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ORIGINAL ARTICLE

A note on "The tanh–coth method combined with the Riccati equation for solving nonlinear coupled equation in mathematical physics"

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Abstract The recent paper "The tanh–coth method combined with the Riccati equation for solving nonlinear coupled equation in mathematical physics" (J. King Saud Univ. Sci. 23 (2011) 127–132) is analyzed. We show that the authors of this paper solved equations with the well known solutions. One of these equations is the famous Riccati equation and the other equation is one for the Weierstrass elliptic function. We present the general solutions of these equations. As this takes place, 19 solutions by authors do not satisfy the equation but the other 29 solutions can be obtained from the general solutions.

In the recent paper Bekir and Cevikel (2011) tried to study the $(2+1)$-dimensional system of two equations in the form

$$u_t + \alpha u_{xxy} + 4\alpha u v_x + 4\alpha u_x v = 0, \tag{1}$$

$$u_x = v_y \tag{2}$$

However, in fact, these authors looked for exact solutions of these equations taking the traveling waves into account

$$u(x,t) = u(\xi), \quad v(x,t) = v(\xi), \quad \xi = x + y - \beta\, t \tag{3}$$

As a result using variables (3) the authors transform equations (1) and (2) to the system of nonlinear ordinary differential equations in the form:

$$-\beta u' + \alpha u''' + 4\alpha u v' + 4\alpha u' v = 0 \tag{4}$$

$$u' = v'. \tag{5}$$

After integrating both equations of system (4) and (5), the authors omitted constants of integration and obtained the system

$$\alpha u'' - \beta u + 4\alpha u^2 = 0, \tag{6}$$

$$u = v. \tag{7}$$

In fact, taking arbitrary constants into account we have the system of equations in the form

$$\alpha u'' + 4\alpha u^2 - Cu - C_2 = 0 \tag{8}$$

$$u = v - C_1, \tag{9}$$

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where $C = \beta - 4\alpha C_1$ and C_1, C_2 are arbitrary constants. Authors used the tanh–coth method for finding some exact solutions of system (8) and (9). However the first equation of system (8) and (9) (or system (6) and (7)) has the well known general solution. Let us show this fact.

After multiplying on u' Eq. (8) can be integrated with respect to ξ again. In this case we have the equation in the form

$$(u')^2 = -\frac{8}{3}u^3 + D_1u^2 + D_2u + D_3, \tag{10}$$

where $D_1 = \frac{C}{\alpha}$, $D_2 = \frac{2C_2}{\alpha}$ and D_3 are arbitrary constants. The general solution of (10) was found more than one century ago and may be expressed via the Weierstrass elliptic function (see, for example, Polyanin and Zaitsev (2003); Kudryashov and Sinelshchikov (2012)). We can see it if we substitute $u = -\frac{3}{2}\wp(\xi) + \frac{D_1}{8}$ into Eq. (10). In this case we obtain the following equation for the Weierstrass elliptic function

$$(\wp')^2 = 4\wp^3 - g_2\wp - g_3$$
$$g_2 = \frac{2D_2}{3} + \frac{D_1^2}{12}, \qquad g_3 = -\frac{4D_3}{9} - \frac{D_1^3}{216} - \frac{D_2D_1}{18} \tag{11}$$

As a result we have solution of Eq. (8) in the form

$$u = \frac{C}{8\alpha} - \frac{3}{2}\wp(\xi - \xi_0, g_2, g_3) \tag{12}$$

where

$$g_2 = \frac{4C_2}{3\alpha} + \frac{C^2}{12\alpha^2}, \quad g_3 = -\frac{4D_3}{9} - \frac{C^3}{216\alpha^3} - \frac{C_2C}{9\alpha^2} \tag{13}$$

We checked all solutions by Bekir and Cevikel (2011) and obtained that 19 solutions from this paper: u_3, u_4, u_{10}, u_{11}, u_{12}, u_{13}, u_{16}, u_{17}, u_{23}, u_{24}, u_{25}, u_{26}, u_{40}, u_{41}, u_{42}, u_{43}, u_{44}, u_{47} and u_{48} do not satisfy Eq. (6) and sequently these solutions are wrong. The other 29 solutions of Eq. (6) of Bekir and Cevikel (2011) can be obtained from solution (12). For example, when D_2 and D_3 in (11) are equal to zero, solution of (6) can be transformed to the form:

$$u = -\frac{3D_1}{8} + \frac{3D_1}{8}\tanh^2\left(\sqrt{-\frac{D_1}{4}}(\xi - \xi_0)\right). \tag{14}$$

Assuming $D_1 = -1$ and $\xi_0 = 0$, we obtain

$$u = \frac{3}{8}\left(1 - \tanh^2\left(\frac{\xi}{2}\right)\right) = \frac{3}{8}\frac{1}{\cosh^2\left(\frac{\xi}{2}\right)} = \frac{3}{8}\mathrm{sech}^2\left(\frac{\xi}{2}\right) \tag{15}$$

which coincides with u_2 of paper by Bekir and Cevikel (2011) (see formula (31)).

For $D_1 = 4$ we have

$$u = -\frac{3}{2}(1 + \tan^2(\xi - \xi_0)) = -\frac{3}{2}\frac{1}{\cos^2(\xi - \xi_0)}$$
$$= -\frac{3}{2}\sec^2(\xi - \xi_0) \tag{16}$$

For $\xi_0 = 0$ the obtained solution coincides with u_9 in work by Bekir and Cevikel (2011) (see formula (38)). If $\xi_0 = \frac{\pi}{2}$ our solution coincides with u_8 in Bekir and Cevikel (2011) (see formula (37)). So it is not hard to make sure that all 29 ''new'' solutions found by Bekir and Cevikel (2011) are contained in the solution (14) and consequently in general solution (12).

Bekir and Cevikel (2011) also considered the Riccati equation in the form

$$Y' = A + BY + CY^2, \tag{17}$$

where A, B and C are constants. Authors applied the tanh–coth method at $B = 0$ to look for exact solution of Eq. (17). However one can note that Eq. (17) also has the general solution. In the case $B = 0$ the general solution of Eq. (17) takes the form

$$Y = \sqrt{\frac{A}{C}}\tan(\sqrt{AC}(D + \xi)), \tag{18}$$

where D is an arbitrary constant. It is not hard to see that all 12 specific solutions (formulae (10) in Bekir and Cevikel (2011), which are used for constructing the solutions of Eq. (6), are contained in (18). For example, if we take $A = -C = \frac{1}{2}$ we have $Y_1 = \tanh\left(\frac{\xi}{2}\right)$ in the case $D = 0$ and we obtain $Y_1 = \coth\left(\frac{\xi}{2}\right)$ in the case of $D = -i\pi$. For $A = 1$, $C = 4$ we obtain both solutions Y_8 which differs only in the phase of the argument (i.e. arbitrary constant D).

In conclusion, let us note that idea by Bekir and Cevikel (2011) using the simplest equation method for finding exact solutions was published in paper by Kudryashov (1990, 2005). We have to point out that the authors of work Bekir and Cevikel (2011) made many mistakes discussed in papers Kudryashov (2009a,b), Kudryashov and Loguinova (2009), Kudryashov and Soukharev (2009), Kudryashov (2010), Kudryashov and Ryabov (2010), Kudryashov et al. (2010), Parkes (2009, 2010a,b,c).

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