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### ORIGINAL ARTICLE

# Kinetic analysis of nonisothermal decomposition of  $(Mg_5(CO_3)_4(OH)_2.4H_2O/5Cr_2O_3)$  crystalline mixture

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#### **KEYWORDS**

Thermal decomposition; Model fitting; Model free; Isokinetic; Kinetic parameter; MgCr2O4 spinel

Abstract Pure MgCr<sub>2</sub>O<sub>4</sub> spinel was synthesized from crystalline mixture of  $(Mg_5(CO_3)_{4}(O-))$  $H$ <sub>2</sub>·4H<sub>2</sub>O/5Cr<sub>2</sub>O<sub>3</sub>) by heating at 900 °C for 27 h. TG, DTA, FT-IR and XRPD techniques were used to follow the reactions and identify the products. Nonisothermal kinetics of thermal decomposition of un-irradiated and  $\gamma$ -irradiated physical crystalline mixtures were studied in static air. The kinetic parameters were obtained through model-fitting and model-free methods, and artificial isokinetic relationship (IKR) for multi-step processes. The results show that the decomposition for both un-irradiated and  $\gamma$ -irradiated mixtures proceed through two steps with different reaction mechanisms. The first is a third-order reaction  $(F_3)$  mechanism followed by one-dimensional diffusion  $(D_1)$  as a second step.

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#### 1. Introduction

The tremendous increase in the development of the industrial and transportation sectors have led to consumption of huge quantities of fuel which results in an increase in the amount of hazardous gases such as  $NO<sub>x</sub>$ , CH and  $CO<sub>x</sub>$  emitted to the environment. Low cost transition metal mixed oxides such as spinels are active in catalytic total oxidation processes aimed at limiting of air pollution, which often is carried out on the

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more expensive noble metal-based catalysts. Spinels with  $AB_2O_4$  formula such as pirochromite  $MgCr_2O_4$  have additional important technological applications such as magnetic materials, high temperature ceramics, combustion catalysis, catalytic support, strength agents, sensor elements and inter connection materials for solid oxide fuel cells [\(Shimizu et al.,](#page-9-0) [1990; Gengembre et al., 1999; Docherty et al., 2001\)](#page-9-0).  $MgCr<sub>2</sub>O<sub>4</sub>$ is normal spinel has space group F3dm, with 56 atoms per unit cell  $(Z = 8)$ , Mg and Cr ions occupy the tetrahedral and octahedral site, respectively [\(Bhatta and Nayak, 2002](#page-8-0)).

Thermal decomposition of various compounds are of major importance because of their frequent applications in calcinations metallurgy and in the production of large-surface materials for sorbents and catalysis ([Hartman et al., 1994](#page-8-0)). Many studies have been carried out on the effect of  $\gamma$ -irradiation on the thermal decomposition of inorganic solids ([Monshi](#page-8-0) [et al., 1998; Mahfouz et al., 2000, p. 59\)](#page-8-0). In general, the effects with increasing dose are in changing of the induction period and acceleration of the decomposition process i.e. the decrease in time or temperature required to complete the reaction in case of the pre-irradiated material which were attributed to formation of additional nucleation site and reactive center ([Mahfouz et al., 2000, p. 363\)](#page-8-0).

In the present work, we report the kinetic studies of the thermal decomposition of crystalline mixtures  $(Mg_5(CO_3)_{4}(O-))$ H)<sub>2</sub>·4H<sub>2</sub>O/5Cr<sub>2</sub>O<sub>3</sub>) before and after  $\gamma$ -irradiation by applying model-fitting and model-free kinetic approaches to the nonisothermal thermoanalytical data. Formation of magnesium chromite from above materials is also reported. TG, DTA, FT-IR and XRPD techniques were used to follow the reactions and identify the products.

#### 2. Experimental

Powder of heavy magnesium carbonate hydrate  $Mg_5(CO_3)_4(O H$ <sub>2</sub> $\cdot$ 4H<sub>2</sub>O and Cr<sub>2</sub>O<sub>3</sub> were obtained commercially from (BDH reagent grade) and were used without any further purifications. 1:5 molar ratio mixtures of the starting materials were calcined at different temperature in the range  $(100-900 \degree C)$ for 9 h each. For irradiation, samples were encapsulated under vacuum in glass vials and were exposed to successively increasing doses of  $\gamma$ -irradiation at constant intensity using Co-60  $\gamma$ ray cell 220 (Nordion MDS, Ontario, Canada) at a dose rate of  $10^2$  kGy/h. The source was calibrated against Fricke ferrous sulfate dosimeter, and the dose rate in the irradiated samples was calculated by applying appropriate corrections on the basis of photon mass attenuation and energy-absorption coefficient for the sample and the dosimeter solution [\(Spinks and](#page-9-0) [Woods, 1990](#page-9-0)). IR spectra were recorded as KBr pellets using a Perkin–Elmer 1000 FT-IR spectrometer. XRPD measurements were carried out on a Jeol D8030 X-ray diffractometer using a nickel filter (Cu K $\alpha$   $\lambda$  = 1.5418 Å). The thermal decomposition of physical mixture was followed by nonisothermal (dynamic) thermogravimetric techniques using TGA-7 (Perkin–Elmer) thermogravimetric analyzer. Dynamic experiments were performed at heating rate 5, 10, 15, 20 and 25  $\mathrm{°C}$ / min.

#### 3. Result and discussion

#### 3.1. X-ray powder diffraction (XRPD) analysis

[Fig. 1](#page-2-0) shows XRPD patterns of the calcined mixtures at different temperatures for different duration times. All phases in the mixtures were identified by FARHAN program ([Al-Farhan,](#page-8-0) [1999\)](#page-8-0). The XRPD pattern of the mixture heated at 300  $\degree$ C displays only the peaks of  $Cr_2O_3$ , indicating that the heavy magnesium carbonate hydrate had converted to amorphous lower carbonate as documented by FT-IR measurement (see [Fig. 2](#page-2-0)). The XRPD of the mixture calcined at 500  $\degree$ C shows only the characteristic peaks of  $Cr_2O_3$  and MgO. No notable changes in XRPD of the mixtures heated up to  $600^{\circ}$ C were detected compared with that heated at 500  $^{\circ}$ C. The XRPD of the mixture calcined at 625 °C are dominated by the peaks of  $Cr_2O_3$ and MgO, with the main characteristic peaks of  $MgCr<sub>2</sub>O<sub>4</sub>$  just starting to show up in the XRPD.  $MgCr<sub>2</sub>O<sub>4</sub>$  becomes the dominant phase in the mixture calcined at 900 °C for 9 h.  $MgCr_2O_4$ is the only phase present in the XRPD pattern of the mixture heated at  $900 °C$  for 27 h. The same result was obtained from the counterpart of irradiated mixture.

#### 3.2. FT-IR analysis

[Fig. 2](#page-2-0) shows the FT-IR spectra of the starting materials before and after  $\gamma$ -irradiation. The spectrum of un-irradiated heavy magnesium carbonate hydrate (a) shows bands of the carbonate anion at  $1120 \text{ cm}^{-1}$  (symmetric stretching), 1485 and 1423 cm<sup>-1</sup> (asymmetric stretching), 745 and 714 cm<sup>-1</sup> (symmetric bend), and 854, 886 and 797  $cm^{-1}$  (asymmetric bend). The two main bands at  $3515$  and  $3450 \text{ cm}^{-1}$  are due to lattice water vibration and the 3649 cm<sup>-1</sup> is due to OH<sup>-</sup> stretching ([White, 1971\)](#page-9-0). Spectrum (c) shows five main bands at 637, 569, 443, 415 and 305  $cm^{-1}$  which are due to lattice vibration of un-irradiated  $Cr_2O_3$ . Neither disappearance nor appearance of new bands was observed as result of  $\gamma$ -irradiation up to  $10^3$  kGy total  $\gamma$ -ray dose as well as seen in (b and d, respectively).

The FT-IR spectra for the calcined mixtures are shown in [Fig. 3](#page-3-0). The spectra of the mixture calcined at 300  $\degree$ C shows main bands at 643, 573, 442, 413 and 305  $\text{cm}^{-1}$  which are due to Mg–O and Cr–O lattice vibrations, and the weak band at 858 cm<sup>-1</sup> is due to carbonate anion. At 500 °C, the spectra displays, in addition to, lattice vibration bands of MgO and  $Cr_2O_3$ , two bands at 957 and 834 cm<sup>-1</sup> which are due to Cr= $O$  stretching of amorphous  $\alpha$ -MgCrO<sub>4</sub> ([Roy et al.,](#page-8-0) [1969\)](#page-8-0). At 625 °C, the bands of  $\alpha$ -MgCrO<sub>4</sub> completely disappeared from the spectra, and the characteristic band of  $MgCr_2O_4$  at 429 cm<sup>-1</sup> ([Williey et al., 1993](#page-9-0)) is not detectable, although it present and it was confirmed by XRPD. However, the 429  $\text{cm}^{-1}$  band is observable in FT-IR spectra of the mixture heated at  $650 \degree C$ . The spectra of the mixture calcined at 900 °C for 27 h displays only the bands of  $MgCr_2O_4$ , and the characteristic band of  $Cr_2O_3$  at 305 cm<sup>-1</sup> is completely disappeared.

#### 3.3. TG and DTA analysis

[Fig. 4](#page-3-0) shows TG and DTA curves for thermal decomposition of heavy magnesium carbonate in a pure form and in physical mixture with chromium(III)oxide.

[Fig. 4a](#page-3-0) shows typical TG–DTA curves of the thermal decomposition of pure heavy magnesium carbonate hydrate. The TG of pure salt showed three overlapped decomposition steps. The first decomposition step was in the range of (100– 250)  $\degree$ C attributed to loss of water of crystallization, this decomposition was accompanied by an endothermic peak at  $230 \degree C$ . The second decomposition step was in the range of  $(370-440)$  °C due to loss of hydroxyl water and was accompanied by an endothermic peak at 430  $^{\circ}$ C. The third decomposition step was in the range of  $(440-500)$  °C due to decomposition of  $MgCO<sub>3</sub>$  and was accompanied by sharp endothermic peak at 480 °C. Very sharp exothermic peak was detected at 460 °C attributed to the crystallization of  $MgCO<sub>3</sub>$  from the amorphous lower carbonate ([Criado et al., 1979; Khan et al., 2001; Sawada](#page-8-0) [et al., 1979a, p. 32, 1979b, p. 33, 1979c, p. 34\)](#page-8-0).

[Fig. 1b](#page-2-0) shows the TG and DTA curves of crystalline mixture. The DTA show an endothermic peak detected at almost the same temperature range as for pure salt for loss of water of crystallization. A more broad second endothermic peak was detectable at 440 °C due to loss of hydroxyl water and starting to form the chromate intermediate. Two endothermic peaks were detectable at 490  $\mathrm{^{\circ}C}$  and 500  $\mathrm{^{\circ}C}$  due to the decomposition of chromate intermediate and formation of amorphous

<span id="page-2-0"></span>

Figure 1 XRPD patterns of un-irradiated  $(Mg_5(CO_3)(OH)_2/H_2O)$  and the calcined un-irradiated mixtures at different temperatures for different time duration.



Figure 2 FT-IR spectra of un-irradiated (a and c) and irradiated (b and d) of  $Mg_5(CO_3)_2(OH)_24H_2O$  and  $Cr_2O_3$ , respectively.

<span id="page-3-0"></span>

Figure 3 FT-IR spectra of un-irradiated calcined mixtures at different temperatures for different time duration.



Figure 4 TG/DTA curves of  $Mg_5(CO_3)_4(OH)_2$ <sup>4</sup>H<sub>2</sub>O (a), physical crystalline mixtures before (b) and after (c) irradiation.

magnesium chromite. The decomposition behavior of  $\gamma$ -irradiated mixture is similar to that of un-irradiated mixture as we seen in [Fig. 1c](#page-2-0). Thus,  $\gamma$ -irradiation has no apparent effect on the decomposition behavior of  $\gamma$ -irradiated physical mixture with  $10^2$  kGy total dose.

of the investigated crystalline mixture and formation of chromite spinel could be suggested.

#### 4. Kinetic studies

Based on the foregoing discussion of XRPD, FT-IR, TG and DTA analysis the following [Scheme 1](#page-4-0), for decomposition Heterogeneous solid-state reaction can empirically be described by a single-step kinetic equation:

<span id="page-4-0"></span>

Scheme 1 The proposed reaction mechanism or the thermal decomposition of the investigated crystalline mixture and formation of chromite spinel.

$$
\frac{d\alpha}{dt} = k(T)f(\alpha) \tag{1}
$$

where t is the time  $(\min^{-1})$ , T is the temperature (K), k is the reaction rate constant,  $\alpha$  is the extent of conversion  $\alpha = \{W_0 - W/W_0 - W_\infty\}$ ;  $W_0$  is the initial weight of the sample (mg),  $W$  is the sample weight (mg) at any temperature  $T$ and  $W_{\infty}$  is the final sample weight (mg)),  $d\alpha/dt$  is the reaction rate (min<sup>-1</sup>) and  $f(x)$  is the reaction model. The reaction model may take various forms [\(Sharp et al., 1966; Galway and](#page-9-0) [Brown, 1999](#page-9-0)), some of which are given in Table 1.

The rate constant,  $k$ , usually has an Arrhenius temperature dependence

$$
k = A \exp\left(\frac{-E_a}{RT}\right) \tag{2}
$$

where A (min<sup>-1</sup>) is pre-exponential factor,  $E_a$  is the activation energy (kJ mol<sup>-1</sup>) and R is the universal gas constant.

Replacing  $k(T)$  in Eq. (1) with the Arrhenius equation gives

$$
\frac{d\alpha}{dt} = A \exp\left(\frac{-E_a}{RT}\right) f(\alpha) \tag{3}
$$

Under a nonisothermal condition, at constant heating rate  $\beta = dT/dt$  Eq. (3) may be written as

$$
\frac{d\alpha}{dT} = \frac{d\alpha}{dt} \left(\frac{1}{\beta}\right) = \frac{A}{\beta} \exp\left(\frac{-E_a}{RT}\right) f(\alpha)
$$
\n(4)

Upon integration Eq. (4) gives

$$
g(\alpha) = \frac{A}{\beta} \int_0^T \exp\left(\frac{-E_a}{RT}\right) dT \tag{5}
$$

If  $E_a/RT$  is replaced by (x) and integration limits are transformed then Eq. (5) becomes

$$
g(\alpha) = \frac{AE_a}{\beta R} \int_x^{\infty} \frac{e^{-x}}{x^2} dx
$$
 (6)

Eq. (6) can be written as

$$
g(\alpha) = \frac{AE_a}{\beta R} P(x) = \frac{A}{\beta} I(Ea, T)
$$
\n(7)

The exponential integral  $(P(x))$  has no analytical solution but has many approximations ([Khawam and Flanagan, 2005\)](#page-8-0).

Kinetic parameters can be obtained from nonisothermal rate laws by both model-fitting and isoconversional (modelfree) methods.

Force-fitting experimental data to different model function  $f(x)$  is denoted as model-fitting methods, after the  $f(x)$  model has been selected from the best linear fit for a series of temperature,  $k(T)$  is evaluated ([Rodante et al., 2002](#page-8-0)).

**Table 1** Algebraic expressions of  $f(x)$  and  $g(x)$  for the reaction models considered in the present work.

| Symbol         | No.            | Reaction model                                  | $f(\alpha)$   | $\mathcal{Z}$                           |
|----------------|----------------|---|---|---|
|                | $D_1$          | One-dimensional diffusion                       | $1/2\alpha$   | $\alpha^2$                              |
|                | D <sub>2</sub> | Two-dimensional diffusion (bi-dimensional       | $1/[-\ln(1-\alpha)]$                                    | $(1 - \alpha) \ln(1 - \alpha) + \alpha$ |
|                |                | particle shape) Valensi equation                |   |   |
| 3              | $D_3$          | Three-dimensional diffusion (tri-dimensional    | $3(1-\alpha)^{1/3}/2[(1-\alpha)^{-1/3}-1]$              | $[1-(1-\alpha)^{1/3}]^2$                |
|                |                | particle shape) Jander equation                 |   |   |
| $\overline{4}$ | $D_4$          | Three-dimensional diffusion (tri-dimensional    | $3/2[(1-\alpha)^{-1/3} - 1]$                            | $(1-2\alpha/3)-(1-\alpha)^{2/3}$        |
|                |                | particle shape) Ginstling-Brounshtein           |   |   |
| 5              | R <sub>2</sub> | Phase-boundary controlled reaction (contracting | $2(1-\alpha)^{1/2}$                                     | $[1-(1-\alpha)^{1/2}]$                  |
|                |                | area, <i>i.e.</i> , bi-dimensional shape)       |   |   |
| 6              | $R_3$          | Phase-boundary controlled reaction (contracting | $3(1-\alpha)^{2/3}$                                     | $[1-(1-\alpha)^{1/3}]$                  |
|                |                | volume, <i>i.e.</i> , tri-dimensional shape)    |   |   |
|                | $F_1$          | First-order (Mampel)                            | $(1-\alpha)$  | $[-\ln(1-\alpha)]$                      |
| 8              | F <sub>2</sub> | Second-order                                    | $(1 - \alpha)^2$  | $(1 - \alpha)^{-1} - 1$                 |
| 9              | $F_3$          | Third-order                                     | $(1 - \alpha)^3$  | $(1/2)[(1-\alpha)^{-2}-1]$              |
| 10             | $F_{3/2}$      | Three-halves order                              | $(1 - \alpha)^{3/2}$                                    | $2[(1-\alpha)^{-1/2}-1]$                |
| 11             | A <sub>2</sub> | Avrami-Eroféev ( $n = 2$ )                      | $2(1-\alpha)[-ln(1-\alpha)]^{1/2}]$                     | $[-\ln(1-\alpha)]^{1/2}$                |
| 12             | A <sub>3</sub> | Avrami-Eroféev ( $n = 3$ )                      | $3(1-\alpha)[-ln(1-\alpha)]^{2/3}$                      | $[-\ln(1-\alpha)]^{1/3}$                |
| 13             | $A_{3/2}$      | Avrami-Eroféev ( $n = 1.5$ )                    | $(3/2)(1 - \alpha)$ [-ln(1 - $\alpha$ )] <sup>1/3</sup> | $[-\ln(1-\alpha)]^{2/3}$                |
| 14             | $A_4$          | Avrami-Eroféev ( $n = 4$ )                      | $4(1 - \alpha) [-\ln(1 - \alpha)]^{3/4}$                | $[-\ln(1-\alpha)]^{1/4}$                |

Model-free isoconversional methods allow for estimating the activation energy as function of  $\alpha$  without choosing the reaction model, the basic assumption of these methods is that the reaction rate of constant extent of conversion  $\alpha$  depends only on the temperature. Hence, constant  $E_a$  value can be expected in the case of single stage decomposition, while for multi-step process  $E_a$  varies with  $\alpha$  due to the variation in the relative contributions of single steps to the overall reaction rate ([Khawam and Flanagan, 2005; Vyazovkin and Sbirrazzuoli,](#page-8-0) [2002\)](#page-8-0).

#### 5. Model-fitting approach

There are several nonisothermal model-fitting methods. One of the most popular is the Coats and Redfern method (CR method), which utilizes the asymptotic series expansion in approximating  $P(x)$  Eq. [\(7\),](#page-4-0) producing the following equation:

$$
\ln \frac{g(\alpha)}{T^2} = \ln \left( \frac{AR}{\beta E_a} \left[ 1 - \left( \frac{2RT^*}{\beta E_a} \right) \right] \right) - \frac{E_a}{RT}
$$
 (8)

where  $T^*$  is the mean experimental temperature.

The other method proposed by Clark and Kennedy (CK method) is based on the expression,  $T = \beta t + T^0$ , where  $T^0$ is initial temperature.

The basic equation is:

$$
\frac{\beta g(\alpha)}{T - T^0} = A \exp\left(-\frac{E_a}{RT}\right) \tag{9}
$$

Taking the logarithm of both sides of the equation:

$$
\ln \frac{\beta g(\alpha)}{T - T^0} = \ln A - \frac{E_a}{RT}
$$
\n(10)

Plotting the left-hand side of both Eqs. (8) and (10) against  $1/T$  should give straight lines for the reaction models listed in [Table 1](#page-4-0) ([Vyzovkin and Wight, 1999\)](#page-9-0) and the result are shown in Fig. 5. From the slope and intercept we can determine  $E_a$ and lnA, respectively.

The values of activation energy  $(E_a)$ , pre-exponential factors (lnA) and the coefficients of linear correlations (r) for kinetic models are presented in [Table 2](#page-6-0). The correlation coefficient (r) is sometimes used as a parameter for choosing the best model.

As listed in [Table 2](#page-6-0) for both applied methods (CR and CK methods), it can be found that  $D_1$  is the best model show the linear relationships of the data for both irradiated and un-irradiated mixtures. The values of  $E_a$ , ln A (Arrhenius parameters) and r obtained from CR and CK methods using the selected model for the processes obtained in [Table 2](#page-6-0) are very low. From this fact it may be concluded that the reaction under consideration is a typical complex multi-step reaction with more than one mechanism.

#### 5.1. Model-free approach

The Kissinger–Akahira–Sunose method (KAS method) is based on the Coats–Redfern approximation of  $P(x) \cong$  $(\exp(-x))/x^2$  which transformed Eq. [\(7\)](#page-4-0) to:

$$
\ln\left(\frac{\beta}{T^2}\right) = \ln\frac{AR}{E_a g(\alpha)} - \frac{E_a}{RT}
$$
\n(11)

Another method proposed by Tang (T method) is based on the approximate formula which introduced into Eq. [\(7\)](#page-4-0). Taking the logarithms of both sides, Eq. (12) is obtained as:

$$
\ln\left(\frac{\beta}{T^{1.894661}}\right) = \ln\left[\frac{AE_a}{Rg(\alpha)}\right] + 3.635041 - 1.894661 \ln E_a
$$

$$
- 1.001450 \frac{E_a}{RT}
$$
(12)

A plot of the left-hand sides of both Eqs. (11) and (12) versus  $1/T$  give a group of straight lines at each  $\alpha$  ([Vyzovkin and](#page-9-0) [Wight, 1999](#page-9-0)) as can be seen in [Fig. 6,](#page-7-0) which give apparent activation energy from the slope for a particular  $\alpha$  without considering a selected model.

The Vyazovkin isoconversional method (VYZ method) (Vyzovkin and Wight, 1999; Janković et al., 2007) is a nonisothermal method which utilizes an accurate nonlinear Senium-Yang approximation of  $P(x)$  (Eq. [\(7\)](#page-4-0)), which leads to:

$$
\Omega = \left| \sum_{i=1}^{n} \sum_{i \neq j}^{n} \frac{I(E_{a,x}, T_{a,i}) \beta_j}{I(E_{a,x}, T_{a,j}) \beta_i} \right| \tag{13}
$$

$$
I(Ea, \alpha, T\alpha) = \int_0^{T_{ai}} \exp\left(\frac{-E_{a,\alpha}}{RT}\right) dT \tag{14}
$$

where *n* the number of heating rates,  $I(E_{a,\alpha}, T_{\alpha})$  the exponential integral  $(P(x))$  that results from heating rate  $\beta$ . The 5th degree Senum-Yang approximation was chosen for our work.

The  $E_{a,x}$  can be determined at any particular value of  $\alpha$  by finding the value of  $E_\alpha$  for which the objective function  $\Omega$  is minimized.

The temperature integral can be evaluated by several approximation. We have used Gorbachev, Agrawal and



Figure 5 (a) CR method and (b) CK method.

<span id="page-6-0"></span>

Sivasubramanian and Cai for integration of Eq. (14) ([Ghoshal](#page-8-0) [and Saha, 2006; Maiti et al., 2006](#page-8-0)).

$$
\int_0^T \exp\left(\frac{-E_x}{RT_{xi}}\right) dT = \frac{RT_{xi}^2}{E_x} \left(\frac{1}{1 + \frac{2RT}{Ex}}\right) \exp\left(\frac{-E_x}{RT}\right) \tag{15}
$$
\n
$$
\int_0^T \exp\left(\frac{-E_x}{RT_{xi}}\right) dT = \frac{RT_{xi}^2}{E_x} \left[\frac{1 - 2RT/E_x}{1 - 5(ER/E_x)^2}\right] \exp\left(\frac{-E_x}{RT_{xi}}\right) \tag{16}
$$

$$
\int_0^T \exp\left(\frac{-E_x}{RT_{xi}}\right) dT = \frac{RT_{xi}^2}{E_x} \left[\frac{E_x/RT_{xi} + 0.66691}{E_x/RT_{xi} + 2.64943}\right] \exp\left(\frac{-E_x}{RT_{xi}}\right)
$$
\n(17)

[Fig. 7](#page-7-0) shows the variation of apparent activation energy  $E_{\alpha\alpha}$ , as a function of the extent of conversion  $\alpha$  of the data for un-irradiated and  $\gamma$ -irradiated mixtures. From [Fig. 7](#page-7-0) we can recognized that the dependence of  $E_{a\alpha}$  on  $\alpha$  is almost the same by using different isoconversional methods. And these differences in the values of  $E_{a,x}$  could be due to the approximation of temperature integral that were used in the derivations of relations that ground KAS, T and VYZ methods ([Vyzovkin and Wight, 1999\)](#page-9-0). Also, [Fig. 7](#page-7-0) shows that, the values of  $E_{\alpha\alpha}$  for y-irradiated mixture are reduced compared to un-irradiated one. The reducing of the values of activation energy of the decomposition reaction could be attributed to the formation of additional nucleation sites and reactive centers. From the dependence of  $E_{a,x}$  on  $\alpha$  we can assumed that there are at least two steps, the first step at  $\alpha$  < 0.3 show decrease of activation energy by increase extant of conversion which is lie to dehydration process, the second step at  $\alpha > 0.3$  which is show an increase of the activation energy by increase extent of conversion which lie to decarbonation and formation of the magnesium chromite.

#### 5.2. Compensation effect

Use of the artificial isokinetic relationship (IKR) that occurs on fitting various reaction models to the same set of nonisothermal kinetic data can be used to evaluate  $\ln A$ .

$$
\ln A_j = a + bE_{aj} \tag{18}
$$

where *j* refers to one of the possible models  $f_i(\alpha)$  assumed to describe the process. The parameters of Eq. (18) are  $a = \ln k_{\rm iso}$ and  $b = 1/RT_{iso}$  ([Agrwal, 1986\)](#page-8-0). [Fig. 8](#page-7-0) shows the artificial isokinetic relationship for the process obtained by CK method. The values of a, b,  $k_{\text{iso}}$ ,  $T_{\text{iso}}$  of Eq. (18) obtained by (CK) model-fitting are given in [Table 3](#page-7-0).

It can be seen that the isokinetic temperatures  $(T_{\text{iso}})$ lying in the region of the experimental temperature and this indicates that the reaction model  $f(x)$  was properly chosen.

Once the correlation parameters  $a$  and  $b$  have been evaluated, the  $E_{a,x}$  values are substituted for  $E_{a,i}$  in Eq. (18) to estimate the corresponding  $\ln A_\alpha$  values and obtaining the dependence of  $\ln A_\alpha$  on  $\alpha$  for multi-step processes ([Vyzovkin](#page-9-0) [and Wight, 1999\)](#page-9-0).

[Fig. 9](#page-7-0) shows the variation of the  $\ln A_\alpha$  as a function of extent of conversion  $\alpha$ , which was obtained by different isoconversional methods for the process.

<span id="page-7-0"></span>

Figure 6 (a) KAS method and (b) T method.



Figure 7 The activation energy plotted as a function of the extent of the conversion.



Figure 8 The isokinetic relationships obtain by CK method.

Table 3 Artificial isokinetic parameter obtained by CK method.

| Method                      |          | a (min <sup>-1</sup> ) b (mol kJ <sup>-1</sup> ) $k_{\text{iso}}$ (min <sup>-1</sup> ) $T_{\text{iso}}$ (K) r |        |        |       |  |  |  |  |
|-----------------------------|----------|---|--------|--------|-------|--|--|--|--|
| KC                          | $-3.396$ | 0.158   | 0.0335 | 759.82 | 0.896 |  |  |  |  |
| $KC$ (irradiation) $-3.488$ |          | 0.159   | 0.0306 | 753.63 | 0.905 |  |  |  |  |



Figure 9 Dependence of the  $\ln A_\alpha$  on extent of conversion, estimated from Eq. [\(18\)](#page-6-0) for the different isoconversional methods.

It can be seen that the  $\ln A_\alpha$  shows the same dependence on  $\alpha$  as the apparent activation energy  $E_{a,\alpha}$  in Fig. 7. This behavior supported the conclusion mentioned above.

#### 5.3. Simulation

Simulated data are the only data for which Arrhenius parameters and reaction models are known exactly [\(Vyazovkin,](#page-9-0) [2000\)](#page-9-0). The data were simulated according to the scheme of two reaction mechanisms

$$
\left.\begin{array}{c}\nA_{(s)} \rightarrow B_{(s)} + C_{(g)} \\
B_{(s)} \rightarrow D_{(s)} + E_{(g)}\n\end{array}\right\}\n\tag{19}
$$

The overall reaction rate of these processes is:

<span id="page-8-0"></span>

Figure 10 Surface plot of activation energy for simulated process before irradiation (a) and after irradiation.

$$
\frac{d\alpha}{dt} = \frac{1}{2} \left( \frac{d\alpha_1}{dt} + \frac{d\alpha_2}{dt} \right) = \frac{1}{2} \left[ k_1(T)f(\alpha_1) + k_2(T)f(\alpha_2) \right]
$$
(20)

The Arrhenius parameters of the individual steps were taken so that  $A_1 = 10^{31}$  min<sup>-1</sup>,  $E_1 = 260$  kJ mol<sup>-1</sup>,  $A_2 = 10^{10}$  min<sup>-1</sup> and  $E_2 = 80 \text{ kJ mol}^{-1}$ . The values were chosen to make rates of two steps are comparable within the working range temperature.

Integration of Eq. (20) for nonisothermal condition has give rise to reaction dependence of  $\alpha$  versus T.

By assumption that, the processes contain at least two steps with different reaction mechanism. The first mechanism is third-order reaction  $(F_3)$  model, and the second mechanism is diffusion in one-dimensional  $(D_1)$  model. Then the effective activation energy of the overall process can be written as:

$$
E_{a,x} = -R \left[ \frac{d \ln(d\alpha/dt)}{dT^{-1}} \right]_x = \frac{E_1 k_1(T) f(\alpha_1) + E_2 k_2(T) f(\alpha_2)}{k_1(T) f(\alpha_1) + k_2(T) f(\alpha_2)}
$$
  
= 
$$
\frac{E_1 k_1(T) f(1-\alpha_1)^3 + E_2 k_2(T) f(\frac{1}{2}\alpha_2)}{k_1(T) f(1-\alpha_1)^3 + k_2(T) f(\frac{1}{2}\alpha_2)}
$$
(21)

This is clearly a function of both temperature an extent of conversion. Substitution of depends of a  $\alpha$  versus T for various  $\beta$ into Eq. (21) allows surface plots of the effective activation energy as function of  $\alpha$  and  $\beta$  to be obtained [\(Sbirrazzuoli et al.,](#page-9-0) [2000](#page-9-0)). These plots are shown in Fig. 10.

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## التحليل الحركي للانحلال عند تغير درجات الحرارة للخليط البلوري  $(Mg_5(CO_3)_4(OH)_2.4H_2O/5Cr_2O_3)$

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ا**لكلمات المفتاحية:** التحلل الحر ار ي، النموذج المقيد، النموذج الحر ، ثبوت الحر كية، الدو ال الحر كية، سبينل2010<sub>.MgCr</sub>.

ملخص البحث. تم تحضير سبينل نقي من رفع درجة حرارة الخليط البلوري 3Cr2O3,4(OH)2.4H2O/ 5Cr2O3) عند 0° 000 لمدة 27 ساعة. استخدمت تقنيات TG و DTA و XRPD لتتبع التفاعل والتعرف على النواتج. تم در اسة حركية الانحلال الحراري عند تغير درجات الحرارة للخليط البلوري الفيزيائي غير المشعع والمشعع في المهواء الثابت, وتقدير الدوال الحركية من خلال تقريبات النماذج المقيدة والحرة وعلاقة ثبوت الحركية للعمليات عديدة الخطوات <sub>(IKR)</sub>. أظهرت النتائج أن انحلال الخليطين (غير المشعع والمشعع) يتم بخطوتين متتابعتين بميكانيكيتين مختلفتين٬ الأولـي ميكانيكيـة التفاعل من الرتبة الثالثة <sub>(F3).</sub> والثانية ميكانيكية الانتشار في بعد واحد <sub>(D)</sub>.