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Soft topological rings

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ABSTRACT

In this paper, we produce and examine several new notions such as soft topological rings, soft topological subrings and soft topological ideals.

1. Introduction

In our life and all disciplines, we face many problems with uncertainties. To deal with the lack of certainty and solve these problems, many theories have recently developed like vague sets (Gau and Buehrer, 1993), fuzzy sets (Zadeh, 1965) and rough sets (Pawlak, 1982). These approaches were regarded as the most famous mathematical instruments to modeling decision makers. However all these approaches have their challenges, and the causes of these complications posed by these methods are probably due to the inadequacy of parameters. Molodtsov (1999) proposed the soft set theory to administer uncertainties, where his approach includes enough parameters. Accordingly, many of those difficulties facing us become easier to solve by applying the soft sets theory. Moreover, many authors studied the relationship between all of these theories such as (Meng et al., 2011; Xiao and Zou, 2014; Aktaş and Çağman, 2016; Zhang et al., 2018; Zhan and Wang, 2018).

On the other hand, algebraic structures of groups and rings have recently been studied by using fuzzy sets and soft sets, for example, Liu et al. (2012) worked on fuzzy rings. Also, Acar et al. (2010) introduced the concept of soft rings, which later extended to fuzzy soft rings by Inan and Öztürk (2012). Further, semigroups and semirings are studied by using fuzzy sets and soft sets in (Feng et al., 2008; Zhan and Davvaz, 2016; Yousafzai et al., 2017). Recently, many research papers have emerged discussing the applications of soft set theory, like (Ma et al., 2018; Zhan et al., 2018; Zhan and Alcantud, 2018; Zhan et al., 2017; Zhan et al., 2017; Ma et al., 2017).

Shabir and Naz (2011) developed the notion of soft topology and soft topological spaces, which have been studied by many authors like (Shabir and Naz, 2011; Çağman et al., 2011; Aygünoğlu and Aygün, 2012; Nazmul and Samanta, 2013; Şenel and Çağman, 2011; Babitha and John, 2015; Kandil et al., 2017). Thereafter, many authors worked on the combination of algebraic constructions and soft topological structures. For example, as a straightforward extension of the familiar concepts of topological groups (Pontrjagin, 1939) and topological rings (Warner, 1993). Nazmul and Samanta (2010) initiated the idea of soft topological groups. Later Hida (2014) added improvements to the concept of soft topological groups. In the same time, Nazmul and Samanta (2015, 2014) returned with the latest version of their work of soft topological groups.

Tahat et al. (2018) introduced the concept of soft topological soft rings by applying soft topological structures on a soft ring and Shah and Shaheen (2014) initiated the concept of a soft

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topological ring by applying the topological structures on a soft ring. In this paper, we will produce a wholly different definition for soft topological rings. Our notion depends on the soft topological structures over the rings directly, rather than on the topological structures over the subrings which are induced by every individual parameter of soft rings.

Our motive is to complete the gaping in the studies of the connections between the soft topological space and the rings theory by studying the combination between the rings and the soft topological spaces and introduce the concept of soft topological rings.

2. Preliminaries

Throughout this paper, X, Y and Z are assumed to be initially universal sets and E is assumed to be a nonempty set of parameters.

2.1. Soft sets and soft rings

In this subsection, we introduce some basic concepts and results, which we will use in the next part of this paper.

Definition 2.1 Molodtsov, 1999.

(i) A soft set F_A over X is defined to be a mapping $F_A : A \longrightarrow P(X)$, where $A \subseteq E$.
(ii) The support of a soft set F_A is defined to be the following subset of A

$$Supp(F_A) = \{a \in A | F_A(a) \neq \phi\}.$$

Remark 2.2 (*see Maji et al., 2003*).

(i) We put $F_E = F$.
(ii) Let A, B, C are nonempty subsets of E. For each $a \in A$ we put $(a, B) := \{(a, b) | b \in B\}$. So, we have $(a, B) \cup (a, C) = (a, B \cup C)$. Sometimes we deal with the soft set F_A over X as the subset $\bigcup_{a \in A} (a, F_A(a))$ of $A \times X$ or we put $F_A = \{(a, F_A(a)) | a \in A\}$.
(iii) We denote the class of all soft sets over X by $S(X)$.
(iv) If $F_A \in S(X)$ such that $F_A(a) = \phi, \forall a \in A$, then F_A is called a null soft set over X and it is denoted by $\tilde{\phi}_A$. We put $\tilde{\phi}_E = \tilde{\phi}$.
(v) If $F_A \in S(X)$ such that $F_A(a) = X, \forall a \in A$, then F_A is called an absolute soft set over X and it is denoted by $\widetilde{X}_A$. We put $\widetilde{X}_E = \widetilde{X}$.
(vi) Let $K \subseteq X$. A soft set $\widetilde{K}_A \in S(X)$ is defined by $\widetilde{K}_A(a) = K, \forall a \in A$. We put $\widetilde{K}_E = \widetilde{K}$.

Definition 2.3 (*Maji et al., 2003; Feng et al., 2008*). Let $F_A, G_B \in S(X)$. Then

(i) F_A is said to be a soft subset of G_B if and only if $A \subseteq B$ and $F_A(a) \subseteq G_B(a), \forall a \in A$. In this case we write $F_A \tilde{\subseteq} G_B$.
(ii) F_A is said to be soft equal to G_B if and only if $G_A \tilde{\subseteq} G_B$ and $G_B \tilde{\subseteq} F_A$. In this case we write $F_A \tilde{=} G_B$.
(iii) The intersection of F_A and G_B is defined to be the soft set $(F \cap G)_{A \cap B} \in S(X)$, such that

$$(F \cap G)_{A \cap B}(e) = F_A(e) \cap G_B(e), \forall e \in A \cap B.$$

We put $(F \cap G)_{A \cap B} = F_A \tilde{\cap} G_B$.
(iv) The union of F_A and G_B is defined to be the soft set $(F \cup G)_{A \cup B} \in S(X)$, such that

$$(F \cup G)_{A \cup B}(e) = \begin{cases} F_A(e) \cup G_B(e), & \text{if } e \in A \cap B \\ F_A(e), & \text{if } e \in A \setminus B, \\ G_B(e), & \text{if } e \in B \setminus A \end{cases} \quad \forall e \in A \cup B.$$

We put $(F \cup G)_{A \cup B} = F_A \tilde{\cup} G_B$.

Note that for $F_A, \tilde{\phi}_A \in S(X)$, we have $F_A = \tilde{\phi}_A$ if and only if $Supp(F_A) = \phi$.

Definition 2.4. Suppose that X is a ring and $F, H \in S(X)$. The soft sets $F \tilde{\cdot} H, F \tilde{+} H, F \tilde{-} H, -F \in S(X)$ are defined as follows, for all $e \in E$:

(i) $(F \tilde{\cdot} H)(e) = F(e) \cdot H(e) = \{xy | x \in F(e), y \in H(e)\}$.
(ii) $(F \tilde{+} H)(e) = F(e) + H(e) = \{x + y | x \in F(e), y \in H(e)\}$.
(iii) $(F \tilde{-} H)(e) = F(e) - H(e) = \{x - y | x \in F(e), y \in H(e)\}$.
(iv) $-F(e) = -(F(e)) = \{-x | x \in F(e)\}$.

Definition 2.5 Kharal and Ahmad, 2011. Let $F_A \in S(X)$ and $G_B \in S(Y)$. Let $\mu : X \rightarrow Y$ and $\varphi : A \rightarrow B$ be two mappings.

(i) (φ, μ) is called a mapping from F_A to G_B, denoted by $(\varphi, \mu) : F_A \rightarrow G_B$, if and only if

$$\mu(F_A(a)) = G_B(\varphi(a)), \forall a \in A.$$

(ii) The image of F_A under μ with respect to φ is defined to be the soft set $(\mu(F_A))_{\varphi(A)} \in S(Y)$, such that

$$(\mu(F_A))_{\varphi(A)}(b) = \bigcup_{\varphi(a) = b} \mu(F_A(a)), \forall b \in \varphi(A).$$

(iii) The inverse image of G_B under μ with respect to φ is defined to be the soft set $(\mu^{-1}(G_B))_A \in S(X)$, such that $(\mu^{-1}(G_B))_A(a) = \mu^{-1}(G_B(\varphi(a))), \forall a \in A$.

Remark 2.6. Let $F_A \in S(X)$ and $G_B \in S(Y)$. Let $\mu : X \rightarrow Y$ and $\varphi : A \rightarrow B$ be two mappings.

(i) If $\varphi \times \mu : A \times X \rightarrow B \times Y$ such that $(\varphi \times \mu)(a, x) = (\varphi(a), \mu(x)), \forall a \in A, x \in X$ and φ is injective then we have $(\mu(F_A))_{\varphi(A)} = (\varphi \times \mu)(F_A)$. This means that the image of F_A under μ with respect to φ is the image of F_A under $\varphi \times \mu$, when φ is injective.
(ii) $(\mu^{-1}(G_B))_A = (\varphi \times \mu)^{-1}(G_B)$.
(iii) If (φ, μ) is a mapping from F_A to G_B, then $(\mu(F_A))_{\varphi(A)} = G_B|_{\varphi(A)}$ (therestractionof$G_B$over$\varphi(A)$) and $(\mu^{-1}(G_B))_A = F_A$ if μ is injective.
(iv) If $A = B$ and $\varphi = id_A$ (identity on A), then $(\mu(F_A))_{\varphi(A)}(a) = \mu(F_A(a)), \forall a \in A$.

Definition 2.7 Acar et al., 2010. Suppose that X is a ring. Then $F_A \in S(X)$ is called a soft ring (resp. soft ideal) over X if and only if $F_A(a)$ is a subring (resp. an ideal) of $X, \forall a \in A$.

Definition 2.8 Shabir and Naz, 2011. Let $F_A \in S(X)$ and $x \in X$. If $x \in \bigcap_{a \in A} F_A(a)$, then we say that x is a soft element in F_A and write $x \tilde{\in} F_A$.

Definition 2.9 Babitha and Sunil, 2010. Let $F_A \in S(X)$ and $G_B \in S(Y)$. The Cartesian product of F_A and G_B is defined to be the soft set $(F_A \tilde{\times} G_B) \in S(X \times Y)$, such that $(F_A \tilde{\times} G_B)(a,b) = F_A(a) \times G_B(b), \forall (a,b) \in A \times B$.

2.2. Soft topology

Throughout this subsection, we recall some basic concepts and results, for soft topological spaces. From now on, we consider that all soft sets are defined on the set of parameters E and all mappings are defined with respect to the identity on E, and we denote the mapping (id_E, f) shortly by f.

Definition 2.10 Shabir and Naz, 2011. Let $\tau \subseteq S(X)$. Then

(1) τ is called a soft topology on X if
  (i) $\widetilde{X}, \tilde{\phi} \in \tau$,
  (ii) τ is closed under finite intersection,
  (iii) τ is closed under arbitrary union.
(2) Let τ be a soft topology on X. Then the pair (X, τ) is called a soft topological space (in short S.T.S).
(3) Let (X, τ) be a S.T.S. Then a soft set $F \in S(X)$ is called a soft open set if and only if $F \in \tau$.

Note that a soft topology on X is a topology on $S(X)$.

Definition 2.11 Shabir and Naz, 2011. A soft topological space (X, τ) is called a soft indiscrete (soft discrete) space over X if and only if $\tau = \left\{\tilde{\phi}, \widetilde{X}\right\} (\tau = S(X))$. In this case τ is called a soft indiscrete (soft discrete) topology on X.

Example 2.12 Aygünoğlu and Aygün, 2012. Suppose that $\mathbb{R}$ is the set of all real numbers and $E = \mathbb{R}^+$ (the set of all positive real numbers). Let $\lambda \in E$ and $F_\lambda \in S(\mathbb{R})$, such that is $F_\lambda(e) = (e - \lambda, e + \lambda), \forall e \in E$. Let $\tau = \{F_\lambda | \lambda \in E\}$. Then $(\mathbb{R}, \tau)$ is a soft topological space.

Definition 2.13 Shabir and Naz, 2011. Suppose that (X, τ) is a S.T.S and $x \in X$.

(i) A soft set $F_x \in S(X)$ is called a soft neighborhood (shortly S. Nhd) of x if there exists $F \in \tau$, such that $x \tilde{\in} F \tilde{\subseteq} F_x$.
(ii) A soft neighborhood F_x of x in (X, τ) is called a soft open neighborhood (shortly S.O.Nhd) if $F_x \in \tau$.

Definition 2.14 Hida, 2014. Suppose that (X, τ) and (Y, υ) are two S.T.S and f is a mapping from X to Y.

(i) f is called soft continuous if and only if for any $x \in X$ and any S.O.Nhd $U_{f(x)}$ of $f(x)$, there exists a S.O.Nhd U_x of x such that $f(x) \tilde{\in} f(U_x) \tilde{\subseteq} U_{f(x)}$.
(ii) f is called soft open, if f satisfies the condition $F \in \tau \Rightarrow f(F) \in \upsilon$.
(iii) f is called soft homeomorphism if f is bijective and both of f and f^{-1} are soft continuous.

Definition 2.15 Nazmul and Samanta, 2014. Suppose that (X, τ) and (Y, υ) are two S.T.S. The collection of all unions of soft sets in $\{F \tilde{\times} G | F \in \tau, G \in \upsilon\}$ is a soft topology on $X \times Y$ and it is called soft product topology on $X \times Y$ and denoted by $\tau \tilde{\times} \upsilon$. The soft topological space $(X \times Y, \tau \tilde{\times} \upsilon)$ is called soft product topological space.

Proposition 2.16 Nazmul and Samanta, 2014. *Suppose that (X, τ) and (Y, υ) are two S.T.S. Then the projection mappings $\mathrm{proj}_X : (X \times Y, \tau \tilde{\times} \upsilon) \to (X, \tau)$ and $\mathrm{proj}_Y : (X \times Y, \tau \tilde{\times} \upsilon) \to (Y, \upsilon)$ are soft continuous and soft open. Also, $\tau \tilde{\times} \upsilon$ is the smallest soft topology on $X \times Y$ for which the projection mappings are soft continuous.*

Proposition 2.17 Nazmul and Samanta, 2014. *Suppose that $(X, \tau), (Y, \upsilon)$ and (Z, κ) are soft topological spaces. Then a mapping $f : (Z, \kappa) \to (X \times Y, \tau \tilde{\times} \upsilon)$ is soft continuous if and only if the mappings $\mathrm{proj}_Y \circ f$ and $\mathrm{proj}_X \circ f$ are soft continuous.*

Proposition 2.18 Nazmul and Samanta, 2014. *Suppose that (X, τ), (Y, υ) and (Z, κ) are soft topological spaces. If $f : X \to Y$ and $g : Y \to Z$ are soft continuous, then the mapping $g \circ f$ is soft continuous.*

Definition 2.19 Shabir and Naz, 2011. *A base β of a soft topological space (X, τ) is defined to be a family of soft open sets such that each soft open set in τ is a union of some elements in β .*

Note that if $\beta = \{B_i | i \in I\}$ is a family of soft sets over X such that β is closed under finite soft intersections, then the family β generates a soft topology on X in the form $\tau = \left\{\tilde{\phi}, \widetilde{X}\right\} \cup \left\{\tilde{\bigcup} V | V \subseteq \beta\right\}$, which called a soft topology generated by the base β.

Example 2.20. Let $E = \{e | e \in \mathbb{Z}, e \geqslant 1\}$ and $\beta = \{U_x | x \in \mathbb{Z}\}$, where $U_x \in S(\mathbb{Z})$ is defined by $U_x(e) = \{x + ne | n \in \mathbb{Z}\}, \forall e \in E$. Note that $U_x(e_1 e_2) \subseteq U_x(e_1) \cap U_x(e_2)$, for all $x \in \mathbb{Z}$ and $e_1, e_2 \in \mathbb{Z}$. So, β is a base of the soft topological space of the soft neighborhoods of each $x \in \mathbb{Z}$ and generates a soft topology on $\mathbb{Z}$.

Let (X, τ) be a S.T.S and $G \subset X$. Then it is clear that $\tau_G = \left\{\widetilde{G} \tilde{\cap} F | F \in \tau\right\}$ is a soft topology on G.

Definition 2.21 Hussain and Ahmad, 2011. Let (X, τ) be a S.T.S and $G \subseteq X$. The soft topology τ_G on G is called a soft relative topology on G and the soft topological space (G, τ_G) is called a soft a soft subspace of (X, τ).

Proposition 2.22. *Suppose that (X, τ) is a S.T.S and $H \subseteq G \subseteq X$. Then, $\tau_H = (\tau_G)_H$.*

Proof. By Definition 2.21, $\tau_H = \left\{\widetilde{H} \tilde{\cap} F | F \in \tau\right\}$ and $\tau_G = \left\{\widetilde{G} \tilde{\cap} F | F \in \tau\right\}$. Therefore,

$$\begin{aligned}(\tau_G)_H &= \left\{\left(\widetilde{G} \tilde{\cap} F\right) \tilde{\cap} \widetilde{H} | F \in \tau\right\} \\ &= \left\{\left(\widetilde{G} \tilde{\cap} \widetilde{H}\right) \tilde{\cap} F | F \in \tau\right\} \\ &= \left\{\widetilde{H} \tilde{\cap} F | F \in \tau\right\} \qquad (\text{Since} H \subseteq G) \\ &= \tau_H.\end{aligned}$$

□

Proposition 2.23 Shabir and Naz, 2011. *Suppose that (X, τ) is a S.T. S. Then, the family $\tau^e = \{F(e) | F \in \tau\}$ produces a topology on X for each parameter $e \in E$.*

Proposition 2.24 Nazmul and Samanta, 2013. *Suppose that (X, τ) is a S.T.S. The family*

$$\tau^* = \{F \in S(X) | F(e) \in \tau^e, \forall e \in E\}$$

is a soft topology on X and $[\tau^*]^e = \tau^e, \forall e \in E$.

Proposition 2.25 Nazmul and Samanta, 2012. *The intersection of two S.T.S is S.T.S. However the union of two S.T.S is not necessary to be a S.T.S.*

Definition 2.26 Arnautov et al., 1996. Let X be an additive group. The topological space (X, τ) is called a topological group and denoted by T.G if the mapping

$$\begin{aligned} f : (X \times X, \tau \times \tau) &\to (X, \tau) \\ (x, y) &\mapsto x - y \end{aligned}$$

is continuous.

Definition 2.27 Arnautov et al., 1996. Let X be a ring. The topological space (X, τ) is called a topological ring and denoted by T.R if the following conditions are satisfied:

(i) The mapping

$$\begin{aligned} (X \times X, \tau \times \tau) &\to (X, \tau) \\ (x, y) &\mapsto x - y \end{aligned}$$

is continuous.

(ii) The mapping

$$\begin{aligned} (X \times X, \tau \times \tau) &\to (X, \tau) \\ (x, y) &\mapsto xy \end{aligned}$$

is continuous.

In 2010, Nazmul and Samanta (2010) initiated the idea of soft topological groups. Later in 2014, Shah and Shaheen (2014), Shah and Shaheen (2014) and Hida (2014) added improvements to the concept of soft topological groups. In the same time, Nazmul and Samanta (2014) returned with the latest version of their work of soft topological groups as follows:

Definition 2.28 Nazmul and Samanta, 2014. Let X be an additive group. The soft topological space (X, τ) is called a soft topological group and denoted by S.T.G if the following conditions are satisfied:

(i) The mapping

$$\begin{aligned} g : (X \times X, \tau \tilde{\times} \tau) &\to (X, \tau) \\ (x, y) &\mapsto x + y \end{aligned}$$

is soft continuous.

(ii) The mapping

$$\begin{aligned} j : (X, \tau) &\to (X, \tau) \\ x &\mapsto -x \end{aligned}$$

is soft continuous.

Proposition 2.29 Nazmul and Samanta, 2014. *Suppose that (X, τ) is a S.T.G. Then, the mapping $h : (X \times X, \tau \tilde{\times} \tau) \to (X \times X, \tau \tilde{\times} \tau)$ defined by $h : (x, y) \mapsto (x, -y), \forall x, y \in X$ is soft continuous.*

Theorem 2.30. *Let τ be a soft topology on a group X. Then (X, τ) is a S.T.G if and only if the mapping*

$$\begin{aligned} k : (X \times X, \tau \tilde{\times} \tau) &\to (X, \tau) \\ (x, y) &\mapsto x - y \end{aligned}$$

is soft continuous.

Proof. $[\Rightarrow]$ Suppose that (X, τ) is a S.T.G. Therefore, the additive mapping $g : (X \times X, \tau \tilde{\times} \tau) \to (X, \tau)$ which defined by $g(x, y) = x + y$ for all $x, y \in X$ is soft continuous. Also, the mapping $j : (X, \tau) \to (X, \tau)$ which defined by $j(y) = -y$ for all $y \in X$ is soft continuous.

Moreover, from Proposition 2.29, the mapping $h : (X \times X, \tau \tilde{\times} \tau) \to (X \times X, \tau \tilde{\times} \tau)$ which defined by $h(x, y) = (x, -y)$ for all $x, y \in X$ is soft continuous.

Since $k = g \circ h$, then by using Proposition 2.18, the mapping k is soft continuous.

$[\Leftarrow]$ Let $k : (X \times X, \tau \tilde{\times} \tau) \to (X, \tau)$, such that $k(x, y) = x - y$ for all $x, y \in X$ be a soft continuous mapping. Since X is a group, then we have the identity element 0 in X. Now, in particular, and without loss of generality pick $x = 0$ and y an arbitrary element in X. Then k is soft continuous at $(0, y)$. Therefore, by the definition of soft continuity 2.14, for any S.O.Nhd U_{-y} of $-y$, there must be a S.O.Nhd U_0 of 0 and a S.O.Nhd U_y of y, such that $U_0 \tilde{-} U_y \tilde{\subseteq} U_{-y}$.

In particular, we have $-U_y \tilde{\subseteq} U_{-y}$, which shows that the inverse mapping $j(y) = -y$ for all $y \in X$ is a soft continuous mapping. Also, from Proposition 2.29, the mapping $h : (X \times X, \tau \tilde{\times} \tau) \to (X \times X, \tau \tilde{\times} \tau)$ which defined by $h(x, y) = (x, -y)$ for all $x, y \in X$ is soft continuous. Therefore, $g = k \circ h$ is a soft continuous mapping from $(X \times X, \tau \tilde{\times} \tau)$ to (X, τ). Then (X, τ) is a S.T.G.

Shah and Shaheen (2014) initiated the concept of a soft topological ring by applying the topological structures on a soft ring as the following:

Definition 2.31 Shah and Shaheen, 2014. *Suppose that τ is a topology defined on a ring X and $F \in S(X)$. The pair (F, τ) is called a soft topological ring over X if*

(i) F is a soft ring over X.
(ii) The mapping $(x, y) \to x - y$ from $(F(e) \times F(e), \tau^e \times \tau^e)$ to $(F(e), \tau^e)$ is continuous, $\forall e \in E$.
(iii) The mapping $(x, y) \to xy$ from $(F(e) \times F(e), \tau^e \times \tau^e)$ to $(F(e), \tau^e)$ is continuous, $\forall e \in E$.

In this paper, we will use the concept of a soft topological ring which is defined by Shah and Shaheen (2014) in the name of a topological soft ring (T.S.R) and in Section 3 we will show the difference between our notion and the notion of Shah and Shaheen (2014).

Definition 2.32 Tahat et al., 2018. Suppose that $F \in S(X)$ is a soft ring and (X, τ) is a soft topological space. Then (F, τ) is called soft topological soft ring over X and denoted by S.T.S.R if the following conditions are satisfied

(i) The mapping $(x, y) \mapsto xy$ from $(F \tilde{\times} F, \tau_F \tilde{\times} \tau_F)$ to (F, τ_F) is soft continuous;
(ii) The mapping $(x, y) \mapsto x + y$ from $(F \tilde{\times} F, \tau_F \tilde{\times} \tau_F)$ to (F, τ_F) is soft continuous;
(iii) The mapping $x \mapsto -x$ from (F, τ_F) to (F, τ_F) is soft continuous.

3. Soft topological ring

Through this section, we will propose the notion of soft topological soft rings and study their properties. From now on, X and Y denote unitary commutative rings.

Definition 3.1. Let τ be a soft topology on X. Then τ is called a ring soft topology on X if the following conditions hold:

(i) The mapping $(x, y) \mapsto xy$ from $(X \times X, \tau \tilde{\times} \tau)$ to (X, τ) is soft continuous;

(ii) The mapping $(x,y) \mapsto x+y$ from $(X \times X, \tau\tilde{\times}\tau)$ to (X,τ) is soft continuous;
(iii) The mapping $x \mapsto -x$ from (X,τ) to (X,τ) is soft continuous.

The soft topological space (X,τ), where τ is a ring soft topology on X is called a soft topological ring and denoted by S.T.R.

Remark 3.2. In the literature, the terminology of a soft ring is used to refer to a soft set F over a ring X such that $F(e)$ is a subring of X, for every $e \in E$. The concept of a soft topological ring F on X was studied by Shah and Shaheen (2014). They have carried out a topological structure and continuity of topology on the subring $F(e)$, for every $e \in E$ to introduce the concept of a soft topological ring via the old idea of a topological ring. So we think it would have been better if he had named his concept as a topological soft ring because he combined a topology with a soft ring. In contrast, our concept introduced via a soft continuity on a soft topology on a ring. Literally; our concept based on applying soft topological structures on a ring. Therefore, the concept of soft topological rings in the sense of our definition (Definition 3.1) is totally different from that in the sense of Definition 2.31.

Remark 3.3. Suppose that (X,τ) is a S.T.S. If (X,τ) is a S.T.R, then (X,τ) is a S.T.G.

Concerning soft neighborhoods, we can redefine the soft topological ring as in the fallowing proposition.

Proposition 3.4. *Suppose that τ is a soft topology on X. Then the soft topological space (X,τ) is a S.T.R if and only if all the following conditions are satisfied:*

(i) *For all $x, y \in X$, and every S.O.Nhd U_{xy} of xy, there must be a S.O.Nhd U_x of x and a S.O.Nhd U_y of y, such that $U_x \tilde{\cdot} U_y \tilde{\subseteq} U_{xy}$.*
(ii) *For all $x, y \in X$, and every S.O.Nhd U_{xy} of xy, there must be a S.O.Nhd U_x of x and a S.O.Nhd U_y of y, such that $U_x \tilde{+} U_y \tilde{\subseteq} U_{x+y}$.*
(iii) *For all $x \in X$, and every S.O.Nhd U_{-x} of $-x$, there must be a S.O.Nhd U_x of x such that $-U_x \tilde{\subseteq} U_{-x}$.*

Proof.

[$\Rightarrow$] Suppose that (X,τ) is a S.T.R. Then the mapping $f : (X \times X, \tau\tilde{\times}\tau) \to (X,\tau)(resp. g : (X \times X, \tau\tilde{\times}\tau) \to (X,\tau))$, which defined by $f(x,y) = xy(resp. g(x,y) = x+y)$ is soft continuous. Let $x,y \in X$ and $U_{xy}(resp. U_{x+y})$ be an arbitrary S.O. Nhd of $f(x,y) = xy(resp. g(x,y) = x+y)$. It follows from Definition 2.14 that, for every $(x,y) \in X \times X$ and every S.O.Nhd $U_{f(x,y)}(resp. U_{g(x,y)})$ of $f(x,y)(resp. g(x,y))$, there must be a S.O. Nhd $U_{(x,y)}$ of (x,y) such that $xy \tilde{\in} f(U_{(x,y)}) \tilde{\subseteq} U_{f(x,y)}$ $(resp. x+y \tilde{\in} g(U_{(x,y)}) \tilde{\subseteq} U_{g(x,y)})$. Now, $U_{(x,y)}$ is a soft open set in $\tau\tilde{\times}\tau$, which mean that there exist in τ a S.O.Nhd U_{x_i} of $x_i \in X$ and a S.O.Nhd U_{y_i} of $y_i \in X$, where $i \in I$ and I an index set, such that $U_{(x,y)} = \bigcup_{i \in I} U_{x_i} \tilde{\times} U_{y_i}$. That implies, there must be $i \in I$ such that $x \tilde{\in} U_{x_i}$ and $y \tilde{\in} U_{y_i}$. So, $U_{x_i} \tilde{\times} U_{y_i} \in \tau\tilde{\times}\tau$ and $U_{x_i} \tilde{\times} U_{y_i} \tilde{\subseteq} U_{(x,y)}$ and since $U_{x_i} \tilde{\cdot} U_{y_i} = f(U_{x_i} \tilde{\times} U_{y_i}) \tilde{\subseteq} f(U_{(x,y)})$ $\tilde{\subseteq} U_{f(x,y)}(resp. (U_{x_i} \tilde{+} U_{y_i}) = g(U_{x_i} \tilde{\times} U_{y_i}) \tilde{\subseteq} g(U_{(x,y)}) \tilde{\subseteq} U_{g(x,y)})$.
Thus, condition (i) (resp. (ii)) of Definition 3.1 is satisfied.
Now, since the inverse mapping $j(x) = -x, \forall x \in X$ from (X,τ) to (X,τ) is soft continuous, it follows from Definition 2.14 that, for every $x \in X$ and for every S.O.Nhd $U_{j(x)}$ of $j(x)$, there must be a S.O.Nhd U_x of x such that $x \tilde{\in} j(U_{(x)}) \tilde{\subseteq} U_{j(x)}$. This implies that $-U_x \tilde{\subseteq} U_{-x}$, which satisfies the condition (iii) of Definition 3.1.

[$\Leftarrow$] Suppose that the conditions (i), (ii) and (iii) are satisfied. For all $x,y \in X$, and for every S.O.Nhd $U_{xy}(resp. U_{x+y})$ of xy $(resp. of x+y)$, there must be a S.O.Nhd U_x of x and a S.O. Nhd U_y of y, such that $U_x \tilde{\cdot} U_y \tilde{\subseteq} U_{xy}(resp. U_x \tilde{+} U_y \tilde{\subseteq} U_{x+y})$. But $U_x \tilde{\cdot} U_y = f(U_x \tilde{\times} U_y)(resp. U_x \tilde{+} U_y = g(U_x \tilde{\times} U_y))$, where f and g are defined as in the first direction. Since $x \tilde{\in} U_x$ and $y \tilde{\in} U_y$, then $U_x \tilde{\times} U_y$ is a S.O.Nhd in $\tau\tilde{\times}\tau$ contains (x,y). So, by Definition 2.14, the mapping f (resp. g) is soft continuous. The condition (iii) of Definition 3.1, follows directly from the condition (iii) and Definition 2.14.

□

Theorem 3.5. *Suppose that τ is a soft topology on X. Then the soft topological space (X,τ) is a S.T.R if and only if*

(i) The mapping

$$\begin{aligned} f : (X \times X, \tau\tilde{\times}\tau) &\to (X,\tau) \\ (x,y) &\mapsto xy \end{aligned}$$

is soft continuous.
(ii) The mapping

$$\begin{aligned} k : (X \times X, \tau\tilde{\times}\tau) &\to (X,\tau) \\ (x,y) &\mapsto x-y \end{aligned}$$

is soft continuous.

Proof.

[$\Rightarrow$] Let (X,τ) be a S.T.R. Then it follows, from Definition 3.1, that

$$\begin{aligned} f : (X \times X, \tau\tilde{\times}\tau) &\to (X,\tau) \\ (x,y) &\mapsto xy \end{aligned}$$

is soft continuous. Since (X,τ) is S.T.G, it follows, from Theorem 2.30, that

$$\begin{aligned} k : (X \times X, \tau\tilde{\times}\tau) &\to (X,\tau) \\ (x,y) &\mapsto x-y \end{aligned}$$

is also soft continuous. So, we have (i) and (ii).
[$\Leftarrow$] Let (i) and (ii) be satisfied. It follows, from (ii) and Theorem 2.30, that (X,τ) is a S.T.G. So, it follows, from (i), that (X,τ) is a S.T.R.

□

Example 3.6. Let $X = \mathbb{Z}$ and $E = \mathbb{Z}^+$. Let τ be the soft topology on X defined as in Example 2.20. Note that, for all $x,y \in X$ and $e \in E$, we have

$$U_x(e) - U_y(e) = U_{x-y}(e) \quad \text{and} \quad U_x(e)U_y(e) \subseteq U_{xy}(e).$$

Then,

$$U_x \tilde{-} U_y \tilde{\subseteq} U_{x-y} \quad \text{and} \quad U_x \tilde{\cdot} U_y \tilde{\subseteq} U_{xy}.$$

Therefore, (X,τ) is a S.T.R.

Theorem 3.7. *Suppose that (X,τ) is a S.T.R. Then (X,τ^e) is a T.R for each $e \in E$.*

Proof. Suppose that (X,τ) is a S.T.R. Then by Theorem 3.5, the mappings

$$f:(X\times X,\tau\tilde{\times}\tau)\to(X,\tau)\quad\text{and}\quad k:(X\times X,\tau\tilde{\times}\tau)\to(X,\tau)$$
$$(x,y)\mapsto xy\qquad (x,y)\mapsto x-y$$

are soft continuous. From the definition of soft continuity 2.14, it follows that, for all $x,y\in X$, and every S.O.Nhd U_{xy} (resp. U_{x-y}) of xy (resp. $x-y$), there must be a S.O.Nhd U_x of x and a S.O.Nhd U_y of y, such that

$$U_x\tilde{\cdot}U_y\,\tilde{\subseteq}\,U_{xy}\quad(\text{resp.}U_x\tilde{-}U_y\,\tilde{\subseteq}\,U_{x-y}).$$

Then,

$$U_x(e)U_y(e)\subseteq U_{xy}(e)\quad(\text{resp.}U_x(e)-U_y(e)\subseteq U_{x-y}(e))\qquad\forall e\in E.$$

Since $\tau^e=\{U(e)|U\in\tau\}$, then $U_x(e),U_{xy}(e),U_y(e)$and$U_{x-y}(e)$ are open sets in τ^e. That satisfied the continuity of subtraction and multiplication mappings in Definition 2.27. Therefore, (X,τ^e) is a T.R for all $e\in E$. □

Proposition 3.8. *Suppose that (X,τ) is a S.T.S such that (X,τ^e) is a T.R, for all $e\in E$. Then (X,τ^*) is a S.T.R.*

Proof. Suppose that (X,τ^e) is a T.R, for all $e\in E$. So, for any $x,y\in X$ and arbitrary open neighborhood $U_{x-y}(e)$ of $x-y$ (resp. $U_{xy}(e)$) of xy there must be open neighborhoods $U_x(e),U_y(e)$ of x and y respectively, such that

$$U_x(e)U_y(e)\subseteq U_{xy}(e)\quad(\text{resp.}U_x(e)-U_y(e)\subseteq U_{x-y}(e)),\qquad\forall e\in E.$$

This implies that

$$U_x\tilde{\cdot}U_y\,\tilde{\subseteq}\,U_{xy}\quad(\text{resp.}U_x\tilde{-}U_y\,\tilde{\subseteq}\,U_{x-y}).$$

Since $\tau^*=\{U\in S(X)|U(e)\in\tau^e,\forall e\in E\}$, we have $U_x,-U_y,U_{xy}$ andU_{x-y} are soft open sets in τ^*. So, the conditions (i) and (ii) of Theorem 3.5 are satisfied. Then (X,τ^*) is a S.T.R.

Proposition 3.9. *Suppose that (X,τ) is a S.T.S. If $\widetilde{\{x\}},\widetilde{\{y\}}\in\tau,\forall x,y\in X$, then (X,τ) is a S.T.R.*

Proof. To prove that (X,τ) is a S.T.R it is enough to show that the conditions of Proposition 3.4 are satisfied.

(i) For all $x,y\in X$, and every S.O.Nhd U_{xy} of xy, there must be a S.O.Nhd $U_x\tilde{=}\widetilde{\{x\}}$ of x and a S.O.Nhd $U_y\tilde{=}\widetilde{\{y\}}$ of y, such that $U_x\tilde{\cdot}U_y\tilde{=}\widetilde{\{x\}}\tilde{\cdot}\widetilde{\{y\}}\tilde{=}\widetilde{\{xy\}}\,\tilde{\subseteq}\,U_{xy}$.
(ii) For all $x,y\in X$, and every S.O.Nhd U_{x+y} of $x+y$, there must be a S.O.Nhd $U_x\tilde{=}\widetilde{\{x\}}$ of x and a S.O.Nhd $U_y\tilde{=}\widetilde{\{y\}}$ of y, such that $U_x\tilde{+}U_y\tilde{=}\widetilde{\{x\}}\tilde{+}\widetilde{\{y\}}\tilde{=}\widetilde{\{x+y\}}\,\tilde{\subseteq}\,U_{x+y}$.
(iii) For all $x\in X$, and every S.O.Nhd U_{-x} of $-x$, there must be S.O.Nhd $U_x\tilde{=}\widetilde{\{x\}}$ of x such that $-U_x\tilde{=}-\widetilde{\{x\}}\tilde{=}\widetilde{\{-x\}}\,\tilde{\subseteq}\,U_{-x}$.

Therefore, (X,τ) is a S.T.R. □

Remark 3.10. Any ring furnished with a soft discrete or soft indiscrete topology is a soft topological ring. It is easy to verify that any ring satisfies the conditions (i) and (ii) of Definition 3.1 in both soft topologies. In this manner, any ring can be considered as a soft topological ring in the soft discrete or soft indiscrete topology.

Proposition 3.11. *Let (X,τ) be a S.T.R. Then*

(i) *For each $x\in X$, the mapping $a_x:y\mapsto y+x$ from (X,τ) to (X,τ) is a soft homeomorphism.*

(ii) *For each $x\in X$, the mapping $m_x:y\mapsto yx$ from (X,τ) to (X,τ) is soft continuous. Moreover, if x is invertible, then m_x is a soft homeomorphism.*

Proof.

(i) To show that $a_x:y\mapsto y+x$ is soft continuous, we need to show that for any soft neighborhood F_{y+x} of $y+x$ there exist a soft open neighborhood F_y of y such that $a_x(F_y)\,\tilde{\subseteq}\,F_{y+x}$. Now, since (X,τ) is a S.T.R, then for any $x,y\in X$ and an arbitrary soft open neighborhood F_{y+x} of $y+x$ there exist soft open neighborhoods F_y and F_x of y and x, respectively, such that $F_y\tilde{+}F_x\,\tilde{\subseteq}\,F_{y+x}$. Hence we have $a_x(F_y)\tilde{=}F_y+x\,\tilde{\subseteq}\,F_y\tilde{+}F_x\,\tilde{\subseteq}\,F_{y+x}$, where $(F_y+x)(e)=F_y(e)+x,\forall e\in E$. So, a_x is soft continuous. Note that a_x is bijective, for all $x\in X$, and $(a_x)^{-1}=a_{-x}$. So, $(a_x)^{-1}$ is also soft continuous. Therefore a_x is a soft homeomorphism, for all $x\in X$.
(ii) To show that $m_x:y\mapsto yx$ is soft continuous, we need to show that for any soft open neighborhood F_{yx} of yx there exist a soft open neighborhood F_y of y such that $m_x(F_y)\,\tilde{\subseteq}\,F_{yx}$. Now, since (X,τ) is a S.T.R then for any $x,y\in X$ and an arbitrary soft open neighborhood F_{yx} of yx, there exist soft open neighborhoods F_y and F_x of y and x, respectively, such that $F_y\tilde{\cdot}F_x\,\tilde{\subseteq}\,F_{yx}$. Hence we have $m_x(F_y)\tilde{=}F_yx\,\tilde{\subseteq}\,F_y\tilde{\cdot}F_x\,\tilde{\subseteq}\,F_{yx}$, where $(F_yx)(e)=F_y(e)x,\forall e\in E$. So, m_x is a soft continuous. Now if x is invertible, then m_x is bijective and $(m_x)^{-1}=m_{x^{-1}}$. So, $(m_x)^{-1}$ is a soft continuous. Thereforem_x is a soft homeomorphism.

□

Note that if τ and υ are two soft topologies on X, then the soft intersection $\tau\tilde{\cap}\upsilon$ of τ and υ is defined by $\tau\tilde{\cap}\upsilon=\{V\tilde{\cap}W|V\in\tau,W\in\upsilon\}$, (see (Shabir and Naz, 2011)).

Definition 3.12. Suppose that (S,τ) and (T,υ) are two S.T.R, where S and T are subrings of X. Then the intersection of (S,τ) and (T,υ) is defined as $(S,\tau)\tilde{\cap}(T,\upsilon)$.

Theorem 3.13. *Suppose that (S,τ) and (T,υ) are two S.T.S each of them is a S.T.R, where S and T are subrings of X. Then $(S,\tau)\tilde{\cap}(T,\upsilon)=(S\cap T,\tau\tilde{\cap}\upsilon)$ is a S.T.R.*

Proof. Suppose that (S,τ) and (T,υ) are two S.T.S each of them is a S.T.R, where S and T are subrings of X. Let $x,y\in S\cap T$, then by the definition of soft topological ring, for any arbitrary soft open neighborhoods $U_{x-y}\in\tau$ and $V_{x-y}\in\upsilon$ of $x-y$ (resp. $U_{xy}\in\tau$ and $V_{xy}\in\upsilon$ of xy), there must be soft open neighborhoods $U_x\in\tau$ and $V_x\in\upsilon$ of x, also there must be soft open neighborhoods $U_y\in\tau$ and $V_y\in\upsilon$ of y, such that

$$x-y\tilde{\in}U_x\tilde{-}U_y\,\tilde{\subseteq}\,U_{x-y}\ \text{and}\ x-y\tilde{\in}V_x\tilde{-}V_y\,\tilde{\subseteq}\,V_{x-y}.\ (\text{resp.}xy\tilde{\in}U_x\tilde{\cdot}U_y\,\tilde{\subseteq}\,U_{xy}\ \text{and}\ xy\tilde{\in}V_x\tilde{\cdot}V_y\,\tilde{\subseteq}\,V_{xy}).$$

This implies that

$$x-y\tilde{\in}(U_x\tilde{-}U_y)\tilde{\cap}(V_x\tilde{-}V_y)\,\tilde{\subseteq}\,U_{x-y}\tilde{\cap}V_{x-y}.$$
$$(\text{resp.}xy\tilde{\in}(U_x\tilde{\cdot}U_y)\tilde{\cap}(V_x\tilde{\cdot}V_y)\,\tilde{\subseteq}\,U_{xy}\tilde{\cap}V_{xy}).$$

Which implies

$$x-y\tilde{\in}(U\tilde{\cap}V)_x\tilde{-}(U\tilde{\cap}V)_y\,\tilde{\subseteq}\,(U\tilde{\cap}V)_{x-y}.$$
$$\left(\text{resp.}xy\tilde{\in}(U\tilde{\cap}V)_x\tilde{\cdot}(U\tilde{\cap}V)_y\,\tilde{\subseteq}\,(U\tilde{\cap}V)_{xy}\right).$$

So, the subtraction (resp. multiplication) mapping is soft continuous. Therefore, $(S \cap T, \tau \tilde{\cap} \upsilon)$ is a S.T.R. □

Remark 3.14. Let $\{(X_i, \tau_i); i \in I\}$ be a nonempty collection of soft topological rings, where I is an index set, X_i is a subring of $X, \forall i \in I$. Then $\tilde{\bigcap}_{i \in I}(X_i, \tau_i) = \left(\bigcap_{i \in I} X_i, \tilde{\bigcap}_{i \in I} \tau_i\right)$ is a S.T.R.

Theorem 3.15. *Suppose that $F \in S(X)$ is a soft ring. Then, (F, τ) is a S.T.S.R over X if the soft topological space (X, τ) is a S.T.R.*

Proof. Suppose that (X, τ) is a S.T.R. So, for every $x, y \in X$, and arbitrary S.O.Nhd V_{x-y} (resp. V_{xy}) of $x - y$ (resp. xy) there must be a S.O.Nhd $V_x \in \tau$ of x and a S.O.Nhd $V_y \in \tau$ of y such that $x - y \tilde{\in} V_x \tilde{-} V_y \tilde{\subseteq} V_{x-y}$ (resp. $xy \tilde{\in} V_x \tilde{\cdot} V_y \tilde{\subseteq} V_{xy}$). Since F is a soft ring over $X, F(e)$ is a subring of X, for all $e \in E$. Also, for any $x, y \tilde{\in} F$ we have $x, y \in X$.

Let $x, y \tilde{\in} F$. Then for any arbitrary S.O.Nhd V_{x-y} (resp. V_{xy}) of $x - y$ (resp. xy) there must be a S.O.Nhd $V_x \in \tau$ of x and a S.O.Nhd $V_y \in \tau$ of y, such that $x - y \tilde{\in} V_x \tilde{-} V_y \tilde{\subseteq} V_{x-y}$ (resp. $xy \tilde{\in} V_x \tilde{\cdot} V_y \tilde{\subseteq} V_{xy}$). That implies $x - y \tilde{\in} F \tilde{\cap} V_x \tilde{-} F \tilde{\cap} V_y \tilde{\subseteq} F \tilde{\cap} V_{x-y}$ (resp. $xy \tilde{\in} F \tilde{\cap} V_x \tilde{\cdot} F \tilde{\cap} V_y \tilde{\subseteq} F \tilde{\cap} V_{x-y}$). Since the soft open sets of τ_F are in the form $F \tilde{\cap} V$, where $V \in \tau$, that implies $F \tilde{\cap} V_x$, $F \tilde{\cap} V_y$ and $F \tilde{\cap} V_{x-y}$ are soft open sets in τ_F. Therefore, (F, τ) is a S.T.S.R over X. □

The converse of Theorem 3.15 is not true as will be shown in the following example:

Example 3.16. Let $X = \mathbb{Z}_6, E = \{e_1, e_2\}$ and let F be a soft set over X defined by $F(e_1) = \{\bar{0}, \bar{2}, \bar{4}\}$ and $F(e_2) = \{\bar{0}\}$. Let $\tau = \left\{\tilde{\phi}, \{(e_1, \{\bar{0}\}), (e_2, \{\bar{0}\})\}, \{(e_1, \{\bar{2}\}), (e_2, \{\bar{0}, \bar{1}\})\}, \{(e_1, \{\bar{0}, \bar{2}\}), (e_2, \{\bar{0}, \bar{1}, \bar{2}\})\}, \tilde{\mathbb{Z}}_6\right\}$. Since $F(e_1)$ and $F(e_2)$ are subrings of X, then F is a soft ring over X.

$$\tau_F = \left\{\tilde{\phi}, \{(e_1, \{\bar{0}\}), (e_2, \{\bar{0}\})\}, \{(e_1, \{\bar{2}\}), (e_2, \{\bar{0}\})\}, \{(e_1, \{\bar{0}, \bar{2}\}), (e_2, \{\bar{0}\})\}, F\right\}.$$

Note that the only soft element belongs to F is $\bar{0}$. So, it is clear that the mapping $(x, y) \mapsto x - y$ from $(F \tilde{\times} F, \tau_F \tilde{\times} \tau_F)$ to (F, τ_F) is soft continuous. Therefore, (F, τ) is a S.T.S.R over X. On the other hand, we claim that (X, τ) is not S.T.R. To show that pick $x = \bar{3}$ and $y = \bar{1}$. Note that $\bar{2} \tilde{\in} \{(e_1, \{\bar{0}, \bar{2}\}), (e_2, \{\bar{0}, \bar{1}, \bar{2}\})\}$. (X, τ) to be a S.T.G there must be a S.O.Nhd V_3 of $\bar{3}$ and a S.O.Nhd V_1 of $\bar{1}$, such that $x - y \tilde{\in} V_3 - V_1 \tilde{\subseteq} V_2 = \{(e_1, \{\bar{0}, \bar{2}\}), (e_2, \{\bar{0}, \bar{1}, \bar{2}\})\}$. But the only S.O.Nhd of $\bar{3}$ and $\bar{1}$ is $\tilde{\mathbb{Z}}_6$. Moreover, $\tilde{\mathbb{Z}}_6 \tilde{-} \tilde{\mathbb{Z}}_6 = \tilde{\mathbb{Z}}_6 \tilde{\nsubseteq} \{(e_1, \{\bar{0}, \bar{2}\}), (e_2, \{\bar{0}, \bar{1}, \bar{2}\})\}$. Therefore, (X, τ) is not a S.T.R.

Theorem 3.17 Tahat et al., 2018. *Suppose that $F \in S(X)$ is a soft ring and (X, τ) is a soft topological space. Then $\left(F(e), (\tau^e)_{F(e)}\right)$ is a T.R for each $e \in E$, if (F, τ) is a S.T.S.R over X.*

Theorem 3.18 Tahat et al., 2018. *If (F, τ) is a S.T.S.R over X. Then, (F, τ^e) is a T.S.R over X for each $e \in E$.*

The following figure shows the relationship between S.T.R, S.T.S.R, T.S.R and T.R.

3.1. Soft topological subring

Definition 3.19. Suppose that (X, τ) and (Y, υ) are two S.T.R. Then (X, τ) is called a soft topological subring of (Y, υ), denoted by $(X, \tau) \tilde{\leqslant} (Y, \upsilon)$ if the following conditions are satisfied:

(i) X is a subring of Y.
(ii) $\tau = \upsilon_X$.

Remark 3.20. A ring soft topology on a ring X clearly induces a ring soft topology on any subring of X, and unless the contrary is indicated, we shall assume that a subring of a soft topological ring is furnished with its induced soft topology.

Example 3.21. Let $X = \mathbb{Z}_8$ and $E = \{e_1, e_2\}$. Also, let

$F = \{\bar{0}, \bar{4}\}$,	$H = \{\bar{0}, \bar{2}, \bar{4}, \bar{6}\}$,	
$F_1 = \{(e_1, \{\bar{0}\}), (e_2, \{\bar{0}\})\}$, $F_4 = \{(e_1, \{\bar{0}, \bar{4}\}), (e_2, \{\bar{0}, \bar{4}\})\}$and	$F_2 = \{(e_1, \{\bar{0}, \bar{2}, \bar{4}, \bar{6}\}), (e_2, \{\bar{0}, \bar{1}, \bar{4}\})\}$, $F_5 = \{(e_1, \{\bar{0}, \bar{4}, \bar{6}\}), (e_2, \{\bar{0}, \bar{4}\})\}$.	$F_3 = \{(e_1, \{\bar{4}\}), (e_2, \{\bar{4}\})\}$,

Let $\tau = \left\{\tilde{\phi}, \tilde{X}, F_1, F_2, F_3, F_4\right\}$. Then τ represents a soft topology on X.

$\tau_F = \left\{\tilde{\phi}, \{(e_1, \{\bar{0}\}), (e_2, \{\bar{0}\})\}, \left\{(e_1, \bar{4}), (e_2, \{\bar{4}\}\right\}, \tilde{F}\right\}$. It is obvious that, the subtraction and multiplication mappings in Theorem 3.5 are soft continuous. Therefore, (F, τ_F) is a S.T.R over X. Again, if $\upsilon = \left\{\tilde{\phi}, \tilde{X}, F_1, F_2, F_3, F_4, F_5\right\}$. Then, υrepresents a soft topology on X.

It is obviously that, the subtraction and multiplication mappings in Theorem 3.5 are soft continuous. Therefore, (H, υ_H) is a S.T.R over X. Since F is a subring of H and

$$\tau_F = \left((\upsilon)_H\right)_F = \left\{\tilde{\phi}, \{(e_1, \{\bar{0}\}), (e_2, \{\bar{0}\})\}, \left\{(e_1, \bar{4}), (e_2, \{\bar{4}\}\right\}, \tilde{F}\right\}.$$

Then $(F, \tau_F) \tilde{\leqslant} (H, \upsilon_H)$.

Theorem 3.22. Let (X, τ) be a S.T.R. If H is a subring of X, then (H, τ_H) is a S.T.R and $(H, \tau_H) \tilde{\leqslant} (X, \tau)$.

Proof. Follows directly from Proposition 2.22 and Theorem 3.5. □

3.2. Soft Topological Ideals

Through this section, we introduce the notion of soft topological ideals over X and study some of their properties.

Definition 3.23. Suppose that (X, τ) is a soft topological ring and I is an ideal in X. Then the soft topological space (I, τ_I) is called a soft topological ideal in (X, τ), denoted by S.T.I if and only if

(i) For all $x, y \in I$, the mapping $(x, y) \mapsto x - y$ from $(I \times I, \tau_I \tilde{\times} \tau_I)$ to (I, τ_I) is soft continuous,
(ii) For all $y \in I$ and $z \in X$, the mapping $(z, y) \mapsto zy$ from $(X \times I, \tau \tilde{\times} \tau_I)$ to (I, τ_I) is soft continuous and

(iii) For all $y \in I$ and $z \in X$ the mapping $(y, z) \mapsto yz$ from $(I \times X, \tau_I \tilde{\times} \tau)$ to (I, τ_I) is soft continuous.

In Definition 3.23 if (I, τ_I) satisfies only conditions (i) and (ii), then it is called left soft topological ideal over X and denoted by L-S.T.I. Also if (I, τ_I) satisfies only conditions (i) and (iii), then it is called right soft topological ideal in X and denoted by R-S.T.I.

Example 3.24. In Example 3.21 the set F is an ideal of X and it is easy to verify that the soft topological space (F, τ_F) satisfies the conditions (i), (ii) and (iii) of Definition 3.23. Therefore, (F, τ_F) is an S.T.I in (X, τ).

The next theorem can be easily handled by Definition 3.23.

Theorem 3.25. Every S.T.I in a S.T.R is a soft topological subring.

Remark 3.26. The converse of Theorem 3.25 is not true as will explain in the following example.

Example 3.27. Let $X = M_2(\mathbb{Z}_2)$. ${}_yI = \{x \in X | yx = 0\}$, for all $y \in X$. Then ${}_yI$ is a subring of X, for all $y \in X$. Let τ be the soft discrete topology on X. Then $\tau_{{}_yI}$ is the soft discrete topology on ${}_yI$.

It is easy to verify that the soft topological space $({}_yI, \tau_{{}_yI})$ satisfies the conditions (i) and (ii) of Theorem 3.5. Therefore $({}_yI, \tau_{{}_yI})$ is a S.T. R. Let $y = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$. Then,

${}_yI = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \right\}$. Note that ${}_yI$ is a left ideal in X. But it is not a right ideal in X. So, ${}_yI$ is not an ideal in X.

4. Applications and studies proposed

Note that if $X = \mathbb{R}$and $F \in S(x)$ is a soft set defined such that for all $e \in E, F(e)$ is a non-empty bounded subset of $\mathbb{R}$, then F is called a soft real set. And if F is a singleton soft set over $\mathbb{R}$, then F is called a soft real number (see (Das and Samanta, 2012)). Also, we call $F \in S(\mathbb{Q})$ (resp. $F \in S(\mathbb{Z})$) a soft rational number (resp. integer), if $F(e)$ is a singleton set for each $e \in E$.

We will use the notions $\hat{\mathbb{R}}, \hat{\mathbb{Q}}$ and $\hat{\mathbb{Z}}$ to denote the set of all soft real numbers, soft rational numbers, and soft integers, respectively.

Also, we will use the notations $\hat{n}, \hat{m}, \hat{k}, \cdots$to denote soft real numbers or soft integers. If $\hat{n}(a) = \{n\}, \forall e \in E$, then $\hat{n}$ is denoted by $\tilde{n}$. For instance, let $E = \{e_1, e_2, e_3\}$, then $\tilde{1} = \{(e_1, \{1\}), (e_2, \{1\}), (e_3, \{1\})\}$.

Definition 4.1 Das and Samanta, 2012. Suppose that $\hat{n}, \hat{m} \in \hat{\mathbb{R}}, k \in \mathbb{N}$ and $\{\hat{n}_j\}_{j \in J}$ is a class of soft real numbers.

(i) The addition of $\hat{n}$ and $\hat{m}$ is defined to be the soft real number $\hat{n} + \hat{m}$, such that $(\hat{n} + \hat{m})(i) = \{n_i + m_i\}$, foreach$i \in E$.
(ii) The multiplication of $\hat{n}$ and $\hat{m}$ is defined to be the soft real number $\hat{n} \cdot \hat{m}$, such that $(\hat{n} \cdot \hat{m})(i) = \{n_i \cdot m_i\}$, foreach$i \in E$.
(iii) The division of $\hat{n}$ by $\hat{m}$, where $m_i \neq 0, \forall i \in E$, is defined to be the soft real number $\frac{\hat{n}}{\hat{m}}$, such that $(\frac{\hat{n}}{\hat{m}})(i) = \left\{\frac{n_i}{m_i}\right\}$, foreach$i \in E$.
(iv) $\hat{n}^{\hat{m}}$ is a soft real number, such that $\hat{n}^{\hat{m}}(i) = \{(n_i)^{m_i}\}$, foreach$i \in E$.

The multiplication of $\{\hat{n}_j\}_{j \in J}$ is denoted by $\prod_{j \in J} \hat{n}_j$. And the soft real number $\hat{n} \cdot \hat{n} \cdots \hat{n}$ (k times) is denoted by $\hat{n}^k$. Note that $\hat{n}^k = \hat{n}^{\tilde{k}}$.

Note that $\hat{\mathbb{Z}}, \hat{\mathbb{Q}}$ and $\hat{\mathbb{R}}$ are commutative rings with zero element $\tilde{0}$ and one element $\tilde{1}$ with respect to addition and multiplication of soft real numbers.

Definition 4.2. Let $\widehat{X} \subseteq \hat{\mathbb{R}}$. A function $d : \widehat{X} \times \widehat{X} \to \widehat{\mathbb{R}^+ \cup \{0\}}$ is called a soft metric if and only if for all $\hat{x}, \hat{y}, \hat{z} \in \widehat{X}$, the following conditions are satisfied:

(i) $d(\hat{x}, \hat{y}) \geqslant \tilde{0}$ and $d(\hat{x}, \hat{y}) = \tilde{0} \iff \hat{x} = \hat{y}$,
(ii) $d(\hat{x}, \hat{y}) = d(\hat{y}, \hat{x})$,
(iii) $d(\hat{x}, \hat{y}) \leqslant d(\hat{y}, \hat{z}) + d(\hat{z}, \hat{y})$,

The pair $\left(\widehat{X}, d\right)$ is called a soft metric space if d is a soft metric.

Definition 4.3. A function $|\,| : \hat{\mathbb{R}} \to \widehat{\mathbb{R}^+ \cup \{0\}}$ is called an absolute value on $\hat{\mathbb{R}}$ if and only if it satisfies the following conditions, for all $\hat{m}, \hat{n} \in \hat{\mathbb{R}}$:

(i) $(|\hat{m}|)_i = 0 \iff m_i = 0$,
(ii) $|\hat{m}\hat{n}| = |\hat{m}|\,|\hat{n}|$and
(iii) $|\hat{m} + \hat{n}| \leqslant |\hat{m}| + |\hat{n}|$.

Remark 4.4.
Let $|\,|$be an absolute value on $\mathbb{R}$. Then the function $|\,| : \hat{\mathbb{R}} \to \widehat{\mathbb{R}^+ \cup \{0\}}$ defined by $(|\hat{n}|)_i = |n_i|$, for all $i \in E$ and $\hat{n} \in \hat{\mathbb{R}}$, is an absolute value on $\hat{\mathbb{R}}$

Definition 4.5. The open ball of center $\hat{a} \in \hat{\mathbb{R}}$ and radius $\hat{r} \in \hat{\mathbb{R}}$ is the set $B(\hat{a}, \hat{r}) = \{\hat{x} \in \hat{\mathbb{R}} : d(\hat{x}, \hat{a}) < \hat{r}\}$.

Theorem 4.6. *The soft metric space* $(\hat{\mathbb{R}}, d)$ *is a soft topological ring, where d is a soft metric defined such that*

$$\begin{aligned} d : \hat{\mathbb{R}} \times \hat{\mathbb{R}} &\to \widehat{\mathbb{R}^+ \cup \{0\}} \\ d(\hat{x}, \hat{y}) &\mapsto |\hat{x} - \hat{y}| \end{aligned}$$

Proof. To show that $(\hat{\mathbb{R}}, d)$ is a soft topological ring we must verify that the conditions (i) and condition (ii) of Theorem 3.5 are satisfied

Let $\hat{x}, \hat{y} \in \hat{\mathbb{R}}$ and $B(\hat{x} - \hat{y}, \hat{\epsilon})$ be an arbitrary S.ONhd of $x - y$, where $\hat{\epsilon} > \tilde{0}$. Our claim that there exist soft open S.ONhds $B\left(\hat{x}, \hat{\delta}_1\right)$ of x and $B\left(\hat{y}, \hat{\delta}_2\right)$ of y, such that $f\left(B\left(\hat{x}, \hat{\delta}_1\right), B\left(\hat{y}, \hat{\delta}_2\right)\right) \subseteq B(\hat{x} - \hat{y}, \hat{\epsilon})$. To prove the claim, choose $\hat{\delta}_1 = \hat{\delta}_2 = \hat{\epsilon}/2$ and since $\hat{x} - \hat{y} \in B(\hat{x} - \hat{y}, \hat{\epsilon})$ then $\{\hat{x} - \hat{y} \in \hat{\mathbb{R}} | d(\hat{x} - \hat{y}, \hat{x_0} - \hat{y_0}) < \hat{\epsilon}\}$. It follows that

$$|\hat{x} - \hat{y} - (\hat{x_0} - \hat{y_0})| = |\hat{x} - \hat{x_0} + \hat{y}_0 - \hat{y}| \leqslant |\hat{x} - \hat{x_0}| + |\hat{y} - \hat{y}_0| < \hat{\delta}_1 + \hat{\delta}_2 = \hat{\epsilon}$$

Therefore, the conditions (i) of Theorem 3.5 is satisfied.

Let $\hat{x}, \hat{y} \in \hat{\mathbb{R}}$ and $B(\hat{x}\hat{y}, \hat{\epsilon})$ be an arbitrary S.ONhd of $x + y$, where $\hat{\epsilon} > \tilde{0}$. Our claim that there exist soft open S.ONhds $B\left(\hat{x}, \hat{\delta}_1\right)$ of x and $B\left(\hat{y}, \hat{\delta}_2\right)$ of y, such that $f\left(B\left(\hat{x}, \hat{\delta}_1\right), B\left(\hat{y}, \hat{\delta}_2\right)\right) \subseteq B(\hat{x}\hat{y}, \hat{\epsilon})$. To prove the claim, choose $\hat{\delta}_1\hat{\delta}_2 = \hat{\epsilon}$. Since $\hat{x}\hat{y} \in B(\hat{x}\hat{y}, \hat{\epsilon})$ then $\{\hat{x}\hat{y} \in \hat{\mathbb{R}} | d(\hat{x}\hat{y}, \hat{x_0}\hat{y_0}) < \hat{\epsilon}\}$. It follows that

$$\begin{aligned} d(\hat{x}\hat{y},\hat{x_0}\hat{y_0}) = & \ |\hat{x}\hat{y} - \hat{x_0}\hat{y_0}| \\ \leqslant & \ |\hat{x}||\hat{y} - \hat{y_0}| + |\hat{y_0}||\hat{x} - \hat{x_0}| \\ \leqslant & \ (|\hat{x_0}| + |\hat{x} - \hat{x_0}|)|\hat{y} - \hat{y_0}| + |\hat{y_0}||\hat{x} - \hat{x_0}| \\ = & \ |\hat{x_0}||\hat{y} - \hat{y_0}| + |\hat{x} - \hat{x_0}||\hat{y} - \hat{y_0}| + |\hat{y_0}||\hat{x} - \hat{x_0}| \\ < & \ |\hat{x} - \hat{x_0}||\hat{y} - \hat{y_0}| \\ < & \ \hat{\delta}_1\hat{\delta}_2 = \hat{\epsilon} \end{aligned}$$

Therefore, the conditions (ii) of Theorem 3.5 is satisfied. □

4.1. Suggested studies

1. It is well known that the ring of p-adic numbers $\mathbb{Z}_p$ with their metric space is a topological ring having numerous applications in algebra and number theory. This topological ring is a compact, complete and metrizable space. However, Das and Samanta (2012) produced the set of soft real numbers and discussed its properties. In addition, a soft metric space has been produced by Das and Samanta (2013). This makes us wonder if the ring of p-adic numbers with their p-adic soft metric space is a S.T.R? If so, what are the conditions leading us to generalize applications of $\mathbb{Z}_p$ in algebra and number theory in the aspect of the soft theory?
2. The Haar measure is exclusively applicable to the locally compact topological groups. A group having a Haar measure means we can import topics from measure theory and analyze it on this group, namely Fourier analysis. Nonetheless, many papers discussed locally soft compactness of soft sets like Aygünoğlu and Aygün (2012) and Bayramov and Gunduz (2013). Hence, can the Haar measure be defined as a generalization of topological groups in terms of soft theory aspect? If so, what are the conditions leading us to generalize the Haar measure to our concept of S.T.G?

5. Discussion

In literature (Shah and Shaheen, 2014), initiated the concept of topological soft rings, as we have named it in Remark 3.2, by applying the topological structures on the soft rings. (Tahat et al., 2018) introduced the idea of soft topological soft rings by examining the soft topological structures on the soft rings. But no one has studied the combination between the rings and the soft topological spaces.

Therefore, we have produced the concept of soft topological rings to complete the gaping in the studies of the connections between the soft topological space and the rings theory.

Recently, we noted that many rings appeared in soft settings where we can't study these types of rings in the standard topological space, but we must examine it in the aspect of soft topological space where their elements are soft sets. So, if we discuss these types of rings as soft topological rings under some circumstances, then we can study the soft topological properties like the separation axioms and soft compactness on these rings as they are soft topological rings. Therefore, the concept of soft topological rings is essential to those who study the soft topological structures over rings.

6. Conclusions

We have produced the concept of soft topological rings by analyzing the soft topological structures over the rings directly. Also, we have examined the relationship between the our new notion S.T.R and the notions of (S.T.S.R, (Tahat et al., 2018)), (T.S.R, (Shah and Shaheen, 2014)) and (T.R, (Warner, 1993)), as we have illustrated the in Fig. (1). Moreover, we have discussed the subsystems

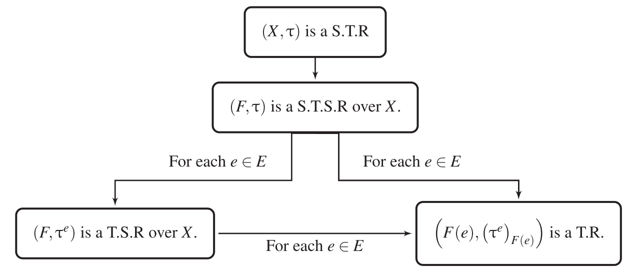

Fig. 1. Please provide a caption for Fig. 1.

of the S.T.R by producing the concepts of soft topological subrings and soft topological ideals.

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Declaration of Competing Interest

We declare that no potential conflict of interest was reported by the authors.

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