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Comments on: Effects on magnetic field in squeezing flow of a Casson fluid between parallel plates

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ABSTRACT

Ahmed et al. (2017) examined the dynamics of a Casson fluid squeezed between two parallel plates where the fluid was also subjected to a constant magnetic field. Therein they claimed to derive a fourth-order boundary value problem from the general governing equations whose solution could be used to form the fluid's velocity components. The purpose of the present note is to place the ideas in Ahmed et al. (2017) under scrutiny. In doing so we raise some doubts regarding some of the forms therein.

1. Introduction

The study of fluid flow with moving boundaries continues to draw the attention of research communities, with some recent contributions to be found in Tisdell (2023a,b). Moreover, Ahmed et al. (2017) [Effects on magnetic field in squeezing flow of a Casson fluid between parallel plates, Journal of King Saud University - Science, 29(1), 119–125, 2017] examined the dynamics of a Casson fluid squeezed between two parallel plates where the fluid was also subjected to a constant magnetic field. Therein they claimed to derive a fourth-order boundary value problem from the general governing equations whose solution could be used to form the fluid's velocity components. The purpose of the present note is to place the ideas in Ahmed et al. (2017) under scrutiny. In doing so we raise some doubts regarding some of the forms therein.

2. Problem formulation

Let us briefly reintroduce the model and the equations under consideration, drawing on the literature of Wang (1976, 1978) and Ahmed et al. (2017) where additional details can be found.

Consider the dynamics of an incompressible, squeezed Casson-type fluid in the xy-plane, with x representing the horizontal axis and y the vertical axis. Two plates parallel to the x-axis are positioned at $\pm h(t) = \pm l\sqrt{1-\alpha t}$ above and below the center line $y = 0$, where $y = \pm l$ signifies their positions at time $t = 0$, and α is a constant of dimension [1/time] that designates the unsteadiness of the plates (Wang, 1976, p. 579). For $\alpha > 0$ the plates are moving towards each other for $0 \leq t \leq t_T$, eventually meeting at the terminal instant $t_T = 1/\alpha$; whilst for $\alpha < 0$, the plates move away from each other for all $t \geq 0$.

The system is exposed to a uniform magnetic field oriented perpendicularly to the plates. It is assumed that there is no external electric field, and any influence of magnetic or electric fields generated by the motion of the electrically conducting fluid is considered insignificant.

The gap between the plates is assumed to be much smaller than their diameter D, so any end effects can be disregarded. The lateral velocity of the fluid is proportional to the distance from the center when considering continuity (Wang, 1976, p. 579).

Ahmed et al. (2017, p. 120) drew on the governing equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left(1+\frac{1}{\gamma}\right)\left(2\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial y \partial x}\right) - \frac{\sigma\beta^2}{\rho}u, \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \nu\left(1+\frac{1}{\gamma}\right)\left(2\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 u}{\partial y \partial x}\right). \tag{3}$$

In the above context, u and v correspond to the velocity components of the fluid along the x- and y-axes, respectively. The pressure of the fluid is denoted by P, while ν represents its kinematic viscosity. The parameter γ characterizes the Casson fluid behavior, and β quantifies the magnetic field strength, expressed as $\beta = \beta_0/h(t)$ (Noor et al., 2020, p. 96), where β_0 is the magnetic field's initial magnitude in the system.

Ahmed et al. (2017, p. 121) established the boundary conditions of the system, namely:

$$u = 0,\ v = v_w = \frac{dh}{dt},\ \text{at } y = h(t); \tag{4}$$

$$\frac{\partial u}{\partial y} = 0, \quad v = 0, \quad \text{at } y = 0; \tag{5}$$

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and introduced the following expressions:

$$\eta = \frac{y}{h(t)}, \quad u = \frac{\alpha l^2 x}{2h^2(t)} F'(\eta), \quad v = -\frac{\alpha l^2}{2h(t)} F(\eta), \tag{6}$$

where F is a sufficiently smooth, unknown function to be determined and $h(t) := l\sqrt{1-\alpha t}$.

3. Regarding the forms of Ahmed et al.

3.1. The derived boundary value problem

Ahmed et al. substituted the forms (6) into the governing equations in (1)–(3) and used cross-differentiation to obtain

$$\left(1+\frac{1}{\gamma}\right)F''''(\eta) - S\Big(\eta F(\eta) + 3F''(\eta) + F'(\eta)F''(\eta) - F(\eta)F'''(\eta)\Big) - M^2 F''(\eta) = 0, \tag{7}$$

and Ahmed et al. used (4) and (5) to form

$$F(0)=0,\ F''(0)=0,\ F(1)=1,\ F''(1)=0, \tag{8}$$

where they defined the squeeze number as $S := \alpha l^2/2\upsilon$ and M was termed as the magnetic number, see Ahmed et al. (2017, p. 121).

Upon reexamining Ahmed et al.'s derivation of the boundary value problem (7)–(8), we believe it should be of the form

$$\left(1+\frac{1}{\gamma}\right)F''''(\eta) - S\Big(\eta F'''(\eta) + 3F''(\eta) + F'(\eta)F''(\eta) - F(\eta)F'''(\eta)\Big) - M^2 F''(\eta) = 0, \tag{9}$$

$$F(0)=0,\ F''(0)=0,\ F(1)=1,\ F'(1)=0, \tag{10}$$

where the squeeze number is corrected to $S := \alpha l^2/2\nu$. Observe that (9)–(10) amends some errors, including: the term in the differential equation has been corrected to $\eta F'''(\eta)$; one of the right-hand boundary conditions is now corrected to $F'(1) = 0$; and υ has been replaced with ν in the definition of the squeeze number S. We will justify these corrections in the following subsection.

3.2. A corrected boundary value problem

Let us justify our forms in (9)–(10) by following the same derivation process of Ahmed et al..

Cross-differentiation, that is, differentiation of (3) with respect to x minus the differentiation of (2) with respect to y produces

$$\frac{\partial^2 v}{\partial x\partial t} - \frac{\partial^2 u}{\partial y\partial t} + \frac{\partial}{\partial x}\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) - \frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right)$$
$$= \nu\left(1+\frac{1}{\gamma}\right)\left(2\frac{\partial^3 v}{\partial x^3} + \frac{\partial^3 v}{\partial x\partial y^2} + \frac{\partial^3 u}{\partial x\partial y\partial x} - 2\frac{\partial^3 u}{\partial y\partial x^2} - \frac{\partial^3 u}{\partial y^3} - \frac{\partial^3 v}{\partial y^2\partial x}\right) + \frac{\sigma\beta^2}{\rho}\frac{\partial u}{\partial y}.$$

Using the transformations (6) on the above, noting that

$$\frac{\partial v}{\partial x} = 0 \quad \text{and} \quad \frac{\partial^2 u}{\partial x^2} = 0,$$

we thus obtain the equation

$$\left(-\frac{\alpha^2 x}{4l(1-\alpha t)^{5/2}}\eta F'''(\eta) - \frac{3\alpha^2 x}{4l(1-\alpha t)^{5/2}}F''(\eta)\right) + \frac{\alpha^2 x}{4l(1-\alpha t)^{5/2}}F(\eta)F''(\eta) - \frac{\alpha^2 x}{4l(1-\alpha t)^{5/2}}F''(\eta)F'(\eta)$$
$$= \nu\left(1+\frac{1}{\gamma}\right)\left(-\frac{\alpha x}{2l^3(1-\alpha t)^{5/2}}F''''(\eta)\right) + \frac{\sigma\beta^2}{\rho}\frac{\alpha x}{2l(1-\alpha t)^{3/2}}F''(\eta).$$

If we thus multiply both sides of the previous expression by

$$\frac{2l^3(1-\alpha t)^{5/2}}{\alpha x \nu}$$

and substitute in the squeeze number $S = \alpha l^2/2\nu$ then we obtain the equation

$$\left(1+\frac{1}{\gamma}\right)F''''(\eta) - \left(\frac{\beta^2 l^2 \sigma(1-\alpha t)}{\rho\nu}\right)F''(\eta) = S\Big(\eta F'''(\eta) + 3F''(\eta) + F'(\eta)F''(\eta) - F(\eta)F'''(\eta)\Big). \tag{11}$$

Observe in (11) that

$$M^2 = \frac{\beta^2 l^2\sigma(1-\alpha t)}{\rho\nu} = \frac{\beta^2\sigma h^2(t)}{\rho\nu} = \frac{\beta_0^2 l^2\sigma}{\rho\nu}$$

producing the non-dimensionalized constant $M = \beta_0 l\sqrt{\sigma/(\rho\nu)}$ known as the Hartmann number.

The boundary conditions in (4), (5), when transformed by (6), yield

$$u = 0 \text{ at } y = h(t) \Rightarrow \frac{\alpha l^2 x}{2h^2(t)}F'\left(\frac{h(t)}{h(t)}\right) = 0, \ \text{ so } \ F'(1) = 0;$$

$$v = \frac{dh}{dt} \text{ at } y = h(t) \Rightarrow -\frac{\alpha l^2}{2h(t)}F\left(\frac{h(t)}{h(t)}\right) = \frac{dh}{dt}, \text{ so } -\frac{\alpha l^2}{2h(t)}F(1) = -\frac{\alpha l^2}{2h(t)}, \text{ and } F(1) = 1;$$

$$\frac{\partial u}{\partial y} = 0 \text{ at } y = 0 \Rightarrow \frac{\alpha x}{2l(1-\alpha t)^{3/2}}F''\left(\frac{0}{h(t)}\right) = 0, \text{ so } F''(0) = 0;$$

$$v = 0 \text{ at } y = 0 \Rightarrow -\frac{\alpha l^2}{2h(t)}F'\left(\frac{0}{h(t)}\right) = 0, \text{ so } F(0) = 0.$$

As we can see, appropriate boundary conditions have now been derived and one of the conditions at the right-hand end point has been corrected.

4. Conclusion

In this commentary we examined some of the forms presented by Ahmed et al. (2017) in their investigation of squeezing flow. We discovered some inconsistencies with the resultant differential equation and one of the boundary conditions therein. We reconsidered and corrected the derivation of the boundary value problem.

CRediT authorship contribution statement

Joshua Liam Lam: Writing – review & editing, Writing – original draft, Validation, Investigation, Formal analysis, Data curation. **Christopher C. Tisdell:** Writing – review & editing, Writing – original draft, Validation, Supervision, Project administration, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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