Journal of King Saud University – Science 31 (2019) 506–510

Contents lists available at ScienceDirect

Journal of King Saud University – Science

journal homepage: www.sciencedirect.com

Original article

Thermodynamics analysis of an internal heat generating fluid of a variable viscosity reactive couette flow

A.R. Hassan

Department of Mathematical Sciences, University of South Africa, Pretoria 0003, South Africa
Department of Mathematics, Tai Solarin University of Education, Ijagun, Ogun State, Nigeria

ARTICLE INFO

Article history:
Received 5 July 2017
Accepted 15 May 2018
Available online 28 May 2018

Keywords:
Reactive fluids
Couette flow
Internal heat generation
Adomian Decomposition Method (ADM)

ABSTRACT

The present investigation examined the influence of heat source on a variable viscosity reactive Couette fluid flow. The upper plate moves with constant velocity while the lower plate is kept fixed. The heat source is considered to be a linear function of temperature while the fluid viscosity depends on temperature. The coupled set of differential equations regulating the fluid regime are obtained using the technique of Adomian Decomposition Method (ADM). Graphical and tabular representations showed the impacts of heat source parameter by increasing the fluid motion and temperature. Finally, the outcome of the solutions compared with previously obtained results without the influence of internal energy parameter significantly showed the effects.

1. Introduction

Many investigative studies like in Akhtar et al. (2008) explained that fluid behaves uniquely under different circumstances. Recently, appreciable awareness has been given to the analysis of variable viscosity reactive fluid flow due to its significance in many industrial and engineering applications, especially in fluid machinery involving moving parts where hydromagnetic lubrication is in use as mentioned in Kobo and Makinde (2010). The study in Hassan and Gbadeyan (2015) further ascertained the importance of taking precautions of a reacting material undergoing an exothermic reaction where heat is being produce in accordance with chemical reaction. Many studies have been conducted on the impact of heat source on various types of fluid moving under different situations, extensively in Patil and Kulkarni (2008), Jha and Ajibade (2009), Perekattu and Balaji (2009), Chen (2010), El-Amin (2004), Bagai and Nishad (2012), Oztop and Bilgen (2006), Ben-Nakhi and Chamkha (2007), D~i Marcello et al. (2010), Cortell (2005), Jawdat and Hashim (2010), Bartella and Nield (2012) and Hayat et al. (2015) investigated so much on the significance of internally generated heat caused within fluid flow due to interaction between the surfaces, porosity, chemical reactions, thermal boundary conditions, temperature and pressure gradients, to mention few.

However, the importance of this study is on how to utilize heat generation or absorption in different fluids moving within channels which has been a major concern to many researchers because of its necessity in industrial and engineering applications to avoid hazards and also to increase productivity. Notably, Hassan and Gbadeyan, 2015 did not consider the impact of the heat source, but noted that the consequence of hydromagnetic reactive flows that are commonly associated with heat generation cannot be totally neglected. Other recent significant studies involving the internal heat generation can be found in Hassan and Gbadeyan (2015), Hassan and Maritz (2016), Hassan and Maritz (2016) and Hassan and Maritz (2017).

The present study is motivated to examine the impact of internal heat source on a variable viscosity reactive Couette fluid flow which was done in Kobo and Makinde, 2010 by neglecting the influence of internal heat source. The velocities at the upper (in motion) and lower plates of the channel are respectively constant and stationary. The internal heat generation is a linear function of temperature while the fluid viscosity is temperature dependent. The analytical solutions of the coupled sets of differential equations regulating the fluid flow are secured by applying the technique of Adomian Decomposition Method (ADM) with suitable estimation by comparing with previously obtained results in literature to show the significance. The choice of this method is due to

Peer review under responsibility of King Saud University.

Production and hosting by Elsevier

E-mail address: anthonyhassan72@yahoo.co.uk

https://doi.org/10.1016/j.jksus.2018.05.015
1018-3647/

the fact that the method does not require any use of guess value but reliable and a better alternative way in providing solutions to differential and integral equations in rapidly convergent series. Few studies to show the effectiveness of ADM with other methods are found in Gul et al. (2015), Gul et al. (2016), Hassan and Gbadeyan (2015), Gbadeyan and Hassan (2012), Wazwaz and El-Sayed (2001), Hassan and Fenuga (2011), Ghani et al. (2016) and Gul et al. (2014).

The plan of the present study formulates the problem in Section 2. In Section 3, the solutions to coupled set of equations are obtained with the use of ADM and the expressions are used to present tables and graphs in Section 4 while the concluding remark is presented in Section 5.

2. Problem Formulation

The below figure (Fig. 1) shows the configuration of the fluid under consideration.The fluid is viscous, incompressible, reactive and steady in the x-direction between two plates that are parallel with width, H and length, L. For Couette flow, the upper plate in motion moves with constant velocity while the lower plate is kept fixed.

The temperature dependent viscosity $(\overline{\mu})$, the chemical reaction kinetic (G) and the heat source (q), are respectively expressed in Arrhenius chemical kinetics following Kobo and Makinde (2010), Hassan and Gbadeyan (2015) and Gbadeyan and Hassan (2012)

$$\overline{\mu} = \mu_0 e^{\frac{E}{RT}}, \qquad G = QC_0Ae^{-\frac{E}{RT}} \qquad \text{and} \qquad q = Q_0(\overline{T} - T_0) \tag{1}$$

such that μ_0 stands for the fluid reference dynamic viscosity at a very large temperature (i.e. as $T \to \infty$), C_0 denotes the initial concentration of the reactant species, E represents the activation energy, R denotes universal gas constant and T represents the fluid temperature. Also, Q, A, Q_0 and T_0 respectively represent the heat of reaction term, reaction rate constant, dimensional heat generation and the ambient temperature of the wall.

By disregarding the utilization of the reactant to its minimum, the velocity and energy equations regulating the flow in dimensional form is given as:

$$\frac{d}{d\overline{y}}\left(\overline{\mu}\frac{d\overline{u}}{d\overline{y}}\right) = 0 \tag{2}$$

$$k\frac{d^2\overline{T}}{d\overline{y}^2} + \overline{\mu}\left(\frac{d\overline{u}}{d\overline{y}}\right)^2 + QC_0Ae^{-\frac{E}{RT}} + Q_0(\overline{T} - T_0) = 0 \tag{3}$$

and the entropy generation is given as

$$S^m = \frac{k}{T_0^2}\left(\frac{d\overline{T}}{d\overline{y}}\right)^2 + \frac{\overline{\mu}}{T_0}\left(\frac{d\overline{u}}{d\overline{y}}\right)^2 \tag{4}$$

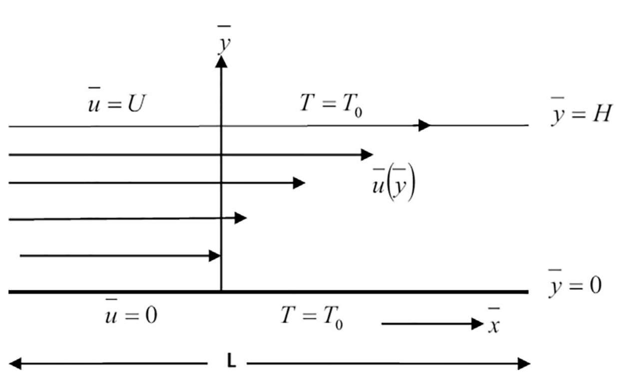

Fig. 1. Geometry of the Problem.

with the following boundary conditions:

$$\overline{u}(0) = \overline{T}(0) = 0, \qquad \overline{u}(1) = 1 \qquad \text{and} \qquad \overline{T}(1) = 0 \tag{5}$$

The under–listed non–dimensional variables and parameters are introduced:

$$y = \frac{\overline{y}}{L}, \quad x = \frac{\overline{x}}{L}, \quad u = \frac{\overline{u}}{U}, \quad \mu = \frac{\overline{\mu}}{\mu_0}e^{-\frac{E}{RT_0}}, \quad T = \frac{E(\overline{T} - T_0)}{RT_0^2}, \quad \beta = \frac{RT_0}{E},$$
$$Br = \frac{\mu_0 EU^2}{kRT_0^2}e^{-\frac{E}{RT_0}}, \quad \lambda = \frac{QC_0EAL^2}{kRT_0^2}e^{-\frac{E}{RT_0}}, \quad \text{and} \quad \delta = \frac{Q_0L^2}{k} \tag{6}$$

The velocity scale is represented with U, k stands for thermal conductivity and u is the fluid velocity. Additional parameters comprise, Br, β, λ and δ which respectively stand for Brinkman number, activation energy, Frank – Kamenettski and the internal heat generation. It is worthy to note that, for all Couette flow, the flow is driven by a constant velocity at the upper plate of the channel such that the axial pressure gradient is not applicable.

Therefore, using (6), the dimensionless forms of Eqs. (2)–(5) becomes:

$$\frac{d}{dy}\left(\mu\frac{du}{dy}\right) = 0 \tag{7}$$

$$\frac{d^2T}{dy^2} + \lambda e^{\frac{T}{1+\beta T}} + \mu Br\left(\frac{du}{dy}\right)^2 + \delta T = 0 \tag{8}$$

and the entropy generation rate

$$N_s = \frac{S^m L^2 E^2}{kR^2T_0^2} = \left(\frac{dT}{dy}\right)^2 + \mu\frac{Br}{\Omega}\left(\frac{du}{dy}\right)^2 \tag{9}$$

with the appropriate boundary conditions stated below as:

$$u(0) = T(0) = 0 \ \text{ for the fixed lower plate} \quad \text{and}$$
$$u(1) = 1 \ \text{ and } \ T(1) = 0 \ \text{ for the upper plate with uniform motion.} \tag{10}$$

where $\mu = e^{-\frac{T}{1+\beta T}}$

Integrating (7) with the boundary conditions, we obtain

$$\frac{du}{dy} = a_0 e^{\frac{T}{1+\beta T}} \tag{11}$$

where a_0 is a constant that will be determined when $a_0 = u'(0)$.

Substituting (11) into (8), we obtain

$$\frac{d^2T}{dy^2} + \gamma e^{\frac{T}{1+\beta T}} + \delta T = 0 \tag{12}$$

where $\gamma = \lambda + a_0^2 Br$. It is important to note that when $\delta = 0$, the result is equivalent to Kobo and Makinde, 2010 and we thereby compare our results where Adomian Decomposition Method (ADM) is used to that of Kobo and Makinde, 2010 where perturbation method (PM) and numerical approach (NM) were used. Hence, the coupled Eqs. (11) and (12) are obtained by using ADM.

3. Method of solution

The governing Eqs. (11) and (12) with the boundary conditions (10) are obtained using the ADM technique. Literature is rich for this method in Gul et al. (2015), Gul et al. (2016), Hassan and Gbadeyan (2015), Gbadeyan and Hassan (2012), Wazwaz and El-Sayed (2001), Hassan and Fenuga (2011) and Ghani et al. (2016). The results from this method has been shown to be an alternative and reliable approximations that converge very rapidly. It is necessary for convenience to commence with the solution of (12) using the series solution (ADM). Therefore, we respectively integrate (12) and (11) twice and once to give the followings:

$$T(y) = b_0 y - \gamma \int_0^y \int_0^y e^{\frac{T(y)}{1+\beta T(y)}} dY dY + \delta \int_0^y \int_0^y T(y) dY dY \tag{13}$$

$$u(y) = a_0 \int_0^y e^{\frac{T(y)}{1+\beta T(y)}} dY \tag{14}$$

where $b_0 = T'(0)$ and can be resolved by using other boundary conditions. The technique of the series solution ADM demands a solution in the form of the following expressions:

$$T(y) = \sum_{n=0}^{\infty} T_n(y) \quad \text{and} \quad u(y) = \sum_{n=0}^{\infty} u_n(y) \tag{15}$$

Where the respective components $T_0, u_0, T_1, u_1, T_2, u_2 \ldots, T_k, u_k$ are to be determined, then (13) and (14) become

$$T(y) = b_0 y - \gamma \int_0^y \int_0^y e^{\frac{\left(\sum_{n=0}^{\infty} T_n(y)\right)}{1+\beta\left(\sum_{n=0}^{\infty} T_n(y)\right)}} dY dY + \delta \int_0^y \int_0^y \left(\sum_{n=0}^{\infty} T_n(y)\right) dY dY \tag{16}$$

$$u(y) = a_0 \int_0^y e^{\frac{\left(\sum_{n=0}^{\infty} T_n(y)\right)}{1+\beta\left(\sum_{n=0}^{\infty} T_n(y)\right)}} dY \tag{17}$$

We let the nonlinear term in (16) and (17) be represented by

$$\sum_{n}^{\infty} A_n = e^{\frac{\left(\sum_{n=0}^{\infty} T_n(y)\right)}{1+\beta\left(\sum_{n=0}^{\infty} T_n(y)\right)}} \tag{18}$$

such that (16) and (17) becomes

$$T(y) = b_0 y - \gamma \int_0^y \int_0^y \left(\sum_{n}^{\infty} A_n\right) dY dY + \delta \int_0^y \int_0^y \left(\sum_{n=0}^{\infty} T_n(y)\right) dY dY \tag{19}$$

$$u(y) = a_0 \int_0^y \left(\sum_{n}^{\infty} A_n\right) dY \tag{20}$$

whose components $A_0, A_1, A_2, \ldots$, are called Adomian polynomials such that

$$A_0 = e^{\frac{T_0(y)}{\beta T_0(y)+1}}$$
$$A_1 = \frac{T_1(y) e^{\frac{T_0(y)}{\beta T_0(y)+1}}}{(\beta T_0(y)+1)^2}$$
$$A_2 = \frac{e^{\frac{T_0(y)}{\beta T_0(y)+1}} \left(T_1(y)^2 (-2\beta - 2\beta^2 T_0(y) + 1) + 2T_2(y)(\beta T_0(y)+1)^2\right)}{2(\beta T_0(y)+1)^4}, \ldots \tag{21}$$

The respective zeroth component of (19) and (20) following the modification of Hassan and Gbadeyan (2015) and Wazwaz and El-Sayed (2001) as follows:

$$T_0(y) = 0,$$
$$u_0(y) = 0, \tag{22}$$

$$T_1(y) = b_0 y - \gamma \int_0^y \int_0^y \left(\sum_{n}^{\infty} A_0\right) dY dY + \delta \int_0^y \int_0^y \left(\sum_{n=0}^{\infty} T_0(y)\right) dY dY,$$
$$u_1 = a_0 \int_0^y \left(\sum_{n}^{\infty} A_0\right) dY, \tag{23}$$

$$T_{n+1}(y) = -\gamma \int_0^y \int_0^y \left(\sum_{n}^{\infty} A_n\right) dY dY + \delta \int_0^y \int_0^y \left(\sum_{n=0}^{\infty} T_n(y)\right) dY dY$$
$$u_{n+1}(y) = a_0 \int_0^y \left(\sum_{n}^{\infty} A_n\right) dY, \qquad n \geqslant 1 \tag{24}$$

The Eqs. (22)–(24) are programmed in the Mathematica software package to secure alternative approximate solutions for the coupled equations as:

$$T(y) = \sum_{n=0}^{k} T_n(y) \quad \text{and} \quad u(y) = \sum_{n=0}^{k} u_n(y) \tag{25}$$

However, the entropy generation rate from (9) can be described to be the rate of disturbance created from the interaction of the fluid flow within both upper and lower plates. From Hassan and Gbadeyan (2015), Makinde and Maserumule (2008) and Wood (1975), the irreversibility ratio denoted by Bejan (Be) number can be calculated using the approximate solutions of (25) in this form as:

$$Be = \frac{N_1}{N_s} = \frac{N_1}{N_1 + N_2} \tag{26}$$

where

$$N_1 = \left(\frac{dT}{dy}\right)^2 \quad \text{and} \quad N_2 = \mu \frac{Br}{\Omega} \left(\frac{du}{dy}\right)^2 \tag{27}$$

Other physical interest includes the rate of heat transfer across the channel which is given as:

$$Nu = -\frac{dT}{dy}\Big|_{y=1} \tag{28}$$

where Nu is the Nusselt number.

Also, the skin friction (C_f) at the upper wall of the channel is given by

$$C_f = \frac{du}{dy}\Big|_{y=1} \tag{29}$$

4. Discussion of results

The tabular and graphical representations of the impact of heat source on a variable viscosity Couette reactive fluid flow and other important flow parameters are presented and discussed. The choice of values for each parameter is in agreement with Kobo and Makinde (2010) and Makinde and Maserumule (2008) in order to significantly show the effect of internal heat energy produced within the flow regime.

Table 1 shows the computation of rapid convergence of the series solutions (ADM) with few iterations and gives approximate values for the constants a_0 and b_0 respectively obtained for velocity and temperature distributions. This significantly showed the

Table 1
Rapid convergence of the series solutions.

	$\delta = \gamma = 0.5, \beta = 0.1$	
n	a_0	b_0
0	0	0
1	1	0.25
2	0.948617	0.275
3	0.954687	0.27345
4	0.954779	0.27344
5	0.954733	0.27344
6	0.954735	0.27344
7	0.954735	0.27344

Table 2
Comparison of solutions with different methods.

	$\beta = 0.1, \gamma = 0.5, \delta = 0$					
y	$T(y)PM$ Kobo and Makinde, 2010	$T(y)ADM$	Absoluteerror	$T(y)NM$ Kobo and Makinde, 2010	$T(y)ADM$	Absoluteerror
0	0	0	0	0	0	0
0.1	0.02360044943	0.02360044953	9.6869×10^{-11}	0.02360039520	0.02360044953	5.4327×10^{-8}
0.2	0.04208380022	0.04208380042	1.9758×10^{-10}	0.04208374384	0.04208380042	5.6578×10^{-8}
0.3	0.05535532038	0.05535532067	2.8517×10^{-10}	0.05535525228	0.05535532067	6.8385×10^{-8}
0.4	0.06334613033	0.06334613062	2.9403×10^{-10}	0.06334604947	0.06334613062	8.1154×10^{-8}
0.5	0.06601440811	0.06601440855	4.3937×10^{-10}	0.06601432697	0.06601440855	8.1579×10^{-8}
0.6	0.06334613023	0.06334613068	4.5001×10^{-10}	0.06334604947	0.06334613068	8.1210×10^{-8}
0.7	0.05535532037	0.05535532077	4.0026×10^{-10}	0.05535525228	0.05535532077	6.8490×10^{-8}
0.8	0.04208380020	0.04208380055	3.4951×10^{-10}	0.04208374384	0.04208380055	5.6710×10^{-8}
0.9	0.02360044944	0.02360044963	1.9469×10^{-10}	0.02360039520	0.02360044963	5.4435×10^{-8}
1	0	0	0	0	0	0

efficient and effective used of this series solutions (ADM) as another alternative method of obtaining results of differential equations with nonlinear terms.

Table 2 shows the comparison between Adomian Decomposition Method (ADM) and results of Kobo and Makinde, 2010 where perturbation (PM) and numerical method (NM) were used. The numerical results of both methods were compared by setting the internal heat generation parameter to be zero, that is, $\delta = 0$ which was not considered in Kobo and Makinde (2010) and Makinde and Maserumule (2008). It evidently showed that they are almost identical with an average difference of order 10^{-10} with perturbation method (PM) and average difference of order 10^{-8} with the numerical method (NM). This evidently showed that the series solutions (ADM) is also an alternative and better method of obtaining solutions to various differential equations with or without nonlinear terms with no exact solution with few iterations as shown in Table 1.

Also, Table 3 shows the computation of Nusselt numbers and skin friction for variation in the values of (γ) and (δ). It is clearly discovered that the Nusselt number becomes greater with respect to rising values of (γ) and (δ) while the reverse is observed in the case of the skin friction, this is due to the rising behaviour in the fluid motion due to increase in (γ) and (δ), hence bring reduction in the skin friction.

The impact of heat source parameter (δ) on the momentum distribution is displayed in Fig. 2. It is detected that the boundary conditions are satisfied where the velocity of the lower plate remains zero and gradually rises to 1 at the upper moving plate. Also, the impact of heat source is noticed across the channel as the fluid velocity rises together with the rising values of (δ) which is caused due to the presence of a relatively low resistance of flow in the presence of shear force within the channel. The temperature profile showing the impact of heat source parameter (δ) is also displayed in Fig. 3. On a normal note, a rise in the value of (δ) brings about another rise in the temperature of the fluid which is caused by the interaction of particles present in the fluid flow within the channel.

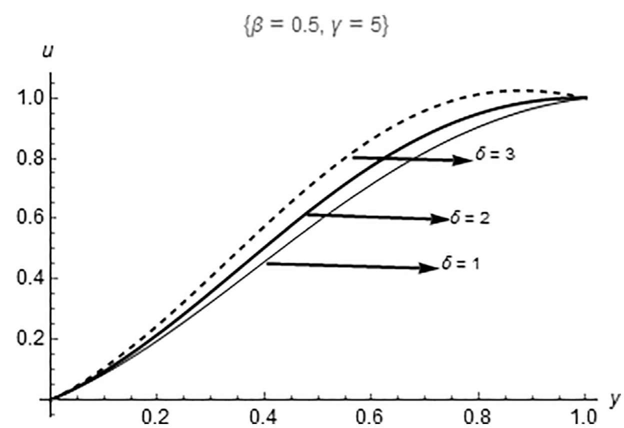

Fig. 2. Impact of δ on $u(y)$.

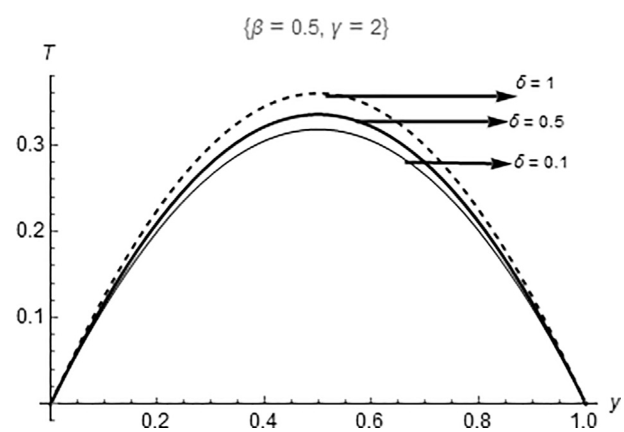

Fig. 3. Impact of δ on $T(y)$.

The entropy generation rate in the flow regime with respect to the impact of heat source parameter (δ) is shown in Fig. 4. It is clearly seen that the rate of disturbance are more active at the wall

Table 3
Effects of different parameters on Nusselt numbers and Skin friction.

β	γ	δ	$Nu\|_{y=1}$	$C_f\|_{y=1}$
0.5	0.1	0.1	0.0508503	0.991568
0.5	0.1	0.2	0.0512887	0.991482
0.5	0.1	0.3	0.0517362	0.991394
0.5	0.1	0.1	0.0508503	0.991568
0.5	0.2	0.1	0.1025770	0.983109
0.5	0.3	0.1	0.1552080	0.974622

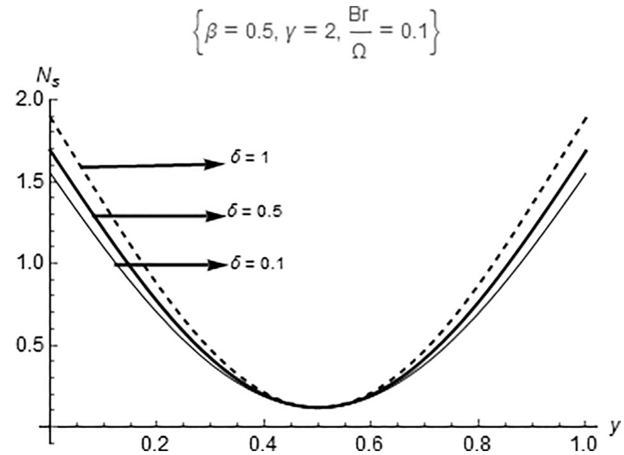

Fig. 4. Effect of δ on N_s.

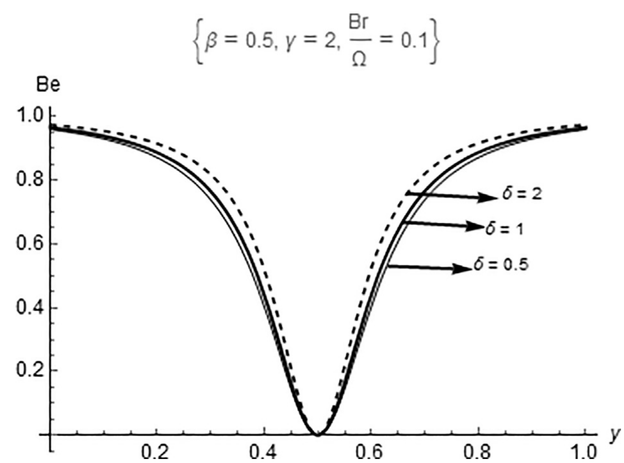

Fig. 5. Effect of δ on *Be*.

surfaces and otherwise at the centreline of the fluid flow. However, the rate of entropy generation rises with rising values of (δ). However, Fig. 5 displays the Bejan number versus the channel width. It is noticed that heat transfer dominates at the lower and upper plate surfaces while the fluid friction irreversibility is controlled around the centreline of the channel with rising values of δ.

5. Conclusion

The influence of internal heat source on a variable viscosity reactive Couette fluid flow was investigated. The results were compared with Kobo and Makinde, 2010 where the effect of heat source was not accounted for. It is assumed that the lower plate is fixed while the upper plate moves with a constant velocity. The coupled nonlinear dimensionless differential equations regulating the fluid flow are determined by employing the technique of Adomian Decomposition Method (ADM). The result from ADM and previously obtained results from Kobo and Makinde, 2010 showed evidently that they are almost identical with a general difference of order 10^{-10} with perturbation method and general difference of order 10^{-8} with numerical results. The graphical representations also show significantly the impact of heat source on the fluid flow system that can be used to provide safety precautions and increase productivity in industries and engineering applications.

References

Akhtar, W., Fetecau, C., Tigou, V., Fetecau, C., 2008. Flow of maxwell fluid between two sides wall induced by a constantly accelerating plates. Int. J. Z. Angew, Math. Phys. 1007/S00033008-7129-8.

Bagai, S., Nishad, C., 2012. Free convection in a non-newtonian fluid along a horizontal plate embedded in a porous media with internal heat generation. Int. Commun. Heat Mass Transfer 39, 537–540.

Bartella, A., Nield, D.A., 2012. On the rayleigh-bernard poiseuille problem with internal heat generation. Int. J. Therm. Sci. Heat Mass Transf. 57, 1–16.

Ben-Nakhi, A., Chamkha, A.J., 2007. Conjugate natural convection around a finned pipe in a square enclose with internal heat generation. Int. J. Heat Mass Transf. 50, 2260–2271.

Chen, C., 2010. On analytic solution of mhd flow and heat transfer for two types of viscoelastic fluid over a stretching sheet with energy dissipation, internal heat source and thermal radiation. Int. J. Heat Mass Transf. 53, 4264–4273.

Cortell, R., 2005. Flow and heat transfer of a fluid through a porous medium over a stretching surface with internal heat generation/absorption and suction/blowing. Fluid Dyn. Res. 37, 231–245.

D~i Marcello, V., Cammi, A., Luzzi, L., 2010. A generalized approach to heat transfer in pipe flow with internal heat generation. Chem. Eng. Sci. 65, 1301–1310.

El-Amin, M.F., 2004. Combined effect of internal heat generation and magnetic field on free convection and mass transfer flow in a micro polar fluid with constant suction. J. Magn. Magn. Mater. 270, 130–135.

Gbadeyan, J.A., Hassan, A.R., 2012. Multiplicity of solutions for a reactive variable viscous couette flow under arrhenius kinetics. Math. Theory Model. 2 (9), 39–49.

Ghani, F., Gul, T., Islam, S., Shah, R.A., Khan, I., Sharida, S., Nasir, S., Khan, M.A., 2016. Unsteady magnetohydrodynamics thin film flow of a third grade fluid over an oscillating inclined belt embedded in a porous medium. Therm. Sci. 5, 875–887.

Gul, T., Islam, S., Shah, R.A., Khan, I., Shafie, S., 2014. Thin film flow in mhd third grade fluid on a vertical belt with temperature dependent viscosity. PloS One 9 (6), e97552.

Gul, T., Islam, S., Shah, R.A., Khalid, A., Khan, I., Shafie, S., 2015. Unsteady mhd thin film flow of an oldroyd-b fluid over an oscillating inclined belt. PloS One 10 (7), e0126698.

Gul, T., Ghani, F., Islam, S., Shah, R.A., Khan, I., Nasir, S., Sharidan, S., 2016. Unsteady thin film flow of a fourth grade fluid over a vertical moving and oscillating belt. Propul. Power Res. 5 (3), 223–235.

Hassan, A.R., Fenuga, O.J., 2011. Flow of a maxwell fluid through a porous medium induced by a constantly accelerating plate. J. Nigerian Assoc. Math. Phys. 19, 249–254.

Hassan, A.R., Gbadeyan, J.A., 2015. A reactive hydromagnetic internal heat generating fluid flow through a channel. Int. J. Heat Technol. 33 (3), 43–50.

Hassan, A.R., Gbadeyan, J.A., 2015. Entropy generation analysis of a reactive hydromagnetic fluid flow through a channel. U. P. B. Sci. Bull. Series A 77 (2), 285–296.

Hassan, A.R., Maritz, R., 2016. The analysis of a reactive hydromagnetic internal heat generating poiseuille fluid flow through a channel. SpringerPlus 5 (1), 1–14.

Hassan, A.R., Maritz, R., 2016. The analysis of a variable-viscosity fluid flow between parallel porous plates with non-uniform wall temperature. Italian J. Pure Appl. Math. 36, 1–12.

Hassan, A.R., Maritz, R., 2017. The effect of internal heat generation on a steady hydromagnetic poiseuille fluid flow between two parallel porous plates. Kragujevac J. Sci 39, 37–46.

Hayat, T., Ashraf, M.B., Alsaedi, A., Shehzad, S.A., 2015. Convective heat and mass transfer effects in three-dimensional flow of maxwell fluid over a stretching surface with heat source. J. Central South Univ. 22 (2), 717–726.

Jawdat, J.M., Hashim, I., 2010. Low prandtl number chaotic convection in porous media with uniform internal heat generation. Int. Commun. Heat Mass Transf. 37, 629–636.

Jha, B.K., Ajibade, A.O., 2009. Free convective flow of heat generating/absorbing fluid between vertical porous plates with periodic heat input. J. Int. Commun. Heat Mass Transf. 36, 624–631.

Kobo, N.S., Makinde, O.D., 2010. Second law analysis for a variable viscosity reactive couette flow under arhenius kinetics. Math. Prob. Eng. 1–15, 2010.

Makinde, O.D., Maserumule, R.L., 2008. Thermal criticality and entropy analysis for a variable viscosity couette flow. Phys. Scr. 78, 1–6.

Oztop, H., Bilgen, E., 2006. Natural convection in differentially heated and partially divided square cavities with internal heat generation. Int. J. Heat Fluid Flow 27, 466–475.

Patil, P.M., Kulkarni, P.S., 2008. Effects of chemical reaction on free convective flow of a polar fluid through porous medium in the presence of internal heat generation. Int. J. Therm. Sci. 7, 1043–1054.

Perekattu, G.K., Balaji, C., 2009. On the onset of natural convection in differentially heated shallow fluid layers with internal heat generation. Int. J. Heat Mass Transf. 52, 4254–4263.

Wazwaz, A.M., El-Sayed, S.M., 2001. A new modification of the adomian decomposition method for linear and nonlinear operators. Appl. Maths Comput. 122, 393–405.

Wood, L.C., 1975. Thermodynamics of Fluid Systems. Oxford University Press, Oxford.