Original article

Gaussian models for anthropogenic CO_2 emissions consistent with prescribed climate targets

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A B S T R A C T

Objectives: The purpose of this paper is to provide global and national climate policy makers with smooth patterns of carbon dioxide (CO_2) emissions that better fit prescribed climate targets, in comparison with existing mitigation models.

Methods: Based on an accessible mathematical analysis, a linearly-increasing relative rate of reduction is considered for the emissions, and therefore, Gaussian modelling appears as a perfect tool for such an improvement.

Results: Among the designed models, a flexible pattern, composed of a half-bell-shaped decline preceded by a parabolic slowdown, is found to be ideal for bringing the emissions to 'zero' as soon as possible without direct removal of CO_2. It is shown, in particular, that a global mitigation, based on this pattern, consistent with the 1.5 °C target and starting in 2020, will help to achieve a global 'zero' emission in 2050, as urged by the United Nations (UN), earlier in the mid 2040s, or later in the mid-late 2050s for more feasibility with an average annual reduction in the range 2.46–3.19 $GtCO_2$ (which includes each of the EU and USA annual records of about 2.8 $GtCO_2$) from a peak projected in the late thirties.

Conclusion: Based on a mathematical approach to CO_2 emissions modelling, the study reveals a parametrised collection of feasible and flexible pathways, with the advantage of bringing the emissions smoothly to an earlier or similar 'zero' timing, with, unlike UN models, no target overshooting nor need for negative-emission technology.

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1. Introduction

The 2015 *Paris Agreement on Climate Change* aimed to 'hold the rise in global annual average temperature above the pre-industrial level well below 2 °C and pursue efforts to limit the increase to 1.5 °C' ((UNF, 2015), Art. 2). Such threshold was known as UN climate target. It also stressed the urgency to 'project global peaking of greenhouse gases emissions as soon as possible along with their rapid reduction' ((UNF, 2015), Art. 4). Recently, the year 2050 was set up by the UN as the deadline for closing 'a resilient neutral-carbon world', as reminded by its Secretary-General at Climate Action Summit held in New York, September 2019 (UN, 2019). Both targets, for global warming and zero-carbon timing, were based on scientific investigations reporting that increasing anthropogenic CO_2 emissions have made this gas significantly surpass the other greenhouse gases over the past three decades (C2ES, 2018; IPCC, 2014). The climate inaction, in particular regarding

these emissions, has been studied also in the realm of physics research (see for example (Pacheco et al., 2014; Perc et al., 2017)). Climate mitigation scenarios would, therefore, include a substantial reduction of CO_2 emissions. For technical details on carbon capture and storage, see for example (Fattahi, 2014) or more recently (Vo Thanh et al., 2019; Vo Thanh et al., 2020b; Vo Thanh et al., 2020a).

Such scenarios have been produced mostly by climatologists using computer simulations. Considered as the best known in literature are the Representative Concentration Pathways (RCPs; IPCC, 2014; Jubb, 2016; Knutti, 2013; Vuuren et al., 2011), the Coupled Carbon Cycle Climate Model Inter-comparison Project (C4MIP); as a part of the Coupled Model Inter-comparison Project (CMIP) providing a set of earth system models involving the carbon cycle (Jones et al., 2016), both adopted by the Intergovernmental Panel on Climate Change (IPCC) (AR5, WG I), and the mixed models; recently developed by a combination of simulation climate and socio-economic models (Rogelj et al., 2018).

In contrast, mathematical modelling of climate mitigation can hardly be found in literature, even though quite recently, future

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trends of global warming and atmospheric CO_2 were projected, based on a mathematical approach, to meet the climate target as defined in the 2015 Paris agreements (see Jaoua, 2020). In the same setting, this work suggests a better match for a given climate target, in a sense that the emissions will be brought smoothly to an earlier or similar 'zero' timing without missing the target or removing CO_2 from the air. The main idea behind this improvement consists of considering a linear relative rate of emissions reduction. This leads to Gaussian models whose integrals, involved in the remaining CO_2 budget equation, can be determined in terms of the well-known tabulated standard normal cumulative distribution. For such models, three categories are considered: Gaussian with no transition, Gaussian with Gaussian transition, then Gaussian with quadratic transition. Whereas the second category will help to advance the 'zero' timing predicted by the first, at the expense of a high emissions peak and the miss of low climate targets, the third pattern will not only cover all climate targets and bring the emissions peak at a lower level, but will also nearly end the emissions earlier than expected from the second and other models commonly in use (e.g. RCPs), with more feasibility due to the transitional slowdown. Considering global emissions, graphical illustrations are presented for comparison purposes. The designed models also apply to national emissions by considering the national estimations of the level of emissions, their rate of growth, as well as the remaining CO_2 budget in the beginning of their mitigation.

The rest of the paper is organized as follows: a definition and integrals of Gaussian models are presented in Section 2, Section 3 is dedicated to the elaboration and discussion of Gaussian models for CO_2 emissions, and the results are summarized in Section 4.

2. Materials and methods

From a mathematical viewpoint, a rapid reduction of CO_2 emissions can be modelled with exponential functions, whose relative rate of change is rather constant. For even faster decline, bell-shaped functions will help with an increasing relative rate of reduction. However, to ensure a smooth transition from the current trend to a rapid decline, a non-linear interpolation will be of great use.

2.1. Quadratic interpolation

Classically, a quadratic interpolation consists of determining a quadratic function using the values that it takes on at exactly three particular values of its variable. The following result provides an original quadratic interpolation using also three given data on the parabola representing the function: its axis of reflection, one of its points (other than the vertex), and the slope of the tangent line at that point. This technique will be used to add a smooth transition to a Gaussian decline of CO_2 emissions.

If a parabola is symmetric about the line: $x = u$, passes through a point (x_0, y_0), with $x_0 \neq u$, and is tangent at this point to the line of slope m, then an equation of this parabola is:

$$y = A(x - u)^2 + B$$
$$A = m/2(x_0 - u)$$
$$B = y_0 - m(x_0 - u)/2 \qquad (1)$$

2.2. Gaussian models and their integration

Gaussian models were introduced in probability theory by C. F. Gauss; considered as one of the greatest mathematicians of all time, for real-valued random variables "normally" distributed with mean μ and variance σ^2. These models are of the form:

$$f_{\mu,\sigma}(t) = Ke^{-(t-\mu)^2/2\sigma^2} \quad (K > 0).$$

Compared with exponential models, which are of the form $Ke^{-\alpha(t-\mu)}$ ($\alpha > 0$), they decrease much more rapidly over the interval $(\mu + \alpha\sigma, \infty)$. Indeed, whereas the relative rate of decline remains constant (α) for the latter, that of the former ($\frac{t-\mu}{\sigma}$) increases indefinitely from its minimum level α. Such a difference will help to design more suitable models by either advancing the 'zero' timing of CO_2 emissions or lowering and advancing their peak.

The special case where $K = 1/\sqrt{2\pi}$, $\mu = 0$, and $\sigma = 1$ gives the standard Gaussian probability density function f used to define the standard Gaussian cumulative distribution function Φ by:

$$\Phi(x) = p(X \leqslant x) = \int_{-\infty}^{x} f(t)dt,$$

which takes on the particular value $1/2$ at 0 and has 0 and 1 as lower and upper limits. More generally, for a given number between 0 and 1, thus, considered as an intermediate value of the continuous function Φ, an estimation of the corresponding x value is provided in the standard normal distribution table, also called Z table. Conversely, the same table can be used to estimate Φ at a given value of x. On the other hand, the substitution $z = \frac{t-\mu}{\sigma}$ gives a typical integral of a general Gaussian model in terms of specific values of the function Φ as follows:

$$\int_a^b f_{\mu,\sigma}(t)dt = K\sigma\sqrt{2\pi}\left(\Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)\right) \qquad (2)$$

Such formulation will be used to design and refine Gaussian pathways for CO_2 emissions consistent with a prescribed climate target.

2.3. No-mitigation scenario and remaining CO_2 Budget

The consistency of future CO_2 emissions with a prescribed climate target (defined as a target limit to the rise in temperature) and their rapid reduction, as urged by the UN, are crucial in the elaboration of suitable pathways for the emissions. Prior to their modelling, however, three predictions in the beginning of the mitigation (at time $t = t_0$) are needed; their level G_0, their rate of growth α_0, and the remaining CO_2 budget R associated with the climate target. By definition, the CO_2 budget is the total amount of cumulative anthropogenic CO_2 emitted in the atmosphere since the industrial revolution up to the time h when the UN climate target will be hit (under the assumption of no climate mitigation). An explicit formula of this date (h) in terms of the climate target is available in (Jaoua, 2020). To estimate the remaining budget at any time, future emissions need to be modelled explicitly with time under the assumption of no climate policy, which can be done by a linear regression of the annual gas emissions since 2000 using Carbon Dioxide Information Analysis Center (CDIAC) database (Marland et al., 2016). This leads to the following no-climate-policy model G_0 (in $GtCO_2$), applicable from the year 2000 ($t = 0$):

$$G_0(t) \approx \alpha_0 t + \beta_0$$
$$\alpha_0 \approx 0.91, \; \beta_0 \approx 24.47 \qquad (3)$$

Such linear regression was found to be statistically highly significant ($p < 10^{-11}$) and extremely strong ($r^2 \approx 0.98$). As a consequence of Eq. (3), the remaining CO_2 budget $R(t)$, from time t ($0 \leqslant t < h$), consistent with the given climate target, is estimated as follows:

$$R(t) = \int_t^h G_0(x)dx \approx (h - t)(\alpha_0(h + t) + 2\beta_0)/2 \qquad (4)$$

In particular, the remaining CO_2 budgets from 2020, to meet the targets 1.5 °C and 1.8 °C, will be estimated at 1155 and 2929 (GtCO$_2$ respectively, and these represent about 63% and 81% of the corresponding remaining budgets from 2000.

3. Results and discussion

Three categories of models will be designed progressively depending on whether or not transitional emissions will be projected prior to a Gaussian reduction, and if so, whether these emissions will have a Gaussian or quadratic pattern to ensure a smooth transition from a linear to a Gaussian trend.

3.1. Gaussian model without transition

Graphically, the right-half of a suitable bell can be suggested as a possible smooth pathway for CO_2 emissions reduction regardless of the 'zero' timing, which will limit the rise in annual temperature to below the prescribed climate target. This gives a Gaussian model without transition, consistent with that target, defined by:

$$G_1(t) = G_0 e^{-(t-t_0)^2/2\sigma^2}, \quad t \geqslant t_0 \tag{5}$$

where $\sigma = \sqrt{2/\pi}\, R/G_0$ is determined by solving the remaining CO_2 budget equation:

$$\int_{t_0}^{\infty} G_1(t)dt = R,$$

which is equivalent to:

$$G_0 \sigma \sqrt{2\pi}(1 - \Phi(0)) = G_0 \sigma \sqrt{2\pi}(1/2) = R,$$

by using Eq. (2) for $K = G_0$, $\mu = a = t_0$ and $b \to \infty$.

Despite the consistency of the model G_1 with the climate target, the emissions could not be brought close to zero as early as needed; not even before the 23rd century for the 1.8 °C target. But this delay is, in fact, much shorter than the one expected from an exponential model (Fig. 1). Nevertheless, a smooth transitional slowdown will expedite the Gaussian reduction, and thus, shorten and even avoid such delay.

3.2. Gaussian model with Gaussian transition

A portion of the left half of a suitable bell will be of great use to ensure a smooth transition to a Gaussian decline. More precisely, for a given climate target above a specific future warming level l expected to be hit due to a total amount r of unmitigated CO_2 emissions from time t_0, one can suggest the following as a unique mitigated-emissions model G, with Gaussian slowdown and decline, consistent with such a climate target:

$$G(t) = \begin{cases} Ae^{-(t-\mu)^2/2\sigma_1^2}, & t_0 \leqslant t < \mu \\ Ae^{-(t-\mu)^2/2\sigma_2^2}, & t \geqslant \mu \end{cases} \tag{6}$$

$$\mu = t_0 + \sigma_1, \ \sigma_1 = G_0/\alpha_0, \ A = G_0\sqrt{e}$$
$$\sigma_2 = \sqrt{2/\pi e}(R-r)/G_0, \ r = \sqrt{2\pi e}(\Phi(1) - 1/2)G_0^2/\alpha_0$$

To fit the predicted level of emissions at t_0, the inflection time $\mu - \sigma_1$ must be set at t_0, and this gives the relation μ vs. σ_1. The coefficient A can then be determined by taking this into account along with the prediction G_0 at t_0. On the other hand, the transition to the Gaussian trend must occur smoothly at t_0, which means that the rates of growth of the emissions must be the same at t_0^- at t_0^+. In other words, $G_0/\sigma_1 = \alpha_0$, and this gives the parameter σ_1.

To determine the other parameter σ_2, one can split the remaining CO_2 budget into two parts: one (pR) for the decline and the other $((1-p)R)$ for the transition $(0 < p < 1)$. Considering the restriction of the model G to the time interval (μ, ∞), one gets an analogous version of the model G_1 with the parameters pR, A, μ and σ_2 instead of R, G_0, t_0 and σ respectively. Therefore, σ_2 can be derived by analogy from Eq. (5): $\sigma_2 = \sqrt{2/\pi}\, pR/A$. Now, plug in $G_0\sqrt{e}$ for A to get:

$$\sigma_2 = \sqrt{2/\pi e}\, pR/G_0 \tag{7}$$

The exact expression of p will be determined by solving the CO_2 budget equation that corresponds to the transition period:

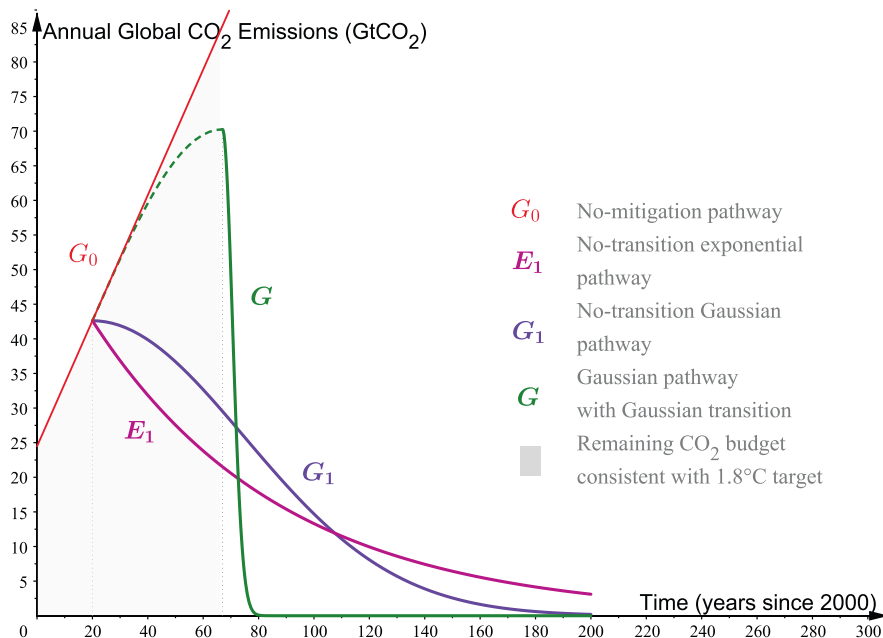$$\int_{t_0}^{\mu} G(t)dt = (1-p)R.$$

Fig. 1. Gaussian and exponential models, for global CO_2 emissions, consistent with the 1.8 °C target (mitigation starting in 2020).

According to Eq. (2) for $K = A$, $\sigma = \sigma_1$, $a = t_0$, and $b = \mu$, this is equivalent to:

$$A\sigma_1\sqrt{2\pi}\left(\Phi(0) - \Phi\left(\frac{t_0 - \mu}{\sigma_1}\right)\right) = (1-p)R.$$

By taking into account the expressions found for A and σ_1 and the relations: $\mu = t_0 + \sigma_1$, $\Phi(0) = 1/2$, and $\Phi(-1) = 1 - \Phi(1)$, one gets: $(1-p)R = r$, where $r = \sqrt{2\pi e}(\Phi(1) - 1/2)G_0^2/\alpha_0$. This gives $p = 1 - (r/R)$, which is in the interval $(0,1)$, due to the assumption made on the climate target. The announced formula for σ_2 follows immediately by plugging in $1 - (r/R)$ for p in Eq. (7). The model G is unique since p is uniquely determined.

The pathway G, as given in Eq. (6), reflects a better mitigation than G_1 does, due to a smooth transition prior to a much faster reduction to almost zero. Indeed, according to G for the 1.8 °C target, the emissions will be brought to nearly zero in the early 2080s if their mitigation starts in 2020. This cannot happen through the pathway G_1 before the 23rd century (Fig. 1) and not even for the lower target 1.5 °C before the 22rd century. Unfortunately, G only fits the climate targets set up above a certain warming level l, which is estimated at about 1.76 °C for a mitigation starting in 2020. Besides, the emissions are expected to peak at a very high level, e.g., around 70.3 GtCO$_2$ for the 1.8 °C target (Fig. 1), thus, about 2.9 times the 2000 record. However, a suitable adjustment of the model G will not only help to include all climate targets, but will also provide an uncountable variety of models that project more appropriate emission peaking and 'zero'-emission timing.

3.3. Gaussian models with quadratic transition

Following the same idea behind the previous modelling, a parameter γ ($0 < \gamma < 1$) is also introduced here to split the remaining budget into two parts: (γR) for the Gaussian decline and $((1 - \gamma)R)$ for the transition, which is, unlike in the model G, quadratic rather than Gaussian. This refinement brings a certain flexibility to the modelling (due to the arbitrariness of γ), which will help to determine optimal pathways for the lowest emissions peak and earliest 'zero' emission. The resulting parametrised model G_γ can be stated as follows:

$$G_\gamma(t) = \begin{cases} A(t-\mu)^2 + B, & t_0 \leqslant t < \mu \\ Be^{-(t-\mu)^2/2\sigma^2}, & t \geqslant \mu \end{cases} \qquad (8)$$

$$\begin{aligned}
A &= -\alpha_0/(2\tau), \ B = G_0 + \alpha_0\tau/2 \\
\mu &= t_0 + \tau, \ \sigma = \gamma R\sqrt{2/\pi}/(G_0 + \alpha_0\tau/2) \\
\tau &= 3(-G_0 + \sqrt{\Delta})/(2\alpha_0), \ \Delta = G_0^2 + (4/3)\alpha_0(1-\gamma)R
\end{aligned}$$

Indeed, let $\mu = t_0 + \tau$, where $\tau > 0$ denotes the total duration of the transition. The coefficients A and B follow immediately from Eq. (1) applied with $(x_0, y_0, u) = (t_0, G_0, \mu)$ and $m = \frac{dG_\gamma}{dt}\big|_{t=t_0} = \frac{dG_0}{dt}\big|_{t=t_0} = \alpha_0$ (for transition smoothness). One can then determine σ using the same argument that led to the coefficient σ_2 in the model G. To find the transition period, one needs to write and solve, for τ, the CO$_2$ budget equation related to the transition phase:

$$\int_{t_0}^{\mu} G_\gamma(t)dt = (A/3)\tau^3 + B\tau = (1-\gamma)R.$$

By taking into account the expressions of A and B then simplifying, one gets the following equation:

$$(\alpha_0/3)\tau^2 + G_0\tau - (1-\gamma)R = 0. \qquad (9)$$

The discriminant Δ of this quadratic equation is given by: $\Delta = G_0^2 + 4\alpha_0(1-\gamma)R/3$ and satisfies the condition: $\Delta > G_0^2 > 0$,

which gives its unique positive solution: $\tau = (-G_0 + \sqrt{\Delta})/(2\alpha_0/3) = 3(-G_0 + \sqrt{\Delta})/(2\alpha_0)$, as announced in Eq. (8).

In the limit case where $\gamma = 0$ (no-mitigation scenario), the model G_γ degenerates into the linear pathway G_0 represented in Fig. 1. However, in the other limit case where $\gamma = 1$ (no-transition scenario), the model is reduced to the 1-phase Gaussian pathway G_1 given in Eq. (5) and graphed in Fig. 1.

Notes

(i) In the setting of moderate climate targets such as 1.8 °C, the flexible pathway G_γ will permit to limit the emissions at a lower level than expected from the model G. For instance, as shown in Table 1, the emissions are projected to peak below 2.5 (2.65) times the record of the year 2000 for $\gamma > .2315$ (.0386), compared to 2.9 times with the model G (see Fig. 1). In addition, the 'zero'-emission timing (i.e., when the emissions will be brought to below 0.01 GtCO$_2$) can be projected earlier than expected from the pathway G, e.g., before the year 2080 for the 1.8 °C target. According to Table 1, this is ensured by any model G_γ for $\gamma < .0839$. As for the class of G_γs, the smaller the γ, the higher and later the peak, and the earlier the zero emission.

(ii) For the 1.5 °C target, the models G_γ, with $\gamma \approx 1$, project low emissions (less than 1 GtCO$_2$) by 2080, similarly to half of the RCP2.6 models ([, Rogelj et al.2018), with the advantage of reducing the emissions (almost immediately) smoothly and nearly ending them by 2090. On the other hand, Fig. 2 shows that the pathway $G_{.1277}$ will bring the emissions to 'zero' in 2050, that is, earlier than expected from the (IPCC) no/limited/higher overshoot models without CO$_2$ removal as required in the latter to meet the climate target (IPCC, 2018). Moreover, $G_{.0566}$ will bring the emissions from a slightly higher peak (≈ 52.65 GtCO$_2$) to 'zero' ($\sim$2046) in only 4 years instead of about 10 years, which explains its lower feasibility (see Fig. 2). When considering a lower target such as 1.4 °C, as shown in Fig. 2, $G_{.1366}$ and $G_{.0506}$ will permit to advance the zero timing to the years 2040 and 2037 respectively, even though the former seems more realistic than the latter due to its longer period between the peak and 'zero' timings (7 years vs. 2 years). More generally, G_γs reflect a better mitigation, not only for projecting a lower (and/or earlier) peaking (compared to G) and/or an earlier 'zero'-emission (compared to IPCC models), but also for slowing the emissions smoothly before their reduction, which will help to ease the decarbonisation of the economy.

(iii) When it comes to the ideal G_γs for a given climate target, their selection depends on specific criteria such as 'zeroing' the emissions before a prescribed year and/or limiting them below a certain level (compared to the record in the year 2000). The selection outcomes based on these criteria are presented in Table 1. According to this table, the G_γs, for .0386 < γ < .0839, are found to be an ideal match for the 1.8 °C target, as they will make the emissions peak below 2.65 times the 2000 level and bring them to nearly zero before the year 2080. For a climate target as close as 1.5 °C (resp. 1.4 °C), compared to a predicted warming of about 1.2 °C for 2020 (see Jaoua, 2020), another range of G_γs (.0593 < γ < .1277 *resp.* .0965 < γ < .1366) is found to be the pattern that will help to limit the emissions below 2.15 times (resp. twice) the 2000 record and reduce them to almost zero before the year 2050 (resp. 2040).

(iv) The feasibility of G_γ can be improved by keeping the highest annual reduction below a sensible level, such as twice the average annual reduction already achieved by each of the EU and USA (since 2005), i.e., $2 \times 2.8 = 5.6$ GtCO$_2$ (see

Table 1

Ideal Gaussian models, with quadratic transition, for global CO_2 emissions consistent with prescribed targets for rise in annual temperature $((\Delta T)^*)$, zero-emission year (z^*), and emissions-peak ratio[a] (ρ^*) (mitigation starting in 2020).

$(\Delta T)^*$	z^*	Ideal G_γ for $z^{*\,b}$	ρ^*	Ideal G_γ for $\rho^{*\,c}$
1.8 °C	2080	$\gamma < .0839$	2.5; (2.65)	$\gamma > .2315 \,(.0386)$
1.5 °C	2050	$\gamma < .1277$	2; (2.15)	$\gamma > .4342 \,(.0593)$
1.4 °C	2040	$\gamma < .1366$	1.9; (2)	$\gamma > .4635 \,(.0965)$

[a] Ratio of the emissions peak to 2000 level.

[b] Pathways G_γ for CO_2 emissions reduced to almost zero (below 0.01 GtCO$_2$) before the year z^*.

[c] Pathways G_γ for CO_2 emissions peaking below ρ^* times 2000 level.
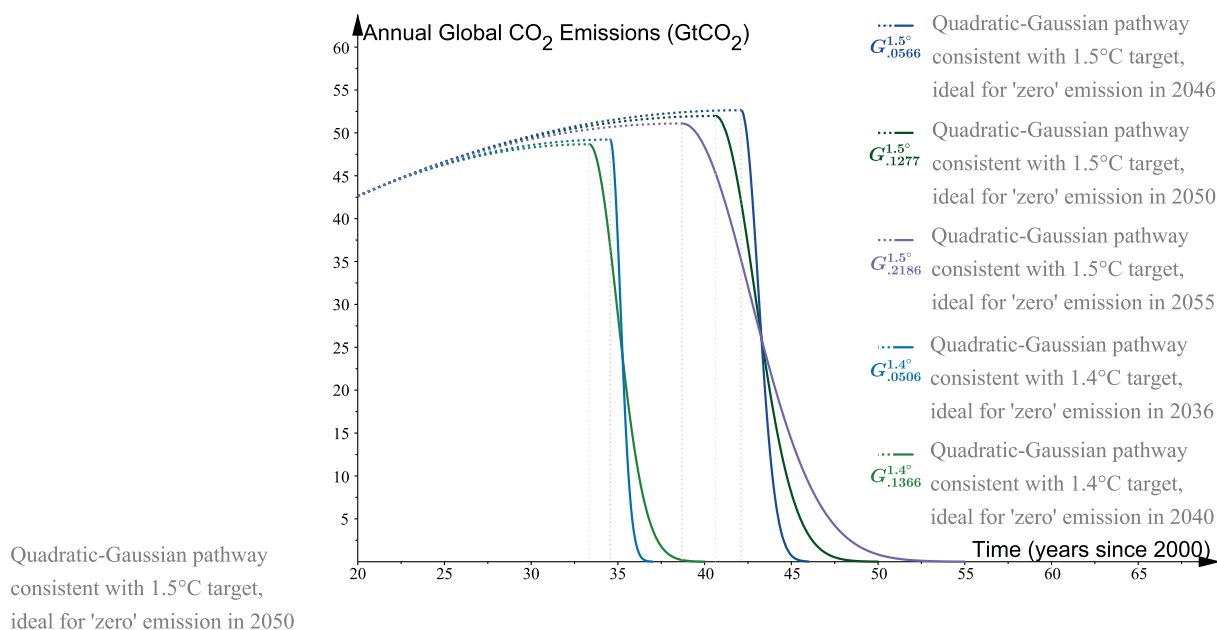

Fig. 2. Gaussian models, with quadratic transition, for global CO_2 emissions, consistent with the climate targets 1.5 °C and 1.4 °C and specific 'zero'-emission timings mitigation starting in 2020.

EESI, 2018). Given the Gaussian trend, the annual reduction will reach its maximum at the inflection time $t = \mu + \sigma$. Therefore, one may consider G_γ realistic whenever it satisfies the following condition:

$$G_\gamma(\mu + \sigma - 1) - G_\gamma(\mu + \sigma) < 6$$

which sets γ in the interval $(.2761, 1)$ for the 1.5 °C target. Among these models, $G_{.2762}$ would be the best fit for the earliest feasible zero-emission which will occur in 2058, with an average annual reduction of less than 2.5 GtCO$_2$. More or less feasible pathways are presented in Table 2 in which three zero-emission timings are considered: 2050, 2055, and 2060.

(v) Limitations of the G_γs: although optimal peaks and zero timings for the emissions can be determined thanks to the flexibility of G_γ, these predictions could be improved with a

lower estimation of the remaining budget of CO_2 emissions. In fact, the estimation used in this study is based on an over-estimation of the date h at which the prescribed climate target would be hit in the no-mitigation scenario, and this is due to the non consideration of further greenhouse gases in the global warming model used to predict this date. On the other hand, the G_γs do not cover any failure of the associated climate action, due for example to the countries which are not part of or will cease their participation in the Paris agreement on climate change. This could ultimately lead to an extended networked version of the models.

4. Conclusions

Refined Gaussian models G_γ are designed to provide climate policy makers with smooth patterns of mitigated CO_2 emissions in order to limit the rise in average temperature to a prescribed target, as close as possible to 1.5 °C, for the global target. Their explicit formulation involves a free parameter $0 < \gamma \leqslant 1$, ensuring their flexibility, along with three predictions in the start of the mitigation: their level, their rate of growth, and the remaining CO_2 budget associated with the climate target. The models apply to global and national scales by using the respective estimates of these predictions.

It is shown that slowing the emissions before reducing them will help to advance their 'zero' timing by few to many decades.

Table 2

Peak (GtCO$_2$) and annual reduction (AR; GtCO$_2$) of CO_2 emissions according to the quadratic-Gaussian model consistent with the 1.5 °C target and prescribed 'zero'-emission year (mitigation starting in 2020).

'Zero' year	2050	2055	2060
Suitable G_γ[a]	$\gamma \approx .1277$	$\gamma \approx .2186$	$\gamma \approx .3062$
Peak (year)	51.97 (2041)	51.0962 (2039)	50.236 (2037)
Highest AR	12.9470 (2043)	7.6832 (2043)	5.3648 (2042)
Average AR	5.5428	3.1912	2.165

[a] Pathway G_γ for CO_2 emissions reduced to almost zero ($\approx$ 0.0099 GtCO$_2$) in the prescribed year.

For instance, when considering a mitigation starting in 2020, the G_γs with very short transition (i.e., for $\gamma \approx 1$) and consistent with the 1.5 °C target will bring the emissions below 1 GtCO$_2$ by 2080, similarly to half of the (IPCC) RCP2.6 models, with the advantage of reducing the emissions (almost immediately) smoothly and nearly ending them by 2090. In comparison with the (IPCC) no/limited/higher overshoot models for the 1.5 °C target, it is found that G_γs with long transition (small γ) will bring the emissions to almost zero (due to a rapid Gaussian reduction) as early as, e.g., 2046 for $\gamma \approx .0566$, 2050 for $\gamma \approx .1277$, and more sensibly, in 2055 for $\gamma \approx .2186$ with an average annual reduction of 3.19 GtCO$_2$, slightly above the EU and USA current records (starting year: 2020, peaking years: 2042, 2041, and 2039 respectively), thus, before the IPCC 1.5 °C-pathways will do or at similar timings, with no need for direct removal of CO$_2$, due to the satisfaction of the remaining CO$_2$ budget (integral) equation.

In sum, half-bell-shaped patterns for CO$_2$ emissions reduction, preceded by a suitable parabolic slowdown, provide more flexibility than the existing models, with the advantage of bringing the emissions smoothly to an earlier or similar 'zero' timing, with, unlike IPCC models, no target overshooting nor need for negative-emission technology.

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Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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