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Original article

On the thermal analysis of magnetohydrodynamic Jeffery fluid via modern non integer order derivative

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A B S T R A C T

The uniqueness of thermal radiation effects on materials has an adhesive role in engineering. This article emphasizes the effects of thermal radiation of Jeffery fluid with magnetic field by employing Caputo-Fabrizio fractional derivative. Analytical solutions for the velocity field and temperature distribution are based on integral transforms (Fourier Sine and Laplace transform) with their inversion. The general solutions have been established in terms of elementary functions and product of convolution theorem satisfying initial and boundary conditions. This analysis is one of the infrequent contributions to elucidate the rheology of Jeffery fluid for free convective problem on oscillating plate. In order to bring physical insights, four models have been prepared for comparison i-e (i) Jeffery fluid with magnetic field (ii) Jeffery fluid without magnetic field (iii) Second grade fluid with magnetic field and (iv) Second grade fluid without magnetic field for several rheological parameters for instance, viscosity v, thermal radiation Rd, thermal conductivity k, Jeffrey fluid parameter λ, Hartmann number H_a, magnetic field M, Prandtl number P_r, Grashof number on fluid flow. At the end, our results suggested that Ordinary Jeffery fluid with magnetic field oscillates more rapidly to remaining models of fluid but fractionalized Jeffery fluid with magnetic field has reciprocal behavior of fluid flow.

1. Introduction

The computational and physico-mathematical analysis in non-Newtonian fluid has diverted the attention of various researchers; this is due to broad applications of certain fluids in engineering such as industrial adhesives and lubricants (Ver Strate and Struglinski, 1991), gels, drugs and creams (Suryawanshi et al., 2012), plastics fabrication (Balmforth and Craster, 1999), and also ecological systems comprising hyper-concentrated sediments, mud flows, contaminant release and oil spills (Jokuty et al., 1995). The conventional Navier-Stokes viscous model (Newtonian) invalidates non-Newtonian fluids due to intrinsic properties. Newtonian fluid is insufficient to describe the dynamics and rheological phenomenon of many fluids for instance, Weissenberg effects, shear-thinning/thickening, stress differences, fading memory, spurt, yield stress, elongation, retardation, relaxation, re-coil, micro-structure and several others. Different models have been proposed to characterize the rheology of non-Newtonian fluid few of them are; Burgers elasto-viscous fluid model (Tripathi and Anwar Bég, 2014), Sisko's model (Sisko, 1958), differential third and second grade fluid models (Reiner-Rivlin) (Sahoo and Poncet, 2011; Sarpkaya and Rainey, 1971), Oldroyd-B fluid model (Khan et al., 2012), Maxwell fluid model (Abro et al., 2015; Abro and Shaikh, 2015), Jeffery's model (Qasim, 2013). The Jeffery fluid model is viscoelastic non-Newtonian fluid that exhibits retardation and relaxation phenomenon. The Jeffery fluid model is simpler linear model in which convective derivative is omitted for time derivative. For this cause, several investigations have been carried on Jeffery fluid few of them are hereby cited. Hayat and et al. have traced out Jeffrey fluid for unsteady mixed convection flow under the existence of thermal radiation over a stretching sheet (Hayat and Mustafa, 2010). The Jeffrey fluid for stagnation point flow on unsteady oscillations has been analyzed by Nadeem et al. (2014).

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Hayat et al. (2012) have investigated the flow of Jeffery fluid in the presence and absence of radiation under the influences of heat source and power law heat flux. Shehzad et al. (2013) worked out on the radiative flow of Jeffrey fluid under impacts of joule heating and thermophoresis with mixed convection. Khan (2015) examined effects the unsteady free convection flow of a Jeffrey fluid and traced out an exact solution. Dalir (2014) investigated heat transfer and convection flow of a Jeffrey fluid with entropy generation over a stretching sheet using Numerical study. Hayat et al. (2010) worked on fractionalized Oscillatory and rotational flows of Jeffrey fluid in porous medium. Idowu et al. (2013) emphasized the impacts of unsteady magnetohydrodynamic flow of Jeffrey fluid with chemical reaction, mass and heat transfer in a horizontal channel. Off course, the study can be continued but we end here by citing few recent studies on magnetic field (Abro and Solangi, 2017; Sheikholeslami and Rokni, 2018; Abro et al., 2018; Atangana and Koca, 2016; Abro et al., 2017), thermal radiation (Abro et al., 2018; Khan and Abro, 2018), Porous medium (Abro et al., 2017; Abro et al., 2017), heat and mass transfer (Atangana, 2016; Sheikholeslami et al., 2018; Abro and Khan, 2017; Sheikholeslami et al., 2018; Abro et al., 2018; Mohyud-Din et al., 2015; Khan et al., 2017), nanofluids (Sheikholeslami et al., 2018; Sheikholeslami et al., 2018; Khan et al., 2017) and modern fractional derivatives (Al-Mdallal et al., 2018; Saad et al., 2017; Abro et al., 2018; Atangana and Baleanu, 2017; Laghari et al., 2017). Motivating by all mentioned models, our aim is to emphasize the effects of oscillating flow of Jeffery fluid for magnetic field and thermal radiation by employing Caputo-Fabrizio fractional derivative. Analytical solutions for the velocity field and temperature distribution are based on integral transforms (Fourier Sine and Laplace transform). The general solutions have been established in terms of elementary functions and product of convolution theorem satisfying initial and boundary conditions. This analysis is one of the infrequent contributions to elucidate the rheology of Jeffery fluid for free convective problem on oscillating plate. In order to bring physical insights, four models have been prepared for comparison i-e (i) Jeffery fluid with magnetic field (ii) Jeffery fluid without magnetic field (iii) Second grade fluid with magnetic field and (iv) Second grade fluid without magnetic field for several rheological parameters for instance, viscosity v, thermal radiation Rd, thermal conductivity k, Jeffrey fluid parameter λ, Hartmann number H_a, magnetic field M, Prandtl number, P_r, Grashof number on fluid flow (see Fig. 1).

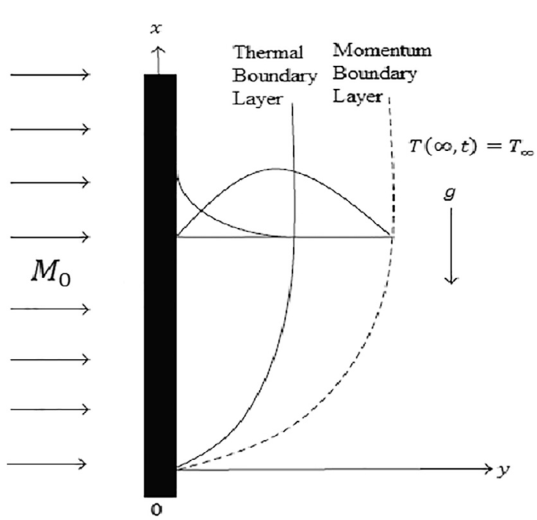

Fig. 1. Geometry of Jeffery Fluid for Vertical Plate.

2. Governing equations

The governing equation which describes unsteady incompressible flow

$$\nabla.\mathbf{V}=0,\quad div\mathbf{T}+\rho\mathbf{g}+\mathbf{J}\times\mathbf{M}=\left((\mathbf{V}.\nabla)\mathbf{V}+\frac{\partial\mathbf{V}}{\partial t}\right)\rho, \tag{1}$$

$$Curl\mathbf{M}=\mathbf{J}\mu_m,\quad Curl\mathbf{E}=-\frac{\partial\mathbf{M}}{\partial t},\quad \nabla.\mathbf{M}=0, \tag{2}$$

In Eqs. (1) and (2), $\mathbf{V}$, $\mathbf{T}$, ρ, $\mathbf{g}$, $\mathbf{J}$, $\mathbf{J}\mu_m$, $\mathbf{E}$ velocity field, Cauchy stress tensor, density of fluid, gravitational force, current density, total magnetic field, magnetic permeability and total electric field current. The constitutive equations for Jeffrey fluid are described below

$$\mathbf{T}=-\mathbf{pI}+\mathbf{S},\quad \frac{(1+\lambda_1)\mathbf{S}}{\mu}=\left(\frac{\partial\mathbf{A_1}}{\partial t}+(\mathbf{V}.\nabla)\mathbf{A_1}\right)\lambda_2+\mathbf{A_1}, \tag{3}$$

where, $-\mathbf{pI}$, $\mathbf{S}$, λ_1, μ, $\mathbf{A_1}$ and λ_2 indeterminate part of the stress, extra tensor, ratio of relaxation to retardation times, dynamic viscosity, Rivlin-Ericksen tensor and retardation time of Jeffery fluid. We assume the stress and velocity field for the oscillating flow as below

$$\mathbf{V}=\mathbf{V}(y,t)=V(y,t)\mathbf{i},\quad \mathbf{S}=\mathbf{S}(y,t), \tag{4}$$

where, $\mathbf{i}$ unit vector along the ??-direction and w is ??-component of velocity $\mathbf{V}$ of the Cartesian coordinate system. The stress field fulfills the assumption $\mathbf{S}(y,0=0)$ which generates the below equation

$$S_{xz}=S_{zx}=S_{yz}=S_{zy}=S_{zz}=S_{xx}=0,\left(1+\lambda_2\frac{\partial}{\partial t}\right)\frac{\partial V}{\partial t}=S_{xy}\frac{(1+\lambda_1)}{\mu}, \tag{5}$$

where, S_{xy} is the non-trivial tangential stress. Assume the impact of thermal radiation along with electrically conducting flow of Jeffery fluid about the vertical plate located in xz -plane. It is assumed at time $t=0^+$, the temperature of plate raised up and the constant temperature T_w is kept up at $t>t_0$. At time $t\leqslant 0$, the fluid and plate are at rest in the constant temperature T_∞ and when $t\leqslant t_0$, lowered at $T_\infty+(T_w-T_\infty)(t/t_0)$. Using usual Boussinesq approximations and implementing above assumptions, the energy and momentum equations for electrically conducting flow of Jeffery fluid in porous medium about the vertical plate as (Khan, 2015; Zin et al., 2016)

$$\left(1+\lambda_2\frac{\partial}{\partial t}\right)\frac{\partial^2V}{\partial y^2}=(1+\lambda_1)\frac{\rho}{\mu}\frac{\partial V}{\partial t}-\sigma M_0^2V+\rho g\beta(T-T_\infty), \tag{6}$$

$$\frac{k}{\rho c_p}\frac{\partial^2V}{\partial y^2}=\frac{\partial T}{\partial t}-\frac{\partial q_r}{\partial y}, \tag{7}$$

while, initial and boundary conditions are

$$T(y,0)=T_\infty,\quad V(y,0)=0,\quad y>0, \tag{8}$$

$$T(0,t)=T_w,\quad t\geqslant t_0,$$
$$V(0,t)=UH(t)\cos(\omega t)\ or\ U\sin(\omega t),\quad t>0, \tag{9}$$

$$T(\infty,t)=T_\infty,\quad V(\infty,t)=0,\quad t>0, \tag{10}$$

where, V, T, ρ, g, c_p, k, T_w, T_∞, q_r are fluid velocity, temperature, constant density, acceleration due to gravity, specific heat capacity, thermal conductivity, wall temperature, free stream temperature and radiation heat flux, respectively. While radiation heat flux can be written by using Rosseland approximation as

$$4\sigma^*\frac{\partial^4T}{\partial y}=-3k_1q_r, \tag{11}$$

where, σ^*, k_1 are Stefan-Boltzmann and absorption coefficient respectively. Temperature differences for flow are assumed sufficiently small and simplifying Eq. (7), we obtained that

$$\frac{\partial^2 T(y,t)}{\partial y^2}\left(1+\frac{16T_\infty^3\sigma^*}{3k_1 k}\right)=\frac{\partial T}{\partial y}\frac{\rho c_p}{k}, \tag{12}$$

Implementing dimensionless numbers and variables,

$$t^*=\frac{U_0^2 t}{v},\ V^*=\frac{V}{U_0},\ y^*=\frac{U_0 y}{v},\ Rd=\frac{16T_\infty^3\sigma^*}{3k_1 k},\ \lambda=\frac{U_0^2\lambda_2}{v},$$

$$T=\frac{T-T_\infty}{T_w-T_\infty}, H_a=\frac{vM_0^2\sigma}{U_0^2\rho},\ P_r=\frac{c_p\mu}{k},\ G_r=\frac{v\beta g(T_w-T_\infty)}{U_0^3}. \tag{13}$$

where, t, w, U, v, y, Rd, T, σ, k_1, k, λ, H_a, M, P_r, G_r are the time, velocity distribution, Non-zero parameter, viscosity, spatial variable, thermal radiation, temperature, Stefan-Boltzmann, absorption coefficient, thermal conductivity, Jeffrey fluid parameter, Hartmann number, magnetic field, Prandtl number, Grashof number respectively. The corresponding governing differential equations and initial and boundary conditions from Eqs. (6)-(10) are termed as

$$(1+\lambda_1)(D_t^\zeta V(y,t)+H_a V(y,t)-G_r T(y,t))=(\lambda D_t^\zeta+1)\frac{\partial^2 V(y,t)}{\partial y^2}, \tag{14}$$

$$\frac{p_r}{(Rd+1)}D_t^\zeta T(y,t)=\frac{\partial^2 T(y,t)}{\partial y^2}, \tag{15}$$

$$T(y,0)=0,\quad V(y,0)=0,\qquad y>0, \tag{16}$$

$$T(0,t)=t,\ \ t>0,\quad V(0,t)=UH(t)\cos(\omega t)\ or\ U\sin(\omega t),\qquad t>0, \tag{17}$$

$$T(\infty,t)=0,\quad V(\infty,t)=0,\qquad t>0. \tag{18}$$

where, D_t^ζ is time-fractional derivatives, represents Caputo-Fabrizio fractional derivative operator (time derivative of non-integer order derivative without singular kernel) of order $0\leqslant\zeta\leqslant1$ as in literature earlier published paper (Caputo and Fabrizio, 2015; Shah and Khan, 2016; Zafar and Fetecau, 2016) defined as

$$D_t^\zeta=\int_0^t\frac{G'(\eta)}{1-\zeta}Exp\left(-\frac{\zeta(t-\eta)}{1-\zeta}\right)d\eta,\ \ \text{for}0\leqslant\zeta\leqslant1. \tag{19}$$

3. Profile of temperature distribution via Fourier Sine transform

Employing Fourier Sine Transform on fractionalized differential equation, Multiplying both sides of (15) $\sqrt{\frac{2}{\pi}}\sin(y\psi)$, integrating the result with respect to y from 0 to infinity (Abro et al., 2016), and considering Eq. $(16)_1$ in mind the initial and boundary condition, we obtain

$$\frac{p_r}{(Rd+1)}D_t^\zeta T_s(\psi,t)=\left(-\psi^2T_s(\psi,t)+\psi\sqrt{\frac{2}{\pi}}T(0,t)\right), \tag{20}$$

where, The image of Fourier Sine transform of $T(y,t)$ is $T_s(\psi,t)$ has satisfy Fourier Sine transform defined as

$$T_s(\psi,t)=\sqrt{\frac{2}{\pi}}\int_0^\infty T(y,t)\sin(y\psi)\,dy, \tag{21}$$

Implementing Laplace transform on Eq. (20) and $(1618)_1$, we get

$$\bar{T}_s(\psi,t)=\sqrt{\frac{2}{\pi}}\frac{(Rd+1)(\delta\zeta+s)}{s^2\{s(P_r\delta+\psi+\psi Rd)+(\psi\delta\zeta+\psi Rd\delta\zeta)\}}, \tag{22}$$

where, utilizing the fact $\delta=\frac{1}{(1-\zeta)}$ we recovered $L\{D_t^\zeta T_s(\psi,t)\}=\frac{\delta s}{s+\delta\zeta}\bar{T}_s(\psi,s)$. Now we rewrite Eq. (22) for more suitable presentation, we have

$$\bar{T}_s(\psi,s)=\frac{1}{\psi}\sqrt{\frac{2}{\pi}}\left[\frac{1}{s^2}-\frac{P_r\delta}{s\{s(P_r\delta+\psi+\psi Rd)+(\psi\delta\zeta+\psi Rd\delta\zeta)\}}\right], \tag{23}$$

Inverting Eq. (23) by Fourier Sine transform, we obtain

$$\bar{T}(y,s)=\frac{2}{\pi}\int_0^\infty\frac{\sin(y\psi)}{\psi}\left[\frac{1}{s^2}-\frac{P_r\delta}{s\{s(P_r\delta+\psi+\psi Rd)+(\psi\delta\zeta+\psi Rd\delta\zeta)\}}\right]d\psi, \tag{24}$$

Finally, in order to trace out the final solution of temperature distribution, we apply inverse Laplace transform on Eq. (24) and employing the fact of integral $\int_0^\infty\frac{\sin(y\lambda_1)}{\lambda_1}d\lambda_1=\frac{\pi}{2},\ y>0$ (Abro, 2016), we attain

$$T(y,t)=t-\frac{2P_r\delta}{\pi}\int_0^\infty\frac{\sin(y\psi)}{\psi(\psi\delta\zeta+\psi Rd\delta\zeta)}\left[1-Exp\left(-\frac{P_r\delta+\psi+\psi Rd}{\psi\delta\zeta+\psi Rd\delta\zeta}\right)t\right]d\psi. \tag{25}$$

4. Profile of velocity field via Fourier Sine transform

The case of cosine oscillation $V(0,t)=UH(t)\cos(\omega t)$ Employing Fourier Sine Transform on fractionalized differential equation, Multiplying both sides of (14) $\sqrt{\frac{2}{\pi}}\sin(y\psi)$, integrating the result with respect to y from 0 to infinity, and considering equation $(16)_2$ in mind the initial and boundary condition, we obtain

$$(1+\lambda_1)(D_t^\zeta V_s(\psi,t)+H_aV_s(\psi,t)-G_rT_s(\psi,t))=(\lambda D_t^\zeta+1)\left(-\psi^2V_s(\psi,t)+\psi\sqrt{\frac{2}{\pi}}V(0,t)\right), \tag{26}$$

Implementing Laplace transform on equations (26) and $(1618)_2$, we get

$$\bar{V}_s(\psi,s)=\sqrt{\frac{2}{\pi}}\frac{U\psi s(\delta\zeta+s)}{(s^2+\omega^2)(ss_1+(\delta\zeta+s)s_2)}+\sqrt{\frac{2}{\pi}}\frac{U\lambda\psi\delta s^2}{(s^2+\omega^2)(ss_1+(\delta\zeta+s)s_2)}+\sqrt{\frac{2}{\pi}}\frac{G_r(Rd+1)(\delta\zeta+s)^2(1+\lambda_1)}{s^2(ss_3+s_4)(ss_1+(\delta\zeta+s)s_2)}, \tag{27}$$

where, $s_1=\delta+\delta\lambda_1+\lambda\delta\psi^2$, $s_2=\psi^2+(H_a+\Phi)(1+\lambda_1)$, $s_3=P_r\delta+\psi+\psi Rd$ and $s_4=\zeta\delta\psi+\psi Rd\zeta\delta$ altering Eq. (27) for more suitable presentation and inverting Eq. (27) by means of Fourier Sine transform, we get

$$\bar{V}(y,s)=\frac{Us}{s^2+\omega^2}+\frac{2}{\pi}\int_0^\infty\frac{\sin(y\psi)}{\psi}\frac{U\psi s(\delta\zeta+s)}{(s^2+\omega^2)(ss_1+(\delta\zeta+s)s_2)}d\psi+\frac{2}{\pi}\int_0^\infty\sin(y\psi)\frac{U\lambda\psi\delta s^2}{(s^2+\omega^2)(ss_1+(\delta\zeta+s)s_2)}d\psi+\frac{2}{\pi}\int_0^\infty\sin(y\psi)\frac{G_r(Rd+1)(\delta\zeta+s)^2(1+\lambda_1)}{s^2(ss_3+s_4)(ss_1+(\delta\zeta+s)s_2)}d\psi, \tag{28}$$

simplifying Eq. (28) by letting $s_5=\frac{\zeta\delta\psi^2-\zeta\delta s_2}{\psi^2-s_1-s_2}$, $s_6=\frac{\zeta\delta s_2}{s_1+s_2}$ and $s_7=\frac{s_4}{s_3}$, we investigated following expression for convolution product as

$$\bar{V}(y,s)=\frac{Us}{s^2+\omega^2}+\frac{2(\psi^2-s_1-s_2)}{\pi(s_1+s_2)}\int_0^\infty\frac{\sin(y\psi)}{\psi}\frac{s(s+s_5)}{(s^2+\omega^2)(s+s_6)}d\psi+\frac{2U\lambda\delta}{\pi(s_1+s_2)}\int_0^\infty\psi\sin(y\psi)\frac{s^2}{(s^2+\omega^2)(s+s_6)}d\psi+\frac{2G_r(1+\lambda_1)(Rd+1)}{\pi(s_1+s_2)s_3}\int_0^\infty\sin(y\psi)\frac{1}{s^2}\{\frac{(\delta\zeta+s_6)^2}{(s_7-s_6)(s-s_6)}+\frac{(\delta\zeta+s_7)^2}{(s_6-s_7)(s-s_7)}\}d\psi, \tag{29}$$

Applying inverse Laplace transform on Eq. (30), we have final form of velocity field in term of integral form as

$$V(y,t)=UH(t)\cos\omega t+\frac{2UH(t)(\psi^2-s_1-s_2)}{\pi(s_1+s_2)}\int_0^\infty\int_0^t\frac{\sin(y\psi)}{\psi}\times\cos\omega(t-q)(s_5-s_6)Exp(-s_6)\,t\,d\psi dq+\frac{2U\lambda\delta s_7}{\pi(s_1+s_2)}\int_0^\infty\int_0^t\psi\sin(y\psi)\cos\omega(t-q)Exp(-s_6)\,t\,d\psi dq+\frac{2G_r(1+\lambda_1)(Rd+1)}{\pi(s_1+s_2)s_3}\int_0^\infty\int_0^t\sin(y\psi)\times(t-q)\left\{\frac{(\delta\zeta+s_6)^2}{(s_7-s_6)}Exp(-s_6)t+\frac{(\delta\zeta+s_7)^2}{(s_6-s_7)}Exp(-s_7)t\right\}d\psi dq. \tag{30}$$

Having an identical procedure, we also investigated the case of sine oscillation $V(0,t)=UH(t)\sin(\omega t)$ by letting the $s_8=\frac{\zeta\delta\psi^2-\zeta\delta s_2}{\psi^2-2s_1}$, we recovered the following expression

$$V(y,t)=UH(t)\sin\omega t+\frac{2UH(t)(\psi^2-2s_1)}{\pi(s_1+s_2)}\int_0^\infty\int_0^t\frac{\sin(y\psi)}{\psi}\times\sin\omega(t-q)(s_8-s_6)Exp(-s_6)\,t\,d\psi dq+\frac{2U\lambda\delta s_7\omega}{\pi(s_1+s_2)}\int_0^\infty\int_0^t\psi\sin(y\psi)\cos\omega(t-q)Exp(-s_6)t\,d\psi dq+\frac{2G_r(1+\lambda_1)(Rd+1)}{\pi(s_1+s_2)s_3}\int_0^\infty\int_0^t\sin(y\psi)\times(t-q)\{\frac{(\delta\zeta+s_6)^2}{(s_7-s_6)}Exp(-s_6)t+\frac{(\delta\zeta+s_7)^2}{(s_6-s_7)}Exp(-s_7)t\}d\psi dq. \tag{31}$$

5. Limiting cases

5.1. Temperature distribution and velocity field without the effects of radiation when $Rd = 0$

The solutions of temperature distribution and velocity field without the effects of radiation cab be recovered by letting $Rd = 0$ in equations (25) and (30), (31) obtained by (Khan, 2015)

$$T(y,t)=t-\frac{2P_r}{\pi}\int_0^\infty\frac{\sin(y\psi)}{\psi^2\zeta}\left[1-Exp(-\frac{P_r\delta+\psi}{\psi\delta\zeta})t\right]d\psi. \tag{32}$$

$$V(y,t)=UH(t)\cos\omega t+\frac{2UH(t)(\psi^2-s_1-s_2)}{\pi(s_1+s_2)}\int_0^\infty\int_0^t\frac{\sin(y\psi)}{\psi}\times\cos\omega(t-q)(s_5-s_6)Exp(-s_6)\,t\,d\psi dq+\frac{2U\lambda\delta s_7}{\pi(s_1+s_2)}\int_0^\infty\int_0^t\psi\sin(y\psi)\cos\omega(t-q)Exp(-s_6)\,t\,d\psi dq+\frac{2G_r(1+\lambda_1)}{\pi(s_1+s_2)s_3}\int_0^\infty\int_0^t\sin(y\psi)\times(t-q)\{\frac{(\delta\zeta+s_6)^2}{(s_7-s_6)}Exp(-s_6)t+\frac{(\delta\zeta+s_7)^2}{(s_6-s_7)}Exp(-s_7)t\}d\psi dq. \tag{33}$$

$$V(y,t)=UH(t)\sin\omega t+\frac{2UH(t)(\psi^2-2s_1)}{\pi(s_1+s_2)}\int_0^\infty\int_0^t\frac{\sin(y\psi)}{\psi}\times\sin\omega(t-q)(s_8-s_6)Exp(-s_6)\,t\,d\psi dq+\frac{2U\lambda\delta s_7\omega}{\pi(s_1+s_2)}\int_0^\infty\int_0^t\psi\sin(y\psi)\cos\omega(t-q)Exp(-s_6)\,t\,d\psi dq+\frac{2G_r(1+\lambda_1)}{\pi(s_1+s_2)s_3}\int_0^\infty\int_0^t\sin(y\psi)\times(t-q)\{\frac{(\delta\zeta+s_6)^2}{(s_7-s_6)}Exp(-s_6)t+\frac{(\delta\zeta+s_7)^2}{(s_6-s_7)}Exp(-s_7)t\}d\psi dq. \tag{34}$$

where, $s_1=\delta+\delta\lambda_1+\lambda\delta\psi^2$, $s_2=\psi^2+(H_a+\Phi)(1+\lambda_1)$, $s_3=P_r\delta+\psi$, $s_4=\zeta\delta\psi$, $s_5=\frac{\zeta\delta\psi^2-\zeta\delta s_2}{\psi^2-s_1-s_2}$, $s_6=\frac{\zeta\delta s_2}{s_1+s_2}$ and $s_7=\frac{s_4}{s_3}$ and $s_8=\frac{\zeta\delta\psi^2-\zeta\delta s_2}{\psi^2-2s_1}$.

5.2. Velocity field without the effects of magnetic field when $H_a = 0$.

The solutions of velocity field without the effects of transverse magnetic field cab be investigated by employing $H_a = 0$ in Eqs. (30) and (31)

$$V(y,t)=UH(t)\cos\omega t+\frac{2UH(t)(\psi^2-s_1-s_2)}{\pi(s_1+s_2)}\int_0^\infty\int_0^t\frac{\sin(y\psi)}{\psi}\times\cos\omega(t-q)(s_5-s_6)Exp(-s_6)\,t\,d\psi dq+\frac{2U\lambda\delta s_7}{\pi(s_1+s_2)}\int_0^\infty\int_0^t\psi\sin(y\psi)\cos\omega(t-q)\times Exp(-s_6)\,t\,d\psi dq+\frac{2G_r(1+\lambda_1)(Rd+1)}{\pi(s_1+s_2)s_3}\int_0^\infty\int_0^t\times\sin(y\psi)\times(t-q)\{\frac{(\delta\zeta+s_6)^2}{(s_7-s_6)}Exp(-s_6)t+\frac{(\delta\zeta+s_7)^2}{(s_6-s_7)}Exp(-s_7)t\}d\psi dq. \tag{35}$$

$$V(y,t)=UH(t)\sin\omega t+\frac{2UH(t)(\psi^2-2s_1)}{\pi(s_1+s_2)}\int_0^\infty\int_0^t\frac{\sin(y\psi)}{\psi}\times\sin\omega(t-q)(s_8-s_6)Exp(-s_6)\,t\,d\psi\,dq+\frac{2U\lambda\delta s_7\omega}{\pi(s_1+s_2)}\times\int_0^\infty\int_0^t\psi\sin(y\psi)\cos\omega(t-q)Exp(-s_6)t\,d\psi dq+\frac{2G_r(1+\lambda_1)(Rd+1)}{\pi(s_1+s_2)s_3}\int_0^\infty\int_0^t\sin(y\psi)\times(t-q)\left\{\frac{(\delta\zeta+s_6)^2}{(s_7-s_6)}Exp(-s_6)t+\frac{(\delta\zeta+s_7)^2}{(s_6-s_7)}Exp(-s_7)t\right\}d\psi dq. \tag{36}$$

where, $s_1=\delta+\delta\lambda_1+\lambda\delta\psi^2$, $s_2=\psi^2$, $s_3=P_r\delta+\psi+\psi Rd$ and $s_4=\zeta\delta\psi+\psi Rd\zeta\delta$, $s_5=\frac{\zeta\delta\psi^2-\zeta\delta s_2}{\psi^2-s_1-s_2}$, $s_6=\frac{\zeta\delta s_2}{s_1+s_2}$, $s_7=\frac{s_4}{s_3}$ and $s_8=\frac{\zeta\delta\psi^2-\zeta\delta s_2}{\psi^2-2s_1}$.

5.3. Velocity field for Second grade fluid with magnetic field when $\lambda_1 = 0$ and $H_a \neq 0$.

The solutions of velocity field with the effects of transverse magnetic field for second grade fluid can be traced out by employing $\lambda_1 = 0$ and $H_a \neq 0$ in Eqs. (30) and (31) obtained by (Samiulhaq et al., 2014)

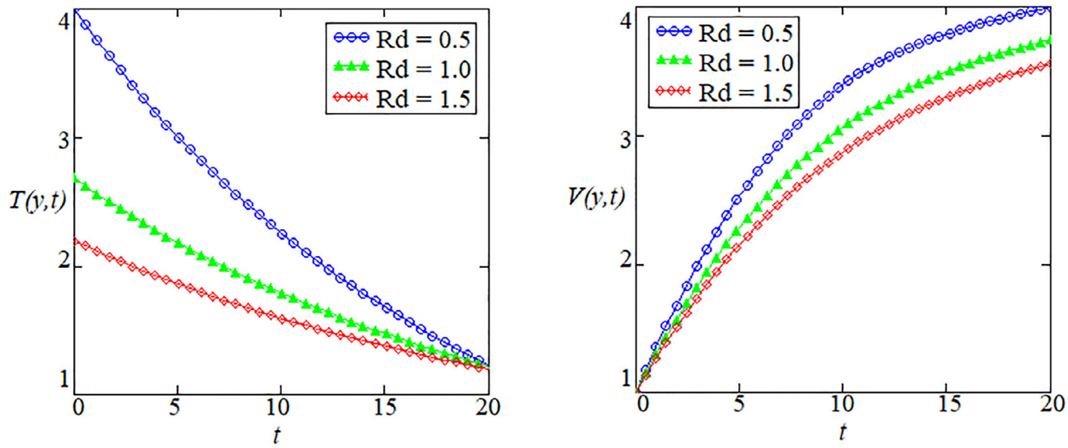

Fig. 2. Effects of thermal radiation on temperature distribution and velocity field.

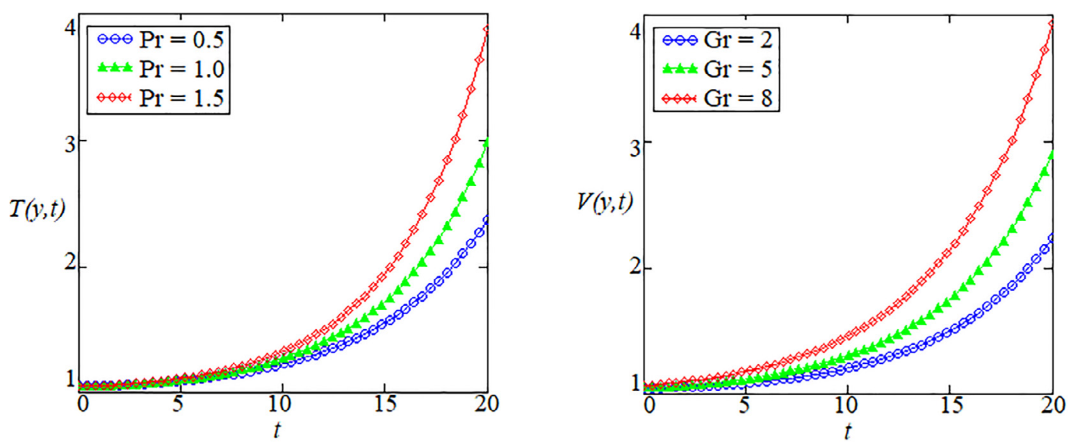

Fig. 3. Effects of Prandtl and Grashof number on temperature distribution and velocity field.

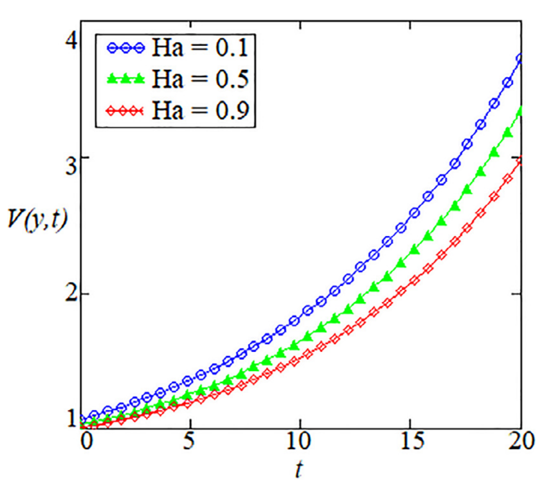

Fig. 4. Effects of Hartman number on the velocity field.

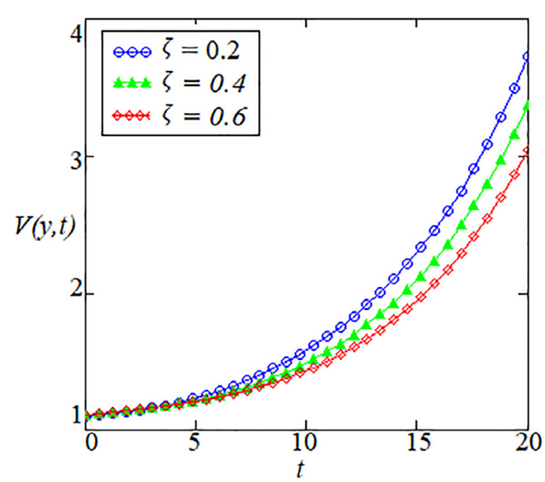

Fig. 5. Effects of Caputo-Fabrizio fractional parameter on the velocity field.

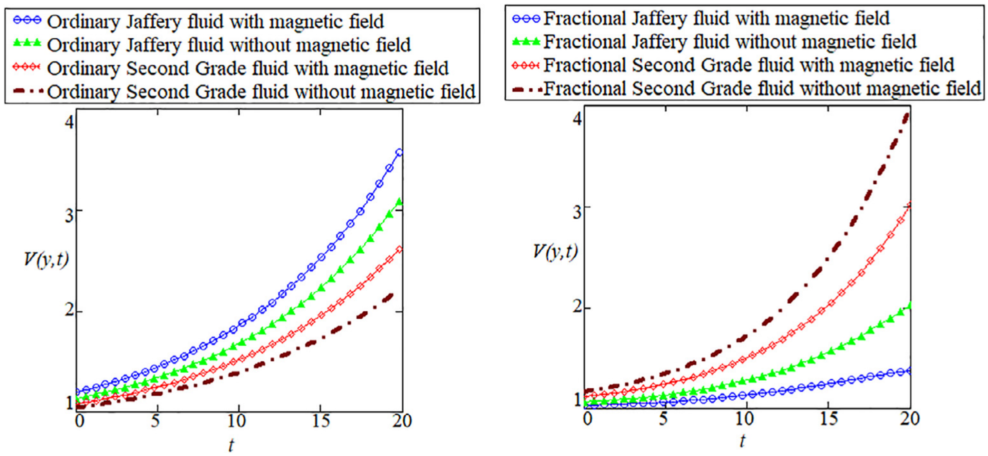

Fig. 6. Comparison of Ordinary Jaffery and Second grade fluid with and without magnetic field on the velocity field.

$$V(y,t) = UH(t)\cos\omega t + \frac{2UH(t)(\psi^2 - s_1 - s_2)}{\pi(s_1+s_2)}\int_0^\infty \int_0^t \frac{\sin(y\psi)}{\psi}\cos\omega(t-q)(s_5 - s_6)Exp(-s_6)\,t\,d\psi dq + \frac{2U\lambda\delta s_7}{\pi(s_1+s_2)}\int_0^\infty \int_0^t \psi\sin(y\psi)\cos\omega(t-q)Exp(-s_6)\,t\,d\psi dq + \frac{2G_r(1+\lambda_1)(Rd+1)}{\pi(s_1+s_2)s_3}\int_0^\infty \int_0^t \sin(y\psi)\times(t-q)\left\{\frac{(\delta\zeta+s_6)^2}{(s_7-s_6)}Exp(-s_6)t + \frac{(\delta\zeta+s_7)^2}{(s_6-s_7)}Exp(-s_7)t\right\}d\psi dq. \tag{37}$$

$$V(y,t) = UH(t)\sin\omega t + \frac{2UH(t)(\psi^2 - 2s_1)}{\pi(s_1+s_2)}\int_0^\infty \int_0^t \frac{\sin(y\psi)}{\psi}\sin\omega(t-q)(s_8 - s_6)Exp(-s_6)t\,d\psi dq + \frac{2U\lambda\delta s_7\omega}{\pi(s_1+s_2)}\int_0^\infty \int_0^t \psi\sin(y\psi)\cos\omega(t-q)Exp(-s_6)\,t\,d\psi dq + \frac{2G_r(1+\lambda_1)(Rd+1)}{\pi(s_1+s_2)s_3}\int_0^\infty \int_0^t \sin(y\psi)\times(t-q)\left\{\frac{(\delta\zeta+s_6)^2}{(s_7-s_6)}Exp(-s_6)t + \frac{(\delta\zeta+s_7)^2}{(s_6-s_7)}Exp(-s_7)t\right\}d\psi dq. \tag{38}$$

where, $s_1 = \delta + \lambda\delta\psi^2$, $s_2 = \psi^2 + (H_a + \Phi)$, $s_3 = P_r\delta + \psi + \psi Rd$ and $s_4 = \zeta\delta\psi + \psi Rd\zeta\delta$. $s_5 = \frac{\zeta\delta\psi^2 - \zeta\delta s_2}{\psi^2 - s_1 - s_2}$, $s_6 = \frac{\zeta\delta s_2}{s_1+s_2}$, $s_7 = \frac{s_4}{s_3}$ and $s_8 = \frac{\zeta\delta\psi^2 - \zeta\delta s_2}{\psi^2 - 2s_1}$It is noted that our general solutions of velocity profile can be calculated for skin friction and Nusselt number for sine and cosine oscillations with and without magnetic field investigated by Nor Athirah (Zin et al., 2016), see Eqs. (37) and (38)]. One can also have solutions in ordinary partial differential equation if Caputo-Fabrizio fractional parameter set as $\zeta = 1$, simply we replaced the ordinary time derivative with the Caputo–Fabrizio fractional order derivative.

6. Discussions and concluding remarks

In this portion, influences of thermal radiation, magnetic field, Caputo–Fabrizio fractional parameter and various physical parameters elucidate the distinct properties of fluid flow on the oscillations on Jeffery fluid. The Caputo–Fabrizio fractionalized partial differential equations are solved by utilizing discrete Laplace and Fourier Sine transforms with their inverses. The general and particularized solutions satisfy initial and boundary conditions as expected. However, the summary of this problem is emphasized with focal points listed below:

I. Fig. 2 is depicted to emphasize the effects of thermal radiation on temperature and velocity profiles. It is observed that profile of temperature is sequestrating while velocity field has scattering behavior of fluid flow over the boundary of plate. This is due to increase in thermal radiation which produces sequestrating and scattering behavior of fluid flow.
II. Fig. 3 is prepared to illustrate the influence on temperature field for viscous and thermal forces and velocity profile for buoyancy forces. It is noted that profiles of temperature and velocity field have quit identical behavior to each other while increase in Prandtl number P_r and Grashof number G_r respectively.
III. The effect of transverse magnetic field M is highlighted in Fig. 4, in which pattern of oscillating is dominant in velocity distribution. Due to applied magnetic field, flow of velocity field is smattering and scattering with monotonic effect.
IV. The Caputo–Fabrizio fractional parameter ζ is emphasized in Fig. 5 to characterize memory properties on temperature distribution and velocity field. The effect of Caputo–Fabrizio fractional parameter ζ seems to be identical qualitatively as expected for both temperature and velocity.
V. Fig. 6 is prepared for comparison of ordinary velocity field and fractionalized velocity field with four models i-e (i) Jeffery fluid with magnetic field (ii) Jeffery fluid without magnetic field (iii) Second grade fluid with magnetic field and (iv) Second grade fluid without magnetic field. It is worth pointing out that ordinary fluid is extremely sequestrating in contrast with fractionalized fluid. Ordinary Jeffery fluid with magnetic field oscillates more rapidly to remaining models of fluid but fractionalized Jeffery fluid with magnetic field has reciprocal behavior of fluid flow. The similar phenomenon can also be assessed for temperature distribution via graphical illustrations as well.

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