Journal of King Saud University – Science 34 (2022) 102056

Contents lists available at ScienceDirect

Journal of King Saud University – Science

journal homepage: www.sciencedirect.com

Original article

A non-linear study of optical solitons for Kaup-Newell equation without four-wave mixing

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ARTICLE INFO

Article history:
Received 29 November 2020
Revised 21 April 2022
Accepted 22 April 2022
Available online 29 April 2022

Keywords:
Nonlinear
Birefringent fibers
Optical solitons
Kaup-Newell Equation
Four-Wave Mixing

ABSTRACT

Nonlinear science is a fundamental science frontier that include studies and the common properties of nonlinear phenomena. This article is devoted to the study of sub-pico second optical pluses in birefringent fibers for Kaup-Newell equation (KNE) without four-wave mixing. Three prominent integrations techniques are successfully implemented on KNE in coupled vector form. Variety of soliton solutions namely dark, bright, periodic singular, singular and bright-singular combo solitons are constructed for the KNE in birefringent fibers. The obtained solutions are reckoned with their respective existence criterion. In addition, two-dimensional and three-dimensional graphs are drawn to exhibit the physical behavior of the obtained solutions.

1. Introduction

Nonlinear PDEs frequently appears in basic laws of nature. In the mathematical physics and other domains of applied sciences, several models obviously arise from solid state physics, plasma physics, ocean hydrodynamics, atmospheric waves, biology, chemistry, mathematical materials sciences, etc (Abbagari et al., 2021; An et al., 2020; Awan et al., 2021; Hosseini et al., 2019; Rehman et al., 2020; Shahen et al., 2020; Tahir and Awan, 2019; Tahir and Awan, 2020; Yepez-Martinez et al., 2018; Yoku et al., 2020). Witout deep understanding of soliton dynamic the modern technology considered to be impossible. The working communication channel such as internet, facebook, cell phones, electronic mail and twitter depend on soliton propagation.

In recent years, the area of soliton propagation in nonlinear optical media have testified a lot of research. These researches have current applications on communication technology reliant on the transmission of optical local pulses. The optical soliton solution is the dynamic area of recent science especially in nonlinear optics. A large number of methods are introduced and applied to find optical soliton solution such as Kudryashov's method, first integral technique, mapping technique, (G'/G)-expansion technique, undetermined coefficient method, functional variable method, generalized Kudryashov's method, modified simple equation method, new extended direct algebraic method and many others (Delgado et al., 2018; Ghanbari and Aguilar, 2019; Martinez et al., 2018; Morales-Delgado et al., 2018; Rehman et al., 2019; Rehman et al., 2019; Rehman et al., 2019; Sedeeg et al., 2019; Tahir et al., 2019).

The KNE (Arshed et al., 2018; Biswas et al., 2018; Biswas et al., 2018; Jawad et al., 2019; Triki et al., 2019) is taken a most popular form of the NLSE. This model is useful to explain the propagation of

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Peer review under responsibility of King Saud University.

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https://doi.org/10.1016/j.jksus.2022.102056
1018-3647/

modulated structures in plasma physics and optical fiber especially Alfven waves.

In this article, we study the nonlinear KNE in birefringent fiber without four-wave mixing (FWM) (Ahmed et al., 2020; Yildirim et al., 2019). FWM expresses a nonlinear optical effect in which four waves interact each other as a result of the third order nonlinearity. There are numerous mathematical analysis to study the NLEEs in birefringent fibers (Awan et al., 2020; Bhrawy et al., 2014; Rehman et al., 2020; Rehman et al., 2020; Rehman et al., 2020; Tahir and Awan, 2020; Tahir et al., 2019). The optical soliton solutions of KNE in birefringent fibers are not much studied in the previous literature. Here, three different techniques namely Kudryashov's method (Rehman et al., 2019; Rehman et al., 2019), undetermined coefficient method (Morales-Delgado et al., 2018; Sedeeg et al., 2019) and modified mapping method (Rehman et al., 2019; Rehman et al., 2020) are adopted to extract solutions of KNE in birefringent without FWM. In this context, the we adhere few recent attempts on analytical as well as numerical approaches of dynamical mathematical models as well (Arshad et al., 2017; Liu et al., 2020; Hosseini et al., 2021; Hussain et al., 2021; Memon et al., 2020; Syed et al., 2021). These integration algorithms are successfully applied to extract dark, bright, singular, bright-singular and periodic singular combo soliton solutions.

The KNE polarization-preserving fibers (An et al., 2020; Hosseini et al., 2019; Rehman et al., 2020; Shahen et al., 2020) is given as

$$p_t + \imath a p_{xx} + b(|p|^2 p)_x = 0. \tag{1}$$

In the model (1), a and b are the coefficients of GVD and the nonlinearity respectively. The KNE in coupled vector form without FWM reads

$$\begin{aligned} &\phi_t + \imath a_1 \phi_{xx} + \lambda_1 (|\phi|^2 \phi)_x + \gamma_1 (|\psi|^2 \psi)_x = 0, \\ &\psi_t + \imath a_2 \psi_{xx} + \lambda_2 (|\psi|^2 \psi)_x + \gamma_2 (|\phi|^2 \phi)_x = 0. \end{aligned} \tag{2}$$

The constants a_i, λ_i and γ_i assure the GVD and nonlinearity.

2. Mathematical Analysis

To solve system of Eqs. (2), we substitute

$$\begin{aligned} \phi(x,t) &= Q_1(\vartheta) e^{\imath \varphi(x,t)}, \\ \psi(x,t) &= Q_2(\vartheta) e^{\imath \varphi(x,t)}. \end{aligned} \tag{3}$$

Here $Q_i(\vartheta)$ represents amplitude, where

$$\vartheta = x - \eta t, \ \ \varphi(x,t) = \theta_0 + \omega t - kx, \tag{4}$$

where k, η, ω and θ_0 sequentially represent frequency, velocity, wave number and phase constant.

By substituting (3) into (2) and then the real and imaginary parts are respectively given as

$$(2ka_i - \eta) Q\prime_i + 3\lambda_i Q\prime_i Q_i^2 + 3\gamma_i Q\prime_i Q_{\bar{i}}^2 = 0, \tag{5}$$

$$a_i Q\prime\prime_i - (a_i k^2 - \omega) Q_i - k\lambda_i Q_i^3 - k\gamma_i Q_{\bar{i}}^3 = 0. \tag{6}$$

By using the balancing condition $Q_i = Q_{\bar{i}}$, then the real and imaginary parts emerged as

$$Q_i(2ka_i - \eta) + Q_i^3(\lambda_i + \gamma_i) = 0, \tag{7}$$

$$a_i Q\prime\prime_i - Q_i(a_i k^2 - \omega) - Q_i^3 k(\lambda_i + \gamma_i) = 0. \tag{8}$$

Eq. (7) gives the soliton velocity as

$$\eta = 2ka_i + Q_i^2(\lambda_i + \gamma_i). \tag{9}$$

In the next subsections, Eq. (8) is solved using the above mentioned integration methods.

2.1. Bright Soliton

Let the solution of (8) in form of bright soliton is taken as

$$Q_i(\vartheta) = A sech^r(B\vartheta), \ \ \vartheta = x - \eta t. \tag{10}$$

Setting (10) into (8), we have

$$-r(r+1)a_i A B^2 sech^{r+2}(B\vartheta) + A sech^r(B\vartheta)[\omega - a_i k^2 + a_i r^2 B^2] - A^3 sech^{3r}(B\vartheta)(\lambda_i + \gamma_i)k = 0. \tag{11}$$

By balancing, we get $r = 1$, comparing the coefficients of linearly independent terms give

$$A = \pm\sqrt{-\frac{2(a_i k^2 - \omega)}{k(\lambda_i + \gamma_i)}}, \ \ B = \pm\sqrt{\frac{a_i k^2 - \omega}{a_i}}. \tag{12}$$

So

$$Q_i(\vartheta) = \pm\sqrt{-\frac{2(a_i k^2 - \omega)}{k(\lambda_i + \gamma_i)}} sech\left(\sqrt{\frac{a_i k^2 - \omega}{a_i}}(x - \eta t)\right). \tag{13}$$

Substituting (13) into (3), we obtain the bright soliton solutions as

$$\phi(x,t) = \pm\sqrt{-\frac{2(a_1 k^2 - \omega)}{k(\lambda_1 + \gamma_1)}} sech\left(\sqrt{\frac{a_1 k^2 - \omega}{a_1}}(x - \eta t)\right) e^{\imath(-kx + \omega t + \theta_0)}, \tag{14}$$

and

$$\psi(x,t) = \pm\sqrt{-\frac{2(a_2 k^2 - \omega)}{k(\lambda_2 + \gamma_2)}} sech\left(\sqrt{\frac{a_2 k^2 - \omega}{a_2}}(x - \eta t)\right) e^{\imath(-kx + \omega t + \theta_0)}. \tag{15}$$

2.2. Dark Soliton

The dark soliton of (8) is taken as

$$Q_i(\vartheta) = A tanh^r(B\vartheta). \tag{16}$$

Setting (16) into (8), we have

$$\begin{aligned} &r(r-1)a_i A B^2 tanh^{r-2}(B\vartheta) + A tanh^r(B\vartheta)[\omega - a_i k^2 - 2a_i r^2 B^2] + \\ &r(r+1)a_i A B^2 tanh^{r+2}(B\vartheta) - A^3 tanh^{3r}(B\vartheta)(\lambda_i + \gamma_i)k = 0. \end{aligned} \tag{17}$$

By balancing we get $r = 1$ and then comparing the coefficients of linearly independent terms give

$$A = \pm\sqrt{-\frac{a_i k^2 - \omega}{k(\lambda_i + \gamma_i)}}, \ \ B = \pm\sqrt{-\frac{a_i k^2 - \omega}{2a_i}}. \tag{18}$$

So

$$Q_i(\vartheta) = \pm\sqrt{-\frac{a_i k^2 - \omega}{k(\lambda_i + \gamma_i)}} tanh\left(\sqrt{-\frac{a_i k^2 - \omega}{2a_i}}(x - \eta t)\right). \tag{19}$$

Substituting (19) into (3), we obtain solutions in the form of dark soliton as

$$\phi(x,t) = \pm\sqrt{-\frac{a_1k^2-\omega}{k(\lambda_1+\gamma_1)}}tanh\left(\sqrt{-\frac{a_1k^2-\omega}{2a_1}}(x-\eta t)\right)e^{\imath(-kx+\omega t+\theta_0)}, \quad (20)$$

$$\psi(x,t) = \pm\sqrt{-\frac{a_2k^2-\omega}{k(\lambda_2+\gamma_2)}}tanh\left(\sqrt{-\frac{a_2k^2-\omega}{2a_2}}(x-\eta t)\right)e^{\imath(-kx+\omega t+\theta_0)}. \quad (21)$$

2.3. Singular Soliton (Type-I)

Let solution of (8) in the form of singular soliton is taken as

$$Q_i(\vartheta) = Acsch^r(B\vartheta). \quad (22)$$

Setting (22) into (8), we have

$$r(r+1)a_iAB^2csch^{r+2}(B\vartheta) + Acsch^r(B\vartheta)[\omega - a_ik^2 + a_ir^2B^2] - A^3csch^{3r}(B\vartheta)(\lambda_i+\gamma_i)k = 0. \quad (23)$$

By balancing we get $r = 1$ and then comparing the coefficients of linearly independent terms give

$$A = \pm\sqrt{\frac{2(a_ik^2-\omega)}{k(\lambda_i+\gamma_i)}}, \quad B = \pm\sqrt{\frac{a_ik^2-\omega}{a_i}}. \quad (24)$$

So

$$Q_i(\vartheta) = \pm\sqrt{\frac{2(a_ik^2-\omega)}{k(\lambda_i+\gamma_i)}}csch\left(\sqrt{\frac{a_ik^2-\omega}{a_i}}(x-\eta t)\right). \quad (25)$$

Substituting (25) into (3), we get the singular soliton solutions as

$$\phi(x,t) = \pm\sqrt{\frac{2(a_1k^2-\omega)}{k(\lambda_1+\gamma_1)}}csch\left(\sqrt{\frac{a_1k^2-\omega}{a_1}}(x-\eta t)\right)e^{\imath(-kx+\omega t+\theta_0)}, \quad (26)$$

and

$$\psi(x,t) = \pm\sqrt{\frac{2(a_2k^2-\omega)}{k(\lambda_2+\gamma_2)}}csch\left(\sqrt{\frac{a_2k^2-\omega}{a_2}}(x-\eta t)\right)e^{\imath(-kx+\omega t+\theta_0)}. \quad (27)$$

2.4. Singular Soliton (Type-II)

The dark soliton of (8) is taken as

$$Q_i(\vartheta) = Acoth^r(B\vartheta). \quad (28)$$

Setting (28) into (8), we have

$$r(r-1)a_iAB^2coth^{r-2}(B\vartheta) + Acoth^r(B\vartheta)[\omega - a_ik^2 - 2a_ir^2B^2] + r(r+1)a_iAB^2coth^{r+2}(B\vartheta) - A^3coth^{3r}(B\vartheta)(\lambda_i+\gamma_i)k = 0. \quad (29)$$

By balancing, we get $r = 1$ and then comparing the coefficients of linearly independent terms provide the same values as in dark soliton ansatz. So

$$Q_i(\vartheta) = \pm\sqrt{-\frac{a_ik^2-\omega}{k(\lambda_i+\gamma_i)}}coth\left(\sqrt{-\frac{a_ik^2-\omega}{2a_i}}(x-\eta t)\right), \quad (30)$$

$$\phi(x,t) = \pm\sqrt{-\frac{a_1k^2-\omega}{k(\lambda_1+\gamma_1)}}coth\left(\sqrt{-\frac{a_1k^2-\omega}{2a_1}}(x-\eta t)\right)e^{\imath(-kx+\omega t+\theta_0)}, \quad (31)$$

$$\psi(x,t) = \pm\sqrt{-\frac{a_2k^2-\omega}{k(\lambda_2+\gamma_2)}}coth\left(\sqrt{-\frac{a_2k^2-\omega}{2a_2}}(x-\eta t)\right)e^{\imath(-kx+\omega t+\theta_0)}. \quad (32)$$

2.5. Kudryashov's Method

Considering the general form of a PDE as

$$G(g, g_t, g_x, g_{tt}, g_{xx}, g_{xt}, \ldots) = 0, \quad (33)$$

where $g(x,t)$ is an unknown function.

Taking following wave transformation

$$g(x,t) = Q_i(\vartheta), \quad \vartheta = x - \eta t, \quad (34)$$

which converts the above PDE into following ODE

$$H(Q_i, Q_i\prime, Q_i\prime\prime, \ldots) = 0. \quad (35)$$

According to the Kudryashov's method, the following finite series states the solution

$$Q_i(\vartheta) = \sum_{j=0}^{N} c_j(R(\vartheta))^j, \quad (36)$$

where c_j are constants and $R(\vartheta)$ satisfies

$$R(\vartheta) = \frac{1}{1+de^{\vartheta}}. \quad (37)$$

Eq. (37) satisfies the following nonlinear differential equation

$$\frac{dR}{d\vartheta} = R(\vartheta)(R(\vartheta)-1). \quad (38)$$

Putting (38) into (36), we get algebraic equations in c_j by taking the coefficients of powers of $R(\vartheta)$ equal to zero.

Thus the solution of Eq. (8) can be expressed as

$$Q_i = c_0 + c_1R(\vartheta), \quad (39)$$

where c_0 and c_1 are constants. Eq. (8) can be rewritten as

$$AQ_i\prime\prime + BQ_i - k(\lambda_i+\gamma_i)Q_i^3 = 0, \quad (40)$$

where

$$A = a_1, \quad B = -(a_ik^2-\omega). \quad (41)$$

Substituting (39) into (40) and comparing the coefficients of alike powers of $R(\vartheta)$ to zero provide algebraic system of equations. After solving the system, the $c_j, j = 0, 1$ are obtained and produce the following results

$$c_0 = \sqrt{\frac{a_i}{2k(\lambda_i+\gamma_i)}}, \quad c_1 = 2\sqrt{\frac{a_i}{2k(\lambda_i+\gamma_i)}}, \quad B = \frac{A}{2}. \quad (42)$$

So

$$Q_i = \sqrt{\frac{a_i}{2k(\lambda_i+\gamma_i)}}[1 + \frac{2}{1+de^{(x-\eta t)}}], \quad (43)$$

$$Q_i = \sqrt{\frac{a_i}{2k(\lambda_i+\gamma_i)}}[1 + \frac{2}{1+d\cosh(x-\eta t)+d\sinh(x-\eta t)}]. \quad (44)$$

Substituting (44) into (3), we obtain see Fig. 1,2

$$\phi(x,t) = \sqrt{\frac{a_1}{2k(\lambda_1+\gamma_1)}}[1 + \frac{2}{1+d\sinh(x-\eta t)+d\cosh(x-\eta t)}]e^{\imath(-kx+\omega t+\theta_0)}, \quad (45)$$

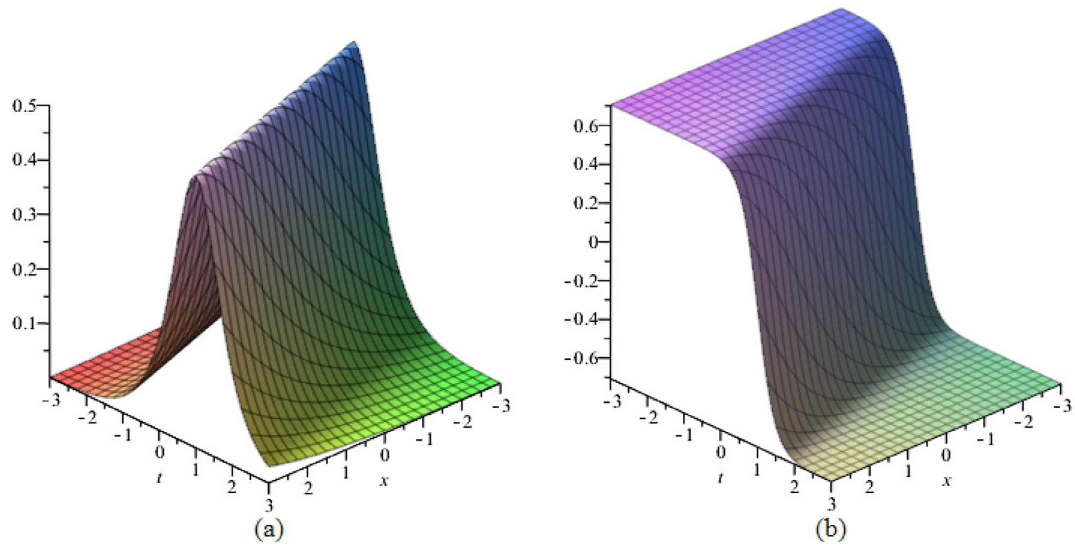

Fig. 1. (a) 3D representation of solution (14) in the form of the bright soliton for $a_1 = \gamma_1 = 1.5, k = -1, \omega = \theta_0 = 1, \lambda_1 = 2.5, \eta = 3$. (b) 3D representation of solution (20) in form of dark soliton for $a_1 == 0.5, k = -1, \gamma_1 = \theta_0 = \omega = 1, \lambda_1 = -2, \eta = 3$..

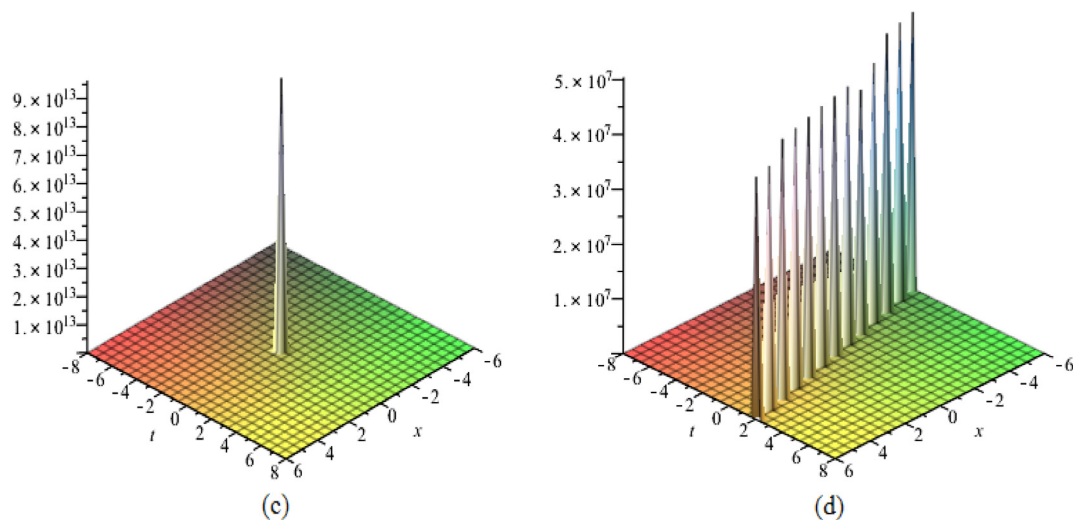

Fig. 2. (c) 3D representation of solution (26) in the form of singular soliton as $a_1 = \gamma_1 = 1.5, k = \omega = \theta_0 = 1, \lambda_1 = 2.5, \eta = 3$. (d) 3D representation of solution (31) in the form of singular soliton for $a_1 = 0.5, k = -1, \gamma_1 = \theta_0 = \omega = 1, \lambda_1 = -2, \eta = 3$.

$$\psi(x,t) = \sqrt{\frac{a_2}{2k(\lambda_2+\gamma_2)}}\left[1 + \frac{2}{1 + d\sinh(x-\eta t) + d\cosh(x-\eta t)}\right]e^{\imath(-kx+\omega t+\theta_0)}. \tag{46}$$

Fig. 3(e) represents combo soliton solution (45) for $a_1 = \gamma_1 = 1.5, k = \theta_0 = \omega = 1, \lambda_1 = 2.5, d = 1.7, \eta = 3$.

2.6. Modified Mapping Technique

Assuming PDE be of the following form

$$G(g, h, g_t, h_t, g_x, h_x, g_{xxx}, h_{xxx}, \ldots) = 0. \tag{47}$$

- Let the solution of (47) is written as

$$g(x,t) = g(\vartheta) = \sum_{i=0}^{n_1} C_i P^i(\vartheta), \qquad h(x,t) = h(\vartheta) = \sum_{i=0}^{n_2} D_i P^i(\vartheta), \tag{48}$$

where $\vartheta = x - \eta t, C_i, D_i$ and η are arbitrary constants. Let the first derivative of P be

$$P\prime^2 = aP^2 + \frac{1}{2}bP^4 + c, \tag{49}$$

with constants a, b, and c.

- Utilizing Eq. (48) in Eq. (47), by which Eq. (47) changes into an ODE and n_1 and n_2 can be calculated by balancing principle.

Thus, the solution of (8) is considered as

$$Q_i(\vartheta) = C_0 + C_1 P(\vartheta) + D_1 P^{-1}(\vartheta), \tag{50}$$

where C_0, C_1 and D_1 are constants.

Eq. (8) can be rewritten as

$$AQ_i'' + BQ_i + CQ_i^3 = 0, \tag{51}$$

where

$$A = a_1,\; B = -(a_i k^2 - \omega),\; C = -k(\lambda_i + \gamma_i). \tag{52}$$

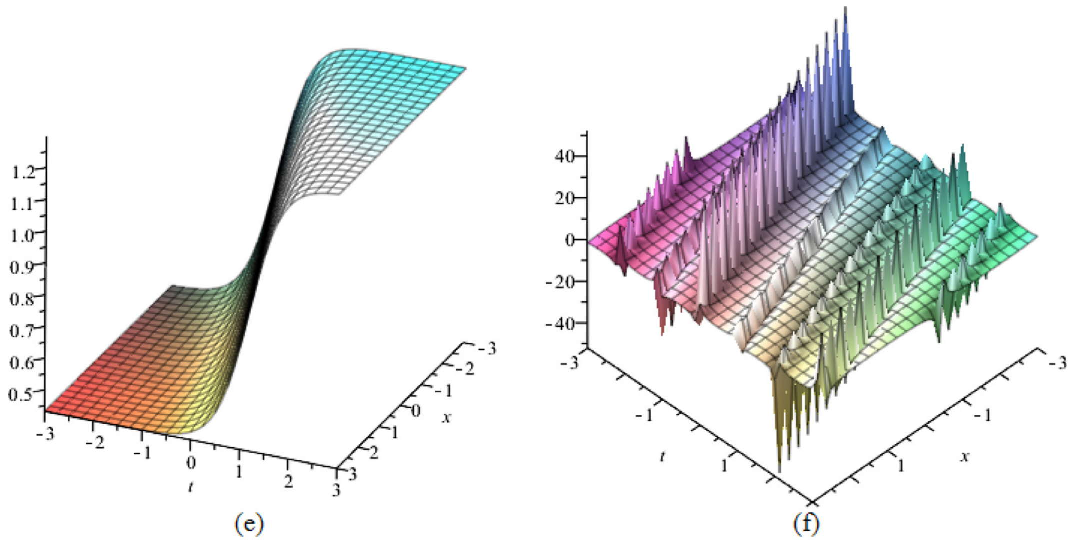

Fig. 3. (e) 3D portrayal of combo optical soliton solution (45). (f) 3D representation of periodic singular solution (57).

Putting (50) into (51) and utilizing (49), we have

$$
\begin{aligned}
&2cAD_1 + CD_1^3 = 0,\\
&3CC_0D_1^2 = 0,\\
&aAD_1 + 3CC_0^2 + 3CC_1D_1^2 + BD_1 = 0,\\
&BC_0 + CC_0^3 + 6CC_0C_1D_1 = 0,\\
&aAC_1 + BC_1 + 3CC_0^2C_1 + 3CC_1^2D_1 = 0,\\
&3CC_0C_1^2 = 0,\\
&bAC_1 + CC_1^3 = 0.
\end{aligned}
$$

From these equations, we get

$$C_0 = 0,\ C_1 = \sqrt{\frac{ba_1}{k(\lambda_i + \gamma_i)}},\ D_1 = \sqrt{\frac{2ca_1}{k(\lambda_i + \gamma_i)}}, \tag{53}$$

with constraint condition

$$aA + 3CC_1D_1 + B = 0. \tag{54}$$

Case-1: $P(\vartheta) = sn(\vartheta)$ or $P(\vartheta) = cd(\vartheta)$, we have $a = -(1 + r^2), b = 2r^2$ and $c = 1$. Now solutions of the (2) are

$$\phi(x,t) = \sqrt{\frac{2a_1}{k(\lambda_1 + \gamma_1)}}[rsn(x - \eta t) + ns(x - \eta t)]e^{\imath(-kx+\omega t+\theta_0)}, \tag{55}$$

and

$$\psi(x,t) = \sqrt{\frac{2a_2}{k(\lambda_2 + \gamma_2)}}[rsn(x - \eta t) + ns(x - \eta t)]e^{\imath(-kx+\omega t+\theta_0)}. \tag{56}$$

When $r \to 0$, then $ns \to csc$, (55) and (56) yield the following singular solutions

$$\phi(x,t) = \sqrt{\frac{2a_1}{k(\lambda_1 + \gamma_1)}}csc(x - \eta t)e^{\imath(-kx+\omega t+\theta_0)}, \tag{57}$$

and

$$\psi(x,t) = \sqrt{\frac{2a_2}{k(\lambda_2 + \gamma_2)}}csc(x - \eta t)e^{\imath(-kx+\omega t+\theta_0)}. \tag{58}$$

When $r \to 1$ then $sn \to tanh$ and $ns \to coth$, (55) and (56) present dark-singular combo solitons as

$$\phi(x,t) = \sqrt{\frac{2a_1}{k(\lambda_1 + \gamma_1)}}[tanh(x - \eta t) + coth(x - \eta t)]e^{\imath(-kx+\omega t+\theta_0)}, \tag{59}$$

and

$$\psi(x,t) = \sqrt{\frac{2a_2}{k(\lambda_2 + \gamma_2)}}[tanh(x - \eta t) + coth(x - \eta t)]e^{\imath(-kx+\omega t+\theta_0)}. \tag{60}$$

Or

$$\phi(x,t) = \sqrt{\frac{2a_1}{k(\lambda_1 + \gamma_1)}}[rcd(x - \eta t) + dc(x - \eta t)]e^{\imath(-kx+\omega t+\theta_0)}, \tag{61}$$

and

$$\psi(x,t) = \sqrt{\frac{2a_2}{k(\lambda_2 + \gamma_2)}}[rcd(x - \eta t) + dc(x - \eta t)]e^{\imath(-kx+\omega t+\theta_0)}. \tag{62}$$

When $r \to 0, dc \to sec$, which gives the following periodic singular solutions

$$\phi(x,t) = \sqrt{\frac{2a_1}{k(\lambda_1 + \gamma_1)}}sec(x - \eta t)e^{\imath(-kx+\omega t+\theta_0)}, \tag{63}$$

and

$$\psi(x,t) = \sqrt{\frac{2a_2}{k(\lambda_2 + \gamma_2)}}sec(x - \eta t)e^{\imath(-kx+\omega t+\theta_0)}. \tag{64}$$

When $r \to 1$, the following solution are obtained

$$\phi(x,t) = \sqrt{\frac{2a_1}{k(\lambda_1 + \gamma_1)}}2e^{\imath(-kx+\omega t+\theta_0)}, \tag{65}$$

and

$$\psi(x,t) = \sqrt{\frac{2a_2}{k(\lambda_2 + \gamma_2)}}2e^{\imath(-kx+\omega t+\theta_0)}. \tag{66}$$

Fig. 3(f) represents the periodic singular solution (57) while Figs. 4(g) and 4(h) represent dark-singular soliton (59) and periodic singular solution (63) respectively, with $a_1 = \gamma_1 = 1.5, k = \theta_0 = \omega = 1, \lambda_1 = 2.5, \eta = 3$.

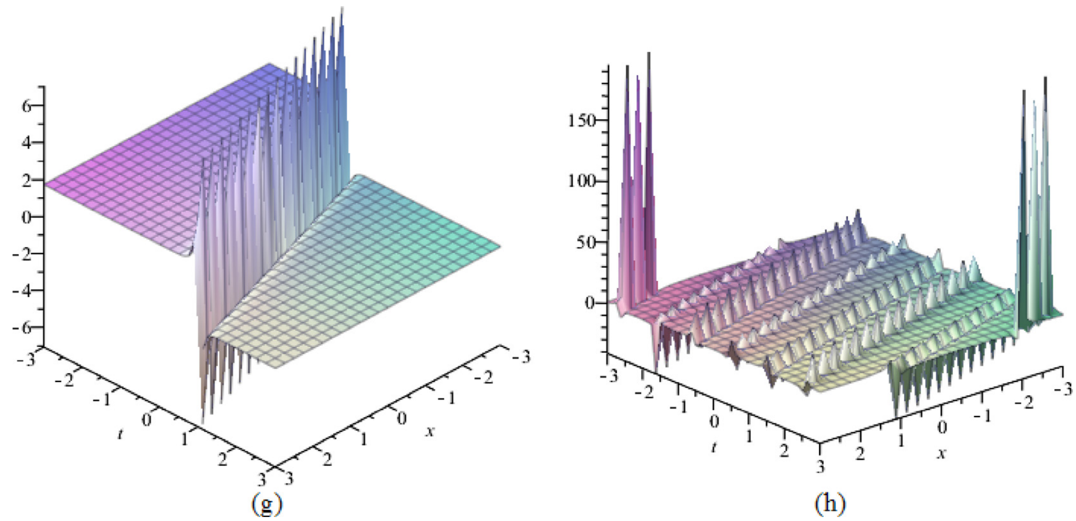

Fig. 4. (g) 3D portrayal of dark-singular soliton solution (59). (h) 3D representation of periodic singular solution (63).

Case-2: When $P(\vartheta) = cn(\vartheta)$, then $a = 2r^2 - 1, b = -2r^2$ and $c = 1 - r^2$. So

$$\phi(x,t) = \sqrt{-\frac{2a_1}{k(\lambda_1+\gamma_1)}}[rcn(x-\eta t) + \sqrt{r^2-1}nc(x-\eta t)]e^{\imath(-kx+\omega t+\theta_0)}, \tag{67}$$

and

$$\psi(x,t) = \sqrt{-\frac{2a_2}{k(\lambda_2+\gamma_2)}}[rcn(x-\eta t) + \sqrt{r^2-1}nc(x-\eta t)]e^{\imath(-kx+\omega t+\theta_0)}. \tag{68}$$

When $r \to 0$, we obtain the same periodic singular solution as (57) and (58).

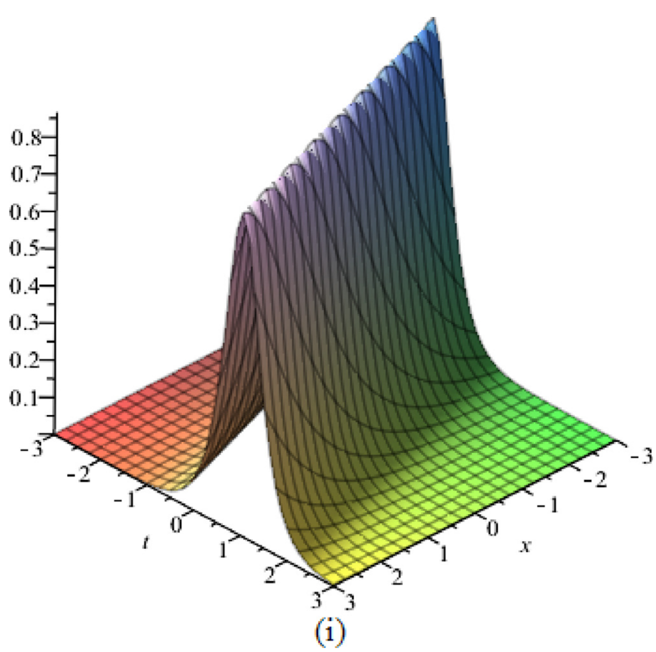

Fig. 5. (i) 3D portrayal of bright soliton solution (69).

When $r \to 1$, the bright soliton solutions are retrieved as

$$\phi(x,t) = \sqrt{-\frac{2a_1}{k(\lambda_1+\gamma_1)}}sech(x-\eta t)e^{\imath(-kx+\omega t+\theta_0)}, \tag{69}$$

and

$$\psi(x,t) = \sqrt{-\frac{2a_2}{k(\lambda_2+\gamma_2)}}sech(x-\eta t)e^{\imath(-kx+\omega t+\theta_0)}. \tag{70}$$

Fig. 5(i) portrays the bright soliton solution (69) with $a_1 = \gamma_1 = 1.5, k = -1, \theta_0 = \omega = 1, \lambda_1 = 2.5, \eta = 3$.

Case-3: When $P(\vartheta) = cs(\vartheta)$, then $a = 2 - r^2, b = 2$ and $c = 1 - r^2$. So

$$\phi(x,t) = \sqrt{-\frac{2a_1}{k(\lambda_1+\gamma_1)}}[rcs(x-\eta t) + \sqrt{1-r^2}sc(x-\eta t)]e^{\imath(-kx+\omega t+\theta_0)}, \tag{71}$$

and

$$\psi(x,t) = \sqrt{-\frac{2a_2}{k(\lambda_2+\gamma_2)}}[rcs(x-\eta t) + \sqrt{1-r^2}sc(x-\eta t)]e^{\imath(-kx+\omega t+\theta_0)}. \tag{72}$$

Substituting $r \to 0$ in (71) and (72) yield the following periodic singular solutions

$$\phi(x,t) = \sqrt{\frac{2a_1}{k(\lambda_1+\gamma_1)}}[cot(x-\eta t)]e^{\imath(-kx+\omega t+\theta_0)}, \tag{73}$$

and

$$\psi(x,t) = \sqrt{\frac{2a_2}{k(\lambda_2+\gamma_2)}}[cot(x-\eta t)]e^{\imath(-kx+\omega t+\theta_0)}. \tag{74}$$

Substituting $r \to 1$ in (71) and (72) yield the following singular soliton solutions

$$\phi(x,t) = \sqrt{\frac{2a_1}{k(\lambda_1+\gamma_1)}}csch(x-\eta t)e^{\imath(-kx+\omega t+\theta_0)}, \tag{75}$$

and

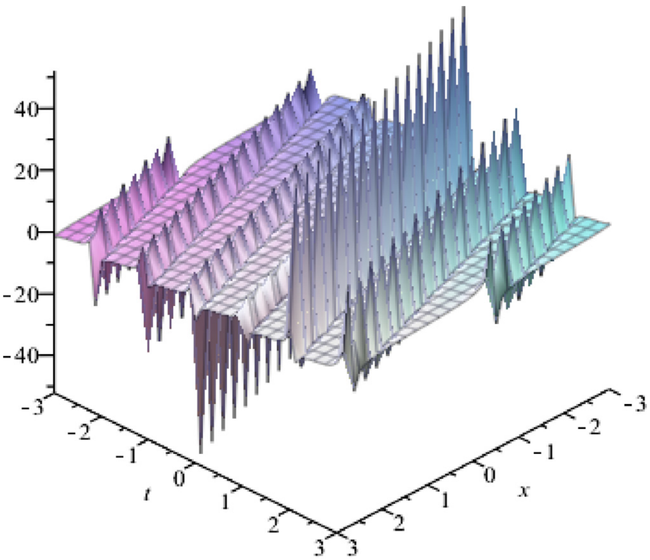

Fig. 6. (j) 3D representation of periodic singular solution (73).

$$\psi(x,t)=\sqrt{\frac{2a_2}{k(\lambda_2+\gamma_2)}}csch(x-\eta t)e^{\imath(-kx+\omega t+\theta_0)}. \quad (76)$$

Figs. 6(j) represents the periodic singular solution (73) for $a_1=\gamma_1=1.5, k=\theta_0=\omega=1, \lambda_1=2.5, \eta=3$.

3. Conclusion

This study takes up the KNE without FWM in birefringent fibers. Three proposed methods are fruitfully applied to recover optical soliton solutions for the present model. Comparing the obtained results with those in earlier study, it is demonstrated that our results are new and not studied earlier. From these integration schemes dark, bright, singular, bright-singular and periodic singular combo soliton solutions are retrieved. These techniques are concise, efficient and the solutions opens up wide opportunities for further studies.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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