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Original article

The exact solutions of the $(2+1)$–dimensional Kadomtsev–Petviashvili equation with variable coefficients by extended generalized $\left(\frac{G'}{G}\right)$-expansion method

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ABSTRACT

In this paper $(2+1)$–dimensional Kadomtsev–Petviashvili (KP) equation with variable coefficients is investigated through the extended generalized $\left(\frac{G'}{G}\right)$–expansion technique. One of the most universal model is KP equation, which is used to explain the ion acoustic waves in plasma physics, to model two dimensional shallow water waves, and in ferromagnetic, Bose–Einstein condensation and string theory. The obtained exact solutions of KP equation are in the form of hyperbolic function, trigonometric function, and rational function. With the aid of symbolic computational software Mathematica, the three dimensional surface plots with corresponding contour plots are provided for the obtained closed from solutions, which are of the form of solitary waves, multi solitons and periodic solitary wave like dynamical structures.

1. Introduction

The study of nonlinear evolution equations and soliton theory is significant for the physical models in different scientific fields such as physics, chemistry, biology, mathematical physics, and plasma physics (Hirota, 2004). The exact solutions of nonlinear partial differential equations plays an important role to understand the dynamical structure of the physical models, to obtain such exact solutions, different types of analytical methods are applied such as generalized exponential rational function method (Kumar, 2021), Lie symmetry approach (Kumar and Niwas, 2021), Kudryashov's method (Alotaibi, 2021), $\left(\frac{G'}{G}\right)$–expansion method (Mohanty and Dev, 2021), generalized $\left(\frac{G'}{G}\right)$–expansion method (Foroutan et al., 2018; Naher and Abdullah, 2013), generalized and improved $\left(\frac{G'}{G}\right)$–expansion method (Mohanty et al., 2022), extended generalized $\left(\frac{G'}{G}\right)$–expansion method (Mohanty et al., 2021) etc.

Invariance analysis frequently employs equations of higher-dimensional nonlinear development. The dissipative long wave equation (Kumar and Rani, 2021), the Pavlov equation (Kumar and Rani, 2020), the Boussinesq equation (Kumar and Rani, 2021), the Sharma-Tasso-Olver equation (Kumar et al., 2021), the Bogoyvlenskip Scheff equation (Kumar and Rani, 2021), and the Kadomtsev–Petviashvili equation (Rani et al., 2021) are a few examples of these equations. These equations have single soliton, multisoliton, periodic soliton, bright soliton, kink wave soliton, and kink wave type solitons, which have several applications in the solitary wave theory.

Out of these equations, the KP equation exists extensively in studying waves in dynamical system (Groves and Sun, 2008). Further, the KP equation exists in the field of dusty plasma (Seadawy and Rashidy, 2018; Samanta et al., 2013; Saha et al., 2015), in Ocean Engineering (Gwinn, 1997).

sIn this paper, we consider a general form of the $(2+1)$-dimensional KP equation with variable coefficients, which is as follows

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$$u_{tx}+f(t)(u_x)^2+f(t)uu_{xx}+g(t)u_{xxxx}+h(t)u_{xx}+q_1(t)u_{yx}+m(t)u_{yy}=0, \tag{1}$$

where $u=u(x,y,t)$, $f(t)$, $g(t)$ are nonlinear and dispersion coefficients respectively, $h(t)$, $q_1(t)$ are perturbed effects and $m(t)$ is the distributive wave velocities along the y–axis direction. Here, we have noticed that our companion researcher have obtained the reduced $(2+1)$–dimensional KP equation with variable coefficient, by putting $h(t)=0$, $q_1(t)=0$ and replacing $m(t)$ with $-m(t)$ in Eq. (1), which provides multiple rogue wave solutions (Liu et al., 2021), Wronskian and Gramian solution (Yao et al., 2008), breather wave solution (Liu et al., 2020) and lump solutions (Jia et al., 2018; Liu et al., 2019).

The symmetry property of the exact solutions of $(2+1)$–dimensional KP equation with variable coefficients is studied by Ma et al. (2013) by using a simple direct method. Borhanifar and Abazari (2011), studied the periodic and solitary wave solutions of the generalized $(2+1)$–dimensional KP equation with constant coefficients by using the $(\frac{G'}{G})$–expansion method. The soliton-like solutions of generalized KP equation with variable coefficients are obtained using the extended hyperbolic function method (Gao, 2001). The breather wave and cross-kink solutions of the $(2+1)$–dimensional KP equation with variable coefficients has been studied by Huang et al. (2020). On the other hand, the exact solutions of the KP equation with constant coefficient are obtained by using different methods such as $\exp(-\Psi(Z))$–expansion method, and extended complex method (Gu and Meng, 2019), $(\frac{G'}{G},\frac{1}{G})$–expansion method (Yang and He, 2013), Lie symmetry analysis (Malik et al., 2021), extended homogeneous balance method (Abdelsalam and Allehiany, 2018).

The extended generalized $(\frac{G'}{G})$–expansion method (Mohanty et al., 2021) is a well defined, simple and effective method, which has been already applied to solve the Schamel Burgers and the Schamel equation with constant coefficients (Mohanty et al., 2021). The idea behind the extended generalized expansion method is that, it is based on the initial assumption solution for the nonlinear evolution equation, which can be a represented by the polynomial of $(\frac{G'}{G})$, where the coefficients of the polynomial are functions of ξ, and $G=G(\xi)$ satisfies the differential equation of the form

$$P_1GG''-P_2GG'-P_3(G')^2-P_4G^2=0, \tag{2}$$

where $P_1\neq 0, P_2, P_3, P_4$ are constants. In present paper, extended generalized $(\frac{G'}{G})$–expansion method is applied to obtain the exact wave solutions of $(2+1)$–dimensional Kadomostev Petviashvili equation with variable coefficients of the form of Eq. (1). In Section 2, the details of the methodology is given and the new exact solution of the Eq. (1) is also presented. The graphical discussion and the remarkable conclusions are given in Section 3 and in Section 4 respectively.

2. The extended $(\frac{G'}{G})$ expansion method and new exact solutions

Consider the nonlinear partial differential equation of the form

$$S(u,u_x,u_y,u_t,u_{xy},u_{xt},u_{yt},u_{xx},u_{yy},\ldots)=0, \tag{3}$$

where subscripts denotes the partial derivatives.

In the extended generalized $(\frac{G'}{G})$ expansion method the assumed solutions of (3) can be written as polynomial of $(\frac{G'}{G})$ with degree m, as follows (Mohanty et al., 2021).

$$u=\sum_{j=0}^{m}b_j(Y)\left(\frac{G'}{G}\right)^j, \tag{4}$$

where $G=G(\xi)$, $\xi=\xi(Y)$ is the traveling wave transformation, $Y=Y(x,y,t)$, $b_j(j=0,1,\ldots,m)$ is undermined functions about x,y,t, to be determined later. The function G satisfies the second order differential equation of the form (2).

The general solution of equation Eq. (2) are given with different conditions in Mohanty et al. (2021), Mohanty et al. (2022), Foroutan et al. (2018), and Naher and Abdullah (2013)

Family-1: When $P_2\neq 0$ and $P_2^2+4P_4(P_1-P_3)>0$,

$$\left(\frac{G'}{G}\right)=\frac{P_2}{2\Omega}+\frac{\sqrt{\Delta}}{2\Omega}\left(\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{2\Omega}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Delta}}{2\Omega}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{2\Omega}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Delta}}{2\Omega}\xi\right)}\right), \tag{5}$$

Family-2: When $P_2\neq 0$ and $P_2^2+4P_4(P_1-P_3)<0$,

$$\left(\frac{G'}{G}\right)=\frac{P_2}{2\Omega}+\frac{\sqrt{-\Delta}}{2\Omega}\left(\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{2\Omega}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Delta}}{2\Omega}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{2\Omega}\xi\right)+C_2\sin\left(\frac{\sqrt{-(\Delta)}}{2\Omega}\xi\right)}\right). \tag{6}$$

Family-3: When $P_2\neq 0$ and $P_2^2+4P_4(P_1-P_3)=0$,

$$\left(\frac{G'}{G}\right)=\frac{P_2}{2\Omega}+\frac{C_2}{C_1+C_2\xi}. \tag{7}$$

Family-4: When $P_2=0$ and $P_4(P_1-P_3)>0$,

$$\left(\frac{G'}{G}\right)=\frac{\sqrt{\Omega P_4}}{\Omega}\left(\frac{C_1\sinh\left(\frac{\sqrt{\Omega P_4}}{\Omega}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Omega P_4}}{\Omega}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Omega P_4}}{\Omega}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Omega P_4}}{\Omega}\xi\right)}\right). \tag{8}$$

Family-5: When $P_2=0$ and $P_4(P_1-P_3)>0$,

$$\left(\frac{G'}{G}\right)=\frac{\sqrt{-\Omega P_4}}{\Omega}\left(\frac{-C_1\sin\left(\frac{\sqrt{-\Omega P_4}}{\Omega}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Omega P_4}}{\Omega}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Omega P_4}}{\Omega}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Omega P_4}}{\Omega}\xi\right)}\right). \tag{9}$$

where $\Omega=P_1-P_3$, $\Delta=P_2^2+4P_4\Omega$ and C_1, C_2 are constants.

In order to construct the exact solutions with arbitrary functions $\xi(x,y,t)$ for the $(2+1)$–dimensional KP Eq. (1), we substitute Eq. (4) into (1). Balancing the highest order derivative term u_{xxxx} with the nonlinear term uu_{xx} in (1), we obtained $m=2$. Thus Eq. (4) can be changed to

$$u=b_0(Y)+b_1(Y)\left(\frac{G'}{G}\right)+b_2(Y)\left(\frac{G'}{G}\right)^2$$

For simplifying the computation, we choose $b_0(Y)=a(y,t)$, $b_1(Y)=b(y,t)$, $b_2(Y)=c(y,t)$, and $\xi=p(y,t)+kx$, where p is function of y,t and k is constant, are to be determined later. Hence the assumed solution of (1) can be written of the form

$$u(x,y,t)=a(y,t)+b(y,t)\left(\frac{G'}{G}\right)+c(y,t)\left(\frac{G'}{G}\right)^2. \tag{10}$$

Now substituting the values of Eq. (10), into Eq. (1), then Eq. (1) is converted in the form of polynomial of $(\frac{G'}{G})$ of degree 6, then collecting the coefficients of same power of $(\frac{G'}{G})$ which is equal to zero, we get the system of partial differential equations is a form

$$\left(\frac{G'}{G}\right)^6: 120gck^4T_1^4+10fc^2k^2T_1^2=0 \tag{11}$$

$$\left(\frac{G'}{G}\right)^5: 24gbk^4T_1^4+336gck^4T_1^3T_2+12fck^2T_1^2b+18fc^2k^2T_1T_2=0 \tag{12}$$

$$\left(\tfrac{G'}{G}\right)^4: 60gbk^4T_1^3T_3 + 240gbk^4T_1^3T_3 + 330gk^4T_1^2T_2^2 + 3fb^2k^2T_1^2 + 21fck^2T_1bT_2 + 16fc^2k^2T_1T_3 + 8fc^2k^2T_2^2 + 6fack^2T_1^2 + 6mcp_y^2T_1^2 + 6q_1ckp_yT_1^2 + 6ckp_tT_1^2 = 0 \quad (13)$$

$$\left(\tfrac{G'}{G}\right)^3: 40gbk^4T_1^3T_3 + 50gbk^4T_1^2T_2^2 + 440gck^4T_1^2T_2T_3 + 130gck^4T_1T_2^3 + 5fb^2k^2T_1T_2 + 18fck^2T_1bT_3 + 9fck^2T_2^2b + 2fabk^2T_1^2 + 14fc^2k^2T_2T_3 + 10fack^2T_1T_2 + 2mbp_y^2T_1^2 + 2q_1bkp_yT_1^2 + 2hbk^2T_1^2 + 10mcp_y^2T_1T_2 + 10q_1ckp_yT_1T_2 + 10hck^2T_1T_2 + 2bp_tkT_1^2 + 10p_tckT_1T_2 + 4mc_yp_yT_1 + 2q_1c_ykT_1 + 2mcp_{yy}T_1 + 2c_tkT_1 = 0 \quad (14)$$

$$\left(\tfrac{G'}{G}\right)^2: 4p_tckT_2^2 + b_tkT_1 + 2c_tkT_2 + mc_{yy} + 3bp_tkT_1T_2 + 8p_tckT_1T_3 + 4hck^2T_2^2 + 2mb_yp_yT_1 + q_1b_ykT_1 + mbp_{yy}T_1 + 4mc_yp_yT_2 + 2q_1c_ykT_2 + 2mcp_{yy}T_2 + 16gck^4T_2^4 + 2fb^2k^2T_2^2 + 6fc^2k^2T_3^2 + 4mcp_y^2T_2^2 + 15gbk^4T_1T_2^3 + 136gck^4T_1^2T_3^2 + 4fb^2k^2T_1T_3 + 4fack^2T_2^2 + 3mbp_y^2T_1T_2 + 3hbk^2T_1T_2 + 8mcp_y^2T_1T_3 + 4q_1ckp_yT_2^2 + 8hck^2T_1T_3 + 60gbk^4T_1^2T_2T_3 + 232gck^4T_1T_2^2T_3 + 15fck^2T_2bT_1 + 3fabk^2T_1T_2 + 8fack^2T_1T_3 + 3q_1bkp_yT_1T_2 + 8q_1ckp_yT_1T_3 = 0 \quad (15)$$

$$\left(\tfrac{G'}{G}\right)^1: mb_{yy} + bp_tkT_2^2 + b_tkT_2 + 2c_tkT_3 + 2bp_tkT_1T_3 + 6p_tckT_2T_3 + gbk^4T_2^4 + mbp_y^2T_2^2 + hbk^2T_2^2 + 2mb_yp_yT_2 + q_1b_ykT_2 + mbp_{yy}T_2 + 4mc_yp_yT_3 + 2q_1c_ykT_3 + 2mcp_{yy}T_3 + 16gbk^4T_1^2T_3^2 + 30gck^4T_2^3T_3 + 3fb^2k^2T_2T_3 + 6fck^2T_3^2b + fabk^2T_2^2 + 2mbp_y^2T_1T_3 + q_1bkp_yT_2^2 + 2hbk^2T_1T_3 + 6mcp_y^2T_2T_3 + 6hck^2T_2T_3 + 22g(t)bk^4T_1T_2^2T_3 + 120gck^4T_1T_2T_3^2 + 2fabk^2T_1T_3 + 6fack^2T_2T_3 + 2q_1bkp_yT_1T_3 + 6q_1ckp_yT_2T_3 = 0 \quad (16)$$

$$\left(\tfrac{G'}{G}\right)^0: 2ckp_tT_3^2 + 16gck^4T_3^3T_1 + 14gck^4T_2^2T_3^2 + gbk^4T_2^3T_3 + hbk^2T_2T_3 + 2q_1ckp_yT_3^2 + 2fack^2T_3^2 + bkp_tT_2T_3 + fabk^2T_2T_3 + 8gbk^4T_1T_3^2T_2 + q_1bkp_yT_2T_3 + b_tkT_3 + ma_{yy} + fb^2k^2T_3^2 + 2hck^2T_3^2 + q_1b_ykT_3 + mbp_{yy}T_3 + 2mcp_y^2T_3^2 + 2mb_yp_yT_3 + mbp_y^2T_2T_3 = 0 \quad (17)$$

where $T_1 = \frac{P_3}{P_1} - 1, T_2 = \frac{P_2}{P_1}, T_3 = \frac{P_4}{P_1}$. Now solving the system of Eqs. (11)–(17) for a, b, c, p and m, the obtained solutions is as follows

$$m(t) = \frac{k(gf_t - fg_t)f}{g^3s_1}, p(y,t) = \frac{s_1g^2y^2}{2f^2} + f_2(t)y + f_3(t),$$
$$(y,t) = -\frac{12k^2gT_2T_1}{f}, c(y,t) = -\frac{12k^2gT_1^2}{f},$$
$$(y,t) = \frac{1}{f^3kg^3s_1}\Big(\Big(-2f_2(t)ys_1fg^3 - f_2(t)^2gf^3\Big)f_t + f_2(t)f^2\Big(2ys_1g^2 + f_2(t)f^2\Big)g_t - 8s_1g^3\Big(\tfrac{1}{8}f_{2t}(t)yf^2 + \tfrac{1}{8}f_{3t}(t)f^2 + \Big(k^3\Big(T_1T_3 + \tfrac{1}{8}T_2^2\Big)g + \tfrac{1}{8}kh + \tfrac{1}{8}f_2(t)q_1\Big)f^2 + \tfrac{1}{8}q_1ys_1g^2\Big)\Big), \quad (18)$$

where $f_2(t), f_3(t)$, are arbitrary differential functions and $f_{2t}(t), f_{3t}(t)$ denotes the partial derivative of $f_2(t)$, and $f_3(t)$ with respect to t accordingly.

New exact solutions of the $(2+1)$–dimensional Kadomtsev–Petviashvili Eq. (1) can be obtained as follows. Substituting Eq. (18) and Eq. (5) into Eq. (10), the hyperbolic solution of Eq. (1) is

$$u = a(y,t) - \frac{12k^2gT_2T_1}{f}\left(\frac{P_2}{2\Omega} + \frac{\sqrt{\Delta}}{2\Omega}\left(\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{2\Omega}(p(y,t)+kx)\right) + C_2\cosh\left(\frac{\sqrt{\Delta}}{2\Omega}(p(y,t)+kx)\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{2\Omega}(p(y,t)+kx)\right) + C_2\sinh\left(\frac{\sqrt{\Delta}}{2\Omega}(p(y,t)+kx)\right)}\right)\right) - \frac{12k^2gT_1^2}{f}\left(\frac{P_2}{2\Omega} + \frac{\sqrt{\Delta}}{2\Omega}\left(\frac{C_1\sinh\left(\frac{\sqrt{\Delta}}{2\Omega}(p(y,t)+kx)\right) + C_2\cosh\left(\frac{\sqrt{\Delta}}{2\Omega}(p(y,t)+kx)\right)}{C_1\cosh\left(\frac{\sqrt{\Delta}}{2\Omega}(p(y,t)+kx)\right) + C_2\sinh\left(\frac{\sqrt{\Delta}}{2\Omega}(p(y,t)+kx)\right)}\right)\right)^2 \quad (19)$$

where $p(y,t) = \frac{s_1g^2y^2}{2f^2} + f_2(t)y + f_3(t)$ for all the solutions of Eq. (1).

Substituting Eq. (18) and Eq. (6) into Eq. (10), the trigonometric solution of Eq. (1) is

$$u = a(y,t) - \frac{12k^2gT_2T_1}{f}\left(\frac{P_2}{2\Omega} + \frac{\sqrt{-\Delta}}{2\Omega}\left(\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{2\Omega}(p(y,t)+kx)\right) + C_2\cos\left(\frac{\sqrt{-\Delta}}{2\Omega}(p(y,t)+kx)\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{2\Omega}(p(y,t)+kx)\right) + C_2\sin\left(\frac{\sqrt{-(\Delta)}}{2\Omega}(p(y,t)+kx)\right)}\right)\right) - \frac{12k^2gT_1^2}{f}\left(\frac{P_2}{2\Omega} + \frac{\sqrt{-\Delta}}{2\Omega}\left(\frac{-C_1\sin\left(\frac{\sqrt{-\Delta}}{2\Omega}(p(y,t)+kx)\right) + C_2\cos\left(\frac{\sqrt{-\Delta}}{2\Omega}(p(y,t)+kx)\right)}{C_1\cos\left(\frac{\sqrt{-\Delta}}{2\Omega}(p(y,t)+kx)\right) + C_2\sin\left(\frac{\sqrt{-(\Delta)}}{2\Omega}(p(y,t)+kx)\right)}\right)\right)^2 \quad (20)$$

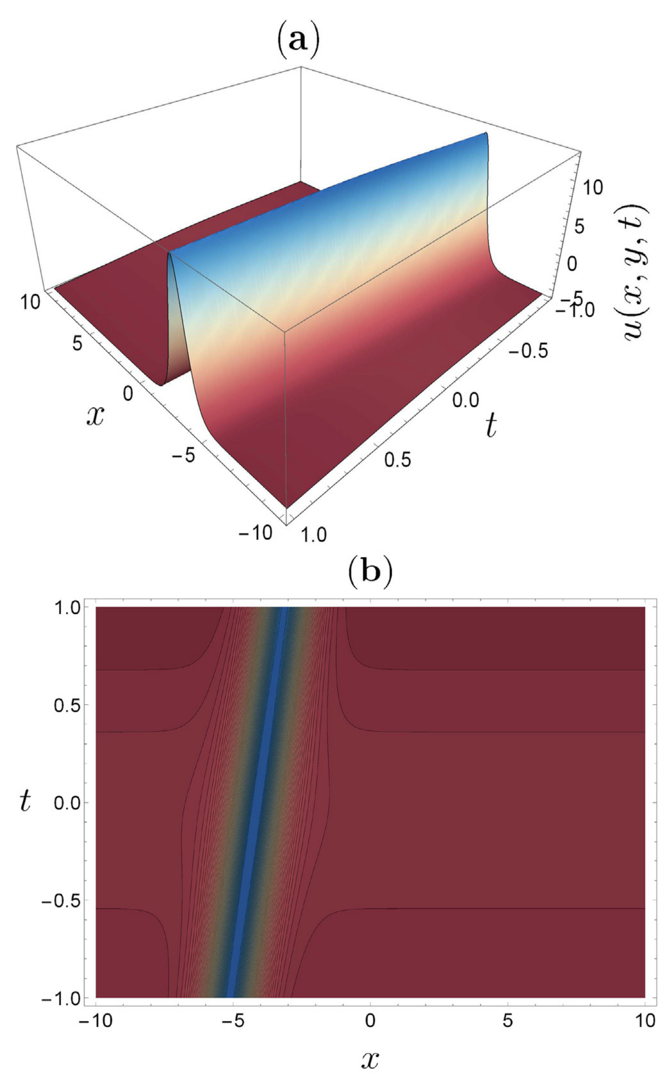

Fig. 1. (a) and (b) are three dimensional surface plot and contour plot of Eq. (19) respectively where the values of the free parameters are taken as $P_1 = 40, P_2 = 1, P_3 = 2, P_4 = 1, C_1 = 1, C_2 = 0, s_1 = 1, k = 8, f(t) = t^2 + 1, g(t) = t^2 + 1, h(t) = t^2 + 1, q_1(t) = t^2 + 1, f_2(t) = t, f_3(t) = 1, y = -8$, and x varies from -10 to $10, t$ varies from -1 to 1.

Substituting Eq. (18) and Eq. (7) into Eq. (10), the rational solution of the Eq. (1) is

$$u = a(y,t) - \frac{12k^2 g T_2 T_1}{f}\left(\frac{P_2}{2\Omega} + \frac{C_2}{C_1 + C_2(p(y,t)+kx)}\right) - \frac{12k^2 g T_1^2}{f}\left(\frac{P_2}{2\Omega} + \frac{C_2}{C_1 + C_2(p(y,t)+kx)}\right)^2. \tag{21}$$

Substituting Eq. (18) and Eq. (8) into Eq. (10), the hyperbolic solution of Eq. (1) is

$$u = a(y,t) - \frac{12k^2 g T_2 T_1}{f}\frac{\sqrt{\Omega P_4}}{\Omega}\left(\frac{C_1 \sinh\left(\frac{\sqrt{\Omega P_4}}{\Omega}(p(y,t)+kx)\right) + C_2 \cosh\left(\frac{\sqrt{\Omega P_4}}{\Omega}(p(y,t)+kx)\right)}{C_1 \cosh\left(\frac{\sqrt{\Omega P_4}}{\Omega}(p(y,t)+kx)\right) + C_2 \sinh\left(\frac{\sqrt{\Omega P_4}}{\Omega}(p(y,t)+kx)\right)}\right) - \frac{12k^2 g T_1^2}{f}\left(\frac{\sqrt{\Omega P_4}}{\Omega}\left(\frac{C_1 \sinh\left(\frac{\sqrt{\Omega P_4}}{\Omega}(p(y,t)+kx)\right) + C_2 \cosh\left(\frac{\sqrt{\Omega P_4}}{\Omega}(p(y,t)+kx)\right)}{C_1 \cosh\left(\frac{\sqrt{\Omega P_4}}{\Omega}(p(y,t)+kx)\right) + C_2 \sinh\left(\frac{\sqrt{\Omega P_4}}{\Omega}(p(y,t)+kx)\right)}\right)\right)^2 \tag{22}$$

Substituting Eqs. (18) and (9) into equation Eq. (10), Thus the trigonometric solutions of the Eq. (1) is

$$u = a(y,t) - \frac{12k^2 g T_2 T_1}{f}\left(\frac{\sqrt{-\Omega P_4}}{\Omega}\left(\frac{-C_1 \sin\left(\frac{\sqrt{-\Omega P_4}}{\Omega}(p(y,t)+kx)\right) + C_2 \cos\left(\frac{\sqrt{-\Omega P_4}}{\Omega}(p(y,t)+kx)\right)}{C_1 \cos\left(\frac{\sqrt{-\Omega P_4}}{\Omega}(p(y,t)+kx)\right) + C_2 \sin\left(\frac{\sqrt{-\Omega P_4}}{\Omega}(p(y,t)+kx)\right)}\right)\right) - \frac{12k^2 g T_1^2}{f}\left(\frac{\sqrt{-\Omega P_4}}{\Omega}\left(\frac{-C_1 \sin\left(\frac{\sqrt{-\Omega P_4}}{\Omega}(p(y,t)+kx)\right) + C_2 \cos\left(\frac{\sqrt{-\Omega P_4}}{\Omega}(p(y,t)+kx)\right)}{C_1 \cos\left(\frac{\sqrt{-\Omega P_4}}{\Omega}(p(y,t)+kx)\right) + C_2 \sin\left(\frac{\sqrt{-\Omega P_4}}{\Omega}(p(y,t)+kx)\right)}\right)\right)^2. \tag{23}$$

3. Graphs and discussion

The solutions obtained for the $(2+1)$–dimensional KP equation with variable coefficients are verified by back substitution in the original equation using the symbolic computational software Mathematica. The graphical representation of some of the obtained solutions are considered by using Mathematica 12. The dynamical structures of the obtained solutions are given as follows Fig. 1–3 illustrates the solitary wave, double soliton, and multisoliton type traveling waves for the KP equation with variable coefficients using the extended generalized expansion method approach through the three-dimensional surface plots and corresponding contour plots. In Fig. 1 the annihilation of surface plot and contour plot of

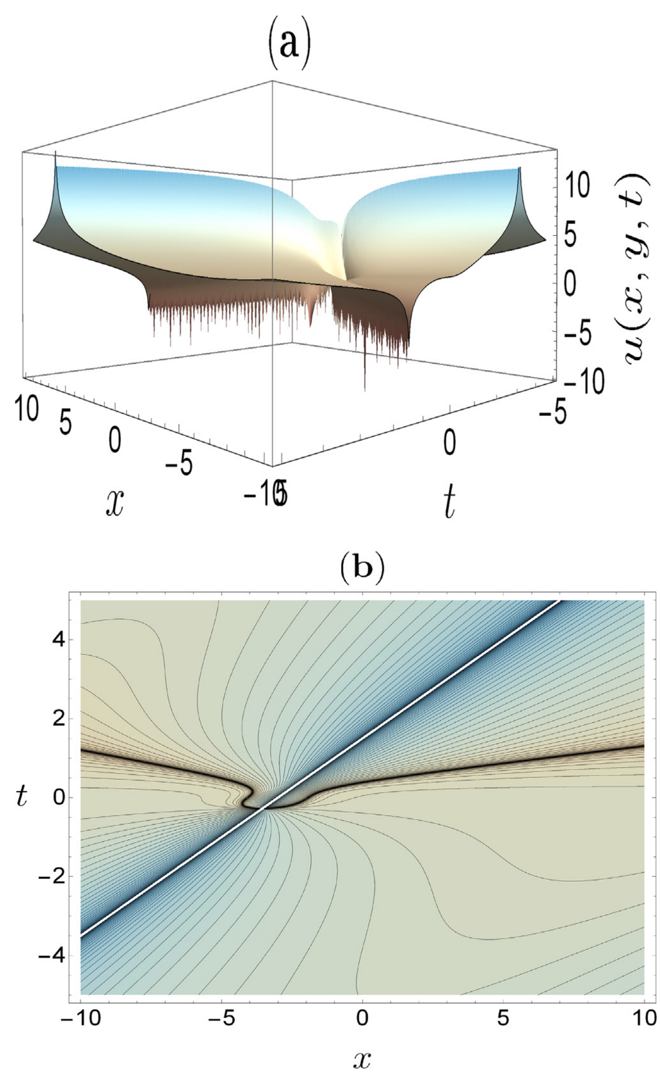

Fig. 2. (a) and (b) are three dimensional surface plot and contour plot of from Eq. (20) respectively, where $P_1 = 30, P_2 = 10, P_3 = 2, P_4 = -1, C_1 = 0, C_2 = 1, s_1 = 1, k = 1, f_2(t) = t, f_3(t) = 1, y = -2, f(t) = t^2 + 1, g(t) = t^2 + 1, h(t) = t^2 + 1, q_1(t) = t^2 + 1$, and x varies from -10 to 10, t varies from -5 to 5.

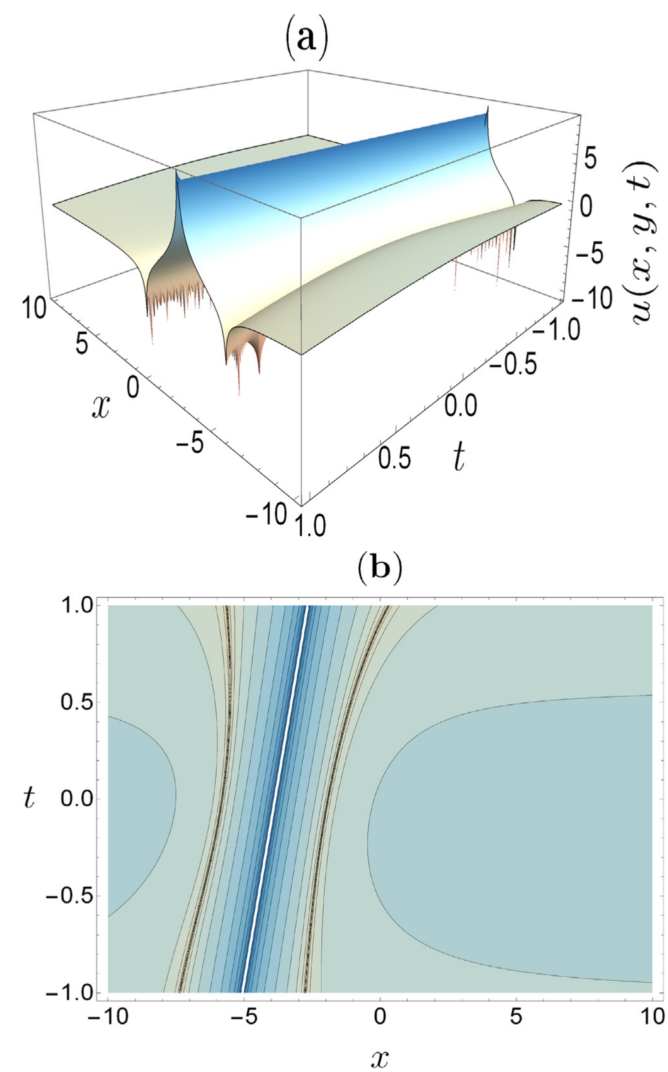

Fig. 3. (a) and (b) are three dimensional surface plot and contour plot of Eq. (21) respectively, where $P_1 = 3, P_2 = 4, P_3 = 5, P_4 = 2, C_1 = 10, C_2 = 30, s_1 = 1, k = 5, f_2(t) = t, f_3(t) = 1, y = -6, f(t) = t^2 + 1, g(t) = t^2 + 1, h(t) = t^2 + 1, q_1(t) = t^2 + 1$, x varies from -10 to 10, t varies from -1 to 1.

solution (19) represent the solitary type traveling wave soliton, and similar type of dynamic structures of solitary tye wave soliton also exists for solution (22). As seen from Fig. 1 with time, there is a smooth transition of the isolated region which resembles a typical classical solitary wave formation in nonlinear dynamics. Fig. 2 shows a typical formation of double layer type structure for such solution (20). Fig. 2 depicts two types of wave propagation: comprehensive and rarefactive. We know that compressive waveform is where the amplitude is positive, and rarefactive when the amplitude is negative. Fig. 2 shows both positive and negative amplitude, comprising both compressive and rarefactive waveforms. The same double layer structure also exists for the solution (23). Fig. 3 provides the information about the multi-soliton structure of solution (21).

4. Conclusion

The exact solutions of $(2+1)$ dimensional KP equation with variable coefficients are obtained, for which the extended generalized $\left(\frac{G'}{G}\right)$–expansion method is employed. The resulting solutions are trigonometric, hyperbolic, and rational functions. Dynamical structures of the obtained solutions are the solitary wave, double layer soliton, and multi soliton structures. The graphical representations of the figures also ensure the typical properties of solitonic characteristics of the nonlinear wave dynamics of plasma physics, which is quite encouraging. We believe this method can also be safely applied in nonlinear plasma physics wherein such a situation arises. The method and solutions obtained may help study degenerate nonlinear plasma physics and compact astronomical phenomena such as non-rotating neutron stars (Chabrier et al., 2006), ultra-intense laser-plasma (Eliezer et al., 2005), and optical fibers (Njikue et al., 2018).

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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