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ORIGINAL ARTICLE

Solitary waves solutions of the MRLW equation using quintic B-splines

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Abstract In this paper, B-spline finite element method is used to solve the Modified Regularized Long Wave (MRLW) equation. The proposed approach involves a collocation method using quintic B-splines at the knot points as element shape. Time integration of the resulting system of ordinary differential equation is effected using the fourth order Runge–Kutta method, instead of the difference method. The resulting system of ordinary differential equations is integrated with respect to time. Three invariants of motion are evaluated to determine the conservation properties of the suggested scheme. The suggested numerical scheme leads to accurate and efficient results. Moreover, interaction two and three solitary waves are studied through computer simulation and the development of the Maxwellian initial condition into solitary waves is also shown.

1. Introduction

Solitary waves are wave packets or pulses, which propagate in nonlinear dispersive media. Due to dynamical balance between the nonlinear and dispersive effects these waves retain a stable waveform. The Regularized Long Wave (RLW) equation of the form:

$$u_t + u_x + uu_x - \delta u_{xxt} = 0, \tag{1}$$

where δ is a positive constant, was originally introduced to describe the behavior of the undular bore by Peregrine (1966). This equation is very important in physics media since it describes phenomena with weak nonlinearity and dispersion waves, including nonlinear transverse waves in shallow water, ion-acoustic and magneto hydrodynamic waves in plasma and phonon packets in nonlinear crystals. The solutions of this equation are kinds of solitary waves named solitons whose shape is not affected by a collision. RLW equation was solved numerically by various forms of finite element methods (Alexander and Morris, 1979; Gardner and Gardner, 1990; Gardner et al., 1995, 1996; Dag, 2000; Khalifa et al., 1979;

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Soliman and Raslan, 2001; Dag et al., 2004; Raslan, 2005; Soliman and Hussien, 2005) such as Galerkin method, least square method and collocation method with quadratic B-splines, cubic B-splines and recently septic splines. Indeed, the RLW equation is special case of the Generalized Long Wave (GRLW) equation, which has the form:

$$u_t + u_x + \mu u^p u_x - \delta u_{xxt} = 0, \tag{2}$$

where μ and δ are positive constants and p is a positive integer. The GRLW equation is studied by authors Zhang (2005) with finite difference method for a Cauchy problem and Kaya and El-Sayed (2003) with adomian decomposition method (ADM). Also, there are other studies on RLW, EWE, and GRLW equations (Ramos, 2007; Lu, 2008; Ramos, 2007; Soliman and Abdou, 2007; Shivamoggi, 2002). In this paper, we consider another special case of the GRLW which is called the Modified Regularized Long Wave (MRLW) equation. This equation was considered by Gardner et al. (1997) using Petrov–Galerkin method with quintic B-splines finite element. Here, a collocation method with quintic B-spline finite elements and uses the fourth order Runge–Kutta method to solve the system of first order ordinary differential equations instead of the finite difference method (Dag, 2000; Khalifa et al., 1979; Soliman and Raslan, 2001; Dag et al., 2004; Raslan, 2005; Soliman and Hussien, 2005; Zhang, 2005; Ramadan et al., 2005; El-Danaf et al., 2005; Hereman et al., 1986) which are accurate and efficient. The interaction of solitary waves and other properties of the MRLW equation are also studied.

2. The Governing equation and direct algebraic method (Gardner et al., 1997; Khalifa et al., 2007a,b; Hereman et al., 1986; Raslan, 2008)

Consider the MRLW equation of the form

$$u_t + u_x + 6u^2 u_x - u_{xxt} = 0, \quad a \leqslant x \leqslant b, \; t \geqslant 0, \tag{3}$$

where the subscripts x and t denote differentiation, is considered with the boundary conditions $u \to 0$ as $x \to \pm\infty$. In this work, periodic boundary conditions on the region $a \leqslant x \leqslant b$ are assumed in the form:

$$\left.\begin{aligned} u(a,t) &= u(b,t) = 0, \\ u_x(a,t) &= u_x(b,t) = 0, \\ u_{xx}(a,t) &= u_{xx}(b,t) = 0, \end{aligned}\right\} \tag{4}$$

and the initial conditions to be used will be prescribed later. To find the traveling wave solution of Eq. (3), we introduce the wave variable $\xi = x - ct$, where c represents the arbitrary constant velocity of the wave traveling in the positive direction on the x axis and $u(x,t) = f(\xi)$. So Eq. (3) takes the form:

$$-cf_\xi(\xi) + f_\xi(\xi) + 6f^2(\xi)f_\xi(\xi) + cf_{\xi\xi\xi}(\xi) = 0. \tag{5}$$

Integrating Eq. (5) gives

$$(1-c)f(\xi) + 2(f(\xi))^3 + cf_{\xi\xi}(\xi) = 0, \tag{6}$$

where the constant of integration equal zero since the solitary wave solution and its derivatives equal zero as $\xi \to \pm\infty$. The linear equation from (6) has the solution in the form $f(\xi) = e^{k\xi}, k = \pm\sqrt{\frac{c-1}{c}}$. We define $g(\xi) = e^{k\xi}$ and let $f(\xi) = \sum_{n=1}^{\infty} a_n (g(\xi))^n$. From Eq. (5), we get the recursion relation (RR)

$$(ck^2n^2 - c + 1)a_n + 2\sum_{m=2}^{n-1}\sum_{l=1}^{m-1} a_{n-m}a_{m-l}a_l = 0, \quad n \geqslant 3, \tag{7}$$

where the coefficients, in general, are of the form

$$\left.\begin{aligned} a_{2n} &= 0, & n \geqslant 1, \\ a_{2n+1} &= \tfrac{a_1^{2n+1}}{2^{2n}(1-c)^n}, & n \geqslant 0, \end{aligned}\right\} \tag{8}$$

and then the exact solutions of Eq. (3) take the forms

$$\left.\begin{aligned} u(x,t) &= \frac{(4-4c)e^{\sqrt{\frac{c-1}{c}}(x-ct)}a_1}{4-4c+e^{2\sqrt{\frac{c-1}{c}}(x-ct)}a_1^2}, \\ u(x,t) &= \frac{(4-4c)e^{\sqrt{\frac{c-1}{c}}(x-ct)}a_1}{(4c-4)e^{2\sqrt{\frac{c-1}{c}}(x-ct)}-a_1^2}, \end{aligned}\right\} \tag{9}$$

In Eq. (9) if we choose $a_1 = \sqrt{4c-4}$ then the solitary wave solution of MRLW equation reduces to:

$$u(x,t) = \sqrt{c-1}\,\mathrm{sec}\,h\left(\sqrt{\frac{c-1}{c}}(x-ct)\right), \tag{10}$$

if we replace c by $c+1$ we get the solution, which is the same solitary wave solution of the MRLW equation appears in other papers (Gardner et al., 1997; Khalifa et al., 2007b) .

$$u(x,t) = \sqrt{c}\,\mathrm{sec}\,h\left(\sqrt{\frac{c}{c+1}}(x-(c+1)t-x_0)\right). \tag{11}$$

Hence, this method may not yield the analytical solutions for many PDEs like in interaction solitary and the Maxwellian initial condition. Therefore, the numerical analysis plays a very important role for obtaining the accurate approximate solutions in these cases and that is our objective in this study. Also, Eq. (3) has three invariants as in the form (Gardner et al., 1997; Khalifa et al., 2007b):

$$\left.\begin{aligned} I_1 &= \textstyle\int_a^b u\,dx, \\ I_2 &= \textstyle\int_a^b (u^2 + u_x^2)dx, \\ I_3 &= \textstyle\int_a^b (u^4 - u_x^2)dx, \end{aligned}\right\} \tag{12}$$

we point out that these invariants help us to test the numerical schemes especially for equations with no analytical solution and during the interaction of solitons.

3. Collocation method for solving MRLW equation

In this section, we apply the method with the function B_j as quintic B-splines. We consider the approximate solution to the solution $u(x,t)$ is given by

$$u_N(x,t) = \sum C_j(t)B_j(x), \tag{13}$$

where $C_j(t)$ are time dependent parameters to be determined at each time level and $B_j(x)$ are the quintic B-splines given by:

$$B_i(x) = \frac{1}{h^5}\begin{cases} (x-x_{i-3})^5, & x_{i-3} \leqslant x \leqslant x_{i-2}, \\ (x-x_{i-3})^5 - 6(x-x_{i-2})^5, & x_{i-2} \leqslant x \leqslant x_{i-1}, \\ (x-x_{i-3})^5 - 6(x-x_{i-2})^5 + 15(x-x_{i-1})^5, & x_{i-1} \leqslant x \leqslant x_i, \\ (-x+x_{i+3})^5 + 6(x-x_{i+2})^5 - 15(x-x_{i+1})^5, & x_i \leqslant x \leqslant x_{i+1}, \\ (-x+x_{i+3})^5 + 6(x-x_{i+2})^5, & x_{i+1} \leqslant x \leqslant x_{i+2}, \\ (-x+x_{i+3})^5, & x_{i+2} \leqslant x \leqslant x_{i+3}, \\ 0, & \text{otherwise}. \end{cases} \tag{14}$$

Then, the discredited equations for the space derivative are derived as

$$\sum(B_j(x) - B_j''(x))\dot{C}_j(t) = -\left(1 + 6\left(\sum C_j(t)B_j(x)\right)^2\right) \times \sum C_j(t)B_j'(x), \quad (15)$$

where x takes the values at the selected collocation knot points for quintic B-spline. The values of $B_j(x)$ and its first and second derivatives at knots points are given in Table 1.

From these equations a system of first order ordinary differential equations can be obtained of the form:

$$A\dot{C}(t) = F(C(t)). \quad (16)$$

Several others studies solved the first order ordinary differential system (16) by using the central difference approximation for C, but in the present studies we solve the system (16) using fourth order Runge–Kutta method.

$$\left.\begin{aligned} AK_1 &= F(C^n), \\ AK_2 &= F\left(C^n + \tfrac{1}{2}K_1\right), \\ AK_3 &= F\left(C^n + \tfrac{1}{2}K_2\right), \\ AK_4 &= F(C^n + K_3), \end{aligned}\right\} \quad (17)$$

and we solved the last equations and using

$$C^{n+1} = C^n + \frac{k(K_1 + 2K_2 + 2K_3 + K_4)}{6}. \quad (18)$$

where K_1, K_2, K_3 and K_4 can be found by solving four systems (17). Once the parameter C has been determined at a specified time we can compute the solution at the required knots the time evolution of the approximate solution $u_N(x,t)$ is determined from that of the vector C^n which is found by repeatedly applying the above procedure once the starting vector C^0 has been computed from the initial condition.

4. Numerical tests and results

In this section we present some numerical tests of our scheme for the solution of MRLW equation for single solitary waves in addition to determining the solution of two and three solitary waves interaction at different time levels. Also to show the development of Maxwellian initial condition into solitary waves. The numerical solution must preserve the conservation laws during propagation.

4.1. Propagation of single solitary waves

To examine the validated and the efficiency of our scheme, we consider two cases in our numerical work, since L_∞-error norm and L_2-error norm are used to compare our numerical solutions with the exact solution of Eq. (3). Also the quantities I_1, I_2 and I_3 are evaluated to measure the conservation properties of the collocation scheme, the analytical values of these

Table 1 The values of quintic B-spline and its first and second derivatives at the knots points.

x	x_{j-3}	x_{j-2}	x_{j-1}	x_j	x_{j+1}	x_{j+2}	x_{j+3}
B_i	0	1	26	66	26	1	0
B_i'	0	$5/h$	$50/h$	0	$-50/h$	$-5/h$	0
B_i''	0	$20/h^2$	$40/h^2$	$-120/h^2$	$40/h^2$	$20/h^2$	0

Table 2 Invariants and errors for single solitary wave for $c = 1, \Delta x = 0.2, \Delta t = 0.1$ and $x_0 = 40, 0 \leqslant x \leqslant 100$.

T	I_1	I_2	I_3	L_2-error norm	L_∞-error norm
0	4.442883	3.298731	1.415311	2.855687E−6	2.145767E−6
1	4.442883	3.298723	1.415301	2.257935E−5	1.645088E−5
2	4.442883	3.298712	1.415290	4.273932E−5	2.598763E−5
3	4.442885	3.298702	1.415280	6.246631E−5	3.492832E−5
4	4.442884	3.298692	1.415270	8.416529E−5	4.494190E−5
5	4.442884	3.298681	1.415260	1.078404E−4	5.537271E−5
6	4.442883	3.298672	1.415250	1.334255E−4	6.842613E−5
7	4.442883	3.298661	1.415240	1.620302E−4	8.213520E−5
8	4.442884	3.298652	1.415230	1.931854E−4	9.953976E−5
9	4.442882	3.298642	1.415219	2.265411E−4	1.171231E−4
10	4.442882	3.298630	1.415209	2.632212E−4	1.369715E−4

Table 3 Invariants and errors for single solitary wave for $c = 1, \Delta x = 0.2, \Delta t = 0.1$ and $x_0 = 40, 0 \leqslant x \leqslant 100$, time $= 10$.

Method	I_1	I_2	I_3	$L_2 \times 10^3$	$L_\infty \times 10^3$
Analytical	4.44288	3.29983	1.41421	0	0
Present	4.44288	3.29863	1.415209	0.26322	0.13697
Gardner et al. (1997)	4.442	3.299	1.413	19.39	9.24
Gardner et al. (1997)	4.440	3.296	1.411	20.3	11.2
Khalifa et al. (2007b)	4.44288	3.29983	1.41420	9.30196	5.43718

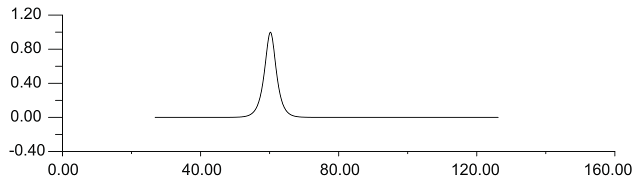

Figure 1 Single solitary wave with $c = 1, \Delta x = 0.2, \Delta t = 0.1$ and $x_0 = 40, 0 \leqslant x \leqslant 100,\ T = 10$.

invariants can be found as (Gardner et al., 1997): $I_1 = \frac{\pi\sqrt{c}}{2}, I_2 = c + \frac{4c}{3}$ and $I_3 = \frac{2c^2}{3} - \frac{4c}{3}$. In the first case, we choose the parameters $c = 1, \Delta x = 0.2, \Delta t = 0.1$ and $x_0 = 40$. The conservation properties and the L_2-error norm and L_∞-error norms. The analytical values for the invariants are $I_1 = 4.44288, I_2 = 3.29983$ and $I_3 = 1.41421$ are illustrated in Table 2 below. Moreover, Table 3 represents the values of the invariants and error norms of the present method at time 10 against the results of Gardner et al. (1997) and Khalifa et al. (2007b).

Also, Table 3 represents the values of the invariants and errors norms of the present method at time 10 against the recorded results of Gardner et al. (1997) and Khalifa et al. (2007b).

We find that our scheme provides good results than others. The motion of solitary wave using our scheme is plotted at time 10 in Fig. 1.

In the second case, we choose the parameters $c = 0.3, \Delta x = 0.2, \Delta t = 0.1$ and $x_0 = 40$ then the amplitude is 0.54772. The analytical values of the invariants are $I_1 = 3.58197, I_2 = 1.34508$ and $I_3 = 0.153723$. The changes of the invariants from the initial variants approach to zero throughout and agree with the analytical values for the three invariants, which indicated that our scheme is satisfactorily conservative. Errors are satisfactorily small, since L_2-error norm $= 1.9193 \times 10^{-5}$ and L_∞-error norm $= 8.970499 \times 10^{-6}$ at time 10. The results for the second case are shown in Table 4.

4.2. Interaction of two solitary waves

Interaction of two positive solitary waves is studied using the initial conditions given by the linear sum of two well-separated solitary waves of various amplitudes:

$$u(x,0) = A_1 \sec h(p_1(x - x_1)) + A_2 \sec h(p_2(x - x_2)), \tag{19}$$

where $A_i = \sqrt{c_i}, p_i = \sqrt{\frac{c_i}{c_i+1}}, i = 1,2, x_i$ and c_i are arbitrary constants. The analytical values of the conservation laws of this case can be found as $I_1 = \frac{\pi\sqrt{c_1}}{p_1} + \frac{\pi\sqrt{c_2}}{p_2}, I_2 = \frac{2c_1}{p_1} + \frac{2c_2}{p_2} + \frac{2p_1c_1}{3} + \frac{2p_2c_2}{3}$ and $I_3 = \frac{4c_1^2}{3p_1} + \frac{4c_2^2}{3p_2} - \frac{2p_1c_1}{3} - \frac{2p_2c_2}{3}$. In our computational work, we choose $c_1 = 4, c_2 = 1, x_1 = 25, x_2 = 55, \delta = 1, \Delta x = 0.2, \Delta t = 0.05$ with interval [0, 250], then the amplitudes are in ratio 2:1, where $A_1 = 2A_2$. The analytical values for the invariants of this case are $I_1 = 11.467698$, $I_2 = 14.629243$ and $I_3 = 22.880466$ and the changes in I_1, I_2

Table 5 Invariants of interaction two solitary waves of MRLW equation $c_1 = 4, c_2 = 1, x_1 = 25, x_2 = 55$, [0, 250].

Time	I_1	I_2	I_3
1	11.467700	14.617920	22.885030
2	11.467700	14.616560	22.879620
3	11.467700	14.615220	22.874220
4	11.467700	14.613870	22.868820
5	11.467770	14.612520	22.863410
6	11.467620	14.611190	22.858020
7	11.467270	14.609970	22.852570
8	11.466890	14.609760	22.846460
9	11.466490	14.613410	22.839010
10	11.466050	14.608160	22.842790

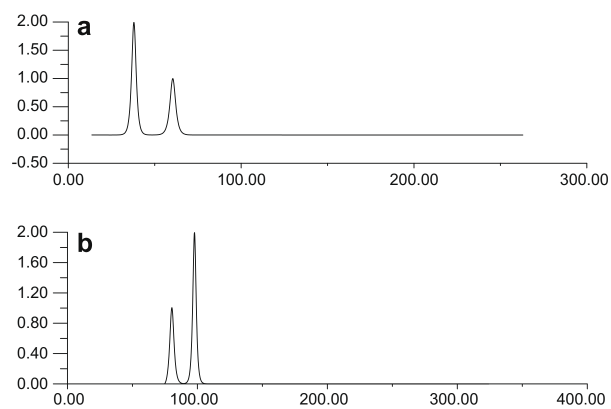

Figure 2 (a) Interaction two solitary waves with $c_1 = 4,\ c_2 = 1,\ x_1 = 25,\ x_2 = 55$, [0, 250] at times $T = 0$. (b) Interaction two solitary waves with $c_1 = 4,\ c_2 = 1,\ x_1 = 25,\ x_2 = 55$, [0, 250] at times $T = 14$.

Table 4 Invariants and errors for single solitary wave for $c = 0.3, \Delta x = 0.2, \Delta t = 0.1$ and $x_0 = 40, 0 \leqslant x \leqslant 100$.

T	I_1	I_2	I_3	L_2-error norm	L_∞-error norm
1	3.581964	1.344973	0.1538265	2.142769E−06	1.221895E−06
2	3.581966	1.344973	0.1538264	4.352044E−06	2.086163E−06
3	3.581964	1.344973	0.1538264	6.752275E−06	3.129244E−06
4	3.581965	1.344973	0.1538264	8.884196E−06	4.291534E−06
5	3.581964	1.344973	0.1538264	1.087434E−05	5.215406E−06
6	3.581965	1.344973	0.1538264	1.292003E−05	6.198883E−06
7	3.581964	1.344972	0.1538264	1.472382E−05	7.271767E−06
8	3.581964	1.344973	0.1538264	1.632736E−05	7.659197E−06
9	3.581960	1.344972	0.1538264	1.784318E−05	8.493662E−06
10	3.581958	1.344973	0.1538264	1.919314E−05	8.970499E−06

and I_3 as seen in Table 5 are small. Also, Fig. 2 shows the computer plot of the interaction of these solitary waves at different time levels, where the simulation is done to $t = 14$.

4.3. Interaction of three solitary waves

The interaction of three MRLW solitary waves having different amplitudes and traveling in the same direction is illustrated. We consider the MRLW equation with initial conditions given by the linear sum of three well-separated solitary waves of various amplitudes:

$$u(x,0) = A_1 \sec h(p_1(x-x_1)) + A_2 \sec h(p_2(x-x_2)) + A_3 \sec h(p_3(x-x_3)), \tag{20}$$

where $A_i = \sqrt{c_i}, p_i = \sqrt{\frac{c_i}{c_i+1}}, i = 1,2,3, x_i$ and c_i are arbitrary constants. The analytical values of the conservation laws of this case can be found as:

$$I_1 = \frac{\pi\sqrt{c_1}}{p_1} + \frac{\pi\sqrt{c_2}}{p_2} + \frac{\pi\sqrt{c_3}}{p_3},$$

$$I_2 = \frac{2c_1}{p_1} + \frac{2c_2}{p_2} + \frac{2c_3}{p_3} + \frac{2p_1c_1}{3} + \frac{2p_2c_2}{3} + \frac{2p_3c_3}{3},$$

$$I_3 = \frac{4c_1^2}{3p_1} + \frac{4c_2^2}{3p_2} + \frac{4c_3^2}{3p_3} - \frac{2p_1c_1}{3} - \frac{2p_2c_2}{3} - \frac{2p_3c_3}{3}.$$

In our computational work, we choose $c_1 = 4, c_2 = 1, c_3 = 0.25, x_1 = 15, x_2 = 45, x_3 = 60$ with interval [0, 250], then the amplitudes are in ratio 4:2:1, where $A_1 = 2A_2 = 4A_3$. The analytical values for the invariants of this case are $I_1 = 14.9801$, $I_2 = 15.8218$ and $I_3 = 22.9923$ and we find from our numerical scheme, that the invariants I_1, I_2 and I_3 for interaction of these solitary waves are sensible constants, comparing with their big amplitudes, the changes are $5 \times 10^{-2}, 5 \times 10^{-3}$ and 1×10^{-2} percent, respectively, for the computer run and the results are recorded in Table 3. Fig. 3 shows details of interaction of these solitary waves at different time levels, and the simulation is done to $t = 10$ (see Table 6).

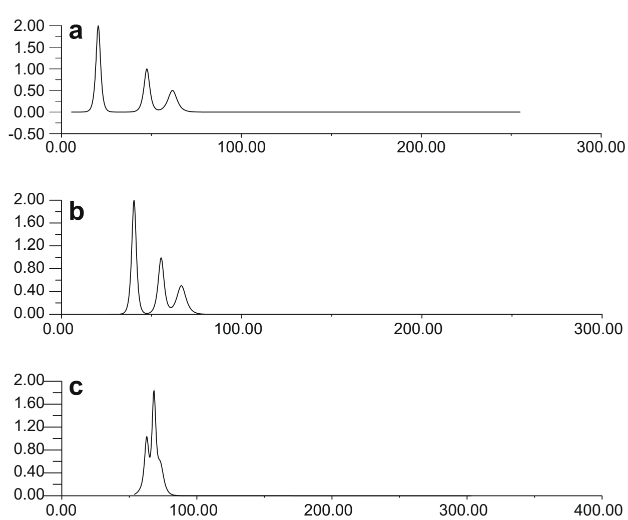

Figure 3 $c_1 = 4, c_2 = 1, c_3 = 0.25, x_1 = 15, x_2 = 45, x_3 = 60$, [0, 250], $T = 1$. (b) $c_1 = 4, c_2 = 1, c_3 = 0.25, x_1 = 15, x_2 = 45, x_3 = 60$, [0, 250], $T = 5$. (c) $c_1 = 4, c_2 = 1, c_3 = 0.25, x_1 = 15, x_2 = 45, x_3 = 60$, [0, 250], $T = 10$.

4.4. The Maxwellian initial condition

The fourth numerical test of our scheme is concerned with the generation of a train of solitary waves from Maxwellian initial condition:

$$u(x,0) = e^{-(x-40)^2}, \tag{21}$$

into a train of solitary waves is examined. In this section, we consider the MRLW equation of the form

$$u_t + u_x + 6u^2u_x - \delta u_{xxt} = 0, \tag{22}$$

as it is known, with the Maxwellian Eq. (21), the behavior of the solution depends on the values of δ. For $\delta \gg \delta_c$, where δ_c is some critical value. The Maxwellian does not break up into solutions but exhibits rapidly oscillating wave packets. When $\delta \approx \delta_c$ the mixed type of solutions is found which consists of a leading soliton and an oscillating tail. For $\delta \ll \delta_c$

Table 6 Invariants of interaction three solitary waves of MRLW equation $c_1 = 4, c_2 = 1, c_3 = 0.25, x_1 = 15, x_2 = 45, x_3 = 60$, [0, 250].

Time	I_1	I_2	I_3
1	14.980110	15.826100	23.012800
2	14.980090	15.824760	23.007380
3	14.980180	15.823430	23.001970
4	14.979930	15.822100	22.996530
5	14.979550	15.820770	22.991110
6	14.979140	15.819470	22.985690
7	14.978740	15.818400	22.980090
8	14.978040	15.818480	22.973750
9	14.974250	15.822740	22.965790
10	14.930390	15.822500	22.964190

Table 7 Computed values I_1, I_2, I_3 for Maxwellian initial condition when $h = 0.1$, $k = 0.1$, [0, 100].

δ	Time	I_1	I_2	I_3
1	2	1.772449	2.506352	−0.3668149
	4	1.772446	2.506235	−0.3666974
	6	1.772447	2.506171	−0.3666326
	8	1.772446	2.506123	−0.3665860
	10	1.772444	2.506092	−0.3665553
0.5	2	1.772451	1.879888	0. 2596494
	4	1.772446	1.879855	0.2596828
	6	1.772449	1.879841	0.2596973
	8	1.772450	1.879834	0.2597050
	10	1.772449	1.879828	0.2597092
0.1	2	1.772452	1.378607	0.7608777
	4	1.772451	1.378577	0.7608364
	6	1.772451	1.378546	0.7607937
	8	1.772451	1.378515	0.7607529
	10	1.772453	1.378483	0.7607117
0.04	2	1.772453	1.302368	0.8343938
	4	1.772453	1.300995	0.8320332
	6	1.772453	1.299635	0.8296967
	8	1.772451	1.298285	0.8273833
	10	1.772440	1.296948	0.8250930

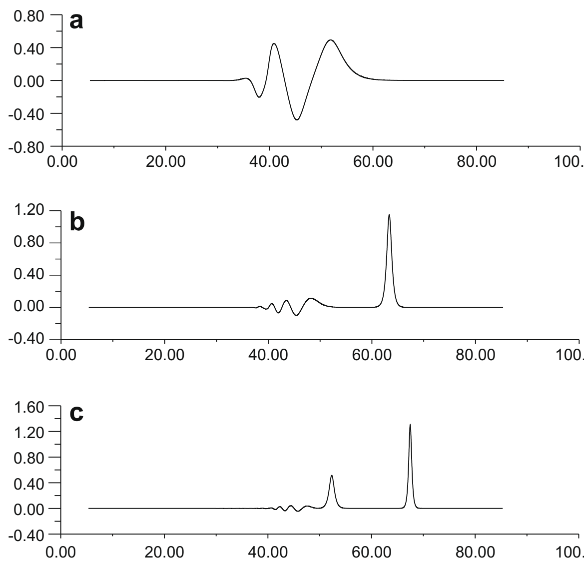

Figure 4 Maxwellian initial condition, $\Delta x = 0.1, \Delta t = 0.1$, $[0, 100], \delta = 1$. (b) Maxwellian initial condition, $\Delta x = 0.1, \Delta t = 0.1, [0, 100], \delta = 0.1$. (c) Maxwellian initial condition, $\Delta x = 0.1$, $\Delta t = 0.1, [0, 100], \delta = 0.04$.

the Maxwellian breaks up into a number of solitons according to the value of δ. The recorded values of the invariants I_1, I_2, I_3 are given in Table 7. The conservation properties are all good.

By decreasing the value of δ, more solitary waves are obtained. When $\delta = 0.1$, a single solitary wave is generated for our scheme, when $\delta = 0.04$ a train of two stable solitary waves is generated, and so on. The total number of solitary waves generated for various values of δ are in agreement with the results found in Gardner et al. (1997) and Khalifa et al. (2007b) (Fig. 4).

5. Conclusion

A numerical method based on collocation method using quintic B-spline finite elements within the collocation method leads to a system of first order differential equations is solved by fourth order Runge–Kutta method, which shows good conservation. The efficiency of the method is tested on the problems of propagation of single solitary wave, interaction of two and three solitary waves and development of train of solitary waves from Maxwellian initial condition. The three invariants of motion are constant in all the computer simulations described here. The problems presented in this paper suggest that the methods should be considered as one of possible ways of solving these kinds of nonlinear partial differential equations.

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