\gamma\nu^2\ln^2(B),\alpha=0,c_0=c-\gamma\ln^2(B)(\lambda^2-4\mu\nu).\right\}$$

The emerging solutions of Eq. (1) are given in detail as

(1) For $\lambda^2-4\mu\nu<0$ and $\nu\neq 0$,

- The trigonometric solutions

$$\psi_1(x,t)=\left\{\gamma\ln^2(B)(\lambda^2-4\mu\nu)\left(\tan_B\left(\frac{1}{2}\eta\sqrt{4\mu\nu-\lambda^2}\right)^2+1\right)\right\}. \tag{47}$$

$$\psi_2(x,t)=\left\{\gamma\ln^2(B)(\lambda^2-4\mu\nu)\left(\cot_B\left(\frac{1}{2}\eta\sqrt{4\mu\nu-\lambda^2}\right)^2+1\right)\right\}. \tag{48}$$

- The combo-trigonometric solutions

$$\psi_3(x,t)=\left\{\gamma\ln^2(B)\left(\lambda^2-4\mu\nu\right)\left(\left(\tan_B\left(\eta\sqrt{4\mu\nu-\lambda^2}\right)\pm\sqrt{pq}\sec_B\left(\eta\sqrt{4\mu\nu-\lambda^2}\right)\right)^2+1\right)\right\}. \tag{49}$$

$$\psi_4(x,t)=\left\{\gamma\ln^2(B)\left(\lambda^2-4\mu\nu\right)\left(\left(\cot_B\left(\eta\sqrt{4\mu\nu-\lambda^2}\right)\pm\sqrt{pq}\csc_B\left(\eta\sqrt{4\mu\nu-\lambda^2}\right)\right)^2+1\right)\right\}. \tag{50}$$

$$\psi_5(x,t)=\left\{\frac{1}{4}\gamma\ln^2(B)\left(\lambda^2-4\mu\nu\right)\left(\left(\cot_B\left(\frac{1}{4}\eta\sqrt{4\mu\nu-\lambda^2}\right)-\tan_B\left(\frac{1}{4}\eta\sqrt{4\mu\nu-\lambda^2}\right)\right)^2+4\right)\right\}. \tag{51}$$

(2) For $\lambda^2 - 4\mu v > 0$ and $v \neq 0$, various exact solutions are constructed.

- The kink solution

$$\psi_6(x,t) = \{-\gamma \ln^2(B)(\lambda^2 - 4\mu v)\left(\tanh_B\left(\frac{1}{2}\eta\sqrt{\lambda^2 - 4\mu v}\right)^2 - 1\right)\} \tag{52}$$

- The singular solution

$$\psi_7(x,t) = \left\{-\gamma \ln^2(B)(\lambda^2 - 4\mu v)\left(\coth_B\left(\frac{1}{2}\eta\sqrt{\lambda^2 - 4\mu v}\right)^2 - 1\right)\right\}. \tag{53}$$

- The complex kink-antikink solution

$$\psi_8(x,t) = \left\{-\gamma \ln^2(B)(\lambda^2 - 4\mu v)\left(-1 + \left(\tanh_B\left(\eta\sqrt{\lambda^2 - 4\mu v}\right) \pm i\sqrt{pq}\,\mathrm{sech}_B\left(\eta\sqrt{\lambda^2 - 4\mu v}\right)\right)^2\right)\right\}. \tag{54}$$

- The mixed singular solution

$$\psi_9(x,t) = \left\{-\gamma \ln^2(B)(\lambda^2 - 4\mu v)\left(\left(\coth_B\left(\eta\sqrt{\lambda^2 - 4\mu v}\right) \pm \sqrt{pq}\,\mathrm{csch}_B\left(\eta\sqrt{\lambda^2 - 4\mu v}\right)\right)^2 - 1\right)\right\}. \tag{55}$$

- The kink-singular solution

$$\begin{aligned}\psi_{10}(x,t) = \{&-\frac{1}{4}\gamma \ln^2(B)\left(\lambda^2 - 4\mu v\right)\left(\coth_B\left(\frac{1}{4}\eta\sqrt{\lambda^2 - 4\mu v}\right)\right.\\ &+ \tanh_B\left(\frac{1}{4}\eta\sqrt{\lambda^2 - 4\mu v}\right)\\ &\left.- 2\right)\left(\coth_B\left(\frac{1}{4}\eta\sqrt{\lambda^2 - 4\mu v}\right) + \tanh_B\left(\frac{1}{4}\eta\sqrt{\lambda^2 - 4\mu v}\right) + 2\right)\}.\end{aligned} \tag{56}$$

(3) For $\mu v > 0$ and $\lambda = 0$.

- The trigonometric solutions

$$\psi_{11}(x,t) = \left\{-4\gamma\mu v \ln^2(B)\left(\tan_B(\eta\sqrt{\mu v})^2 + 1\right)\right\}. \tag{57}$$

$$\psi_{12}(x,t) = \left\{-4\gamma\mu v \ln^2(B)\left(\cot_B(\eta\sqrt{\mu v})^2 + 1\right)\right\}. \tag{58}$$

- The combined form of trigonometric solutions

$$\psi_{13}(x,t) = \left\{-4\gamma\mu v \ln^2(B)\left((\tan_B(2\eta\sqrt{\mu v}) \pm \sqrt{pq}\sec_B(2\eta\sqrt{\mu v}))^2 + 1\right)\right\}. \tag{59}$$

$$\psi_{14}(x,t) = \left\{-4\gamma\mu v \ln^2(B)\left((\cot_B(2\eta\sqrt{\mu v}) \pm \sqrt{pq}\csc_B(2\eta\sqrt{\mu v}))^2 + 1\right)\right\}. \tag{60}$$

$$\psi_{15}(x,t) = \left\{\gamma\mu v \ln^2(B)\left(-\left(\cot_B\left(\frac{1}{2}\eta\sqrt{\mu v}\right) - \tan_B\left(\frac{1}{2}\eta\sqrt{\mu v}\right)\right)^2 - 4\right)\right\}. \tag{61}$$

(4) For $\mu v < 0$ and $\lambda = 0$.

- The kink solutions

$$\psi_{16}(x,t) = \left\{4\gamma\mu v \ln^2(B)\left(\tanh_B(\eta\sqrt{-\mu v})^2 - 1\right)\right\}. \tag{62}$$

- The singular solution

$$\psi_{17}(x,t) = \left\{4\gamma\mu v \ln^2(B)\left(\coth_B(\eta\sqrt{-\mu v})^2 - 1\right)\right\}. \tag{63}$$

- Complexion combo types solutions

$$\psi_{18}(x,t) = \left\{4\gamma\mu v \ln^2(B)\left(-1 + (\tanh_B(2\eta\sqrt{-\mu v}) \pm i\sqrt{pq}\,\mathrm{sech}_B(2\eta\sqrt{-\mu v}))^2\right)\right\}. \tag{64}$$

$$\psi_{19}(x,t) = \left\{4\gamma\mu v \ln^2(B)\left((\coth_B(2\eta\sqrt{-\mu v}) \pm \sqrt{pq}\,\mathrm{csch}_B(2\eta\sqrt{-\mu v}))^2 - 1\right)\right\}. \tag{65}$$

$$\psi_{20}(x,t) = \left\{\gamma\mu v \ln^2(B)\left(\left(\coth_B\left(\frac{1}{2}\eta\sqrt{-\mu v}\right) + \tanh_B\left(\frac{1}{2}\eta\sqrt{-\mu v}\right)\right)^2 - 4\right)\right\}. \tag{66}$$

(5) For $\lambda = 0$ and $v = \mu$.

- The periodic solutions

$$\psi_{21}(x,t) = \left\{-4\gamma\mu^2 \ln^2(B)\left(\tan_B(\mu\eta)^2 + 1\right)\right\}. \tag{67}$$

$$\psi_{22}(x,t) = \left\{-4\gamma\mu^2 \ln^2(B)\left(\cot_B(\mu\eta)^2 + 1\right)\right\}. \tag{68}$$

- Combined periodic solutions

$$\psi_{23}(x,t) = \{-4\gamma\mu^2 \ln^2(B)\left((\tan_B(2\mu\eta) \pm \sqrt{\mathrm{pq}}\sec_B(2\mu\eta))^2 + 1\right)\} \tag{69}$$

$$\psi_{24}(x,t) = \{-4\gamma\mu^2 \ln^2(B)\left((-\cot_B(2\mu\eta) \pm \sqrt{\mathrm{pq}}\csc_B(2\mu\eta))^2 + 1\right)\}. \tag{70}$$

$$\psi_{25}(x,t) = \left\{\gamma\mu^2 \ln^2(B)\left(-\left(\cot_B\left(\frac{\mu\eta}{2}\right) - \tan_B\left(\frac{\mu\eta}{2}\right)\right)^2 - 4\right)\right\}. \tag{71}$$

(6) For $\lambda = 0$ and $v = -\mu$.

- The exact traveling wave solutions

$$\psi_{26}(x,t) = \left\{-4\gamma\mu^2 \ln^2(B)\left(\tanh_B(\mu\eta)^2 - 1\right)\right\}. \tag{72}$$

$$\psi_{27}(x,t) = \left\{-4\gamma\mu^2 \ln^2(B)\left(\coth_B(\mu\eta)^2 - 1\right)\right\}. \tag{73}$$

$$\psi_{28}(x,t) = \left\{-4\gamma\mu^2 \ln^2(B)\left(-1 + (-\tanh_B(2\mu\eta) \pm i\sqrt{pq}\,\mathrm{sech}_B(2\mu\eta))^2\right)\right\}. \tag{74}$$

$$\psi_{29}(x,t) = \left\{-4\gamma\mu^2 \ln^2(B)\left((-\coth_B(2\mu\eta) \pm \sqrt{pq}\,\mathrm{csch}_B(2\mu\eta))^2 - 1\right)\right\}. \tag{75}$$

$$\psi_{30}(x,t) = \left\{\gamma\mu^2 \ln^2(B)\left(4 - \left(\coth_B\left(\frac{\mu\eta}{2}\right) + \tanh_B\left(\frac{\mu\eta}{2}\right)\right)^2\right)\right\}. \tag{76}$$

(7) For $\lambda^2 = 4\mu v$.

- Plane wave solutions

$$\psi_{31}(x,t) = \left\{\frac{4\gamma\mu v(\lambda\xi\ln(B)(\lambda^2 - 4\mu v)(\lambda\xi\ln(B) + 4) - 16\mu v)}{\lambda^4\xi^2}\right\}. \tag{77}$$

(8) For $\mu = 0$ and $\lambda \neq 0$.

- Mixed type hyperbolic solutions

$$\psi_{32}(x,t)=\left\{\frac{4\gamma\lambda^2 p\ln^2(B)(\cosh_B(\lambda\eta)-\sinh_B(\lambda\eta))}{(\cosh_B(\lambda\xi)+p-\mathrm{Sinh}_\delta(\lambda\eta))^2}\right\}. \tag{78}$$

$$\psi_{33}(x,t)=\left\{\frac{4\gamma\lambda^2 q\ln^2(B)(\cosh_B(\lambda\eta)+\sinh_B(\lambda\eta))}{(\cosh_B(\lambda\eta)+q+\sinh_B(\lambda\eta))^2}\right\}. \tag{79}$$

(9) For $\mu=0,\lambda=\chi$ and $v=r\chi$.

- Plane wave solution

$$\psi_{34}(x,t)=\left\{-\frac{4\gamma pr\chi^2\ln^2(B)B^{\eta\chi}(r(p-q)B^{\eta\chi}+p)}{(p-qrB^{\eta\chi})^2}\right\}. \tag{80}$$

Where $\eta=x-ct$, for all above solutions.

4. Results and discussion

A comparison is given of our acquired results to some available published work. Previous work has been done on this dynamical model. Generalizations of the Camassa-Holm and the Dullin-Gottwald-Holm equations were investigated from the perspective of existence of global solutions, criteria for wave breaking phenomena and integrability. They proved the existence and uniqueness of solutions of the Cauchy problem through Kato's technique. In (Mustafa, 2006), the authors determined the existence and uniqueness of low regularity solutions of the governing model. In (Zhou et al., 2013), peakon-antipeakon interaction with the aid of direct computation was constructed. Exact solutions have been discussed by using traveling-wave transformation and the exp-function technique in (Can et al., 2009). Tian et al. (2005) analyzed peaked solution by assuming $2\alpha^2\omega+\gamma=0$. In (Meng et al., 2011), periodic waves solutions have been recovered by integral bifurcation and semi-inverse approaches. The ansatz method to retrieve 1-soliton solution was also discussed. The categorization of bounded traveling wave solutions have been constructed in (da Silva, 2019). In diverse parameter regions, the dynamical deportment of traveling wave solutions and its bifurcations have been given in (Yu et al., 2016). The qualitative approach of planar systems to secure the bounded exact solutions were discussed (Zhong and Deng, 2017). The results obtained are exceptional and unique in comparison to previous findings in the literature. The periodic, kink, singular, anit-kink, combo kink-anti kink, and rational function (plane wave) solutions, which are appeared in Eqs. (47), (52), (56), (62), (64), (74) and (80) as exhibited in Figs. 1–7 respectively. The achievements reported in this article may be valuable in clarifying the true meaning of numerous nonlinear advancement circumstances that develop in various domains of nonlinear sciences.

5. Concluding remarks

In this research work, we examined the new exact traveling wave structures to (1+1)-dimensional DGH system through mathematical technique known as NEDAM. By employing this mentioned norm we recovered several exact in the form of hyperbolic and trigonometric. The results obtained are exceptional and unique in comparison to previous findings in the literature. In addition, 3-dimensional, 2-dimensional, and corresponding contour graphs of earned outcomes are sketched in order to observe their dynamics with the choices of involved parameters. The outcomes retrieved in this article may be valuable in clarifying the true meaning of numerous nonlinear advancement circumstances that develop in various domains of applied sciences.

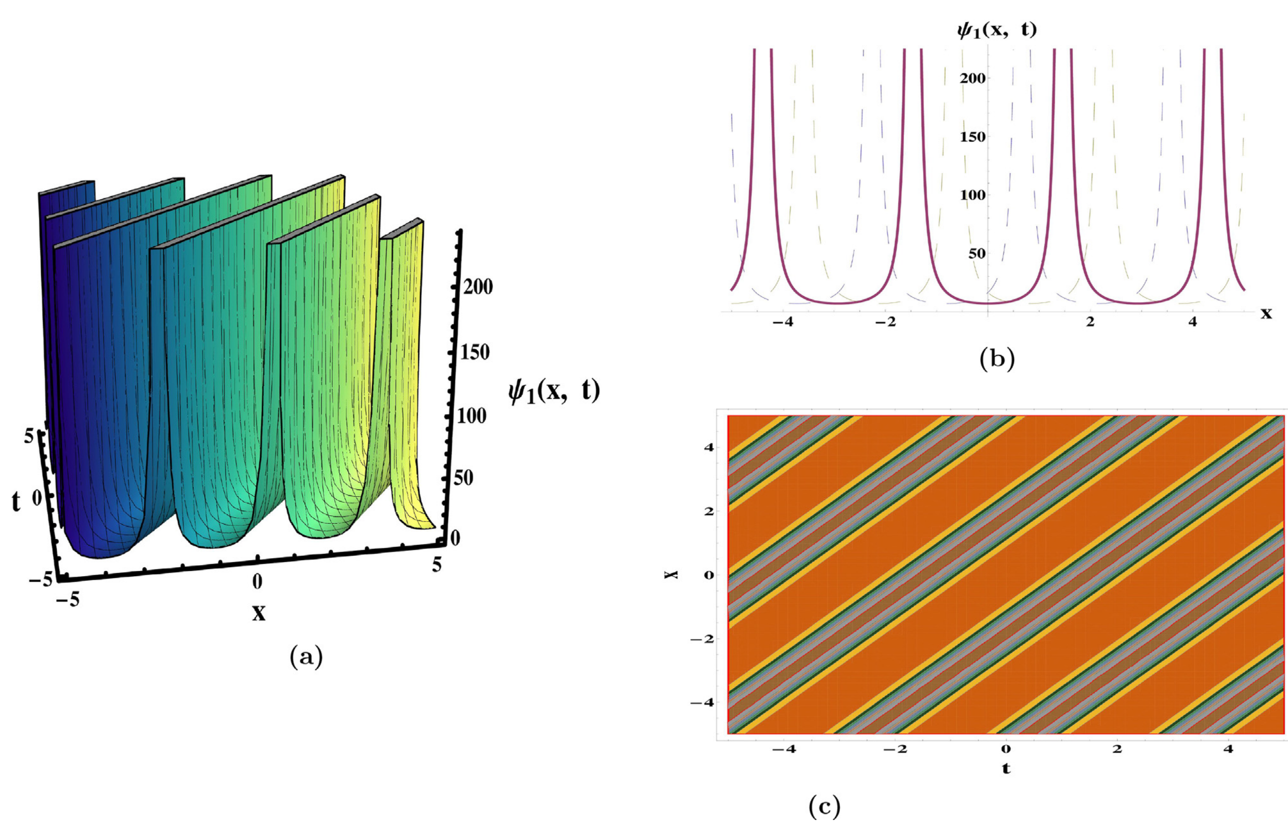

Fig. 1. Plots of solution (47) for the parameters $\gamma=-1.5, c=0.8, v=1.7, \lambda=0.7, \mu=0.75$ and $B=e$.

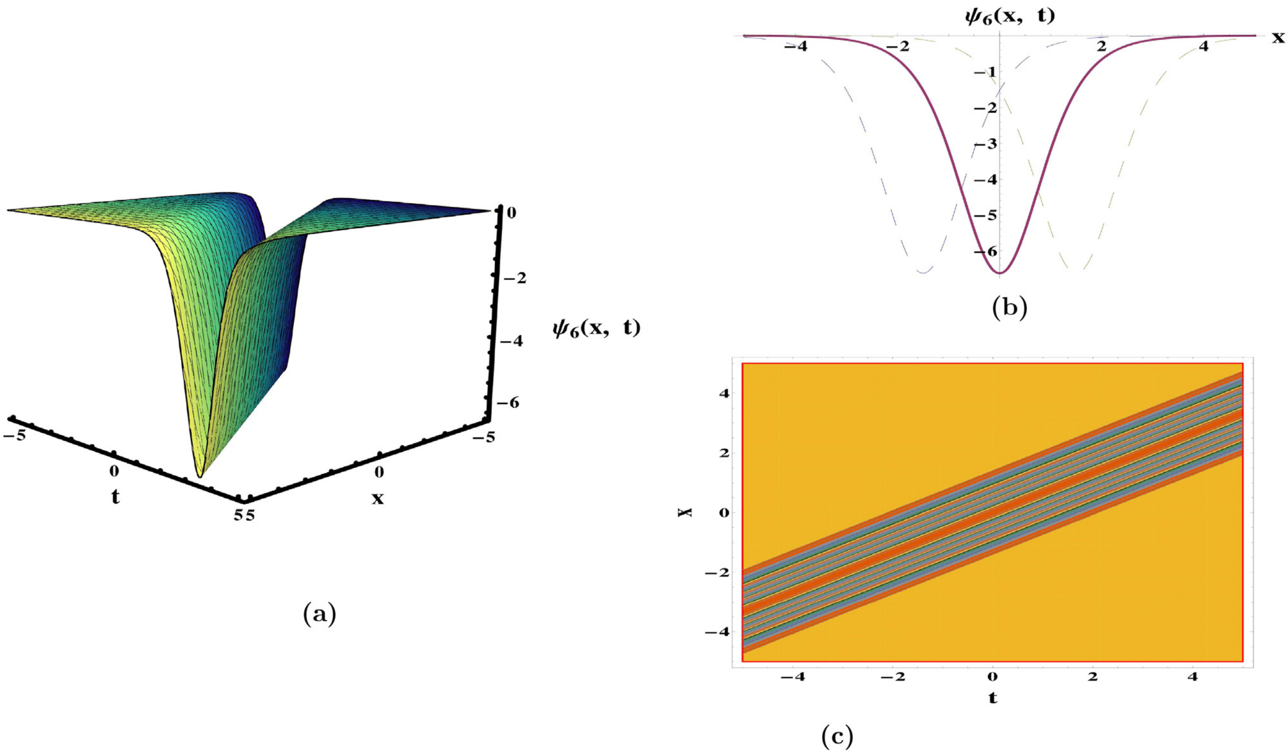

Fig. 2. Plots of solution (52) for the parameters $\gamma = -2, c = 1.5, v = 0.7, \lambda = 1.1, \mu = -0.7.5$ and $B = e$.

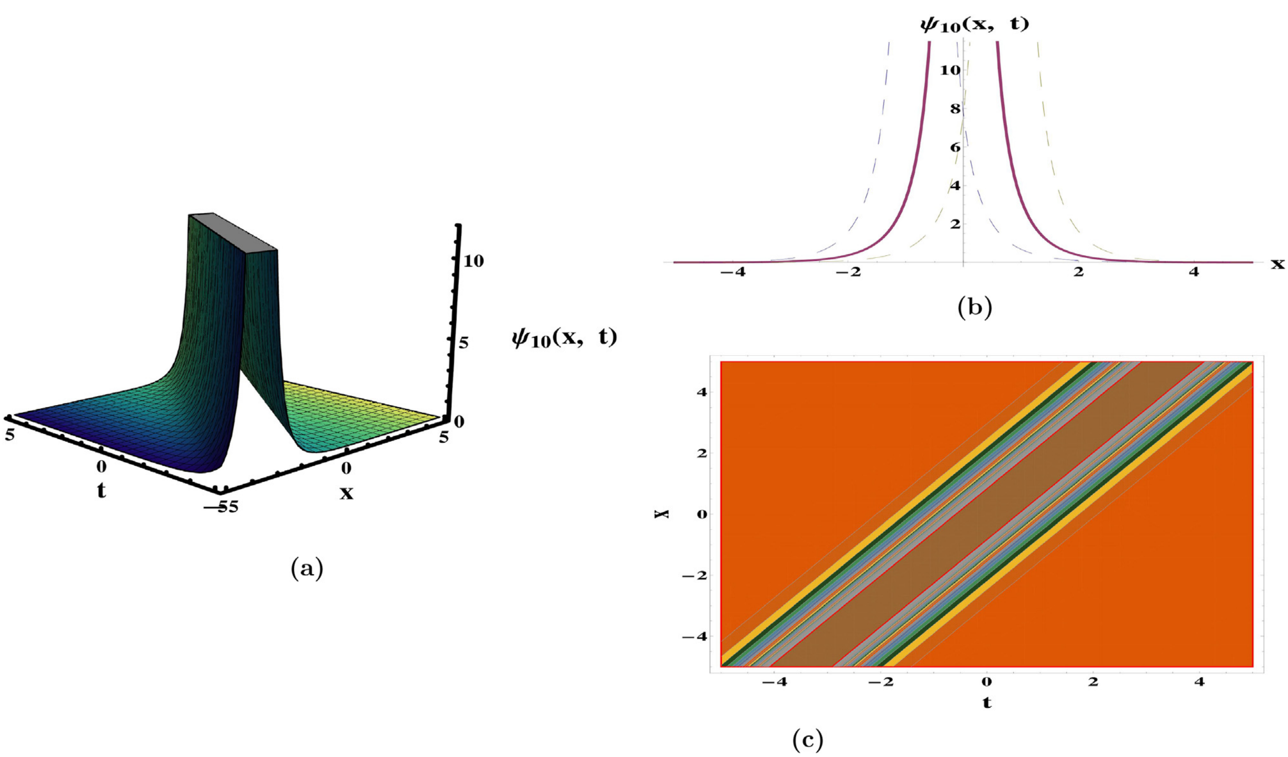

Fig. 3. Plots of solution (56)for the parameters $\gamma = -1.1, c = 0.7, v = 1.2, \lambda = 0.75, \mu = -0.65.5$ and $B = e$.

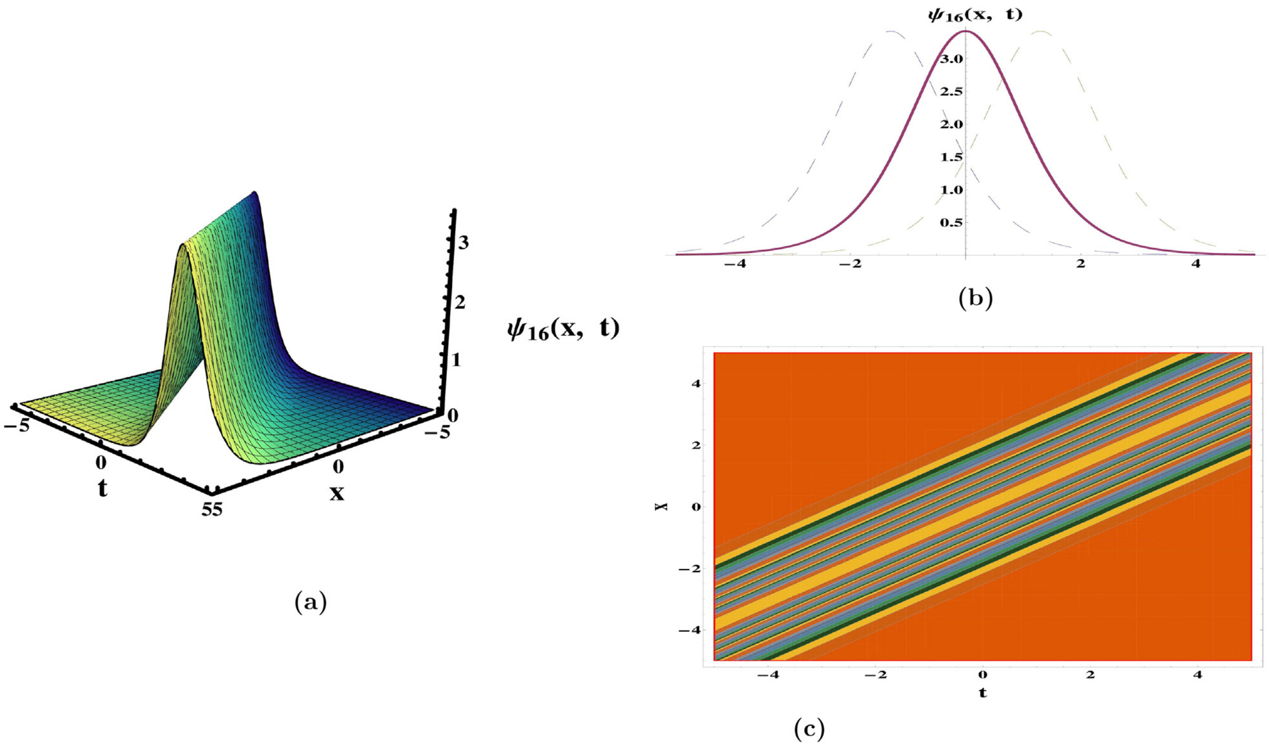

Fig. 4. Plots of solution (62) for the parameters $\gamma = 1.5, c = 1.3, v = 0.6, \mu = -0.9.5$ and $B = e$.

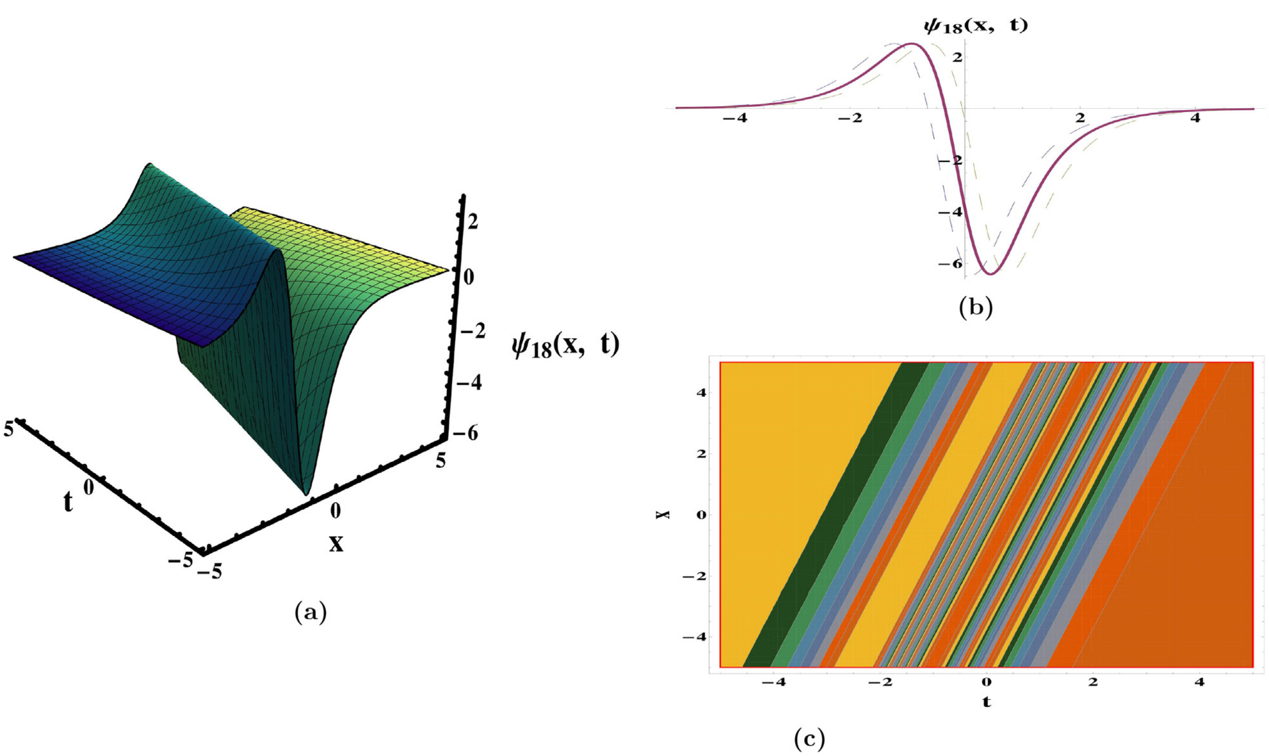

Fig. 5. Plots of solution (64) for the parameters $p = 1.7, q = 1.5, \gamma = 1.4, c = 0.3, v = 0.9, \lambda = 0.7, \mu = -0.5$ and $B = e$.

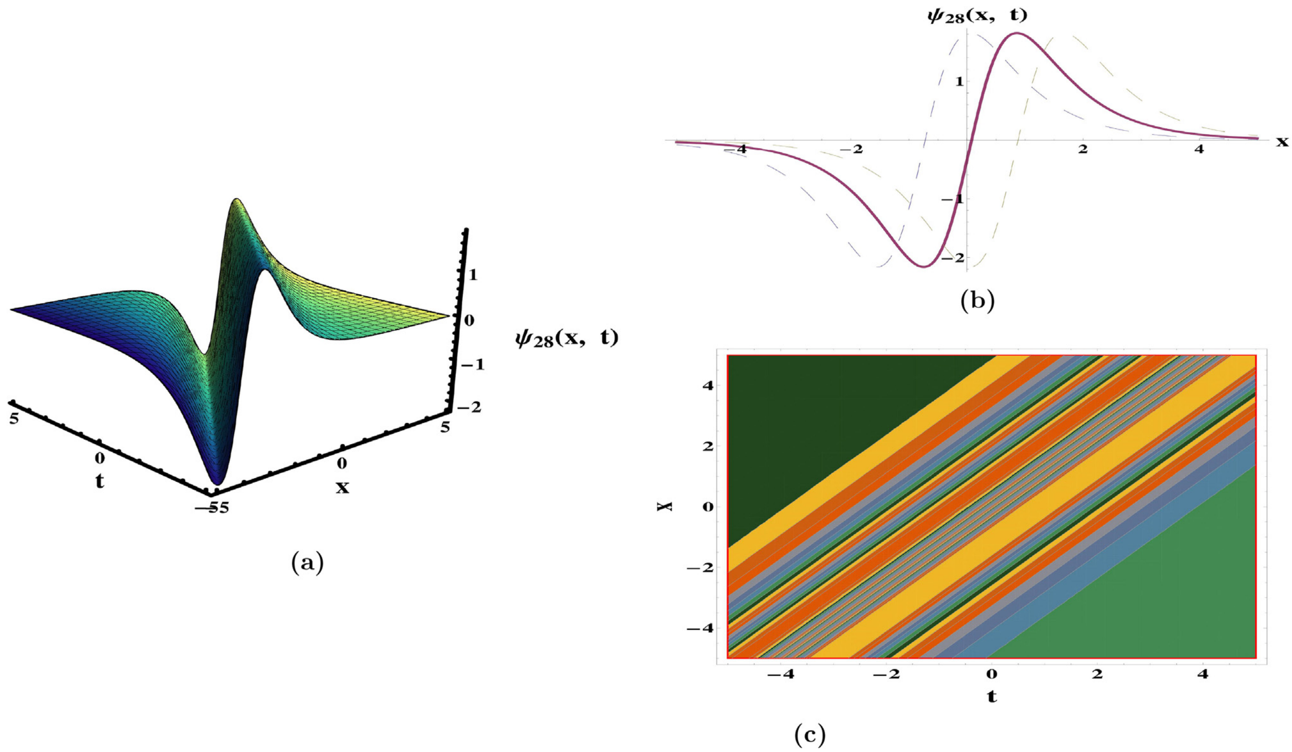

Fig. 6. Plots of solution (74) for the parameters $p = 1.7, q = 0.7, \gamma = 1.5, c = 0.8, \mu = 0.55$ and $B = e$.

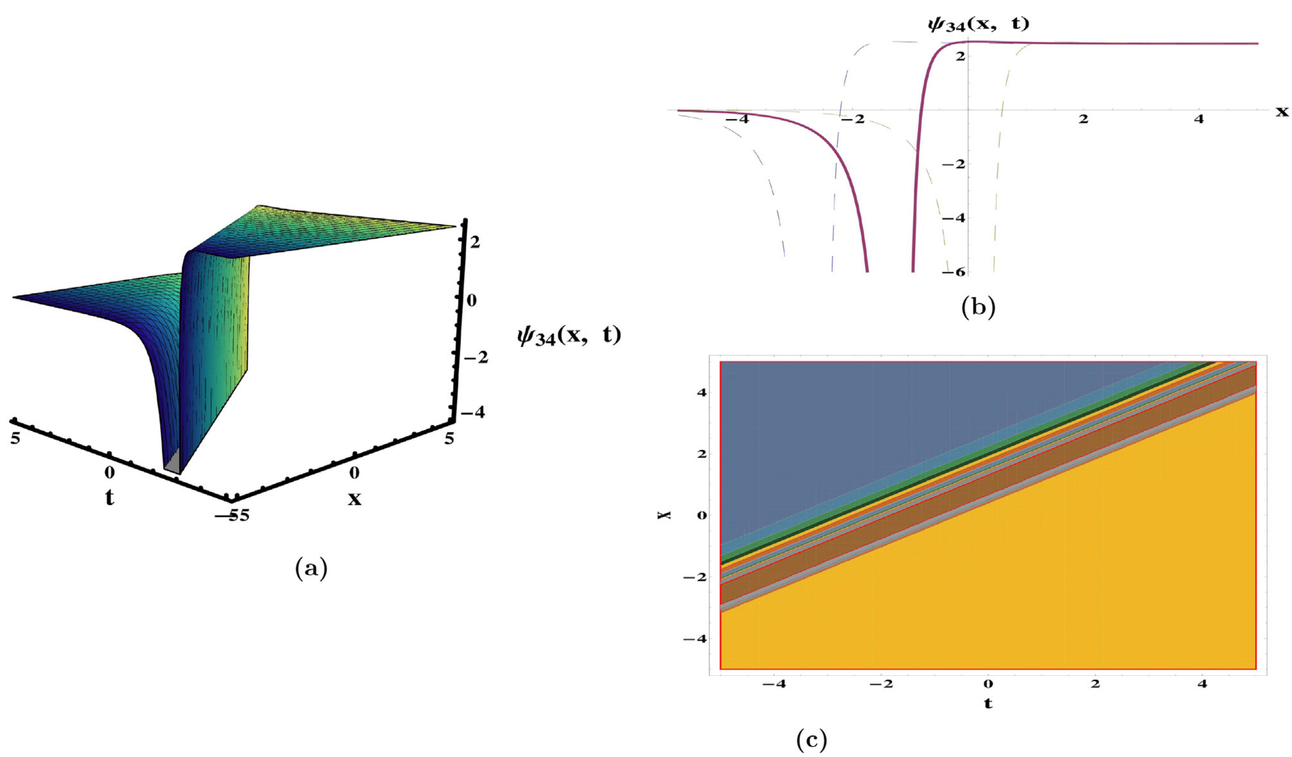

Fig. 7. Plots of solution (80) for the parameters $p = 0.7, q = 1.7, \gamma = 1.5, c = 1.4, r = 2, \lambda = 2.4, \chi = 1.3$ and $B = e$.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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