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ORIGINAL ARTICLE

Travelling waves solution for MHD aligned flow of a second grade fluid with heat transfer: A symmetry independent approach

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Abstract In this work, an approach is implemented for finding exact solutions of an incompressible MHD aligned flow with heat transfer in a second grade fluid. This approach based on travelling wave phenomenon. The partial differential equations (PDEs) are reduced to ordinary differential equations (ODEs) by using wave parameter. The methodology used in this work is independent of perturbation, symmetry consideration and other restrictive assumption. Comparison is made with the results obtained previously.

1. Introduction

It is an established fact that the flow characteristics of non-Newtonian fluids are quite different when compared with the viscous fluids. Therefore, the well-known Navier–Stokes equations (Berker, 1963; Chandna et al., 1982; Chandna and Oku-Ukpong, 1994a; Hui, 1987; Naeem and Jamil, 2006) are inappropriate for the non-Newtonion fluids. There are numerous models of fluids are mainly classified in the literature. The flows of non-Newtonion fluids (Khan et al., 2009; Fetecau et al., 2008; Fetecau et al., 2006) are mainly classified into differential, integral and rate type fluids. Amongst these, the flows of differential type fluids have attracted the mathematician, computer programmers and numerical solvers. The constitutive equations of differential type fluids are very complex. Several authors in fluid dynamics are engaged with equations of motion of second grade fluid (Hayat et al., 2005; Siddiqui et al., 1985; Labropulu, 2003; Siddiqui, 1990; Rajagopal, 1980; Rajagopal and Gupta, 1984; Kaloni and Huschilt, 1984; Siddiqui and Kaloni, 1986; Mohyuddin et al., 2005; Ali and Hasan, 2007; Ali et al., 2007).

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Ting (1963) has studied unsteady flows of a second grade fluid in a bounded region. He studied that the solution exists only if the coefficient of the higher order derivative in the governing equation positive. In 1994, Chandna and Oku-Ukpong (Chandna and Oku-Ukpong, 1994b) studied MHD aligned flow of a second grade fluid by assuming the prescribed form of vorticity. By taking the vorticity to be proportional to the stream function perturbed by a uniform stream, Lin and Tobak (Lin and Tobak, 1986), Benharbit and Siddiqui (Benharbit and Siddiqui, 1992), Labropulu (Labropulu, 2000), they investigated the exact solutions for second grade fluid. Under the slow motion assumption Dolapçi and Pakdemirli (Dolapçi and Pakdemirli, 2004) found the exact solution by Lie group method. Yürüsoy (Yürüsoy, 2004) found exact solutions of steady creeping flow of a second grade fluid with heat transfer via Lie group. Recently, for creeping flow Afify (Afify, 2009) studied the Lie group analysis approach and found the exact solutions for steady MHD aligned second grade fluid with heat transfer.

In this work, the MHD aligned unsteady flow of a second grade fluid with heat transfer is studied. The study of flow for an electrically conducting fluid has applications in many engineering problems such as MHD generators, plasma studied, geothermal energy extractions and electromagnetic propulsion. The electromagnetic propulsion system is closely associated with magneto chemistry, requires a complete understanding of the equation of state and transport properties such as diffusion, stress–shear rate relationship, thermal conductivity and radiation. The investigations of the travelling wave solution of nonlinear equations play an important role in the study of nonlinear physical phenomena. Travelling wave phenomenon, that appears in many areas such as physics (El-Wakil and Abdou, 2008; Yildirim and Gülkanat, 2010), mathematical biology (Mohyud-Din et al., 2010), chemical kinetics, fiber optics, fluid mechanics, etc.

To the best of our knowledge only few studies, which deal with the flows of MHD aligned second grade fluid available in the literature. For the present paper, the method adopted is as follows. In Section 2, we put forward the statement of the problem. A theoretical development of Section 3 is illustrated by solution of the equations of motion of MHD aligned second grade fluid with heat transfer in Section 4. Section 5 synthesis the concluding remarks.

2. Flow development

The equations of motion of an unsteady MHD aligned flow of a second grade fluid with heat transfer is governed by

$$\rho_t + div(\rho V) = 0 \quad \text{(Continuity)} \tag{1}$$

$$\rho\frac{dV}{dt} = div\tau + \Lambda(Curl\ H)\times H + \rho r_1 \quad \text{(Linear momentum)} \tag{2}$$

$$divH = 0 \quad \text{(Solenoidal)} \tag{3}$$

$$H_t = Curl(V\times H) - \frac{1}{\Lambda\delta}Curl(Curl\ H) \quad \text{(Diffusion)} \tag{4}$$

$$\rho\frac{dE}{dt} = \tau.L - div\ q + \rho r_2 \quad \text{(Energy)} \tag{5}$$

The constitutive equations for the stress of a second grade fluid given by Coleman and Noll (Coleman and Noll, 1960)

$$\tau = -pI + \mu L_1 + \alpha_1 L_2 + \alpha_2 L_1^2 \tag{6}$$

$$L_1 = (grad\ V) + (grad\ V)^* \tag{7}$$

$$L_2 = \frac{dL_1}{dt} + L_1(grad\ V) + (grad\ V)^* L_1 \tag{8}$$

Here V is the velocity, p is the fluid pressure function, ρ the density, μ the constant viscosity, grad the gradient operator, '*' the transpose, $\frac{d}{dt}$ the material time derivative, α_1, α_2 are the constant normal stress moduli, L_1 and L_2 are the first two Rivlin–Ericksen tensors, r_1 the body force per unit mass, r_2 is the radiant heating (assumed to be zero), $E = C_p T$ is the specific internal energy, C_p is the specific heat and T is the temperature, $q = -k\nabla T$ is the heat flux vector, k is the constant thermal conductivity.

If an incompressible fluid of second grade is to have motions that are compatible with thermodynamics in the sense of the Clausius–Duhem inequality and the condition that the Helmholtz free energy be a minimum when the fluid is at rest, then the following must be satisfied (Dunn and Fosdick, 1974)

$$\mu \geqslant 0, \quad \alpha_1 \geqslant 0, \quad \alpha_1 + \alpha_2 = 0 \tag{9}$$

In this paper, we shall consider a second grade fluid which under goes isochoric motion in a plane and body force is negligible, we take $V = (u(x,y,t), v(x,y,t), 0)$, $p = p(x,y,t) H = (H_1(x,y,t), H_2(x,y,t), 0)$ and $T = T(x,y,t)$ so that our flow Eqs. (1)–(5) takes the form

$$\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y} \tag{10}$$

$$\begin{aligned}\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u + \beta\left[\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)\right.\\ &\quad + 5\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + u\frac{\partial^3 u}{\partial x^3} + v\frac{\partial^3 u}{\partial y^3} + u\frac{\partial^3 u}{\partial x\partial y^2}\\ &\quad \left. + 2\frac{\partial v}{\partial x}\frac{\partial^2 v}{\partial x^2} + \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y} + \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial x^2} + v\frac{\partial^3 u}{\partial y\partial x^2}\right]\end{aligned} \tag{11}$$

$$\begin{aligned}\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} &= -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\nabla^2 v + \beta\left[\frac{\partial}{\partial t}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)\right.\\ &\quad + 5\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x\partial y} - \frac{\partial u}{\partial x}\frac{\partial^2 v}{\partial x^2} - v\frac{\partial^3 u}{\partial x\partial y^2} + u\frac{\partial^3 v}{\partial x^3} - v\frac{\partial^3 u}{\partial x^3}\\ &\quad \left. + 2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} - \frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial x^2} + \frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial y^2} - u\frac{\partial^3 u}{\partial x^2\partial y}\right]\end{aligned} \tag{12}$$

$$\frac{\partial H_1}{\partial x} + \frac{\partial H_2}{\partial y} = 0 \tag{13}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial H_2}{\partial x} - \frac{\partial H_1}{\partial y}\right) = \nabla^2\left[\frac{1}{\Lambda\delta}\left(\frac{\partial H_2}{\partial x} - \frac{\partial H_1}{\partial y}\right) + vH_1 - uH_2\right] \tag{14}$$

$$\begin{aligned}\rho C_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) &= k\nabla^2 T + \mu\left[\left(\frac{\partial u}{\partial y}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + 2\left(\frac{\partial u}{\partial x}\right)^2 + 2\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}\right]\\ &\quad + \alpha_1\left[\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial t\partial y} + 2\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial t\partial y} + 2v\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial y^2} + \frac{\partial v}{\partial x}\frac{\partial^2 v}{\partial t\partial y} + v\frac{\partial v}{\partial x}\frac{\partial^2 v}{\partial y^2}\right.\\ &\quad \left. + 2\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial t\partial x} + \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial t\partial x} + \frac{\partial v}{\partial x}\frac{\partial^2 v}{\partial t\partial x} + u\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y} + v\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}\right]\end{aligned} \tag{15}$$

3. Travelling waves

Let Lx be a linear form of the independent variables.

$$Lx = L_{\bar{\alpha}} x_{\bar{\alpha}} \tag{16}$$

The representation of a solution is assumed in the form

$$\vartheta(x) = \upsilon(Lx) \tag{17}$$

where the function $\upsilon(\zeta)$ depends on one independent variable ζ. The value of ζ is called a phase of the wave. Fixing the phase $\zeta = Lx$, one obtains the front of the wave, where the values of the dependent variables are constant. Hence, the front of the wave is a plane propagating in the space R^n.

A solution $u(x_1, x_2, \ldots, x_n)$ is called an r-multiple travelling wave, if it has the representation

$$\vartheta(x) = \upsilon(Lx) \tag{18}$$

where Lx be a vector, which coordinates are linear forms of independent variables

$$(Lx)_i = L_{i\bar{\alpha}} x_{\bar{\alpha}}, \quad i = 1, 2, \ldots r, \; r < n \tag{19}$$

The variables $\bar{\zeta} = Lx \in R^r$ are called parameters of the wave. It is note that for an r-multiple travelling wave the rank of the Jacobi matrix of the dependent variables with respect to the independent variables is less or equal then r.

4. Solution

Without loss of generality one can assume that the travelling wave type solution has the representation

$$u = u(\xi), \quad v = v(\xi), \quad p = p(\xi), \quad T = T(\xi),$$
$$H_1 = H_1(\xi), \quad H_2 = H_2(\xi) \tag{20}$$

where $\xi = bx + ay + Ct$ is the wave parameter b, a are constants and C is the constant phase velocity.

On substituting the representation of the solution (20) into (10)–(14), we have

$$bu' + av' = 0 \tag{21}$$

$$\begin{aligned}(C + bu + av)u' &= -\frac{b}{\rho}p' + v(a^2 + b^2)u'' \\ &+ \beta\left[(C + bu + av)(a^2 + b^2)u''' + \frac{2b}{a^2}(a^2 + b^2)^2 u'u''\right] \\ &- \eta H_2(bH_2' - aH_1')\end{aligned} \tag{22}$$

$$\begin{aligned}(C + bu + av)v' &= -\frac{a}{\rho}p' + v(a^2 + b^2)v'' \\ &+ \beta\left[-(C + bu + av)\left(\frac{a^2 + b^2}{a}\right)bu''' + \frac{2}{a}(a^2 + b^2)^2 u'u''\right] \\ &+ \eta H_2(bH_2' - aH_1')\end{aligned} \tag{23}$$

$$bH_1' + aH_2' = 0 \tag{24}$$

$$\begin{aligned}C(bH_2'' - aH_1'') &= (a^2 + b^2)\left[\frac{1}{\Lambda\delta}(bH_2''' - aH_1''') + vH_1'' - uH_2'' + H_1 v'' - H_2 u''\right]\end{aligned} \tag{25}$$

$$(C + bu + av)T' = \frac{k}{\rho C_p}(a^2 + b^2)T'' + \frac{\mu}{\rho C_p}(a^2 + b^2)u'^2 + \frac{\alpha_1}{\rho C_p}\left[a^2(bu + av) + \frac{C}{a^2}(a^2 + b^2)^2\right]u'u'' \tag{26}$$

where $u = \frac{\mu}{\rho}$ and $\beta = \frac{\alpha_1}{\rho}$ are kinematical viscosity and non-Newtonian parameter respectively and $\eta = \frac{\Lambda}{\rho}$

On integrating the Eqs. (21) and (24), we have

$$bu + av = m_1 \tag{27}$$

$$bH_1 + aH_2 = m_2 \tag{28}$$

where m_1, m_2 are arbitrary constants. Here we assumed $m_2 = 0$

On utilizing Eqs. (27) and (28) in Eqs. (21), (22), (25) and (26), we have

$$\begin{aligned}(C + m_1)u' &= -\frac{b}{\rho}p' + v(a^2 + b^2)u'' \\ &+ \beta\left[(C + m_1)(a^2 + b^2)u''' + \frac{2b}{a^2}(a^2 + b^2)^2 u'u''\right] \\ &+ \eta\frac{(a^2 + b^2)}{b}H_1 H_1'\end{aligned} \tag{29}$$

$$\begin{aligned}(C + m_1)v' &= -\frac{a}{\rho}p' + v(a^2 + b^2)v'' \\ &+ \beta\left[-(C + m_1)\left(\frac{a^2 + b^2}{a}\right)bu''' + \frac{2}{a}(a^2 + b^2)^2 u'u''\right] \\ &- \eta\frac{(a^2 + b^2)}{b}H_2 H_2'\end{aligned} \tag{30}$$

$$(a^2 + b^2)H_1''' - CH_1'' = 0 \tag{31}$$

$$(C + m_1)T' = \frac{k}{\rho C_p}(a^2 + b^2)T'' + \frac{\mu}{\rho C_p}(a^2 + b^2)^2 u'^2 + \frac{\alpha_1}{\rho C_p}\left[\frac{U(a^2 + b^2) + a^4 m_1}{a^2}\right]u'u'' \tag{32}$$

On eliminating pressure p from Eqs. (29) and (30), we get

$$u''' - Mu'' - Nu' = 0 \tag{33}$$

where

$$M = \frac{\mu}{\alpha_1(C + m_1)}, \quad N = \frac{\rho}{\alpha_1(a^2 + b^2)}$$

The components of velocity are

$$u(\xi) = m_3 e^{n_1\xi} + m_4 e^{n_2\xi} - \frac{m_5}{N} \tag{34}$$

$$v(\xi) = \frac{m_1}{a} - \frac{b}{a}\left(m_3 e^{n_1\xi} + m_4 e^{n_2\xi} - \frac{m_5}{N}\right) \tag{35}$$

where n_1, n_2 are the roots of the equation $n^2 - Mn - N = 0$

From Eq. (31), the components of magnetic field are

$$H_1(\xi) = m_6 e^{\overline{\frac{C}{(a^2+b^2)}}\xi} + m_7\xi + m_8 \tag{36}$$

$$H_2(\xi) = -\frac{b}{a}\left(m_6 e^{\overline{\frac{C}{(a^2+b^2)}}\xi} + m_7\xi + m_8\right) \tag{37}$$

Substituting Eqs. (34) and (36) into Eq. (29), we obtain

$$\begin{aligned}p(\xi) &= M_1 e^{n_1\xi} + M_2 e^{n_2\xi} + M_3 e^{2n_1\xi} + M_4 e^{2n_2\xi} \\ &+ M_5 e^{(n_1+n_2)\xi} + M_6\left(m_6 e^{\overline{\frac{C}{(a^2+b^2)}}\xi} + m_7\xi + m_8\right)^2 + M_7\end{aligned} \tag{38}$$

Where n_1, n_2 are the roots of the equation

$$M_1 = \frac{m_3\left[-\rho(c + m_1) + (a^2 + b^2)\left\{\mu n_1 + \alpha_1 m_4 n_1^2(C + m_1)\right\}\right]}{b}$$

$$M_2 = \frac{m_4\left[-\rho(c + m_1) + (a^2 + b^2)\left\{\mu n_2 + \alpha_1 m_4 n_2^2(C + m_1)\right\}\right]}{b}$$

$$M_3 = \frac{\alpha_1 n_1^2 m_3^2 (a^2 + b^2)^2}{a^4}$$

$$M_4 = \frac{\alpha_1 n_2^2 m_4^2 (a^2 + b^2)^2}{a^4}$$

$$M_5 = \frac{2\alpha_1 m_3 m_4 n_1 n_2 (a^2 + b^2)}{a^4}$$

$$M_6 = \frac{A(a^2 + b^2)}{2b^2}$$

$$M_7 = \frac{bNm_9 + \rho m_5(C + m_1)}{bN}$$

Utilizing Eqs. (27) and (34) in Eq. (26) and integrating, we obtain

$$T(\xi) = M_9 e^{2n_1\xi} + M_{10}e^{2n_2\xi} + M_{11}e^{(n_1+n_2)\xi} + m_{10}e^{M_8\xi} - \frac{m_9}{M_8} \quad (39)$$

$$M_8 = \frac{\rho C_p(C + m_1)}{k(a^2 + b^2)}$$

$$M_9 = \frac{-n_1\lambda_2^2(a^2 + b^2)}{k(2n_1 - A_7)}\left[\mu(a^2 + b^2) + \frac{n_1\alpha_1\{U(a^2 + b^2) + a^4\lambda_1\}}{2(a^2 + b^2)}\right]$$

$$M_{10} = \frac{-n_2\lambda_3^2(a^2 + b^2)}{k(2n_2 - A_7)}\left[\mu(a^2 + b^2) + \frac{n_2\alpha_1\{U(a^2 + b^2) + a^4\lambda_1\}}{2(a^2 + b^2)}\right]$$

$$M_{11} = \frac{-2\lambda_2\lambda_3 n_1 n_2(a^2 + b^2)}{k(n_1 + n_2 - A_7)}\left[\frac{\mu(a^2 + b^2)}{(n_1 + n_2)} + \frac{\alpha_1\{U(a^2 + b^2) + a^4\lambda_1\}}{2(a^2 + b^2)}\right]$$

The velocity components, pressure, temperature and magnetic field components in the original variables are

$$u(x, y, t) = m_3 e^{n_1(bx+ay+Ct)} + m_4 e^{n_2(bx+ay+Ct)} - \frac{m_5}{N} \quad (40)$$

$$v(x, y, t) = \frac{m_1}{a} - \frac{b}{a}\left(m_3 e^{n_1(bx+ay+Ct)} + m_4 e^{n_2(bx+ay+Ct)} - \frac{m_5}{N}\right) \quad (41)$$

$$\begin{aligned} p(x, y, t) &= M_1 e^{n_1(bx+ay+Ct)} + M_2 e^{n_2(bx+ay+Ct)} + M_3 e^{2n_1(bx+ay+Ct)} \\ &+ M_4 e^{2n_2(bx+ay+Ct)} + M_5 e^{(n_1+n_2)(bx+ay+Ct)} \\ &+ M_6\left(m_6 e^{\frac{C}{(a^2+b^2)}(bx+ay+Ct)} + m_7(bx + ay + Ct) + m_8\right)^2 + M_7 \end{aligned} \quad (42)$$

$$\begin{aligned} T(x, y, t) &= A_8 e^{2n_1(bx+ay+Ct)} + A_9 e^{2n_2(bx+ay+Ct)} + A_9 e^{(n_1+n_2)(bx+ay+Ct)} \\ &+ (\lambda_6(bx + ay + Ct) + \lambda_7)e^{2A_7(bx+ay+Ct)} \end{aligned} \quad (43)$$

$$H_1(x, y, t) = m_6 e^{\frac{C}{(a^2+b^2)}(bx+ay+Ct)} + m_4(bx + ay + Ct) + m_8 \quad (44)$$

$$H_2(x, y, t) = -\frac{b}{a}\left(m_6 e^{\frac{C}{(a^2+b^2)}(bx+ay+Ct)} + m_4(bx + ay + Ct) + m_8\right) \quad (45)$$

5. Concluding remarks

The goal to obtain exact solution of an incompressible MHD aligned flow with heat transfer of a second grade fluid. The methodology (Meleshko, 2004) used in this work is easy for linearizing the flow equations by considering travelling wave phenomenon. The method was used in a direct way without transformation, symmetry consideration and restrictive assumptions. It is observed that exponential type solution have been obtained. Finally, compared the velocity and magnetic field in our work by Ali and Mahmood (Ali and Mahmood, 2005) the results are found to be excellent. This guarantees the correctness of the mathematical calculation.

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