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ORIGINAL ARTICLE

Exp-function method for traveling wave solutions of modified Zakharov–Kuznetsov equation

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Abstract In this paper, we apply the exp-function method to construct generalized solitary and periodic solutions of modified Zakharov–Kuznetsov equation which play a very important role in mathematical physics and engineering sciences. The suggested algorithm is quite efficient and is practically well suited for use in these problems. Numerical results clearly indicate the reliability and efficiency of the proposed exp-function method.

1. Introduction

This paper is devoted to the study of a nonlinear evolution equation which is called the Zakharov–Kuznetsov (ZK) equation and is of the form

$$u_t + u^2u_x + u_{xxx} + u_{xyy} = 0. \quad (1)$$

The ZK equation arises in number of scientific models including fluid mechanics, astrophysics, solid state physics, plasma physics, chemical kinematics, chemical chemistry, optical fiber and geochemistry, see (Mohyud-Din et al., 2008; Tascan et al., 2008; Wazwaz, 2008) and the references therein. The basic motivation of this paper is to extend the application of a very reliable and efficient technique which is called the exp-function method for traveling wave solutions of modified Zakharov–Kuznetsov (ZK) equation. The proposed method was developed by He and Wu (He and Wu, 2006) to seek the solitary, periodic and compacton like solutions of nonlinear differential equations; see (Abdou et al., 2007; El-Wakil et al., 2007; He and Wu, 2006; He and Abdou, 2007; Mohyud-Din et al., 2009, 2010, 2008; Noor et al., 2008a,b; Tascan et al., 2008; Wazwaz, 2008; Wu and He, in press, 2007; Yusufoglu, 2008; Zhou, 2007; Zhou et al., 2008; Zhang, 2007; Zhu, 2007a,b) and the references therein.

2. Exp-function method

We consider the general nonlinear PDE of the type

$$P(u, u_t, u_x, u_y, u_{tt}, u_{xx}, u_{yy}, u_{xt}, u_{xy}, u_{ty} \ldots) = 0. \quad (2)$$

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Using a transformation

$$\eta = kx + \omega y + \rho t, \quad \text{or } \eta = \alpha x + \beta y + \rho t, \tag{3}$$

where k, ω, α, β and ρ are constants, we can rewrite Eq. (3) in the following nonlinear ODE;

$$Q(u, u', u'', u''', \ldots) = 0. \tag{4}$$

According to exp-function method, which was developed by He and Wu (2006), we assume that the wave solution can be expressed in the following form:

$$u(\eta) = \frac{\sum_{n=-d}^{c} a_n \exp[n\eta]}{\sum_{m=-q}^{p} b_m \exp[m\eta]}, \tag{5}$$

where p, q, c and d are positive integers which are known to be further determined, a_n and b_m are unknown constants. We can rewrite Eq. (6) in the following equivalent form:

$$u(\eta) = \frac{a_c \exp[c\eta] + \cdots + a_{-d} \exp[-d\eta]}{b_p \exp[p\eta] + \cdots + b_{-q} \exp[-q\eta]}. \tag{6}$$

This equivalent formulation plays an important and fundamental part for finding the analytic solution of problems. To determine the value of c and p, we balance the linear term of highest order of Eq. (4) with the highest order nonlinear term. Similarly, to determine the value of d and q, we balance the linear term of lowest order of Eq. (4) with lowest order nonlinear term (Abdou et al., 2007; El-Wakil et al., 2007; He and Wu, 2006; He and Abdou, 2007; Mohyud-Din et al., 2009, 2010, 2008; Noor et al., 2008a,b; Tascan et al., 2008; Wazwaz, 2008; Wu and He, in press, 2007; Yusufoglu, 2008; Zhou, 2007; Zhou et al., 2008; Zhang, 2007; Zhu, 2007a,b).

3. Solution procedure

Consider the modified Zakharov–Kuznetsov (ZK) Eq. (1)

$$u_t + u^2 u_x + u_{xxx} + u_{xyy} = 0.$$

Introducing a transformation as $\eta = \alpha x + \beta y + \rho t$, we can covert Eq. (1) into an ODE as

$$\rho u' + \alpha u^2 u' + (\alpha^3 + \alpha\beta^2) u''' = 0. \tag{7}$$

The solution of the Eq. (7) can be expressed as follows:

$$u(\eta) = \frac{a_c \exp[c\eta] + \cdots + a_{-d} \exp[-d\eta]}{b_p \exp[p\eta] + \cdots + b_{-q} \exp[-q\eta]}. \tag{6}$$

To determine the value of c and p, we balance the linear term of highest order of Eq. (7) with the highest order nonlinear term

$$u''' = \frac{c_1 \exp[(7p + c)\eta] + \cdots}{c_2 \exp[8p\eta] + \cdots} \tag{8}$$

and

$$u^2 u' = \frac{c_3 \exp[(p + 3c)\eta] + \cdots}{c_4 \exp[4p\eta] + \cdots} = \frac{c_3 \exp[(5p + 3c)\eta] + \cdots}{c_4 \exp + [8p\eta] \cdots}, \tag{9}$$

where c_i are determined coefficients only for simplicity; balancing the highest order of exp-function in (8) and (9), we have

$$7p + c = 5p + 3c, \tag{10}$$

which in turn gives

$$p = c. \tag{11}$$

To determine the value of d and q, we balance the linear term of lowest order of Eq. (7) with the lowest order nonlinear term

$$u''' = \frac{\cdots + d_1 \exp[(-d - 7q)\eta]}{\cdots + d_2 \exp[-8q\eta]} \tag{12}$$

and

$$u' u'' = \frac{\cdots + d_3 \exp[(-q - 3d)\eta]}{\cdots + d_4 \exp[-4q\eta]} = \frac{\cdots + d_3 \exp[(-3d - 5q)\eta]}{\cdots + d_4 \exp[-8q\eta]}, \tag{13}$$

where d_i are determined coefficients only for simplicity. Now, balancing the lowest order of exp-function in (12) and (13), we have

$$-7q - d = -5q - 3d, \tag{14}$$

which in turn gives

$$q = d. \tag{15}$$

Case 3.1.1. We can freely choose the values of c and d, but we will illustrate that the final solution does not strongly depend upon the choice of values of c and d. For simplicity, we set $p = c = 1$ and $q = d = 1$, then the trial solution, Eq. (6) reduces to

$$u(\eta) = \frac{a_1 \exp[\eta] + a_0 + a_{-1} \exp[-\eta]}{b_1 \exp[\eta] + a_0 + b_{-1} \exp[-\eta]}. \tag{16}$$

Substituting Eq. (16) into (7), we have

$$\frac{1}{A}[c_3 \exp(3\eta) + c_2 \exp(2\eta) + c_1 \exp(\eta) + c_0 + c_{-1} \exp(-\eta) + c_{-2} \exp(-2\eta) + c_{-3} \exp(-3\eta)] = 0, \tag{17}$$

where $A = (b_1 \exp(\eta) + b_0 + b_{-1} \exp(-\eta))^4$, $c_i\,(i = -3, \ldots, 0, \ldots, 3)$ are constants obtained by Maple 11.

Equating the coefficients of $\exp(n\eta)$ to be zero, we obtain

$$\{c_{-3} = 0, c_{-2} = 0, c_{-1} = 0, c_0 = 0, c_1 = 0, c_2 = 0, c_3 = 0\}. \tag{18}$$

Solution of (18) will yield

$$a_{-1} = 0, \quad b_0 = b_0, \quad b_1 = b_1, \quad b_{-1} = \frac{1}{24} \frac{a_0^2}{b_1(\beta^2 + \alpha^2)},$$
$$a_1 = 0, \quad \rho = -\alpha\beta^2 - \alpha^3, \quad a_0 = a_0. \tag{19}$$

We, therefore, obtained the following generalized solitary solution $u(x, y, t)$ of Eq. (1)

$$u(x, y, t) = \frac{a_0}{b_1 e^{(\alpha x + \beta y + \rho t)} + \frac{1}{24} \frac{a_0^2}{b_1(\beta^2 + \alpha^2)} e^{(-\alpha x - \beta y - \rho t)}}, \tag{20}$$

where $\rho = -\alpha\beta^2 - \alpha^3$, and a_0, b_1, α and β are real numbers (see Fig. 3.1).

In case α and β are imaginary numbers, the obtained soliton solution can be converted into periodic solution or compact-

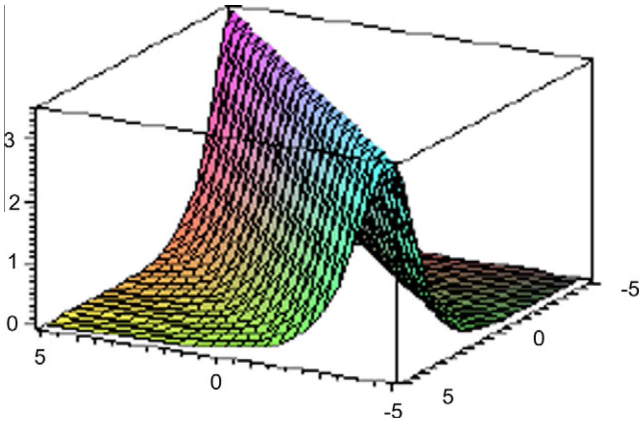

Figure 3.1 Depicts soliton solutions of Eq. (1), when $a_0 = b_1 = \beta = \alpha = 1$.

like solution. Therefore, we write $\alpha = i\omega$ and $\beta = i\theta$, respectively. Consequently, Eq. (20) becomes

$$u(x,y,t) = \frac{a_0}{b_1 e^{(i\omega x + i\theta y + \rho t)} - \frac{a_0^2}{24 b_1(\omega^2+\theta^2)} e^{(-i\omega x - i\theta y - \rho t)}},$$

where $\rho = i(\omega\theta^2 + \omega^3)$, and a_0, b_1, θ and ω are real numbers, consequently

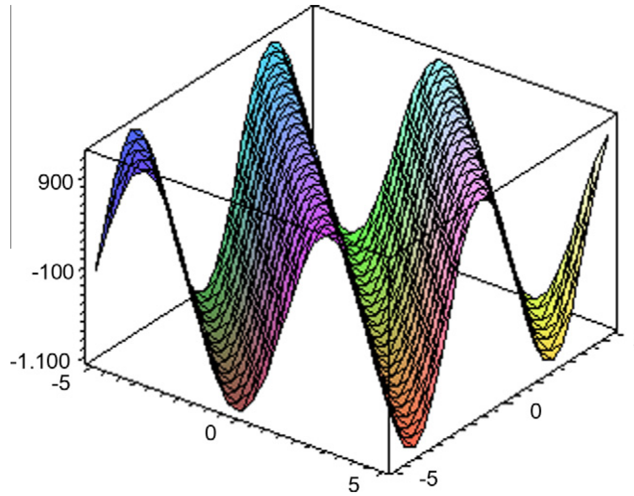

Figure 3.2 Depicts periodic solutions of Eq. (1), when $a_0 = b_1 = \theta = \omega = 1$.

$$a_{-1} = 0, \quad a_2 = 0, \quad b_2 = \frac{1}{96}\frac{a_0^2}{b_{-2}(\beta^2+\alpha^2)}, \quad b_0 = 0, \quad a_{-2} = 0,$$
$$a_1 = 0, \quad b_{-2} = b_{-2}, \quad \rho = -4(\beta^2+\alpha^2)\alpha, \quad a_0 = a_0. \tag{24}$$

$$u(x,y,t) = \frac{\begin{bmatrix} \cos(\omega x + \theta y + t\omega\theta^2 + t\omega^3)\left[576 b_1^2\theta^2 + 576 b_1^2\omega^2 - 24a_0^2\right] \\ + i\sin(\omega x + \theta y + t\omega\theta^2 + t\omega^3)\left[-576 b_1^2\theta^2 - 576 b_1^2\omega^2 - b_1 a_0(\theta^2+\omega^2)\right] \end{bmatrix}}{\begin{bmatrix} \cos(\omega x + \theta y + t\omega\theta^2 + t\omega^3)^2\left[-96 b_1^2 a_0^2(\theta^2+\omega^2)\right] + 576 b_1^4(\theta^4+\omega^4) \\ +1152 b_1^4\theta^2\omega^2 + 48 b_1^2 a_0^2(\theta^2+\omega^2) + a_0^4 \end{bmatrix}}. \tag{21}$$

For periodic or compact-like solutions, the imaginary part in Eq. (21) must be zero, hence

Hence we get the generalized solitary wave solution $u(x,y,t)$ of Eq. (1) as follows:

$$u(x,y,t) = \frac{\cos(\omega x + \theta y + t\omega\theta^2 + t\omega^3)\left[576 b_1^2\theta^2 + 576 b_1^2\omega^2 - 24a_0^2\right]}{\begin{bmatrix} \cos(\omega x + \theta y + t\omega\theta^2 + t\omega^3)^2\left[-96 b_1^2 a_0^2(\theta^2+\omega^2)\right] + 576 b_1^4(\theta^4+\omega^4) \\ +1152 b_1^4\theta^2\omega^2 + 48 b_1^2 a_0^2(\theta^2+\omega^2) + a_0^4 \end{bmatrix}}, \tag{22}$$

which is periodic solution of Eq. (1) (see Fig. 3.2).

Case 3.1.2. If $p = c = 2$, and $q = d = 2$, then Eq. (6) reduces to

$$u(\eta) = \frac{a_2\exp[2\eta] + a_1\exp[\eta] + a_0 + a_{-1}\exp[-\eta] + a_{-2}\exp[-2\eta]}{b_2\exp[2\eta] + +b_1\exp[\eta] + b_0 + b_{-1}\exp[-\eta] + b_{-2}\exp[-2\eta]}. \tag{23}$$

Setting $b_1 = b_{-1} = 0$, the trial-function (23) is simplified as follows:

$$u(\eta) = \frac{a_2\exp[2\eta] + a_1\exp[\eta] + a_0 + a_{-1}\exp[-\eta] + a_{-2}\exp[-2\eta]}{b_2\exp[2\eta] + b_0 + b_{-2}\exp[-2\eta]}.$$

Proceeding as before, we obtain

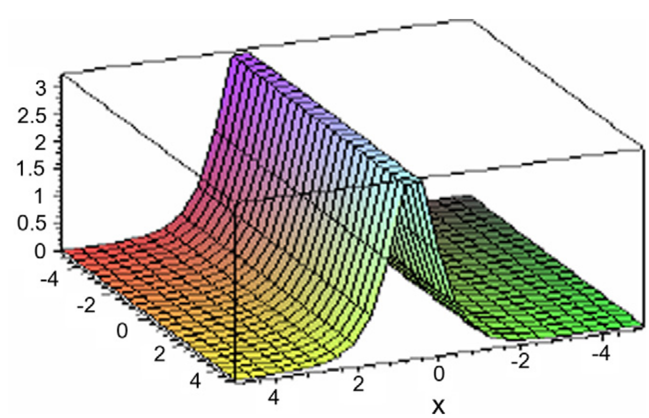

Figure 3.3 Depicts soliton solutions of Eq. (1) in Case 3.1.2, when $a_0 = b_{-2} = \alpha = \beta = \rho = 1$.

$$u(x,y,t) = \frac{a_0}{\frac{1}{96}\frac{a_0^2}{b_{-2}(\beta^2+\alpha^2)}e^{2(\alpha x+\beta y+\rho t)} + b_{-2}e^{-2(\alpha x+\beta y+\rho t)}}, \quad (25)$$

where $\rho = -4(\beta^2 + \alpha^2)$ and a_0, b_{-2}, α and β are real numbers (see Fig. 3.3).

4. Conclusion

In this paper, we applied the exp-function method to obtain the generalized solitary and periodic solutions of the modified Zakharov–Kuznetsov equation. It is concluded that exp-function method is a very effective and powerful mathematical tool for finding solitary and periodic solutions of the nonlinear partial differential equations.

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