Journal of King Saud University – Science 31 (2019) 1478–1484

Contents lists available at ScienceDirect

Journal of King Saud University – Science

journal homepage: www.sciencedirect.com

Original article

The conformable space-time fractional mKdV equations and their exact solutions

Asim Zafar [a], Aly R. Seadawy [b,c,*]

[a] Department of Mathematics, CUI, Vehari Campus, Pakistan
[b] Department of Mathematics, Faculty of Science, Taibah University, Al-Madinah Al-Munawarah, Saudi Arabia
[c] Mathematics Department, Faculty of Science, Beni-Suef University, Beni Suef, Egypt

A R T I C L E I N F O

Article history:
Received 21 October 2018
Revised 31 December 2018
Accepted 16 September 2019
Available online 27 September 2019

Keywords:
Space-time-fractional mKdV equations
Conformable derivatives
Hyperbolic function approach
Exact soliton solutions

A B S T R A C T

In this study, we are interested in exploring the conformable space-time fractional mKdV equations via hyperbolic function approach. A traveling wave transformation and the conformable derivative are used to convert the nonlinear fractional differential equation into a nonlinear ordinary differential equation. Then, the resulting equation is elucidated by utilizing the hyperbolic function approach through Mathematica. A variety of soliton type solutions including, hyperbolic and trigonometric functions, is formulated and the graphical representation for these solutions is given by using MATLAB.

1. Introduction

The Korteweg-deVries (KdV), Boussinesq, Kadomtsev-Petviashvili, and Whitham-Broer-Kaup (WBK) equations are the well-known completely integrable models that describe the propagation of shallow waDepartment of Mathematicster (Wadati, 1973; Ohkuma and Wadati, 1983; Ablowitz and Segur, 1981). The adynamic of shallow water waves in various places like sea beaches are governed bt the KdV and Boussinesq Equations (Korteweg and de Vries, 1895; Yan and Zhang, 2001). The KdV equation has an impact in modeling blood pressure pulses (Schamel, 1973; Mousavian et al., 2011). Furthermore, Wazwaz (Wazwaz, 2017) introduced the nonlinear modified KdV equations of $(3+1)$-dimension and investigate their exact soliton and kink solutions. In particular, Nuruddeen (Nuruddeen, 2018) has explored the soliton solutions for the conformable space-time fractional modified KdV equations of $(3+1)$-dimension.

There are various mathematical approaches to solve PDEs with nonlinear characteristics or fractional derivatives. Some of the commonly used approaches are: The ansatz (Hosseini et al., 2018), modified simple equation (Jawad et al., 2010), the extended trial equation (Mahmood et al., 2014), the first integral (Eslami, 2016), $(\frac{G'}{G})$-expansion (Younis and Zafar, 2014), sine-Gordon expansion (Hosseini et al., 2017). Furthermore, some other excellent works like generalized Kudryashov method (Pandir et al., 2012), a modified form of Kudryashov and functional variable methods (Ayati et al., 2017) have been done by different researchers. In Bekir et al. (2014), Fan (2000) and Zhang (2007), the auxiliary equation, the extended *tanh*-function, the improved $tan(\frac{\phi(\eta)}{2})$-expansion methods and the exp function approach have been explored for discrete and fractional order PDEs as well. Many more in several theoretical works about solitons and their applications (Seadawy, 2017; Khater et al., 2000; Khater et al., 2000; Khater et al., 2006; Khater et al., 2006; Helal and Seadawy, 2009; Helal and Seadawy, 2011; Seadawy, 2011; Seadawy, 2012; Helal and Seadawy, 2012). Moreover, the exp_a function method has been utilized to explore the PDEs in (Ali and Hassan, 2010; Zafar, 2018; Seadawy and El-Rashidy, 2016; Ul-Haq Tariq and Seadawy, 2017; Seadawy, 2017; Seadawy, 2017; Seadawy and Manafian, 2018; Selima et al., 2016). In particular, an efficient approach namely the hyperbolic function approach has been studied in Xie et al. (2001), Bai (2001), Hosseini et al. (2018) and Seadawy et al. (2018) to procure exact solutions. One can observe that aforemented approach produces the soliton type and periodic function solutions of nonlinear evolution equations. The study of fractional calculus has been carried out in Samko et al. (1993) and Chung (2015).

This paper aims to explore the conformable space-time fractional modified KdV equations of $(3+1)$-dimension for exact soliton type solutions via hyperbolic function approach using conformable derivative and the traveling wave transformation.

* Corresponding author at: Department of Mathematics, Faculty of Science, Taibah University, Al-Madinah Al-Munawarah, Saudi Arabia.

E-mail address: aly742001@yahoo.com (A.R. Seadawy).

https://doi.org/10.1016/j.jksus.2019.09.003
1018-3647/

The scheme of this paper is as follows: A brief description of the conformable derivative and the hyperbolic function approach is given in Section 2. Section 3 and 4 illustrate how to utilize this approach for producing new solutions with their graphs. The latter part summarizes results of the current study.

2. Conformable fractional derivative approach

We recall the conformable derivative with some of its properties (Khalil et al., 2014).

Defination 1 Suppose $h:\mathbb{R}_{>0}\to\mathbb{R}$be a function. Then, for all $t>0$,

$$D_t^\alpha(p(t))=\lim_{\varepsilon\to 0}\frac{p(t+\varepsilon t^{1-\alpha})-p(t)}{\varepsilon}$$

is known as $\alpha,\ 0<\alpha\leqslant 1$ order conformable fractional derivative of p. The followings are some useful properties:

$D_t^\alpha(ap+bg)=aD_t^\alpha(p)+bD_t^\alpha(g)$, for all $a,b\in\mathbb{R}$.

$D_t^\alpha(pg)=pD_t^\alpha(g)+gD_t^\alpha(p)$

Let $p:\mathbb{R}_{>0}\to\mathbb{R}$ be an α-differentiable function, g be a differentiable function defined in the range of p.

$$D_t^\alpha(p\circ g(t))=t^{1-\alpha}g'(t)p'(g(t)).$$

On the top of that, the following rules hold.

$D_t^\alpha(t^h)=ht^{h-\alpha}$, for all $h\in\mathbb{R}$.

$D_t^\alpha(\delta)=0$, where δ is constant.

$D_t^\alpha(p/g)=\frac{gD_t^\alpha(p)-pD_t^\alpha(g)}{g^2}$.

Conjointly, if p is differentiable, then $D_t^\alpha(p(t))=t^{1-\alpha}\frac{dp(t)}{dt}$.

2.1. The hyperbolic function approach

The present subsection provides a transitory explanation of hyperbolic function approach in engendering new exact solutions to nonlinear conformable space-time fractional modified KdV equations. For this purpose, suppose that we have a nonlinear conformable space-time FDE that can be presented in the form

$$F(u,D_t^\gamma u,D_x^\gamma u,D_{tt}^{2\gamma}u,D_{xx}^{2\gamma}u,\ldots)=0 \tag{1}$$

The FDE (1) can be changed into the following nonlinear ODE of integer order

$$P(U,U',U'',\ldots,)=0 \tag{2}$$

with the use of following wave variable

$$u(x,t)=U(\eta),\eta=k\frac{x^\gamma}{\gamma}+r\frac{y^\gamma}{\gamma}+s\frac{z^\gamma}{\gamma}-l\frac{t^\gamma}{\gamma}, \tag{3}$$

where k,r,s and l are nonzero arbitrary constants.

Let us try to search a non-trivial solution to the Eq. (2) in the following form (Xie et al., 2001; Bai, 2001; Hosseini et al., 2018; Seadawy et al., 2018)

$$U(\eta)=A_0+\sum_{i=1}^{N}\sinh^{i-1}(\rho)[B_i\sinh(\rho)+A_i\cosh(\rho)] \tag{4}$$

where ρ is some specific functions. By calculating the positive integer N, setting the Eq. (4) in Eq. (2), and comparing coefficients, we will find a set of nonlinear equations whose solution, finally provides explicit exact solutions of the Eq. (1). It is worth mentioning that using the separation of variables techniques on $\frac{d\rho}{d\eta}=\sinh(\rho)$, we find $\sinh(\rho)=\pm\mathrm{csch}(\eta),\cosh(\rho)=-\coth(\eta)$ and $\sinh(\rho)=\pm\imath\mathrm{sech}(\eta)$, $\cosh(\rho)=-\tanh(\eta)$. Accordingly, the solution (4) can be rewritten as

$$U(\eta)=A_0+\sum_{i=1}^{N}(\pm\mathrm{csch})^{i-1}(\eta))[\pm B_i csch(\eta)-A_i\coth(\eta)],$$

and

$$U(\eta)=A_0+\sum_{i=1}^{N}(\pm\imath\mathrm{sech})^{i-1}(\eta)[\pm\imath B_i\mathrm{sech}(\eta)-A_i\tanh(\eta)].$$

Likewise, it is obvious that from $\frac{dw}{d\eta}=\cosh(\rho)$, we find $\sinh(\rho)=-\cot(\eta),\cosh(\rho)=\pm\csc(\eta)$ and $\sinh(\rho)=\tan(\eta),\cosh(\rho)=\pm\sec(\eta)$. Accordingly, the solution (4) can be rewritten as

$$U(\eta)=A_0+\sum_{i=1}^{N}(-\cot)^{i-1}(\eta)[-B_i\cot(\eta)\pm A_i\csc(\eta)],$$

and

$$U(\eta)=A_0+\sum_{i=1}^{N}(\tan^{i-1})(\eta)[B_i\tan(\eta)\pm A_i\sec(\eta)].$$

On similar lines one can see Xian-Lin and Jia-Shi (2008), the extended sinh-Gordon equation expansion method based on the sinh-Gordon equation, was proposed to construct hyperbolic, trigonometric and Jacobi elliptic function solutions of nonlinear evolution equations.

3. The conformable fractional (3 + 1)-dimensional mKdV equation-I

Firstly, we consider the following space-time fractional mKdV equation (Nuruddeen, 2018):

$$D_t^\gamma u+6D_x^\gamma u^3+u_{xyz}^{3\gamma}=0,\ 0<\gamma\leqslant 1. \tag{5}$$

Using the transformation (3), and integrating once w.r.t. η with zero constant of integration, we get

$$-lU+krsU''+6kU^3=0. \tag{6}$$

The balance between U'' and U^3 gives $N=1$, then the nontrivial solution (4) reduces to:

$$U(\eta)=B_1\sinh(\rho)+A_1\cosh(\rho)+A_0. \tag{7}$$

Case-I: $\frac{d\rho}{d\eta}=\sinh(\rho)$ By inserting the above solution in reduced equation Eq. (6) and equating the coefficients of each hyperbolic function to zero, we procure a set of nonlinear algebraic equations and its solution yields the following new exact solutions:

$$A_0=0,A_1=\mp\frac{i\sqrt{rs}}{2\sqrt{3}},B_1=\pm\frac{i\sqrt{rs}}{2\sqrt{3}},l=-\frac{1}{2}krs$$

$$u_{1,2}(\eta)=\mp\frac{i\sqrt{rs}}{2\sqrt{3}}\tanh\left(\frac{\eta}{2}\right) \tag{8}$$

$$A_0=0,A_1=\mp\frac{i\sqrt{rs}}{2\sqrt{3}},B_1=\mp\frac{i\sqrt{rs}}{2\sqrt{3}},l=-\frac{1}{2}krs$$

$$u_{3,4}(\eta)=\mp\frac{i\sqrt{rs}}{2\sqrt{3}}\coth\left(\frac{\eta}{2}\right) \tag{9}$$

where $\eta=k\frac{x^\gamma}{\gamma}+r\frac{y^\gamma}{\gamma}+s\frac{z^\gamma}{\gamma}+\frac{1}{2}krs\frac{t^\gamma}{\gamma}$.

$$A_0=0,A_1=0,B_1=\mp\frac{i\sqrt{rs}}{\sqrt{3}},l=krs$$

$$u_{5,6}(\eta)=\mp\frac{i\sqrt{rs}}{\sqrt{3}}\mathrm{csch}(\eta) \tag{10}$$

$$A_0=0,A_1=\mp\frac{i\sqrt{rs}}{\sqrt{3}},B_1=0,l=-2krs$$

$$u_{7,8}(\eta)=\mp\frac{i\sqrt{rs}}{\sqrt{3}}\coth(\eta) \tag{11}$$

where $\eta=k\frac{x^\gamma}{\gamma}+r\frac{y^\gamma}{\gamma}+s\frac{z^\gamma}{\gamma}-l\frac{t^\gamma}{\gamma}$.

Case-2: $\frac{d\rho}{d\eta}=\cosh(\rho)$ and $N=1$, (7), gives a set of nonlinear algebraic equations and its solution yields the following exact solutions:

$$A_0=0, A_1=\mp\frac{i\sqrt{rs}}{2\sqrt{3}}, B_1=\mp\frac{i\sqrt{rs}}{2\sqrt{3}}, l=\frac{krs}{2}$$

$$u_{1,2}(\eta)=\pm\frac{i\sqrt{rs}}{2\sqrt{3}}\tan\left(\frac{\eta}{2}\right) \quad (12)$$

$$A_0=0, A_1=\mp\frac{i\sqrt{rs}}{2\sqrt{3}}, B_1=\pm\frac{i\sqrt{rs}}{2\sqrt{3}}, l=\frac{krs}{2}$$

$$u_{3,4}(\eta)=\pm\frac{i\sqrt{rs}}{2\sqrt{3}}\cot\left(\frac{\eta}{2}\right) \quad (13)$$

where $\eta=k\frac{x^\gamma}{\gamma}+r\frac{y^\gamma}{\gamma}+s\frac{z^\gamma}{\gamma}-\frac{krs}{2}\frac{t^\gamma}{\gamma}$.

$$A_0=0, A_1=0, B_1=\mp\frac{i\sqrt{rs}}{\sqrt{3}}, l=2krs$$

$$u_{5,6}(\eta)=\mp\frac{i\sqrt{rs}}{\sqrt{3}}\cot(\eta) \quad (14)$$

$$A_0=0, A_1=\mp\frac{i\sqrt{rs}}{\sqrt{3}}, B_1=0, l=-krs$$

$$u_{7,8}(\eta)=\pm\frac{i\sqrt{rs}}{\sqrt{3}}\csc(\eta) \quad (15)$$

where $\eta=k\frac{x^\gamma}{\gamma}+r\frac{y^\gamma}{\gamma}+s\frac{z^\gamma}{\gamma}-l\frac{t^\gamma}{\gamma}$.

3.1. The conformable fractional (3 + 1)-dimensional mKdV equation-II

The equation-II can be read as (Nuruddeen, 2018)

$$D_t^\gamma u+6D_y^\gamma u^3+u_{xyz}^{3\gamma}=0,\ 0<\gamma\leqslant 1. \quad (16)$$

Using the transformation (3), and integrating once w.r.t. η, we get

$$-lU+krsU''+6rU^3=0. \quad (17)$$

Case-I: $\frac{d\rho}{d\eta}=\sinh(\rho)$ Through balancing the terms U'' and U^3, we select $N=1$. Then, by inserting (7) in (17), and equating coefficients of hyperbolic functions to zero in the resulting equation, a nonlinear algebraic set of equations is obtained and its solution yields the following exact solutions to (16).

$$A_0=0, A_1=\mp\frac{i\sqrt{ks}}{2\sqrt{3}}, B_1=\pm\frac{i\sqrt{ks}}{2\sqrt{3}},\ l=-\frac{1}{2}krs$$

$$u_{1,2}(\eta)=\mp\frac{i\sqrt{ks}}{2\sqrt{3}}\tanh\left(\frac{\eta}{2}\right) \quad (18)$$

$$A_0=0, A_1=\pm\frac{i\sqrt{ks}}{2\sqrt{3}}, B_1=\pm\frac{i\sqrt{ks}}{2\sqrt{3}},\ l=-\frac{1}{2}krs$$

$$u_{3,4}(\eta)=\pm\frac{i\sqrt{ks}}{2\sqrt{3}}\coth\left(\frac{\eta}{2}\right) \quad (19)$$

where $\eta=k\frac{x^\gamma}{\gamma}+r\frac{y^\gamma}{\gamma}+s\frac{z^\gamma}{\gamma}+\frac{1}{2}krs\frac{t^\gamma}{\gamma}$.

$$A_0=0, A_1=0, B_1=\mp\frac{i\sqrt{ks}}{\sqrt{3}},\ l=krs$$

$$u_{5,6}(\eta)=\mp\frac{i\sqrt{ks}}{\sqrt{3}}\operatorname{csch}(\eta) \quad (20)$$

$$A_0=0, A_1=\mp\frac{i\sqrt{ks}}{\sqrt{3}}, B_1=0,\ l=-2krs \quad (21)$$

$$u_{7,8}(\eta)=\mp\frac{i\sqrt{ks}}{\sqrt{3}}\coth(\eta) \quad (22)$$

where $\eta=k\frac{x^\gamma}{\gamma}+r\frac{y^\gamma}{\gamma}+s\frac{z^\gamma}{\gamma}-l\frac{t^\gamma}{\gamma}$.

Case-2: By using $\frac{d\rho}{d\eta}=\cosh(\rho)$ and $N=1$ in (4), (7) yields a set of nonlinear equations and its solution gives the following exact solutions:

$$A_0=0, A_1=\mp\frac{i\sqrt{ks}}{2\sqrt{3}}, B_1=\mp\frac{i\sqrt{ks}}{2\sqrt{3}},\ l=\frac{krs}{2}$$

$$u_{1,2}(\eta)=\pm\frac{i\sqrt{ks}}{2\sqrt{3}}\tan\left(\frac{\eta}{2}\right) \quad (23)$$

$$A_0=0, A_1=\mp\frac{i\sqrt{ks}}{2\sqrt{3}}, B_1=\pm\frac{i\sqrt{ks}}{2\sqrt{3}},\ l=\frac{krs}{2}$$

$$u_{3,4}(\eta)=\mp\frac{i\sqrt{ks}}{2\sqrt{3}}\cot\left(\frac{\eta}{2}\right) \quad (24)$$

where $\eta=k\frac{x^\gamma}{\gamma}+r\frac{y^\gamma}{\gamma}+s\frac{z^\gamma}{\gamma}-\frac{krs}{2}\frac{t^\gamma}{\gamma}$.

$$A_0=0, A_1=\mp\frac{i\sqrt{ks}}{\sqrt{3}}, B_1=0,\ l=-krs$$

$$u_{5,6}(\eta)=\pm\frac{i\sqrt{ks}}{\sqrt{3}}\csc(\eta) \quad (25)$$

$$A_0=0, A_1=0, B_1=\pm\frac{i\sqrt{ks}}{\sqrt{3}},\ l=2krs$$

$$u_{7,8}(\eta)=\pm\frac{i\sqrt{ks}}{\sqrt{3}}\cot(\eta) \quad (26)$$

where $\eta=k\frac{x^\gamma}{\gamma}+r\frac{y^\gamma}{\gamma}+s\frac{z^\gamma}{\gamma}-l\frac{t^\gamma}{\gamma}$.

3.2. The conformable fractional (3 + 1)-dimensional mKdV equation-III

In this section, the following conformable space–time fractional mKdV equation (Nuruddeen, 2018), is going to be considered for solutions.

$$D_t^\gamma u+6D_z^\gamma u^3+u_{xyz}^{3\gamma}=0,\ 0<\gamma\leqslant 1. \quad (27)$$

Using the transformation (3), and integrating once w.r.t. η with zero constant of integration, we get

$$-lU+krsU''+6sU^3=0. \quad (28)$$

Case-I: $\frac{d\rho}{d\eta}=\sinh(\rho)$ Through balancing the terms U'' and U^3 we select $N=1$, the non-trivial solution (7) and its derivatives in (28) produce a set of nonlinear algebraic equations. Then the solution of this set yields the following exact solutions for (27).

$$A_0=0, A_1=\mp\frac{i\sqrt{kr}}{2\sqrt{3}}, B_1=\pm\frac{i\sqrt{kr}}{2\sqrt{3}},\ l=-\frac{1}{2}krs$$

$$u_{1,2}(\eta)=\mp\frac{i\sqrt{kr}}{2\sqrt{3}}\tanh\left(\frac{\eta}{2}\right) \quad (29)$$

$$A_0=0, A_1=\pm\frac{i\sqrt{kr}}{2\sqrt{3}}, B_1=\pm\frac{i\sqrt{kr}}{2\sqrt{3}},\ l=-\frac{1}{2}krs$$

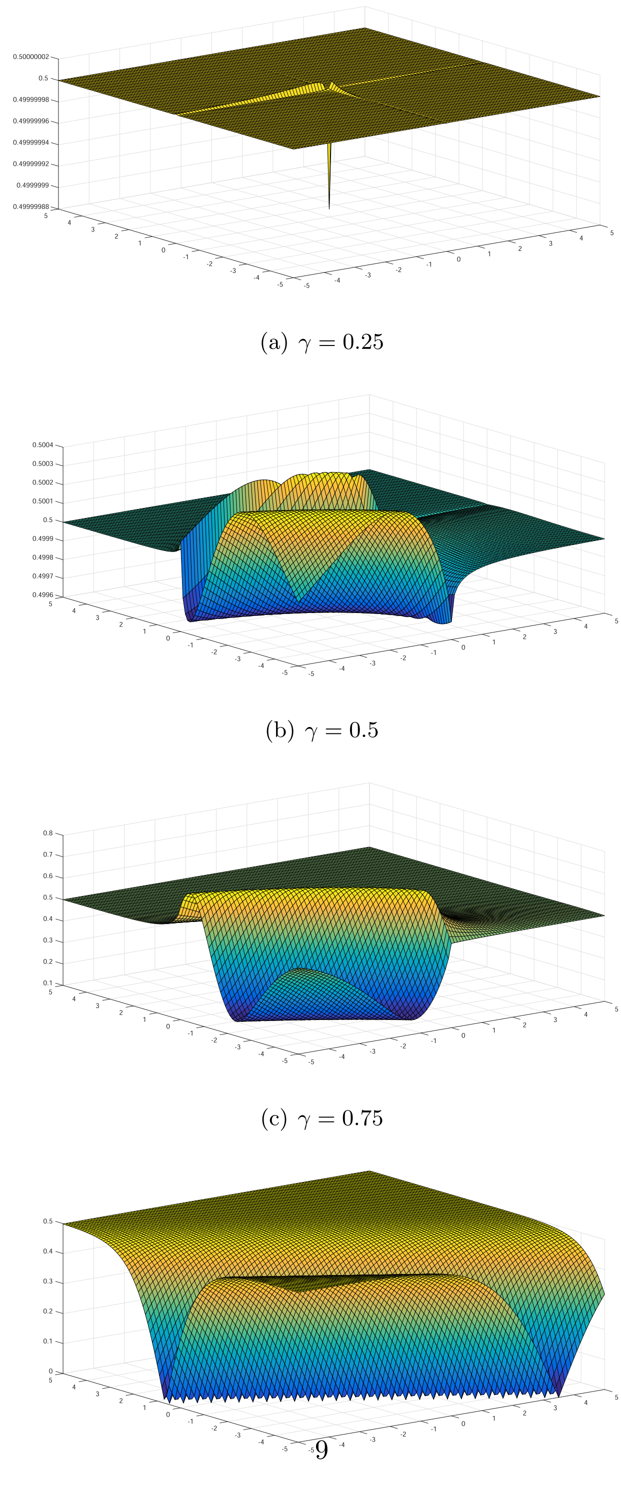
$$u_{3,4}(\eta)=\pm\frac{i\sqrt{kr}}{2\sqrt{3}}\coth\left(\frac{\eta}{2}\right) \quad (30)$$

$$A_0=0, A_1=0, B_1=\mp\frac{i\sqrt{kr}}{\sqrt{3}},\ l=krs$$

$$u_{5,6}(\eta)=\mp\frac{i\sqrt{kr}}{\sqrt{3}}\operatorname{csch}(\eta) \quad (31)$$

$$A_0=0, A_1=\mp\frac{i\sqrt{kr}}{\sqrt{3}}, B_1=0,\ l=-2krs$$

$$u_{7,8}(\eta)=\mp\frac{i\sqrt{kr}}{\sqrt{3}}\coth(\eta) \quad (32)$$



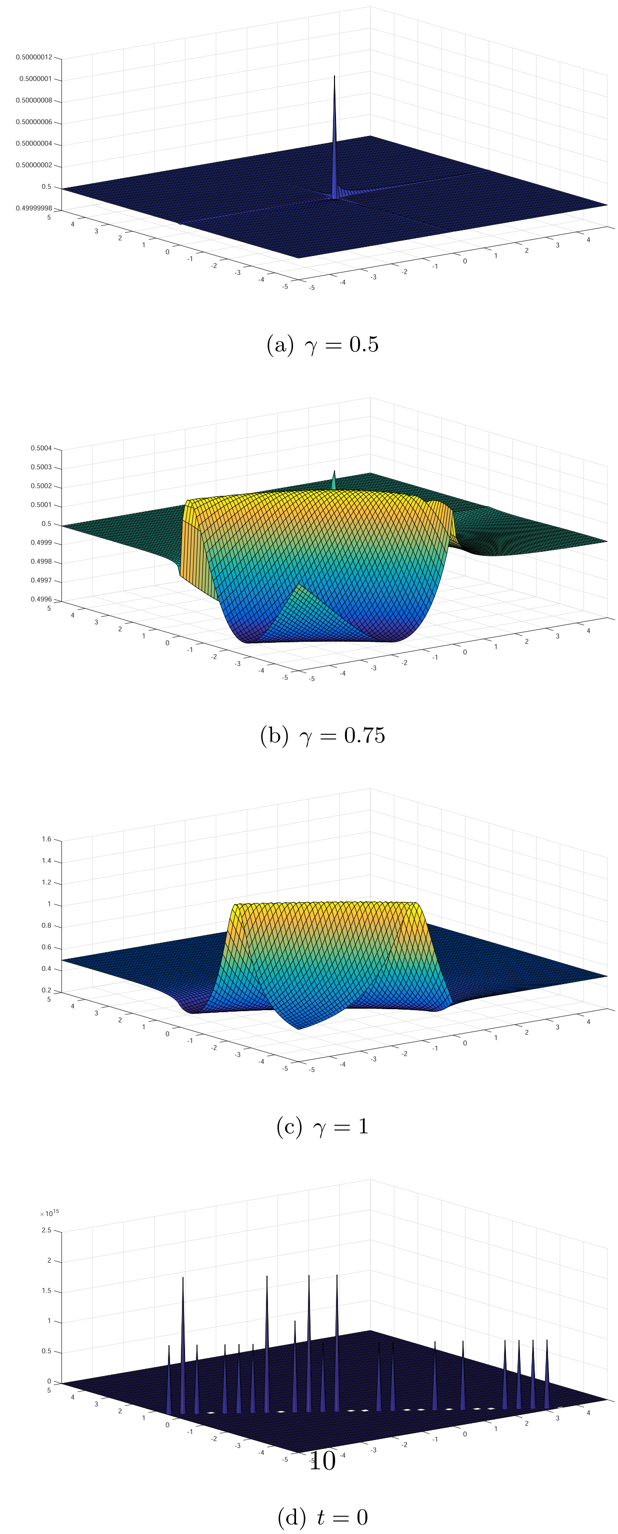
(a) $\gamma = 0.25$

(b) $\gamma = 0.5$

(c) $\gamma = 0.75$

(d) $\gamma = 1$

Fig. 1. Solitary wave profile of $u_{1,2}$ appears in Eq. (8).



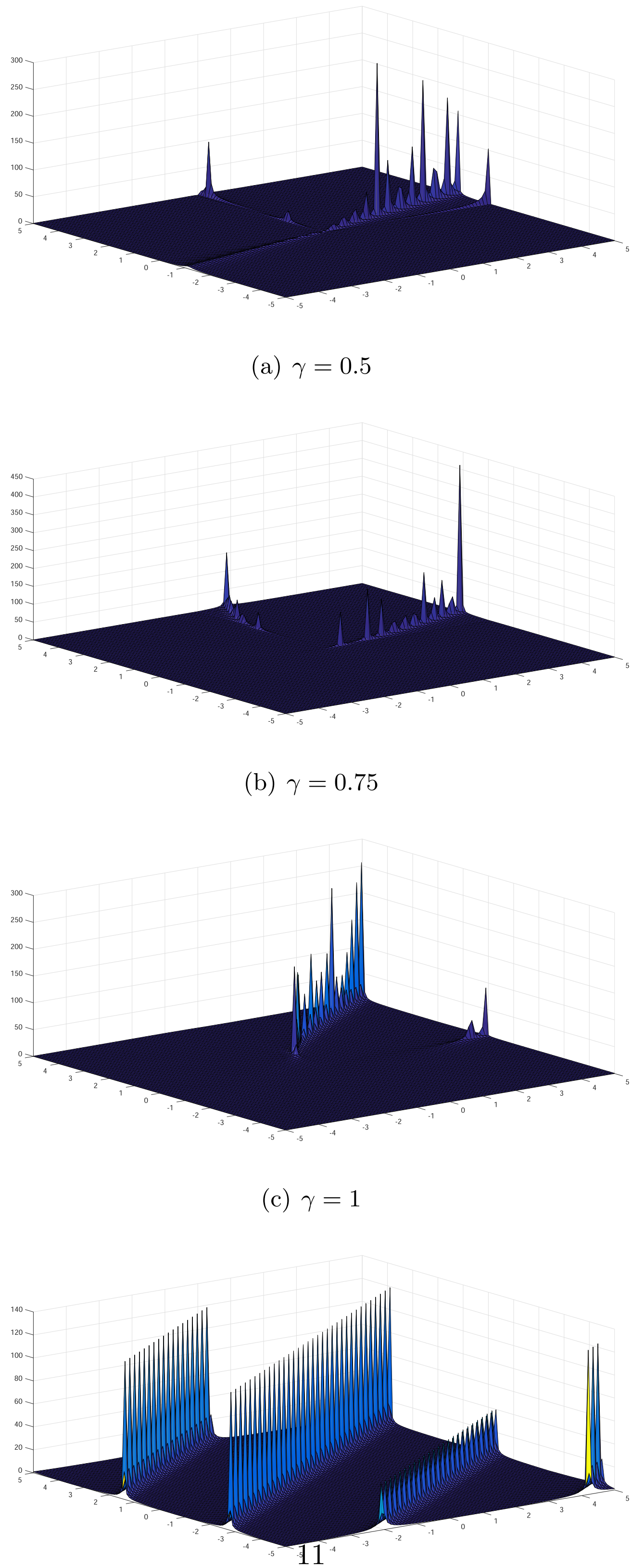
(a) $\gamma = 0.5$

(b) $\gamma = 0.75$

(c) $\gamma = 1$

(d) $t = 0$

Fig. 2. Solitary wave profile of $u_{3,4}$ appears in Eq. (8).



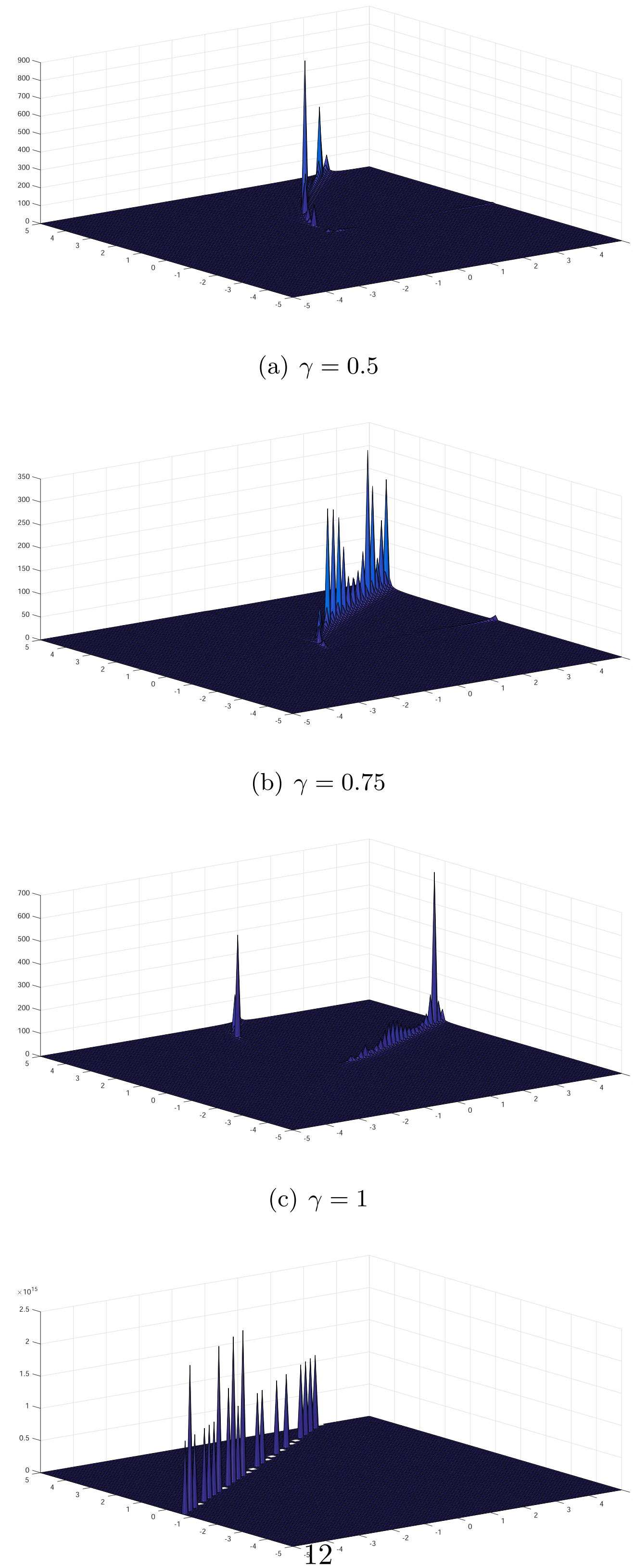
(a) $\gamma = 0.5$

(b) $\gamma = 0.75$

(c) $\gamma = 1$

(d) $t = 0$

Fig. 3. Solitary wave profile of $u_{1,2}$ appears in Eq. (12).



(a) $\gamma = 0.5$

(b) $\gamma = 0.75$

(c) $\gamma = 1$

(d) $t = 0$

Fig. 4. Solitary wave profile of $u_{3,4}$ appears in Eq. (12).

Case-2: $\frac{d\rho}{d\eta} = \cosh(\rho)$ and $N = 1$ in (4), yield a set of nonlinear equations and its solution yields the following exact solutions:

$$A_0 = 0, A_1 = \mp\frac{i\sqrt{kr}}{2\sqrt{3}}, B_1 = \mp\frac{i\sqrt{kr}}{2\sqrt{3}},\ l = \frac{krs}{2}$$

$$u_{1,2}(\eta) = \pm\frac{i\sqrt{kr}}{2\sqrt{3}}\tan\left(\frac{\eta}{2}\right) \tag{33}$$

$$A_0 = 0, A_1 = \mp\frac{i\sqrt{kr}}{2\sqrt{3}}, B_1 = \pm\frac{i\sqrt{kr}}{2\sqrt{3}},\ l = \frac{krs}{2}$$

$$u(\eta) = \pm\frac{i\sqrt{kr}}{2\sqrt{3}}\cot\left(\frac{\eta}{2}\right) \tag{34}$$

$$A_0 = 0, A_1 = 0, B_1 = \mp\frac{i\sqrt{kr}}{\sqrt{3}},\ l = 2krs$$

$$u(\eta) = \mp\frac{i\sqrt{kr}}{\sqrt{3}}\cot(\eta) \tag{35}$$

$$A_0 = 0, A_1 = \mp\frac{i\sqrt{kr}}{\sqrt{3}}, B_1 = 0,\ l = -krs$$

$$u(\eta) = \pm\frac{i\sqrt{kr}}{\sqrt{3}}\csc(\eta) \tag{36}$$

3.3. Graphical representation of the solutions for Eq. (5)

The obtained solutions of Eq. (5) are graphed here for different γ-values corresponding to $k = 1 = r$, and $s = 3$.

Case-I: The Figs. 1 and 2 reveal the two solutions given in Eq. (8) of Eq. (5) for $\gamma = 0.25, 0.5, 0.75$ and 1 respectively.

Case-II: The Figs. 3 and 4 represent two solutions given in Eq. (12) of Eq. (5) for $\gamma = 0.25, 0.5, 0.75$ and 1 respectively.

4. Conclusion

We investigated the exact soliton type solutions, different from those reported in Nuruddeen (2018), for the space-time fractional mKdV equations via hyperbolic function approach. For this purpose, we utilized a wave transformation and the conformable derivative to alter the nonlinear differential equation of fractional order into some nonlinear ordinary differential equation. Then, some real and complex valued solutions are calculated for three types of space-time fractional mKdV equations with the aid of soft computation in MATHEMATICA. Additionally, the graphs of some solutions indicates the well-preserved shape and hight of initial wave prorogations.

References

Ablowitz, Mark J., Segur, Harvey, 1981. Solitons and the Inverse Scattering Transform. Siam. volume 4.

Ali, A.T., Hassan, E.R., 2010. General $\exp_a$ function method for nonlinear evolution equations. Appl. Math. Comput. 217, 451–459.

Ayati, Z., Hosseini, K., Mirzazadeh, M., 2017. Application of kudryashov and functional variable methods to the strain wave equation in microstructured solids. Nonlinear Eng. 6, 25–29.

Bai, Chenglin, 2001. Exact solutions for nonlinear partial differential equation: a new approach. Phys. Lett. A 288 (3), 191–195.

Bekir, A., Adem, C., Cevikel, Ö., 2014. Güner, and Sait San. Bright and dark soliton solutions of the (2+1)-dimenssional evolution equation. Math. Modell. Anal. 19, 118–126.

Chung, W.S., 2015. Fractional newton mechanics with conformable fractional derivative. J. Comput. Appl. Math. 209, 150–158.

Eslami, M., 2016. Trial solution technique to chiral nonlinear schrodinger's equation in (1+2)-dimensions. Nonlinear Dynam. 85 (2), 813–816.

Fan, E., 2000. Extended tanh-function method and its application to nonlinear equation. Phys. Lett. A. 277, 212–218.

Helal, M.A., Seadawy, A.R., 2009. Variational method for the derivative nonlinear schrödinger equation with computational applications. Phys. Scr. 80, 350–360.

Helal, M.A., Seadawy, A.R., 2011. Exact soliton solutions of an d-dimensional nonlinear schrödinger equation with damping and diffusive terms. Z. Angew. Math. Phys. (ZAMP) 62 (2), 839–847.

Helal, M.A., Seadawy, A.R., 2012. Benjamin-feir-instability in nonlinear dispersive waves. Comput. Math. Appl. 64, 3557–3568.

Hosseini, K., Kumar, D., Kaplan, M., Yazdani Bejarbaneh, E., 2017. New exact traveling wave solutions of the unstable nonlinear schrödinger equations. Commun. Theor. Phys. 68 (6), 761.

Hosseini, Kamyar, Mayeli, Peyman, Ansari, Reza, 2018. Bright and singular soliton solutions of the conformable time-fractional Klein-Gordon equations with different nonlinearities. Waves Random Complex Media 28 (3), 426–434.

Hosseini, K., Zabihi, A., Samadani, F., Ansari, R., 2018. New explicit exact solutions of the unstable nonlinear Schrödinger's equation using the $\exp_a$ and hyperbolic function methods. Opt. Quant. Electron. 50 (2), 82.

Jawad, A.J.A.M., Petrovic, M.D., Biswas, A., 2010. Modified simple equation method for nonlinear evolution equations. Appl. Math. Comput. (2), 869–877

Khalil, R., Al Horani, M., Yousef, A., Sababheh, M., 2014. A new definition of fractional derivative. J. Comput. Appl. Math. 264, 65–70.

Khater, A.H., Callebaut, D.K., Seadawy, A.R., 2000. General soliton solutions of an n-dimensional complex ginzburg-landau equation. Phys. Scr. 62, 353–357.

Khater, A.H., Callebaut, D.K., Seadawy, A.R., 2000. General soliton solutions of n-dimensional nonlinear Schrödinger equation. IL Nuovo Cimento 115B, 1303–1312.

Khater, A.H., Callebaut, D.K., Helal, M.A., Seadawy, A.R., 2006. Variational method for the nonlinear dynamics of an elliptic magnetic stagnation line. Eur. Phys. J. D 39, 237–245.

Khater, A.H., Callebaut, D.K., Helal, M.A., Seadawy, A.R., 2006. General soliton solutions for nonlinear dispersive waves in convective type instabilities. Phys. Scr. 74, 384–393.

Korteweg, D.J., de Vries, G., 1895. Xli. on the change of form of long waves advancing in a rectangular canal, and on a new type of long stationary waves. The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science 39 (240), 422–443.

Mahmood, S., Mahmood, S., Mahmood, S., Mahmood, S., 2014. Mathematical analysis of the generalized benjamin and burger-kdv equations via the extended trial equation method. J. Assoc. Arab Univ. Basic Appl. Sci. 16.

Mousavian, S.R., Jafari, H., Khalique, C.M., Karimi, S.A., 2011. New exact-analytical solutions for the mkdv equation. TJMCS 2 (3), 413–416.

Nuruddeen, R.I., 2018. Multiple soliton solutions for the (3+ 1) conformable space–time fractional modified korteweg–de-vries equations. J. Ocean Eng. Sci. 3 (1), 11–18.

Ohkuma, Kenji, Wadati, Miki, 1983. The kadomtsev-petviashvili equation: the trace method and the soliton resonances. J. Phys. Soc. Jpn. 52 (3), 749–760.

Pandir, Y., Gurefe, Y., Misirli, E., 2012. A new approach to kudryashov's method for solving some nonlinear physical models. Int. J. Phys. Sci. 7, 2860–2866.

Samko, G., Kilbas, A.A., Marichev, 1993. Fractional Integrals and Derivatives: Theory and Applications. Gordon and Breach, Yverdon. Gordon and Breach, Yverdon.

Schamel, Hans, 1973. A modified korteweg-de vries equation for ion acoustic waves due to resonant electrons. J. Plasma Phys. 9 (3), 377–387.

Seadawy, A.R., 2011. New exact solutions for the KDV equation with higher order nonlinearity by using the variational method. Comput. Math. Appl. 62, 3741–3755.

Seadawy, A.R., 2012. Exact solutions of a two-dimensional nonlinear schrodinger equation. Appl. Math. Lett. 25, 687–691.

Seadawy, Aly, 2017. The generalized nonlinear higher order of KDV equations from the higher order nonlinear schrodinger equation and its solutions. Optik – Int. J. Light Electron. Opt. 139, 31–43.

Seadawy, Aly, 2017. Two-dimensional interaction of a shear flow with a free surface in a stratified fluid and its a solitary wave solutions via mathematical methods. Eur. Phys. J. Plus 132, 518.

Seadawy, Aly R., 2017. Solitary wave solutions of tow-dimensional nonlinear Kadomtsev-Petviashvili dynamic equation in a dust acoustic plasmas. Pramana J. Phys. 89 (49), 1–11.

Seadawy, A.R., El-Rashidy, K., 2016. Rayleigh-Taylor instability of the cylindrical flow with mass and heat transfer. Pramana – J. Phys. 87, 20.

Seadawy, Aly, Manafian, Jalil, 2018. New soliton solution to the longitudinal wave equation in a magneto-electro-elastic circular rod. Results Phys. 8, 1158–1167.

Seadawy, Aly R., Kumar, D., Hosseini, K., Samadani, F., 2018. The system of equations for the ion sound and langmuir waves and its new exact solutions. Results Phys. 9, 1631–1634.

Selima, Ehab S., Seadawy, Aly R., Yao, Xiaohua, 2016. The nonlinear dispersive Davey-Stewartson system for surface waves propagation in shallow water and its stability. Eur. Phys. J. Plus 131, 425.

Ul-Haq Tariq, Kalim, Seadawy, Aly, 2017. Bistable Bright-Dark solitary wave solutions of the (3+1)-Dimensional Breaking Soliton, Boussinesq equation with dual dispersion and modified Korteweg-de Vries-Kadomtsev-Petviashvili equations and their applications. Results Phys. 7, 1143–1149.

Wadati, Miki, 1973. The modified korteweg-de vries equation. J. Phys. Soc. Jpn. 34 (5), 1289–1296.

Wazwaz, Abdul-Majid, 2017. Exact soliton and kink solutions for new (3+ 1)-dimensional nonlinear modified equations of wave propagation. Open Eng. 7 (1), 169–174.

Xian-Lin, Y.A.N.G., Jia-Shi, T.A.N.G., 2008. Travelting solutions for konopelchenko-dubrovsky equation using an extended sinh-gordon equation expansion method. Commun. Theor. Phys. 50 (5), 1047–1051.

Xie, Fuding, Yan, Zhenya, Zhang, Hongqing, 2001. Explicit and exact traveling wave solutions of Whitham–Broer–Kaup shallow water equations. Phys. Lett. A 285 (1), 76–80.

Yan, Zhenya, Zhang, Hongqing, 2001. New explicit solitary wave solutions and periodic wave solutions for Whitham–Broer–Kaup equation in shallow water. Phys. Lett. A 285 (5), 355–362.

Younis, M., Zafar, Asim, 2014. Exact solutions to nonlinear differential equations of fractional order via $(\frac{G'}{G})$-expansion method. Appl. Math. 5, 1–6.

Zafar, Asim, 2018. Rational exponential solutions of conformable space-time fractional equal-width equations. Nonlinear Eng.

Zhang, S., 2007. Application of exp-function to a KDV equation with variable-coefficients. Phys. Lett. A 365, 448–453.