Journal of King Saud University – Science (2012) **24**, 271–276

King Saud University
Journal of King Saud University – Science
www.ksu.edu.sa
www.sciencedirect.com

ORIGINAL ARTICLE

Optical quasi-solitons by Lie symmetry analysis

Anjan Biswas [a,*], Chaudry Masood Khalique [b]

[a] *Department of Mathematical Sciences, Delaware State University, Dover, DE 19901-2277, USA*
[b] *International Institute for Symmetry Analysis and Mathematical Modeling, Department of Mathematical Sciences, North-West University, Mafikeng Campus, Private Bag X2046, Mmabatho 2735, South Africa*

Received 16 April 2011; accepted 12 May 2011
Available online 18 May 2011

KEYWORDS

Quasi-solitons;
Integrability;
Nonlinearity;
Kerr law;
Power law;
Parabolic law;
Saturable law;
Logarithmic law

Abstract This paper studies optical quasi-solitons by the aid of Lie group analysis. Nine types of nonlinearities are considered here. They are Kerr law, power law, parabolic law, dual-power law, polynomial law, triple-power law, saturable law, exponential law and log law nonlinearity. A closed form solution is obtained in each case.

1. Introduction

The nonlinear Schrödinger's equation (NLSE) plays a very important role in the area of Nonlinear Optics (Biswas and Konar, 2006; Biswas et al., 2008; Biswas and Milovic, 2010; Khalique and Biswas, 2009, 2010; Kohl et al., 2008, 2009; Kudryashov and Loguinova, 2009; Liu et al., 2010; Lü et al., 2008). This equation is the total backbone of the study of solitons that propagate through optical fibers for trans-oceanic and trans-continental distances. There are various kinds of nonlinearities that are studied in this context. In various kinds of optical fibers, there are these various kinds of optical nonlinearities that appear. The details of these types of nonlinearities are given in the book by Biswas and Konar that was published in 2006 (Biswas and Konar, 2006). In this paper, the focus will be on obtaining the solutions for nine types of nonlinear media. The perturbed NLSE that will be studied in this paper appears in the study of optical quasi-solitons.

There are various methods of studying the NLSE that has been developed in the past couple of decades. Some of these well known methods are Adomian decomposition method, He's variational iteration method, He's semi-inverse variational principle, exponential function method and many more. These methods have turned out to be a blessing in this area of research. However, one needs to be careful in applying these methods of integration as pointed out by Kudryashov in 2009 (Kudryashov and Loguinova, 2009). In this paper, the method of Lie symmetry, also known as Lie group analysis will be used to carry out the integration of the perturbed NLSE that governs the study of optical quasi-solitons (Kohl et al., 2008).

2. Mathematical analysis

The dimensionless form of the NLSE that is going to be studied in this paper is given by (Biswas and Konar, 2006; Kohl et al., 2008)

* Corresponding author.
E-mail address: biswas.anjan@gmail.com (A. Biswas).
Peer review under responsibility of King Saud University.

Production and hosting by Elsevier

1018-3647 © 2011 King Saud University. Production and hosting by Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.jksus.2011.05.003

$$iq_t + aq_{xx} + bG(|q|^2)\,q = 0 \tag{1}$$

In (1), G is a real-valued algebraic function, where $G(|q|^2)q : C \mapsto C$. Considering the complex plane C as a two-dimensional linear space R^2, it can be said that the function $G(|q|^2)\,q$ is k times continuously differentiable so that one can write (Biswas and Konar, 2006; Kohl et al., 2009; Kudryashov and Loguinova, 2009)

$$G(|q|^2)\,q \in \bigcup_{m,n=1}^{\infty} C^k((-n,n)\times(-m,m);R^2)$$

In Eq. (1), q is the dependent variable while x and t are the independent variables that represent space and time respectively. The first term in (1) represents the time evolution term while the second term is due to the group velocity dispersion and the third term accounts for nonlinearity that is also known as the non-Kerr law of nonlinearity. This is a nonlinear partial differential equation that is not integrable, in general. The non-integrability is not necessarily related to the nonlinear term in (1). Higher order dispersion, for example, can also make the system non-integrable while it still remains Hamiltonian. The solutions to (1) for particular forms of G are known as *solitons*. These solitons are the outcome of a delicate balance between dispersion and nonlinearity.

The solution of (1) is given in the form (Biswas and Konar, 2006)

$$q(x,t) = Ag[B(x - vt)]\mathrm{e}^{i(-\kappa x + \omega t + \theta)}, \tag{2}$$

where the function g represents the shape of the soliton that depends on the type of nonlinearity $G(s)$ in question. Here, A and B respectively represent the amplitude and width of the soliton while v is the soliton velocity, κ is the soliton frequency, ω is the soliton wave number and θ is the phase constant for the soliton. Thus,

$$\kappa = -v \tag{3}$$

and

$$\omega = \frac{i}{2E}\int_{-\infty}^{\infty}\left(qq_t^* - q^*q_t\right)\mathrm{d}x, \tag{4}$$

where in (4) E is the energy of the soliton that is defined in (5) in the following subsection and q^* denotes the complex conjugate of q.

2.1. Integrals of motion

An important property of NLSE given by (1) is that it has conserved quantities also known as *Integrals of Motion*. In fact, Eq. (1) has three integrals of motion. They are the energy (E), linear momentum (M) and Hamiltonian (H) which are respectively given by (Biswas and Konar, 2006; Kohl et al., 2008)

$$E = \int_{-\infty}^{\infty}|q|^2\,\mathrm{d}x, \tag{5}$$

$$M = i\int_{-\infty}^{\infty}(q^*q_x - qq_x^*)\,\mathrm{d}x \tag{6}$$

and

$$H = \int_{-\infty}^{\infty}\left[a|q_x|^2 - f(I)\right]\mathrm{d}x, \tag{7}$$

where

$$f(I) = \int_0^I F(\xi)\,\mathrm{d}\xi \tag{8}$$

with the intensity $I = |q|^2$. The first conserved quantity is also known as the *wave power* while mathematically, it is known as the L_2 norm. The Hamiltonian is one of the most fundamental notions in mechanics and more generally in the theory of conservative dynamical systems with finite or even infinite degrees of freedom. The most useful approach in the soliton theory of conservative non-integrable Hamiltonian system is a representation on the plane of conserved quantities namely the Hamiltonian-versus-energy diagrams.

3. Perturbation terms

The perturbed NLSE that is going to be studied in this paper is given by

$$iq_t + aq_{xx} + bG(|q|^2)q = kq_x^2q^*, \tag{9}$$

where k is the perturbation parameter. This Eq. (9) appears in the study of optical quasi-solitons (Kohl et al., 2008). In this paper, a different form of solution structure will be obtained. This solution form is given by (Khalique and Biswas, 2009, 2010)

$$q(x,t) = \phi(x)\mathrm{e}^{i\lambda t} \tag{10}$$

On substituting the form (10), Eq. (9) reduces to the ordinary differential equation

$$\lambda\phi + a\phi'' + bG(\phi^2)\phi = k(\phi')^2\phi. \tag{11}$$

Eq. (11) has a single Lie point symmetry, namely $X = \partial/\partial x$ (Khalique and Biswas, 2009, 2010). This symmetry will be used to integrate equation (11) once. It can be easily seen that the two invariants are

$$u = \phi \tag{12}$$

and

$$v = \phi' \tag{13}$$

Treating u as the independent variable and v as the dependent variable, (11) can be rewritten as

$$\frac{\mathrm{d}v}{\mathrm{d}u} = \left(\frac{ku}{a}\right)v - \left\{\frac{\lambda u + bG(u^2)u}{a}\right\}\frac{1}{v}. \tag{14}$$

Integrating (14) yields

$$v^2 = \left(\frac{\mathrm{d}\phi}{\mathrm{d}x}\right)^2 = \frac{\lambda}{k} - \frac{2b}{a}\mathrm{e}^{\frac{k\phi^2}{a}}\int^{\phi} sG(s^2)\,\mathrm{e}^{-\frac{ks^2}{a}}\,\mathrm{d}s + c_1\,\mathrm{e}^{\frac{k\phi^2}{a}}, \tag{15}$$

where c_1 is an arbitrary constant of integration. Now, (15) can be integrated once more to yield

$$x + c_2 = \int \frac{\mathrm{d}\phi}{\left[\frac{\lambda}{k} - \frac{2b}{a}\mathrm{e}^{\frac{k\phi^2}{a}}\int^{\phi} sG(s^2)\,\mathrm{e}^{-\frac{ks^2}{a}}\,\mathrm{d}s + c_1\,\mathrm{e}^{\frac{k\phi^2}{a}}\right]^{\frac{1}{2}}}, \tag{16}$$

where c_2 is the second constant of integration. This equation will be further analyzed depending on the type of nonlinearity in the following subsections.

3.1. Kerr law nonlinearity

The Kerr law of nonlinearity originates from the fact that a light wave in an optical fiber faces nonlinear responses. Even though the nonlinear responses are extremely weak, their effects appear in various ways over long distance of propagation that is measured in terms of light wavelength. The origin of nonlinear response is related to the non-harmonic motion of bound electrons under the influence of an applied field. As a result the Fourier amplitude of the induced polarization from the electric dipoles is not linear in the electric field, but involves higher terms in electric field amplitude (Biswas and Konar, 2006; Kohl et al., 2008, 2009).

For Kerr law,

$$G(s) = s \tag{17}$$

so that the perturbed NLSE modifies to

$$iq_t + aq_{xx} + b|q|^2 q = kq_x^2 q^* \tag{18}$$

In this case Eq. (11) modifies to

$$\lambda\phi + a\phi'' + b\phi^3 = k(\phi')^2\phi \tag{19}$$

so that Eq. (16) simplifies to

$$x + c_2 = \int \frac{\mathrm{d}\phi}{\left[\frac{\lambda}{k} + \frac{b}{k^2}(k\phi^2 + a) + c_1 \mathrm{e}^{\frac{k\phi^2}{a}}\right]^{\frac{1}{2}}}. \tag{20}$$

If $c_1 = 0$, (20) integrates to

$$x + c_2 = \sqrt{\frac{k}{b}} \ln\left[bk\phi + \sqrt{bk(bk\phi^2 + ab + \lambda k)}\right]. \tag{21}$$

3.2. Power law nonlinearity

The power law nonlinearity arises in various materials, including semiconductors. Moreover, this law of nonlinearity arises in nonlinear plasmas that solves the problem of small K-condensation in weak turbulence theory (Biswas and Konar, 2006; Kohl et al., 2008, 2009). In this case,

$$G(s) = s^n \tag{22}$$

and thus the NLSE is

$$iq_t + aq_{xx} + b|q|^{2n} q = kq_x^2 q^*. \tag{23}$$

In (23), it is necessary to have $0 < n < 2$ to prevent wave collapse (Biswas and Konar, 2006; Kohl et al., 2009) and, in particular, $n \neq 2$ to avoid self-focusing singularity (Biswas and Konar, 2006). In this case Eq. (11) modifies to

$$\lambda\phi + a\phi'' + b\phi^{2n+1} = k(\phi')^2\phi \tag{24}$$

so that Eq. (16) simplifies to

$$x + c_2 = \int \frac{\mathrm{d}\phi}{\left[\frac{\lambda}{k} + \mathrm{e}^{\frac{k\phi^2}{a}}\left\{\frac{b}{a}\phi^{2n+2}E_{-n}\left(\frac{k\phi^2}{a}\right) + c_1\right\}\right]^{\frac{1}{2}}}, \tag{25}$$

which cannot be integrated any further. In (25), the $E_n(x)$ is the exponential integral that is defined as

$$E_n(x) = \int_1^\infty \frac{\mathrm{e}^{-xt}}{t^n}\,\mathrm{d}t. \tag{26}$$

3.3. Parabolic law nonlinearity

For the parabolic law,

$$G(s) = s + k_1 s^2, \tag{27}$$

where k_1 is a constant. This law is for constant k_1 is also known as the cubic-quintic nonlinearity. The term with k_1 is large for the case of p-toluene sulfonate crystals. It arises in the nonlinear interaction between Langmuir waves and electrons and describes the nonlinear interaction between the high frequency Langmuir waves and the ion-acoustic waves by pondermotive forces (Biswas and Konar, 2006; Kohl et al., 2008, 2009).

There was little attention paid to the propagation of optical beams in the fifth order nonlinear media, since no analytic solutions were known and it seemed that chances of finding any material with significant fifth order term was slim. However, recent developments have rekindled interest in this area. The optical susceptibility of CdS_xSe_{1-x}-doped glasses was experimentally shown to have a considerable $\chi^{(5)}$, the fifth order susceptibility. It was also demonstrated that there exists a significant $\chi^{(5)}$ nonlinearity effect in a transparent glass in intense femtosecond pulses at 620 nm (Biswas and Konar, 2006; Kohl et al., 2008, 2009).

It is necessary to consider nonlinearities higher than the third order to obtain some knowledge of the diameter of the self-trapping beam. It was recognized in 1960 s and 1970 s that saturation of the nonlinear refractive index plays a fundamental role in the self-trapping phenomenon. Higher order nonlinearities arise by retaining the higher order terms in the nonlinear polarization tensor (Biswas and Konar, 2006; Kohl et al., 2008, 2009).

The form of the perturbed NLSE here is

$$iq_t + aq_{xx} + b(|q|^2 + k_1|q|^4)q = kq_x^2 q^*. \tag{28}$$

In this case Eq. (11) modifies to

$$\lambda\phi + a\phi'' + b(\phi^3 + k_1\phi^5) = k(\phi')^2\phi \tag{29}$$

so that Eq. (16) simplifies to

$$x + c_2 = \int \frac{\mathrm{d}\phi}{\left[\frac{\lambda}{k} + \frac{b}{k^2}\{k\phi^2 + a\} + \frac{bk_1}{k^3}\{k^2\phi^4 + 2ak\phi^2 + 2a^2\} + c_1 \mathrm{e}^{\frac{k\phi^2}{a}}\right]^{\frac{1}{2}}}. \tag{30}$$

For $c_1 = 0$, Eq. (32) reduces to

$$x + c_2 = -\frac{ig_1 g_2 F(R|p)}{h_1 h_2} \tag{31}$$

where

$$R = i\sinh^{-1}\left[\phi\left\{\frac{2bkk_1}{\sqrt{b\left(bk^2 - 4a^2bk_1^2 - 4\lambda k^2 k_1\right)} + 2abk_1 + bk}\right\}^{\frac{1}{2}}\right], \tag{32}$$

$$p = \frac{bk + 2abk_1 + \sqrt{b\left(bk^2 - 4a^2bk_1^2 - 4\lambda k^2 k_1\right)}}{bk + 2abk_1 - \sqrt{b\left(bk^2 - 4a^2bk_1^2 - 4\lambda k^2 k_1\right)}}, \tag{33}$$

$$g_1 = \left[1 - \frac{2bkk_1\phi^2}{bk + 2abk_1 + \sqrt{b\left(bk^2 - 4a^2bk_1^2 - 4\lambda k^2 k_1\right)}}\right]^{\frac{1}{2}}, \tag{34}$$

$$g_2 = \left[1 + \frac{2bkk_1\phi^2}{bk + 2abk_1 + \sqrt{b\left(bk^2 - 4a^2bk_1^2 - 4\lambda k^2 k_1\right)}}\right]^{\frac{1}{2}}, \quad (35)$$

$$h_1 = \frac{\sqrt{2}}{k\sqrt{k}}[bk_1(2a^2 + 2ak\phi^2 + k^2\phi^4) + k\{ab + k(b\phi^2 + \lambda)\}]^{\frac{1}{2}}, \quad (36)$$

$$h_2 = \left[\frac{bkk_1}{bk + 2abk_1 + \sqrt{b\left(bk^2 - 4a^2bk_1^2 - 4\lambda k^2 k_1\right)}}\right]^{\frac{1}{2}} \quad (37)$$

and $F(\psi| k)$ is Jacobi's elliptic function that is defined as

$$u = F(\psi|k) = \int_0^{\psi} \frac{\mathrm{d}t}{\sqrt{1 - k^2\sin^2 t}} = \int_0^{x} \frac{\mathrm{d}t}{\sqrt{(1-t^2)(1-k^2t^2)}}, \quad (38)$$

where k is the elliptic modulus with $0 < k < 1$, ψ is the amplitude and $x = \sin\psi$.

3.4. Dual-power law nonlinearity

This model is used to describe the saturation of the nonlinear refractive index. Also, this serves as a basic model to describe the solitons in photovoltaic-photorefractive materials such as $LiNbO_3$. In this case,

$$G(s) = s^n + k_2 s^{2n} \quad (39)$$

so that the NLSE with dual-power law of nonlinearity is given by (Biswas and Konar, 2006; Kohl et al., 2008, 2009)

$$iq_t + aq_{xx} + b(|q|^{2n} + k_2|q|^{4n})q = kq_x^2 q^* \quad (40)$$

so that Eq. (11) in this case reduces to

$$\lambda\phi + a\phi'' + b(\phi^{2n+1} + k_2\phi^{4n+1}) = k(\phi')^2\phi. \quad (41)$$

Therefore Eq. (16) simplifies to

$$x + c_2 = \int \frac{\mathrm{d}\phi}{\left[\frac{\lambda}{k} + e^{\frac{k\phi^2}{a}}\left\{\frac{b}{a}\phi^{2n+2}E_{-n}\left(\frac{k\phi^2}{a}\right) + \frac{bk_2}{a}\phi^{4n+2}E_{-2n}\left(\frac{k\phi^2}{a}\right) + c_1\right\}\right]^{\frac{1}{2}}}, \quad (42)$$

which once again involves $E_n(x)$, the exponential integral that is defined in (26). Eq. (42) cannot be integrated any further.

3.5. Higher order polynomial law nonlinearity

In this section, the type of nonlinearity that is going to be studied is higher order polynomial law and is given by (Biswas and Konar, 2006)

$$G(s) = s + k_1 s^2 + k_2 s^3 \quad (43)$$

so that the NLSE that is being studied in this section takes the form

$$iq_t + aq_{xx} + (|q|^2 + k_1|q|^4 + k_2|q|^6)q = kq_x^2 q^*. \quad (44)$$

In this case Eq. (11) reduces to

$$\lambda\phi + a\phi'' + b(\phi^3 + k_1\phi_5 + k_2\phi^7) = k(\phi')^2\phi \quad (45)$$

so that Eq. (16) simplifies to

$$x + c_2 = \int \frac{\mathrm{d}\phi}{\sqrt{G_1(\phi; a, b, k, k_1, k_2, \lambda)}}, \quad (46)$$

where

$$G_1(\phi; a, b, k, k_1, k_2, \lambda) = \left[\frac{\lambda}{k} + \frac{b}{k^2}\{k\phi^2 + a\} + \frac{bk_1}{k^3}\{k^2\phi^4 + 2ak\phi^2 + 2a^2\} + \frac{bk_2}{k^4}\{k^3\phi^6 + 3ak^2\phi^4 + 6a^2k\phi^2 + 6a^3\} + c_1 e^{\frac{k\phi^2}{a}}\right]^{\frac{1}{2}}. \quad (47)$$

Eq. (46) cannot be integrated further in a closed form.

3.6. Triple power law nonlinearity

In this section, the type of nonlinearity that is going to be studied is the triple-power law nonlinearity that is given by Biswas and Konar (2006)

$$G(s) = s^n + k_1 s^{2n} + k_2 s^{3n} \quad (48)$$

so that the NLSE takes the form

$$iq_t + aq_{xx} + b(|q|^{2n} + k_1|q|^{4n} + k_2|q|^{6n})q = kq_x^2 q^*. \quad (49)$$

Eq. (11) in this case reduces to

$$\lambda\phi + a\phi'' + b(\phi^{2n+1} + k_1\phi_{4n+1} + k_2\phi^{6n+1}) = k(\phi')^2\phi \quad (50)$$

so that Eq. (16) simplifies to

$$x + c_2 = \int \frac{\mathrm{d}\phi}{\sqrt{G_2(\phi; a, b, k, k_1, k_2, \lambda)}}, \quad (51)$$

where

$$G_2(\phi; a, b, k, k_1, k_2, \lambda) = \left[\frac{\lambda}{k} + e^{\frac{k\phi^2}{a}}\left\{\frac{b}{a}\phi^{2n+2}E_{-n}\left(\frac{k\phi^2}{a}\right) + \frac{bk_1}{a}\phi^{4n+2}E_{-2n}\left(\frac{k\phi^2}{a}\right) + \frac{bk_2}{a}\phi^{6n+2}E_{-3n}\left(\frac{k\phi^2}{a}\right) + c_1\right\}\right]^{\frac{1}{2}} \quad (52)$$

3.7. Saturable law nonlinearity

In the case of short pulses and high input peak power, the field induced change of the refractive index cannot be described by a Kerr type nonlinearity, since it is influenced by higher order nonlinearities. As a consequence, the optically induced refractive index change becomes saturated at higher field strength. This is specially more important in materials with higher nonlinear coefficients, for example, semiconductor doped glasses and organic polymers in which the saturation of nonlinear refractive index changes come to play at moderately high intensities and should be taken into account. Thus, the saturable law of nonlinearity is an important one.

In the case of saturable law nonlinearity, the function $F(s)$ is given by

$$G(s) = \frac{s}{1 + ps}, \quad (53)$$

where p is a non-zero constant. If $p = 0$, saturable law collapses to Kerr law nonlinearity. In this case, the NLSE is given by

$$iq_t + aq_{xx} + \frac{b|q|^2 q}{1+p|q|^2} = kq_x^2 q^*. \tag{54}$$

Thus, in this case (11) reduces to

$$\lambda\phi + a\phi'' + \frac{b\phi^2}{1+p\phi^2} = k(\phi')^2\phi \tag{55}$$

so that Eq. (16) simplifies to

$$x + c_2 = \int \frac{\mathrm{d}\phi}{\left[\frac{\lambda}{k} + \frac{b}{pk} + \frac{b}{ap^2}\,\mathrm{e}^{\left(\frac{k\phi^2}{a}+\frac{k}{ap}\right)} Ei\left(-\frac{pk\phi^2+k}{ap}\right) + c_1\,\mathrm{e}^{\frac{k\phi^2}{a}}\right]^{\frac{1}{2}}}, \tag{56}$$

which cannot be integrated in a closed form any further. Here, in (56), the exponential integral $Ei(x)$ is defined as the exponential integral for $n = 1$ or in other words

$$Ei(x) = E_1(x) = \int_1^\infty \frac{\mathrm{e}^{-xt}}{t}\,\mathrm{d}t. \tag{57}$$

3.8. Exponential law nonlinearity

This case of exponential nonlinearity serves as useful model in homogenous unmagnetized plasmas and laser produced plasmas. When the phase velocity of the slow plasma oscillation is much smaller than the ion thermal velocity, one can obtain the adiabatic or quasistatic electron density under the quasi-neutral approximation. Now, combining the coupling equation that exhibits the slowly varying complex amplitude interacting with the low frequency plasma motion one obtains saturable law of nonlinearity. This saturable law nonlinearity sometimes serves as an alternate model for the saturable law nonlinearity since this law also saturates after a finite time although at a slower rate than the previous model (Biswas and Konar, 2006). In this case

$$G(s) = \frac{1}{r}(1 - \mathrm{e}^{-rs}), \tag{58}$$

where r is a positive constant. The NLSE is given by

$$iq_t + aq_{xx} + \frac{bq}{r}(1 - \mathrm{e}^{-r|q|^2}) = kq_x^2 q^*. \tag{59}$$

Therefore, Eq. (11) in this case reduces to

$$\lambda\phi + a\phi'' + \frac{b\phi}{r}(1 - \mathrm{e}^{-r\phi^2}) = k(\phi')^2\phi \tag{60}$$

and therefore Eq. (16) simplifies to

$$x + c_2 = \int \frac{\mathrm{d}\phi}{\left[\frac{\lambda}{k} + \frac{b}{rk} - \frac{b\,\mathrm{e}^{-r\phi^2}}{r(ar+k)} + c_1\,\mathrm{e}^{\frac{k\phi^2}{a}}\right]^{\frac{1}{2}}} \tag{61}$$

and this cannot be simplified further in terms of any elementary functions.

3.9. Log law nonlinearity

This law arises in various fields of contemporary physics. It allows closed form exact expressions for stationary Gaussian beams (*Gaussons*) as well as for periodic and quasiperiodic regimes of the beam evolution. The advantage of this model is that the radiation from the periodic soliton is absent as the linearized problem has a discrete spectrum only (Biswas and Konar, 2006; Biswas and Milovic, 2010). In this case,

$$G(s) = \log s \tag{62}$$

so that the NLSE with log law nonlinearity is given by Biswas and Milovic (2010)

$$iq_t + aq_{xx} + bq\log|q|^2 = kq_x^2 q^* \tag{63}$$

and therefore (11) reduces to

$$\lambda\phi + a\phi'' + 2b\ln\phi = k(\phi')^2\phi \tag{64}$$

so that (16) reduces to

$$x + c_2 = \int \frac{\mathrm{d}\phi}{\left[\frac{\lambda}{k} + \frac{2b}{k}\ln\phi - \frac{b}{k}\,\mathrm{e}^{\frac{k\phi^2}{a}} Ei\left(-\frac{k\phi^2}{a}\right) + c_1\,\mathrm{e}^{\frac{k\phi^2}{a}}\right]^{\frac{1}{2}}}, \tag{65}$$

where once again $Ei(x)$ denotes the exponential integral and (65) cannot be integrated any further.

4. Conclusions

In this paper the stationary solution for optical quasi-solitons is obtained for nine laws of nonlinearity. The resulting solutions are in quadratures. It is observed that out of these nine forms of the laws of nonlinearity, only two of the laws yield closed form solutions although in the implicit form. In future, the stochastic perturbation terms will be incorportaed and those results will be published elsewhere.

Acknowledgements

The first author is very grateful to Prof. C.M. Khalique and North-West University, Mafikeng campus, for the warm hospitality that he received during his academic visit in November 2009.

References

Biswas, A., Konar, S., 2006. Introduction to Non-Kerr Law Optical Solitons. CRC Press, Boca Raton, FL, USA.

Biswas, A., Milovic, D., 2010. Optical solitons with log law nonlinearity. Communications in Nonlinear Science and Numerical Simulation 15 (12), 3763–3767.

Biswas, A., Milovic, D., Majid, F., Kohl, R., 2008. Optical soliton cooling in a saturable law media. Journal of Electromagnetic Waves and Applications 22, 1735–1746.

Khalique, C.M., Biswas, A., 2009. A Lie symmetry approach to nonlinear Schrödinger's equation with non-Kerr law nonlinearity. Communications in Nonlinear Science and Numerical Simulation 14 (12), 4033–4040.

Khalique, C.M., Biswas, A., 2010. Gaussian soliton solution to nonlinear Schrödinger's equation with log law nonlinearity. International Journal of Physical Sciences 5 (3), 280–282.

Kohl, R., Biswas, A., Milovic, D., Zerrad, E., 2008. Optical soliton perturbation in a non-Kerr law media. Optics and Laser Technology 40 (4), 647–662.

Kohl, R., Milovic, D., Zerrad, E., Biswas, A., 2009. Optical solitons by He's variational principle in a non-Kerr law media. Journal of Infrared Millimeter and Terahertz Waves 30 (5), 526–537.

Kudryashov, N.A., Loguinova, N.B., 2009. Seven common errors in finding exact solutions of nonlinear differential equations. Communications in Nonlinear Science and Numerical Simulation 14 (9-10), 3507–3529.

Liu, W-J., Tian, B., Xao, T., Sun, K., Jiang, Y., 2010. Bright and dark solitons in the normal dispersion regime of inhomogeneous optical fibers: soliton interaction and soliton control. Annals of Physics 325 (8), 1633–1643.

Lü, X., Zhu, H-W., Yao, Z-Z., Meng, X-H., Zhang, C., Zhang, C-Y, Tian, B., 2008. Multisoliton solutions in terms of double Wronskian determinant for a generalized variable coefficient nonlinear Schrödinger equation from plasma physics, arterial mechanics, fluid dynamics and optical communications. Annals of Physics 323 (8), 1947–1955.