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ORIGINAL ARTICLE

Generalized fractional kinetic equations involving generalized Struve function of the first kind

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Abstract In recent paper Dinesh Kumar et al. developed a generalized fractional kinetic equation involving generalized Bessel function of first kind. The object of this paper is to derive the solution of the fractional kinetic equation involving generalized Struve function of the first kind. The results obtained in terms of generalized Struve function of first kind are rather general in nature and can easily construct various known and new fractional kinetic equations.

1. Introduction

The Struve function $H_\nu(z)$ and $L_\nu(z)$ are defined as the sum of the following infinite series

$$H_\upsilon(z) = \left(\frac{z}{2}\right)^{\upsilon+1} \sum_{k=0}^{\infty} \frac{(-1)^k}{\Gamma\left(k+\frac{3}{2}\right)\Gamma\left(k+\upsilon+\frac{1}{2}\right)} \left(\frac{z}{2}\right)^{2k} \tag{1}$$

and

$$L_\upsilon(z) = \left(\frac{z}{2}\right)^{\upsilon+1} \sum_{k=0}^{\infty} \frac{1}{\Gamma\left(k+\frac{3}{2}\right)\Gamma\left(k+\upsilon+\frac{1}{2}\right)} \left(\frac{z}{2}\right)^{2k}. \tag{2}$$

The purpose of this work is to investigate the generalized form of the fractional kinetic equation involving generalized Struve function of the first kind $\mathcal{H}_{p,b,c}(z)$ defined for complex $z \in \mathbb{C}$ and $b, c, p \in \mathbb{C}$ ($\mathcal{R}(p) > -1$) by

$$\mathcal{H}_{p,b,c}(z) := \sum_{k=0}^{\infty} \frac{(-1)^k c^k}{\Gamma\left(p+1+\frac{b}{2}+k\right)\Gamma(k+3/2)} \left(\frac{z}{2}\right)^{2k+p+1}. \tag{3}$$

Details related to the function $\mathcal{H}_{p,b,c}(z)$ and its particular cases can be seen in Baricz (2010, 2008), Mondal and Nisar (2014), Mondal and Swaminathan (2012) and the references therein. In this paper we consider the following transformation

$$\phi_{b,c}(z) = 2^{p+1}\Gamma\left(p+\frac{b}{2}+1\right)\Gamma\left(\frac{3}{2}\right) z^{\frac{1}{2}-\frac{p}{2}} \mathcal{H}_{p,b,c}(\sqrt{z})$$
$$= z + \sum_{k=1}^{\infty} \frac{(-c)^k z^{k+1}}{(\upsilon)_k \left(\frac{3}{2}\right)_k}, \tag{4}$$

where

$$\upsilon = p + \frac{b}{2} + 1 \notin \mathbb{Z}_0^- := \{0, -1, -2, \ldots\}$$

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and

$$(a)_k = \begin{cases} 1 & (k=0) \\ a(a+1)\ldots(a+k-1) & (k \in \mathbb{N} := \{1,2,3,\ldots\}) \end{cases} = \frac{\Gamma(a+k)}{\Gamma(a)} \left(a \in \mathbf{C} \setminus \mathbb{Z}_0^-\right). \tag{5}$$

The importance of fractional differential equations in the field of applied science has gained more attention not only in mathematics but also in physics, dynamical systems, control systems and engineering, to create the mathematical model of many physical phenomena. Especially, the kinetic equations describe the continuity of motion of substance and are the basic equations of mathematical physics and natural science. The extension and generalization of fractional kinetic equations involving many fractional operators were found (Zaslavsky, 1994; Saichev and Zaslavsky, 1997; Haubold and Mathai, 2000; Saxena et al., 2002, 2004, 2006, 2008; Chaurasia and Pandey, 2008; Gupta and Sharma, 2011; Chouhan and Sarswat, 2012; Chouhan et al., 2013; Gupta and Parihar, 2014; Kumar et al., 2015; Choi and Kumar, 2015). In view of the effectiveness and a great importance of the kinetic equation in certain astrophysical problems the authors develop a further generalized form of the fractional kinetic equation involving generalized Struve function of the first kind.

The fractional differential equation between rate of change of reaction was established by Haubold and Mathai (2000), the destruction rate and the production rate are calculated as follows:

$$\frac{dN}{dt} = -d(N_t) + p(N_t), \tag{6}$$

where $N = N(t)$ the rate of reaction, $d = d(N)$ the rate of destruction, $p = p(N)$ the rate of production and N_t denotes the function defined by $N_t(t^*) = N(t - t^*)$, $t^* > 0$.

The special case of (6), for spatial fluctuations or inhomogeneities in $N(t)$ the quantity are neglected, that is the equation

$$\frac{dN}{dt} = -c_i N_i(t) \tag{7}$$

with the initial condition that $N_i(t=0) = N_0$ is the number of density of species i at time $t = 0$ and $c_i > 0$. If we remove the index i and integrate the standard kinetic Eq. (7), we have

$$N(t) - N_0 = -c_0 D_t^{-1} N(t), \tag{8}$$

where ${}_0D_t^{-1}$ is the special case of the Riemann–Liouville integral operator ${}_0D_t^{-\upsilon}$ defined as

$${}_0D_t^{-\upsilon} f(t) = \frac{1}{\Gamma(\upsilon)} \int_0^t (t-s)^{\upsilon-1} f(s)ds, \quad (t > 0, \mathcal{R}(\upsilon) > 0).$$

The fractional generalization of the standard kinetic Eq. (8) is given by Haubold and Mathai (2000) as follows:

$$N(t) - N_0 = -c_0^{\upsilon} D_t^{-1} N(t) \tag{9}$$

and obtained the solution of (9) as follows:

$$N(t) = N_0 \sum_{k=0}^{\infty} \frac{(-1)^k}{\Gamma(\upsilon k + 1)} (ct)^{\upsilon k}. \tag{10}$$

Further, Saxena and Kalla (2008) considered the following fractional kinetic equation:

$$N(t) - N_0 f(t) = -c_0^{\upsilon} D_t^{-\upsilon} N(t) \quad (\mathcal{R}(\upsilon) > 0), \tag{11}$$

where $N(t)$ denotes the number density of a given species at time t, $N_0 = N(0)$ is the number density of that species at time $t = 0$, c is a constant and $f \in L(0, \infty)$.

By applying the Laplace transform to (11) (see Kumar et al., 2015),

$$L\{N(t);p\} = N_0 \frac{F(p)}{1 + c^{\upsilon} p^{-\upsilon}} = N_0 \left(\sum_{n=0}^{\infty} (-c^{\upsilon})^n p^{-n\upsilon} \right) F(p) \quad \left(n \in N_0, \left|\frac{c}{p}\right| < 1 \right), \tag{12}$$

where the Laplace transform (Spiegel, 1965) is defined by

$$F(p) = L\{f(t);p\} = \int_0^{\infty} e^{-pt} f(t)dt \quad (\mathcal{R}(p) > 0). \tag{13}$$

The object of this paper is to derive the solution of the fractional kinetic equation involving generalized Struve function of the first kind. The results obtained in terms of generalized Struve function of the first kind are rather general in nature and can easily construct various known and new fractional kinetic equations.

2. Solution of generalized fractional kinetic equations

In this section, we will investigate the solution of the generalized fractional kinetic equations by considering the generalized Struve function of first kind. The results are as follows.

Theorem 1. *If $d > 0, \upsilon > 0, c, b, l, t \in \mathbf{C}$ and $\mathcal{R}(l) > -1$, then the solution of the equation*

$$N(t) - N_0 \mathcal{H}_{l,b,c}(t) = -d^{\upsilon}{}_0D_t^{-\upsilon} N(t) \tag{14}$$

is given by the following formula

$$N(t) = N_0 \sum_{k=0}^{\infty} \frac{(-c)^k \Gamma(2k+l+2)}{\Gamma\left(l+k+\frac{b}{2}+1\right)\Gamma\left(k+\frac{3}{2}\right)} \left(\frac{t}{2}\right)^{2k+l+1} E_{v,2k+l+2}(-d^{\upsilon} t^{\upsilon}) \tag{15}$$

where the generalized Mittag-Leffler function $E_{\alpha,\beta}(x)$ is given by Mittag-Leffler (1905)

$$E_{\alpha,\beta}(x) = \sum_{n=0}^{\infty} \frac{x^n}{\Gamma(\alpha n + \beta)}.$$

Proof. The Laplace transform of the Riemann–Liouville fractional integral operator is given by Erdélyi et al. (1954), Srivastava and Saxena (2001)

$$L\left\{{}_0D_t^{-\upsilon} f(t);p\right\} = p^{-\upsilon} F(p) \tag{16}$$

where $F(p)$ is defined in (13). Now, applying the Laplace transform to the both sides of (14), gives

$$L\{N(t);p\} = N_0 L\{\mathcal{H}_{l,b,c}(t);p\} - d^{\upsilon} L\left\{{}_0D_t^{-\upsilon} N(t);p\right\}$$

$$N(p) = N_0 \left(\int_0^{\infty} e^{-pt} \sum_{k=0}^{\infty} \frac{(-c)^k}{\Gamma\left(l+1+\frac{b}{2}+k\right)\Gamma\left(k+\frac{3}{2}\right)} \left(\frac{t}{2}\right)^{2k+l+1} dt \right) - d^{\upsilon} p^{-\upsilon} N(p)$$

$$N(p)+d^{\upsilon}p^{-\upsilon}N(p)=N_0\sum_{k=0}^{\infty}\frac{(-c)^k(2)^{-(2k+l+1)}}{\Gamma\left(l+k+\frac{b}{2}+1\right)\Gamma\left(k+\frac{3}{2}\right)}\times\int_0^{\infty}e^{-pt}t^{2k+l+1}dt$$
$$=N_0\sum_{k=0}^{\infty}\frac{(-c)^k(2)^{-(2k+l+1)}}{\Gamma\left(l+k+\frac{b}{2}+1\right)\Gamma\left(k+\frac{3}{2}\right)}\frac{\Gamma(2k+l+2)}{p^{2k+l+2}}$$

$$N(p)=N_0\sum_{k=0}^{\infty}\frac{(-c)^k(2)^{-(2k+l+1)}\Gamma(2k+l+2)}{\Gamma\left(l+k+\frac{b}{2}+1\right)\Gamma\left(k+\frac{3}{2}\right)}\times\left\{p^{-(2k+l+2)}\sum_{r=0}^{\infty}\left[-\left(\frac{p}{d}\right)^{-\upsilon}\right]^r\right\}. \tag{17}$$

Taking Laplace inverse of (17), and by using $L^{-1}\{p^{-\upsilon};t\}=\frac{t^{\upsilon-1}}{\Gamma(\upsilon)},\mathcal{R}(\upsilon)>0$ we have

$$L^{-1}\{N(p)\}=N_0\sum_{k=0}^{\infty}\frac{(-c)^k(2)^{-(2k+l+1)}\Gamma(2k+l+2)}{\Gamma\left(l+k+\frac{b}{2}+1\right)\Gamma\left(k+\frac{3}{2}\right)}L^{-1}\times\left\{\sum_{r=0}^{\infty}(-1)^r d^{\upsilon r}p^{-(2k+l+2+\upsilon r)}\right\}$$

$$N(t)=N_0\sum_{k=0}^{\infty}\frac{(-c)^k(2)^{-(2k+l+1)}\Gamma(2k+l+2)}{\Gamma\left(l+k+\frac{b}{2}+1\right)\Gamma\left(k+\frac{3}{2}\right)}\times\left\{\sum_{r=0}^{\infty}(-1)^r d^{\upsilon r}\frac{t^{(2k+l+1+\upsilon r)}}{\Gamma(\upsilon r+2k+l+2)}\right\}$$
$$=N_0\sum_{k=0}^{\infty}\frac{(-c)^k(2)^{-(2k+l+1)}\Gamma(2k+l+2)}{\Gamma\left(l+k+\frac{b}{2}+1\right)\Gamma\left(k+\frac{3}{2}\right)}t^{2k+l+1}\times\left\{\sum_{r=0}^{\infty}(-1)^r d^{\upsilon r}\frac{t^{\upsilon r}}{\Gamma(\upsilon r+2k+l+2)}\right\}$$
$$=N_0\sum_{k=0}^{\infty}\frac{(-c)^k\Gamma(2k+l+2)}{\Gamma\left(l+k+\frac{b}{2}+1\right)\Gamma\left(k+\frac{3}{2}\right)}\left(\frac{t}{2}\right)^{2k+l+1}\times\left\{\sum_{r=0}^{\infty}(-1)^r d^{\upsilon r}\frac{t^{\upsilon r}}{\Gamma(\upsilon r+2k+l+2)}\right\}$$

$$N(t)=N_0\sum_{k=0}^{\infty}\frac{(-c)^k\Gamma(2k+l+2)}{\Gamma\left(l+k+\frac{b}{2}+1\right)\Gamma\left(k+\frac{3}{2}\right)}\left(\frac{t}{2}\right)^{2k+l+1}E_{\upsilon,2k+l+2}(-d^{\upsilon}t^{\upsilon}).$$

□

Theorem 2. *If $d>0,\upsilon>0,c,b,l,t\in\mathbf{C}$ and $\mathcal{R}(l)>-1$, then the solution of the equation*

$$N(t)-N_0\mathcal{H}_{l,b,c}(d^{\upsilon}t^{\upsilon})=-d^{\upsilon}_0D_t^{-\upsilon}N(t) \tag{18}$$

is given by the following formula

$$N(t)=N_0\sum_{k=0}^{\infty}\frac{(-c)^k\Gamma(2k+\upsilon l+2)}{\Gamma\left(l+k+\frac{b}{2}+1\right)\Gamma\left(k+\frac{3}{2}\right)}\left(\frac{d^{\upsilon}t^{\upsilon}}{2}\right)^{2k+l+1}\times E_{v,2k\upsilon+\upsilon l+2}(-d^{\upsilon}t^{\upsilon}), \tag{19}$$

where $E_{v,2k\upsilon+l+2}(.)$ is the generalized Mittag-Leffler function (Mittag-Leffler, 1905).

Proof. The Laplace transform of the Riemann–Liouville fractional integral operator is given by Erdélyi et al. (1954)

$$L\{{}_0D_t^{-\upsilon}f(t);p\}=p^{-\upsilon}F(p) \tag{20}$$

where $F(p)$ is defined in (13). Now, applying the Laplace transform to the both sides of (18), gives

$$L\{N(t);p\}=N_0L\{\mathcal{H}_{l,b,c}(d^{\upsilon}t^{\upsilon});p\}-d^{\upsilon}L\{{}_0D_t^{-\upsilon}N(t);p\} \tag{21}$$

$$N(p)=N_0\left(\int_0^{\infty}e^{-pt}\sum_{k=0}^{\infty}\frac{(-c)^k}{\Gamma\left(l+1+\frac{b}{2}+k\right)\Gamma\left(k+\frac{3}{2}\right)}\left(\frac{d^{\upsilon}t^{\upsilon}}{2}\right)^{2k+l+1}dt\right)-d^{\upsilon}p^{-\upsilon}N(p)$$

$$N(p)+d^{\upsilon}p^{-\upsilon}N(p)=N_0\sum_{k=0}^{\infty}\frac{(-c)^k\left(\frac{d^{\upsilon}}{2}\right)^{2k+l+1}}{\Gamma\left(l+k+\frac{b}{2}+1\right)\Gamma\left(k+\frac{3}{2}\right)}\int_0^{\infty}e^{-pt}t^{2k\upsilon+\upsilon l+\upsilon}dt$$
$$=N_0\sum_{k=0}^{\infty}\frac{(-c)^k\left(\frac{d^{\upsilon}}{2}\right)^{2k+l+1}}{\Gamma\left(l+k+\frac{b}{2}+1\right)\Gamma\left(k+\frac{3}{2}\right)}\frac{\Gamma(2k\upsilon+\upsilon l+\upsilon+1)}{p^{2k\upsilon+\upsilon l+\upsilon+1}}$$

$$N(p)=N_0\sum_{k=0}^{\infty}\frac{(-c)^k\left(\frac{d^{\upsilon}}{2}\right)^{2k+l+1}\Gamma(2k\upsilon+\upsilon l+\upsilon+1)}{\Gamma\left(l+k+\frac{b}{2}+1\right)\Gamma\left(k+\frac{3}{2}\right)}\left\{p^{-(2k\upsilon+\upsilon l+\upsilon+1)}\sum_{r=0}^{\infty}\left[-\left(\frac{p}{d}\right)^{-\upsilon}\right]^r\right\}. \tag{22}$$

Taking Laplace inverse of (22), and by using $L^{-1}\{p^{-\upsilon};t\}=\frac{t^{\upsilon-1}}{\Gamma(\upsilon)},\mathcal{R}(\upsilon)>0$ we have

$$L^{-1}\{N(p)\}=N_0\sum_{k=0}^{\infty}\frac{(-c)^k\left(\frac{d^{\upsilon}}{2}\right)^{2k+l+1}\Gamma(2k\upsilon+\upsilon l+\upsilon+1)}{\Gamma\left(l+k+\frac{b}{2}+1\right)\Gamma\left(k+\frac{3}{2}\right)}\times L^{-1}\left\{\sum_{r=0}^{\infty}(-1)^r d^{\upsilon r}p^{-(2k\upsilon+\upsilon l+\upsilon+\upsilon r+1)}\right\}$$

$$N(t)=N_0\sum_{k=0}^{\infty}\frac{(-c)^k\left(\frac{d^{\upsilon}}{2}\right)^{2k+l+1}\Gamma(2k\upsilon+\upsilon l+\upsilon+1)}{\Gamma\left(l+k+\frac{b}{2}+1\right)\Gamma\left(k+\frac{3}{2}\right)}\times\left\{\sum_{r=0}^{\infty}(-1)^r d^{\upsilon r}\frac{t^{\upsilon(2k+l+r+1)}}{\Gamma(2k\upsilon+\upsilon l+\upsilon r+\upsilon+1)}\right\}$$
$$=N_0\sum_{k=0}^{\infty}\frac{(-c)^k\left(\frac{d^{\upsilon}}{2}\right)^{2k+l+1}\Gamma(2k\upsilon+\upsilon l+\upsilon+1)}{\Gamma\left(l+k+\frac{b}{2}+1\right)\Gamma\left(k+\frac{3}{2}\right)}\times t^{\upsilon(2k+l+1)}\left\{\sum_{r=0}^{\infty}(-1)^r d^{\upsilon r}\frac{t^{\upsilon r}}{\Gamma(2k\upsilon+\upsilon l+\upsilon r+\upsilon+1)}\right\}$$
$$=N_0\sum_{k=0}^{\infty}\frac{(-c)^k\Gamma(2k\upsilon+\upsilon l+\upsilon+1)}{\Gamma\left(l+k+\frac{b}{2}+1\right)\Gamma\left(k+\frac{3}{2}\right)}\left(\frac{d^{\upsilon}}{2}\right)^{2k+l+1}\times t^{\upsilon(2k+l+1)}\left\{\sum_{r=0}^{\infty}(-1)^r d^{\upsilon r}\frac{t^{\upsilon r}}{\Gamma(2k\upsilon+\upsilon l+\upsilon r+\upsilon+1)}\right\}$$

$$N(t)=N_0\sum_{k=0}^{\infty}\frac{(-c)^k\Gamma(2k\upsilon+\upsilon l+\upsilon+1)}{\Gamma\left(l+k+\frac{b}{2}+1\right)\Gamma\left(k+\frac{3}{2}\right)}\left(\frac{d^{\upsilon}}{2}\right)^{2k+l+1}\times t^{\upsilon(2k+l+1)}E_{v,(2k+l+1)\upsilon+1}(-d^{\upsilon}t^{\upsilon})$$

This completes the proof of Theorem 2. □

Theorem 3. *If $d>0,\upsilon>0,c,b,l,t\in\mathbf{C},a\neq d$ and $\mathcal{R}(l)>-1$, then the solution of the equation*

$$N(t) - N_0\mathcal{H}_{l,b,c}(d^{\upsilon}t^{\upsilon}) = -a^{\upsilon}{}_0D_t^{-\upsilon}N(t) \tag{23}$$

is given by the following formula:

$$N(t) = N_0\sum_{k=0}^{\infty}\frac{(-c)^k\Gamma(2k\upsilon+\upsilon l+\upsilon+1)}{\Gamma\left(l+k+\frac{b}{2}+1\right)\Gamma\left(k+\frac{3}{2}\right)}\left(\frac{d^{\upsilon}}{2}\right)^{2k+l+1} \times t^{\upsilon(2k+l+1)}E_{\nu,(2k+l+1)\upsilon+1}(-a^{\upsilon}t^{\upsilon}). \tag{24}$$

Proof. Applying Laplace transform to the both side of (23) we get

$$L_0\{N(t);p\} = N_0L\{\mathcal{H}_{l,b,c}(d^{\upsilon}t^{\upsilon});p\} - a^{\upsilon}L\{{}_0D_t^{-\upsilon}N(t);p\}$$

$$N(p) = N_0\left(\int_0^{\infty}e^{-pt}\sum_{k=0}^{\infty}\frac{(-c)^k}{\Gamma\left(l+1+\frac{b}{2}+k\right)\Gamma\left(k+\frac{3}{2}\right)}\left(\frac{d^{\upsilon}t^{\upsilon}}{2}\right)^{2k+l+1}dt\right) - d^{\upsilon}p^{-\upsilon}N(p)$$

$$\begin{aligned}N(p) + a^{\upsilon}p^{-\upsilon}N(p) &= N_0\sum_{k=0}^{\infty}\frac{(-c)^k\left(\frac{d^{\upsilon}}{2}\right)^{2k+l+1}}{\Gamma\left(l+k+\frac{b}{2}+1\right)\Gamma\left(k+\frac{3}{2}\right)} \times \int_0^{\infty}e^{-pt}t^{2k\upsilon+\upsilon l+\upsilon}dt \\ &= N_0\sum_{k=0}^{\infty}\frac{(-c)^k\left(\frac{d^{\upsilon}}{2}\right)^{2k+l+1}}{\Gamma\left(l+k+\frac{b}{2}+1\right)\Gamma\left(k+\frac{3}{2}\right)} \times \frac{\Gamma(2k\upsilon+\upsilon l+\upsilon+1)}{p^{2k\upsilon+\upsilon l+\upsilon+1}}\end{aligned}$$

$$N(p) = N_0\sum_{k=0}^{\infty}\frac{(-c)^k\left(\frac{d^{\upsilon}}{2}\right)^{2k+l+1}\Gamma(2k\upsilon+\upsilon l+\upsilon+1)}{\Gamma\left(l+k+\frac{b}{2}+1\right)\Gamma\left(k+\frac{3}{2}\right)} \times \left\{p^{-(2k\upsilon+\upsilon l+\upsilon+1)}\sum_{r=0}^{\infty}\left[-\left(\frac{p}{a}\right)^{-\upsilon}\right]^r\right\}. \tag{25}$$

Taking Laplace inverse of (25), and by using $L^{-1}\{p^{-\upsilon};t\} = \frac{t^{\upsilon-1}}{\Gamma(\upsilon)}, \mathcal{R}(\upsilon) > 0$ we have

$$L^{-1}\{N(p)\} = N_0\sum_{k=0}^{\infty}\frac{(-c)^k\left(\frac{d^{\upsilon}}{2}\right)^{2k+l+1}\Gamma(2k\upsilon+\upsilon l+\upsilon+1)}{\Gamma\left(l+k+\frac{b}{2}+1\right)\Gamma\left(k+\frac{3}{2}\right)} \times L^{-1}\left\{\sum_{r=0}^{\infty}(-1)^r a^{\upsilon r}p^{-(2k\upsilon+\upsilon l+\upsilon+\upsilon r+1)}\right\}$$

$$\begin{aligned}N(t) &= N_0\sum_{k=0}^{\infty}\frac{(-c)^k\left(\frac{d^{\upsilon}}{2}\right)^{2k+l+1}\Gamma(2k\upsilon+\upsilon l+\upsilon+1)}{\Gamma\left(l+k+\frac{b}{2}+1\right)\Gamma\left(k+\frac{3}{2}\right)} \times \left\{\sum_{r=0}^{\infty}(-1)^r a^{\upsilon r}\frac{t^{\upsilon(2k+l+r+1)}}{\Gamma(2k\upsilon+\upsilon l+\upsilon r+1)}\right\} \\ &= N_0\sum_{k=0}^{\infty}\frac{(-c)^k\left(\frac{d^{\upsilon}}{2}\right)^{2k+l+1}\Gamma(2k\upsilon+\upsilon l+2)}{\Gamma\left(l+k+\frac{b}{2}+1\right)\Gamma\left(k+\frac{3}{2}\right)}t^{\upsilon(2k+l+1)} \times \left\{\sum_{r=0}^{\infty}(-1)^r a^{\upsilon r}\frac{t^{\upsilon r}}{\Gamma(2k\upsilon+\upsilon l+\upsilon r+\upsilon+1)}\right\} \\ &= N_0\sum_{k=0}^{\infty}\frac{(-c)^k\Gamma(2k\upsilon+\upsilon l+\upsilon+1)}{\Gamma\left(l+k+\frac{b}{2}+1\right)\Gamma\left(k+\frac{3}{2}\right)}\left(\frac{d^{\upsilon}}{2}\right)^{2k+l+1}t^{\upsilon(2k+l+1)} \times \left\{\sum_{r=0}^{\infty}(-1)^r a^{\upsilon r}\frac{t^{\upsilon r}}{\Gamma(2k\upsilon+\upsilon l+\upsilon r+\upsilon+1)}\right\}\end{aligned}$$

$$N(t) = N_0\sum_{k=0}^{\infty}\frac{(-c)^k\Gamma(2k\upsilon+\upsilon l+\upsilon+1)}{\Gamma\left(l+k+\frac{b}{2}+1\right)\Gamma\left(k+\frac{3}{2}\right)}\left(\frac{d^{\upsilon}}{2}\right)^{2k+l+1} \times t^{\upsilon(2k+l+1)}E_{\nu,(2k+l+1)\upsilon+1}(-a^{\upsilon}t^{\upsilon}).$$

3. Conclusion

In this work we give a new fractional generalization of the standard kinetic equation and derived solutions for the same. From the close relationship of the generalized Struve function of the first kind $\mathcal{H}_p(z)$ with many special functions, we can easily construct various known and new fractional kinetic equations.

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