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Accelerating universe with decreasing gravitational constant

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ABSTRACT

We study the cosmology of dark energy in an extended theory of gravity in Lyra's geometry. By analyzing a possible phenomenological interaction between the matter field with a constant equation-of-state parameter ω_m $(0 \leq \omega_m < 1)$ and the geometric displacement field in Lyra's geometry, we find that the displacement vector field engenders an effective time-dependent gravitational constant G_{eff} constrained in the region $0 < G_{eff}(t) \leq G$. We show that the effective equation-of-state parameter $\omega_{eff}(t)$ evolves in the same way as the effective time-dependent gravitational constant which, by decreasing with time, can give rise to the late-time cosmic acceleration with $\omega = -1$ crossing in a flat Robertson-Walker background without adding dynamical ghost mode.

1. Introduction

Since the discovery of an accelerated expansion of the universe from astrophysical observations (Riess et al., 1998; Perlmutter et al., 1999), cosmologists think that beside the standard matter there must be an unknown fluid with a negative pressure $p_{de} < -(1/3)\rho_{de}$, that drives the universe into acceleration. Such a component, dubbed "dark energy" was completely negligible in most of the past and nowadays dominates entirely the universe. The origin and the exact nature of dark energy are still unknown. Subsequently, the Einstein cosmological constant has been commonly considered as the best candidate to play the role of dark energy; however it faces fine-tuning and coincidence puzzles. The late-time cosmic acceleration may alternatively be driven by a dynamic dark energy which could be a time evolving and spatially dependent scalar field. Lots of such dynamic dark energy models have been proposed and are roughly classified into three categories: quintessence (Peebles and Ratra, 1988; Ratra and Peebles, 1988; Caldwell et al., 1998; Copeland et al., 2006), phantom (Caldwell, 2002) and quintom (Feng et al., 2005; Guo et al., 2005). In quintessence models, a scalar field ϕ with a canonical kinetic energy and a self-interaction potential energy $V(\phi)$ is supposed to be minimally coupled to Einstein gravity. In a flat Robertson-Walker background the quintessence scalar behaves as a perfect fluid with an evolving equation-of-state (EoS) parameter $\omega = p/\rho$ lying in the range $-1 \leq \omega \leq 1$. In phantom models the quintessence is replaced by a ghost scalar with negative kinetic energy yielding $\omega < -1$. The quintom models generally involve two scalar fields, one is a quintessence scalar and another a phantom. Quintom models characterize themselves by the property that their effective EoS parameter can cross the $\omega = -1$ barrier, which enables them to evade the divergence of linear perturbations and makes them to fit the observational data better (Cai et al., 2010). Having $\omega = -1$ crossing in a dynamic dark energy model is bewitching. However, the emergence of a phantom scalar with negative kinetic energy in quintom models brings about great embarrassment on physics. It is worthwhile to study the mechanism of removing the phantom field from the quintom models. In fact, there has lots of attempts to investigate the possibility of $\omega = -1$ crossing in quintessence like models. It has been empirically realized that to have $\omega = -1$ crossing and remove ghost mode at the same time, the model building should be involved in either modifying the general theory of Einstein's relativity or introducing some higher derivative terms for the scalar fields (Cai et al., 2010). For example, in the so-called Galileon cosmology (Deffayet et al., 2009; Nicolis et al., 2009) of a scalar field, the higher derivatives of operators are introduced into the Lagrangian but the equation of motion of the scalar remains of the second order. The Galileon models can have $\omega = -1$ crossing without ghost modes. It goes without saying, however, that these models are very complicated.

Aimed at finding the explanation for the cosmic acceleration, some authors (Sivaram et al., 1975; Pande et al., 2000; Tsagas

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and Maartens, 2000; Vishwakarma, 2001; Rane and Katore, 2014) have constructed homogeneous isotropic cosmological models with variable cosmological constant and time-dependent gravitational constant satisfying the present day observational data. A time variation of the gravitational constant, which characterizes the strength of the gravitational interaction, has been first suggested by Dirac (1937a,b, 1938) and extensively described in literature (Dicke, 1961; Wu and Wang, 1986; Carroll et al., 1992; Kordi, 2009), where the gravitational constant is considered in methods that include studies of the evolutions of clusters of galaxies and of the Sun, observations of lunar occultations, planetary radar-ranging measurements and laboratory experiments. Many other extensions of Einstein theory with variable gravitational constant have been proposed in order to achieve a possible unification of gravitation and elementary particles physics or incorporate Mach's principle in general relativity (Canuto et al., 1977; Uehara and Kim, 1982; Alfonso-Faus, 1986). Results on the time variation of the gravitational constant G usually yield experimental bounds $\left|\dot{G}/G\right| \leq 4.0\times10^{-11}\mathrm{yr}^{-1}$ (Benvenuto et al., 2004). The variability of G and Λ becomes then one of the most striking and unsettled problems in cosmology.

In this paper, we study the $\omega=-1$ crossing possibility in a cosmological model in the framework of the extended theories of gravity (ETG) in Lyra's geometry (Lyra, 1951; Scheibe, 1952; Sen, 1957, 1960; Halford, 1970; Sen and Dunn, 1971; Manoukian, 1972; Hudgin, 1973; Soleng, 1987; Hova, 2013, 2014, Hova et al., 2019) and the references therein. Despite the impossibility for the standard ΛCDM model to have $\omega=-1$ crossing property in Einstein's gravity in pseudo-Riemannian geometry, the existence of a displacement field interacting with the cold dark matter (CDM) in the ETG in Lyra's geometry leads to an effective time-dependent gravitational constant and does probably modify the effective distribution of the cosmic fluids so that the effective EoS parameter may cross the $\omega=-1$ barrier as the effective time-dependent gravitational constant decreases with time.

In Section II, we describe a model, where an effective gravitational constant is derived, and compute the effective equation of stage parameter that depends exclusively on the effective time-dependent gravitational constant. Section III is devoted to the discussions and conclusions are given in Section 4.

2. Model

In Planckian units $c=\hbar=\kappa^2=1$, the Einstein-Hilbert action invariant under both coordinate and gauge transformations in Lyra's geometry which is a modification of Riemannian geometry by the introduction of a nonzero gauge function ψ into the structureless manifold, is described by (Sen, 1957)

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}\left(\psi^2R+\frac{3}{2}\psi^4\phi_\mu\phi^\mu+3\psi^2\phi^\mu\nabla_\mu\psi\right)+\psi^4L_m\right] \tag{1}$$

where R is the Ricci scalar, ϕ^μ is a displacement vector field arising from the introduction of the gauge function ψ and L_m is the matter Lagrangian. Like Riemannian geometry, Lyra's geometry preserves the metric and integrability of length under infinitesimal parallel transfer. Moreover, cosmological theories built within Lyra's geometry solve the entropy and horizon problems, which beset the standard models based on Riemannian geometry, and predict the same effects within observations limits, as far as the classical Solar System, as well as tests based on the linearized form of the field equations (Scheibe, 1952; Sen, 1957, 1960; Halford, 1970; Sen and Dunn, 1971; Manoukian, 1972; Hudgin, 1973; Soleng, 1987). Variation of the action S with respect to the metric tensor components $g_{\mu\nu}$ gives the Einstein gravitational field equations

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+\frac{3}{2}\psi^2\left(\phi_\mu\phi_\nu-\frac{1}{2}g_{\mu\nu}\phi_\lambda\phi^\lambda\right)+3\left(\phi_\mu\nabla_\nu\psi-\frac{1}{2}g_{\mu\nu}\phi^\lambda\nabla_\lambda\psi\right)+\frac{2}{\psi}\left(\nabla_\mu\nabla_\nu\psi+g_{\mu\nu}\psi\right)+\frac{2}{\psi^2}\left(\nabla_\mu\psi\nabla_\nu\psi+g_{\mu\nu}\nabla_\lambda\psi\nabla^\lambda\psi\right)=\psi^2T_{\mu\nu} \tag{2}$$

In the so-called normal-gauge Lyra's geometry, defined by $\psi=1$, the gravitational Eq. (2) reduce to (Sen, 1957)

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+\tau_{\mu\nu}=T_{\mu\nu}, \tag{3}$$

where

$$\tau_{\mu\nu}=\frac{3}{2}\left(\phi_\mu\phi_\nu-\frac{1}{2}g_{\mu\nu}\phi_\lambda\phi^\lambda\right) \tag{4}$$

is an intrinsic geometrical tensor associated with the displacement vector field ϕ_μ, whereas

$$T_{\mu\nu}=(p_m+\rho_m)u_\mu u_\nu+p_mg_{\mu\nu} \tag{5}$$

is the energy–momentum tensor of a perfect fluid with energy ρ_m and pressure $p_m=\omega_m\rho_m(0\leq\omega_m\leq1)$. The vector $u_\mu=(1,0,0,0)$, with $u^\mu u_\mu=-1$, is the 4-velocity of the comoving observer. When the tensor $\tau_{\mu\nu}$ vanishes, we recover the gravitational field equations of general relativity; so the modification of the Einstein's equations is solely due to the intrinsic geometrical vector field ϕ_μ. In the flat Friedmann-Lemaître-Robertson-Walker background $ds^2=-dt^2+a^2(t)d\vec{x}^2$, where $a(t)$ is the scale factor of an expanding universe, the nonvanishing components of the Ricci tensor, and the Ricci scalar are given by

$$R_{00}=-3\left(\dot{H}+H^2\right),\ R_{ij}=\left(\dot{H}+3H^2\right)g_{ij},\ R=6\left(\dot{H}+2H^2\right) \tag{5}$$

where an overdot denotes differentiation with respect to the time coordinate t and $H=\dot{a}/a$ is the Hubble rate. Throughout this paper we are to consider a time-like displacement vector

$$\phi_\mu=(\phi(t),0,0,0). \tag{6}$$

Thus, the gravitational field equations (3) reduce to the Friedmann equations

$$3H^2+\frac{3}{4}\phi^2(t)=\rho_m, \tag{7}$$

$$-\left(2\dot{H}+3H^2\right)+\frac{3}{4}\phi^2(t)=\omega_m\rho_m, \tag{8}$$

while applying the Bianchi identities gives

$$\dot{\rho}_m+3H(1+\omega_m)\rho_m=\frac{3}{4}\left(\phi\dot{\phi}+3H\phi^2\right). \tag{9}$$

Due to the interaction of matter field with the displacement field during the evolution of the universe, the matter energy ρ_m does not conserve separately; there may be transfer of energy between matter fields and the displacement field. In (Hova, 2013) an interaction was built between ϕ and the matter energy ρ_m by encoding Eq. (10) into two conservation equations. Here, we are to consider the first Friedmann equation (8) to find the connection between ϕ, H and ρ_m. Assuming therefore the geometrical quantities H and ϕ are both real, we can recast Eq. (8) in the form

$$3\left(H+\frac{i}{2}\phi\right)\left(H-\frac{i}{2}\phi\right)=\rho_m. \tag{10}$$

By defining the complex dimensionless quantity $\mathcal{Z}$ and its conjugate $\mathcal{Z}^*$ by

$$\mathcal{Z} = \sqrt{\frac{3}{\rho_m}}\left(H + \frac{i}{2}\phi\right) \text{ and } \mathcal{Z}^* = \sqrt{\frac{3}{\rho_m}}\left(H - \frac{i}{2}\phi\right) \text{ with } i = \sqrt{-1}, \tag{11}$$

Eq. (11) is recast as

$$\mathcal{Z}\mathcal{Z}^* = |\mathcal{Z}|^2 = 1 \tag{12}$$

In terms of $\mathcal{Z}$ and its conjugate $\mathcal{Z}^*$ the Hubble rate and the displacement field read

$$H = \sqrt{\frac{\rho_m}{3}}\frac{\mathcal{Z} + \mathcal{Z}^*}{2} \tag{13}$$

and

$$\phi = 2\sqrt{\frac{\rho_m}{3}}\frac{\mathcal{Z} - \mathcal{Z}^*}{2i} \tag{14}$$

respectively. As for every complex number the general form of $\mathcal{Z}(t)$ is parametrized as

$$\mathcal{Z}(t) = \beta(t)e^{i\zeta(t)} \tag{15}$$

where $\zeta(t)$ and $\beta(t)$ are two real functions of time (with $\beta(t) > 0$). In the present case, $|\mathcal{Z}|^2 = 1$, we then have $\beta(t) = 1$, and H and ϕ transform according to

$$H(t) = \frac{1}{\sqrt{3}}\sqrt{\rho_m(t)}cos[\zeta(t)] \tag{16}$$

and

$$\phi(t) = \frac{2}{\sqrt{3}}\sqrt{\rho_m(t)}sin[\zeta(t)]. \tag{17}$$

As suggested by observations, both H and ρ_m are positive definite; the sign of H is determined therefore by that of $\cos[\zeta(t)]$, and we must have $\cos[\zeta(t)] > 0$ whatever the sign of $\sin[\zeta(t)]$ (ϕ can be positive or negative). However, depending on $\sin[\zeta(t)]$, the displacement field ϕ could vanish yielding a decelerating expanding universe. So, to avoid a decelerated expansion, we require that $\sin[\zeta(t)] \neq 0$, that is $\zeta(t) \neq k\pi$ with$k \in \mathbb{Z}$.

Combining Eqs. (17) and (18) yields a relationship between H and ϕ:

$$\phi(t) = 2tan[\zeta(t)]H(t). \tag{18}$$

Thus, $\zeta(t)$ appears as playing the role of interaction between H and ϕ. In view of (19), Eq. (10) becomes

$$\dot{\rho}_m(t) + 3H\left[1 + \omega_m - (1 - \omega_m)tan^2[\zeta(t)]\right]\rho_m(t) = 2\dot{\zeta}(t)tan[\zeta(t)]\rho_m(t). \tag{19}$$

Using the e-folding number $x = \ln a$, Eq. (18) may be written as a conservation equation of the matter

energy ρ_m in the form

$$\frac{d\rho_m(x)}{dx} + 3\left(1 + \widehat{\omega}_m(x)\right)\rho_m(x) = 0, \tag{20}$$

where we introduced the quantity

$$\widehat{\omega}_m(x) = \omega_m - (1 - \omega_m)tan^2[\zeta(x)] - \frac{2}{3}tan[\zeta(x)]\frac{d\zeta(x)}{dx}, \tag{21}$$

which appears as an evolving EoS parameter of the background matter. The parameter $\widehat{\omega}_m$ depends on time via the function ζ and is *a priori* nonzero even if the background matter is pressureless, that is when $\omega_m = 0$. So, through the interaction, a naturally pressureless matter could get pressure as the universe evolves.

To find the effective EoS parameter, ω_{eff}, that controls the different phases of the expanding universe, let us rewrite the Friedmann equation (8) (or equivalently Eq. (17)) in the form

$$3H^2 = cos^2[\zeta(x)]\rho_m(x) = \rho_{eff}(x). \tag{22}$$

where we defined an effective energy density $\rho_{eff}(x)$ as

$$\rho_{eff}(x) = cos^2[\zeta(x)]\rho_m(x) \tag{23}$$

The effective energy density $\rho_{eff}(x)$ satisfies the conservation equation

$$\frac{d\rho_{eff}(x)}{dx} + 3\left(1 + \omega_{eff}(x)\right)\rho_{eff}(x) = 0 \tag{24}$$

with an effective EoS parameter

$$\omega_{eff}(x) = \omega_m - (1 - \omega_m)tan^2[\zeta(x)]. \tag{25}$$

On the other hand, by introducing explicitly the gravitational constant G, the Friedmann equation (8) reads

$$3H^2 = 8\pi Gcos^2[\zeta(x)]\rho_m(x) = 8\pi G_{eff}(x)\rho_m(x), \tag{26}$$

where we define an effective time-dependent gravitational constant, $G_{eff}(x)$, by

$$G_{eff}(x) = Gcos^2[\zeta(x)]. \tag{27}$$

The effective time-dependent gravitational constant $G_{eff}(x)$ arises as a direct consequence of the displacement field. It is constrained within the range $0 < G_{eff}(x) \leq G$. A vanishing $G_{eff}(x)$ is strongly avoided since it will lead to a static universe, $a(t) = a_0 = constant$, devoid of any gravitational forces. Using a dimensionless quantity

$$\widehat{g}(x) \equiv \frac{G_{eff}(x)}{G} = cos^2[\zeta(x)] \tag{28}$$

representing the effective gravitational constant, the parameters ω_m and ω_{eff} may be expressed completely by means of $\widehat{g}(x)$ as follows:

$$\widehat{\omega}_m(x) = 1 - \frac{(1 - \omega_m)}{\widehat{g}(x)} + \frac{1}{3}\frac{1}{\widehat{g}(x)}\frac{d\widehat{g}(x)}{dx} \tag{29}$$

and

$$\omega_{eff}(x) = 1 - \frac{(1 - \omega_m)}{\widehat{g}(x)}. \tag{30}$$

3. Discussions

The EoS parameters$\widehat{\omega}_m(x)$ and $\omega_{eff}(x)$ coincide when the function ζ is constant, in particular $\omega_{eff} = \widehat{\omega}_m = -1 + 2\omega_m$ for $\zeta = \pi/4$, leading to an accelerating universe when $\omega_m < 1/3$. Moreover, a pressureless matter will yield the cosmological constant boundary since $\omega_{eff} = \widehat{\omega}_m = -1$. The difference between $\widehat{\omega}_m(x)$ and $\omega_{eff}(x)$ is due to the variation of the effective gravitational constant that will be responsible for the late-time accelerated expansion of the universe. Equation (31) shows that both $\omega_{eff}(x)$ and $\widehat{g}(x)$ either increase or decrease. As observations indicate that the dark energy EoS parameter is decreasing with time, the possible accelerated expansion of the universe will therefore be the result of the time-dependent gravitational constant that decreases following the expansion of the universe. In view of (29) and (31) we do have $\omega_{eff}(x) \leq \omega_m$, especially $\omega_{eff}(x) = \omega_m$ when $\widehat{g}(x) = 1$. When the background fluid is a stiff matter ($\omega_m = 1$), the total energy fluid will also behave as a stiff matter during the evolution of the universe since $\omega_{eff}(x) = \omega_m = 1$ whatever the variation of $\widehat{g}(x)$: the universe will be permanently decelerating. And $\omega_{eff}(x)$ vanishes when $\widehat{g}(x) = 1 - \omega_m$. As shown in Tables 1 and 2, when $3(1 - \omega_m)/4 < \widehat{g}(x) \leq 1$, the expansion is decelerated, and starts accelerating when $\widehat{g}(x) = 3(1 - \omega_m)/4$. The universe then crosses the cosmological constant boundary (quintom scenario) for

Table 1
Particular values of $\hat{g}$ and ω_{eff} for $0 \le \omega_m \le 1$.

$\omega_m = 1$(stiff matter)	$\omega_{eff}(x) = 1 \forall x$	whatever $\hat{g}(x)$
$0 \le \omega_m < 1$	$\omega_{eff}(x_m) = \omega_m$	$\hat{g}(x_m) = 1$
$0 \le \omega_m < 1$	$\omega_{eff}(x_{ac}) = -\frac{1}{3}$	$\hat{g}(x_{ac}) = \frac{3}{4}(1-\omega_m)$
$0 \le \omega_m < 1$	$\omega_{eff}(x_0) = -1$	$\hat{g}(x_0) = \frac{1}{2}(1-\omega_m)$

Table 3
Evolution of the statefinder $\{s, r\}$: $\{s_{min}, r_{min}\}$ correspond to $\{s, r\}$ at the minimum.

$0 \le \omega_{eff}(x) < 1$	$\frac{1}{2} \le q < 2$	$s > 1$	$r > 1$
$-1 < \omega_{eff}(x) < 0$	$-1 < q < \frac{1}{2}$	$s_0 < s \le s_{min} < 1$	$-\frac{7}{2} < r_{min} \le r < r_0$
$\omega_{eff}(x_0) = -1$	$q = -1$	$s_0 = \frac{1}{3}\frac{d\omega_{eff}(x_0)}{dx} \le 0$	$r_0 = 1 - \frac{3}{2}\frac{d\omega_{eff}(x_0)}{dx} \ge 1$
$\omega_{eff}(x) < -1$	$q < -1$	$s < s_0$	$r > r_0$

$\hat{g}(x) = (1-\omega_m)/2$ and evolves therefore into a phantom regime. For a positive definite Hubble parameter and to avoid a permanently decelerating expanding universe or a static universe, we need to have $\cos\zeta(x) > 0$ with $\zeta(x) \neq k\pi$ for $k \in \mathbb{Z}$, and $\omega_m \neq 1$ $(0 \le \omega_m < 1)$.

Now we will be interesting in the geometrical diagnostic of the model under consideration, we thus introduce the so-called statefinder pair $\{r, s\}$, defined by (Alam et al., 2003; Sahni et al., 2003):

$$r \equiv \frac{\dddot{a}}{aH^3}, \quad s \equiv \frac{r-1}{3\left(q-\frac{1}{2}\right)}, \tag{31}$$

that probes the expansion dynamics of the universe through higher derivatives of the expansion factor $\dddot{a}$ and is a natural companion to the deceleration parameter $q = -\ddot{a}/\left(aH^2\right)$. Trajectories in the $r-s$ plane corresponding to different cosmological models exhibit qualitatively different behaviors. The spatially flat ΛCDM scenario corresponds to a fixed point $\{r_\Lambda, s_\Lambda\} = \{1, 0\}$ in the diagram. Departure of a given dark energy model from this fixed point provides a good way of establishing the distance of the model from ΛCDM. Using Eqs. (23) and (25), one has:

$$q(x) = \frac{1}{2} + \frac{3}{2}\omega_{eff}(x) \tag{32}$$

and

$$r(x) = 1 + \frac{9}{2}\omega_{eff}(x)s(x) \; with \; s(x) = 1 + \omega_{eff}(x) - \frac{1}{3\omega_{eff}(x)}\frac{d\omega_{eff}(x)}{dx} \tag{33}$$

Since $\omega_{\rm eff}(x)$ decreases as the universe evolves, $\frac{d\omega_{\rm eff}(x)}{dx}$ is then negative. Today's values $\{r_0, s_0\}$ are

$$r_0 = 1 - \frac{3}{2}\frac{d\omega_{eff}(x_0)}{dx} \; and \; s_0 = \frac{1}{3}\frac{d\omega_{eff}(x_0)}{dx} \tag{34}$$

that differ from those of the ΛCDM model by the term $d\omega_{eff}(x_0)/dx$; so $\{s_0, r_0\} = \{s_\Lambda, r_\Lambda\} = \{0, 1\}$ if the effective EoS parameter $\omega_{eff}(x)$ has a minimum at $x_0 = 0$ without a phantom regime or reaches an inflection point at $x_0 = 0$ with existence of a phantom regime: we then have $r_0 \ge 1$ and $s_0 \le 0$. Table 3 shows different evolution phases of $r(s)$, $r(q)$ and $s(q)$. The parameter $r \ge r_0 \ge 1$ when $s \le s_0 \le 0$ and $q \le -1$; for $-1 < q < 1/2$, $r(s)$ reaches a minimum at $\{s_{min}, r_{min}\}$ such that $s_0 < s \le s_{min} < 1$ and $-7/2 < r_{min} \le r < r_0$. Finally, for $1/2 \le q < 2$, both $s > 1$ and $r > 1$.

As our model is based on a modification of general relativity, it is important to constrain the different functions so that general relativity Solar System tests remain valid. All the cosmological quantities depending on time via the function $\zeta(x)$ or equivalently $\hat{g}(x)$, a necessary condition to have a realistic gravitational theory is

$$\left|\frac{\dot{G}_{eff}}{G_{eff}}\right| \le 4.0 \times 10^{-11} yr^{-1} \Longleftrightarrow \left|\frac{\dot{\hat{g}}}{\hat{g}}\right| \le 4.0 \times 10^{-11} yr^{-1} \tag{35}$$

Using equations (28) and (29) one has

$$\left|\frac{\dot{\hat{g}}}{\hat{g}}\right| = 2\left|\dot{\zeta}tan\zeta\right| = 2\left|\dot{\zeta}\right|\sqrt{\frac{1}{\hat{g}} - 1} \tag{36}$$

and the constraint (35) then becomes

$$\left|\dot{\zeta}\right|\sqrt{\frac{1}{\hat{g}} - 1} \le 2.0 \times 10^{-11} yr^{-1} \tag{37}$$

4. Conclusions

In this work we have constructed within Lyra's geometry a cosmological model involving a matter energy, with equation of state $0 \le \omega_m \le 1$, interacting with the displacement field. It has been shown that the late-time accelerated expansion of the universe could be explained for $\omega_m \neq 1$ by a nonzero effective time-dependent gravitational constant, resulting from the displacement field. A quintom scenario could even appear without a ghost mode involved in the model when the effective gravitational constant takes some particular value.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper

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Table 2
Variations of $\hat{g}$ and ω_{eff} and corresponding epochs of cosmic expansion.

$x \le x_{ac}$	$-\frac{1}{3} \le \omega_{eff}(x) \le \omega_m$	$\frac{3}{4}(1-\omega_m) \le \hat{g}(x) \le 1$	Decelerated expansion
$x_{ac} < x < 0$	$-1 < \omega_{eff}(x) \le -\frac{1}{3}$	$\frac{1}{2}(1-\omega_m) < \hat{g}(x) < \frac{3}{4}(1-\omega_m)$	Accelerated expansion
$x = x_0 = 0$	$\omega_{eff}(x_0) = -1$	$\hat{g}(x) = \frac{1}{2}(1-\omega_m)$	Quintom scenario
$x > 0$	$\omega_{eff}(x) < -1$	$0 < \hat{g}(x) < \frac{1}{2}(1-\omega_m)$	Phantom universe

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