



ORIGINAL ARTICLE

A new analytical method for solving systems of linear integro-differential equations

Hossein Aminikhah *

Department of Mathematics, School of Mathematical Sciences, University of Guilan, P.O. Box 1914, Rasht, Iran

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Abstract In this paper, we introduce a new modification of homotopy perturbation method (NHPM) to obtain exact solutions of systems of linear integro-differential equations. Theoretical considerations are discussed. Some examples are presented to illustrate the efficiency and simplicity of the method.

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1. Introduction

Integro-differential equation has attracted much attention and solving this equation has been one of the interesting tasks for mathematicians. These equations have been found to describe various kind of phenomena such as wind ripple in the desert, nono-hydrodynamics, dropwise consideration and glass-forming process (Bo et al., 2007; Sun et al., 2007; Wang et al., 2007; Xu et al., 2007).

The homotopy perturbation method is a powerful device for solving functional equations. The method has been used by many authors to handle a wide variety of scientific and

engineering applications to solve various functional equations. In this method the solution is considered as the summation of an infinite series which converges rapidly to the accurate solutions. Considerable research works have been conducted recently in applying this method to a class of linear and nonlinear equations. This method was further developed and improved by He and applied to nonlinear oscillators with discontinuities (He, 1999), nonlinear wave equations (He, 2000), boundary value problems (He, 2004), limit cycle and bifurcation of nonlinear problems (He, 2003), and many other subjects (He, 2004, 2005, 2006, 2005). It can be said that He's homotopy perturbation method is a universal one, and is able to solve various kinds of nonlinear functional equations. For examples it was applied to nonlinear Schrödinger equations Biazar and Ghazvini (2007), to nonlinear equations arising in heat transfer (Ganji, 2006), to the quadratic Ricatti differential equation (Abbasbandy, 2006), and to other equations (Odibat and Momani, 2008; Siddiqui et al., 2008; Ganji and Sadighi, 2007; Golbabai and Javidi, 2007; Golbabai and Keramati, 2008; Shakeri and Dehghan, 2008; Beléndez et al., 2008). Biazar and Ghazvini (2009) and Biazar and Ghazvini (2008) employed He's homotopy perturbation method to compute an approximation to the solution of system of Volterra integral equations and nonlinear Fredholm integral equation of second

* Tel./fax: +98 131 3233509.

E-mail address: hossein.aminikhah@gmail.com



kind. In Mohyud-Din et al. (2010), Raftari and Yildirim (2010), Yildirim et al. (2010) and Yildirim and Gülkanat (2010), some recent non-perturbative methods have been studied to solve various nonlinear problems.

In this article a new homotopy perturbation method is introduced to obtain exact solutions of systems of integro-differential equations. To demonstrate this method, some examples are given.

2. New homotopy perturbation method (NHPM) for systems of integro-differential equations

A system of integro-differential can be considered in general as follows:

$$\frac{dx(t)}{dt} = \mathbf{F}(t, x(t)) + \int_{t_0}^t \mathbf{K}(s, t, x(s))ds, \quad t_0 \geq 0, \tag{1}$$

$$x(t_0) = x_0,$$

where

$$x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T,$$

$$\mathbf{K}(s, t, x(t)) = (k_1(s, t, x(s)), k_2(s, t, x(s)), \dots, k_n(s, t, x(s)))^T.$$

If $\mathbf{K}(s, t, x(t))$ and $\mathbf{F}(t, x(t))$ be linear, the system (1) can be represented as the following simple form:

$$\frac{dx_i}{dt} = f_i(t) + \sum_{j=1}^n \left(w_{ij}(t)x_j(t) + \int_{t_0}^t k_{ij}(s, t)x_j(s)ds \right), \tag{2}$$

$$x_i(t_0) = \alpha_i, \quad i = 1, 2, \dots, n.$$

For solving system (2), by new homotopy perturbation method, we construct the following homotopy

$$(1-p) \left(\frac{dX_i}{dt} - x_{i,0} \right) + p \left(\frac{dX_i}{dt} - f_i(t) - \sum_{j=1}^n \left(w_{ij}(t)x_j(t) + \int_{t_0}^t k_{ij}(s, t)x_j(s)ds \right) \right) = 0, \tag{3}$$

or equivalently,

$$\frac{dX_i}{dt} = x_{i,0} - p \left(x_{i,0} - f_i(t) - \sum_{j=1}^n \left(w_{ij}(t)x_j(t) + \int_{t_0}^t k_{ij}(s, t)x_j(s)ds \right) \right). \tag{4}$$

Applying the inverse operator, $L^{-1} = \int_{t_0}^t (\cdot)dt$ to both sides of Eq. (4), we obtain

$$X_i(t) = \alpha_i + \int_{t_0}^t x_{i,0}(s)ds - p \left(\int_{t_0}^t x_{i,0}(s)ds - \int_{t_0}^t f_i(s)ds - \sum_{j=1}^n \left(\int_{t_0}^t w_{ij}(s)x_j(s)ds + \int_{t_0}^t \int_{t_0}^\tau k_{ij}(s, \tau)x_j(s)dsd\tau \right) \right). \tag{5}$$

Suppose the solutions of system (5) have the following form:

$$X_i(t) = X_{i,0}(t) + pX_{i,1}(t) + p^2X_{i,2}(t) + \dots, \quad i = 1, 2, \dots, n. \tag{6}$$

where $X_{ij}(t)$, $i = 1, 2, \dots, n$ and $j = 0, 1, 2, \dots$ are functions which should be determined.

Now suppose that the initial approximations to the solutions $X_{i,0}(t)$ or $x_{i,0}(t)$ have the form

$$X_{i,0}(t) = x_{i,0}(t) = \sum_{j=0}^\infty \alpha_{ij}P_j(t), \quad i = 1, 2, \dots, n, \tag{7}$$

where α_{ij} are unknown coefficients and $P_0(x), P_1(x), P_2(x), \dots$ are specific functions.

Substituting (6) into (5) and equating the coefficients of p with the same power leads to

$$p^0 : X_{i,0}(t) = \alpha_i + \sum_{j=0}^\infty \alpha_{ij} \int_{t_0}^t P_j(s)ds,$$

$$p^1 : X_{i,1}(t) = - \sum_{j=0}^\infty \alpha_{ij} \int_{t_0}^t P_j(s)ds + \int_{t_0}^t f_i(s)ds + \sum_{j=1}^n \left(\int_{t_0}^t w_{ij}(s)X_{j,0}(s)ds + \int_{t_0}^t \int_{t_0}^\tau k_{ij}(s, \tau)X_{j,0}(s)dsd\tau \right),$$

$$p^2 : X_{i,2}(t) = \sum_{j=1}^n \left(\int_{t_0}^t w_{ij}(s)X_{j,1}(s)ds + \int_{t_0}^t \int_{t_0}^\tau k_{ij}(s, \tau)X_{j,1}(s)dsd\tau \right),$$

$$\vdots$$

$$p^i : X_{i,j-1}(t) = \sum_{j=1}^n \left(\int_{t_0}^t w_{ij}(s)X_{j,j-1}(s)ds + \int_{t_0}^t \int_{t_0}^\tau k_{ij}(s, \tau)X_{j,j-1}(s)dsd\tau \right),$$

$$\vdots$$

$$\tag{8}$$

Now if these equations be solved in a way that $X_{i,1}(t) = 0$, then Eq. (8) result in $X_{i,2}(t) = X_{i,2}(t) = \dots = 0$, therefore the exact solution can be obtained by using

$$x_i(t) = X_{i,0}(t) = \alpha_i + \sum_{j=0}^\infty \alpha_{ij} \int_{t_0}^t P_j(s)ds. \tag{9}$$

It is worthwhile to note that if $f_i(t)$ and $x_{i,0}(t)$ are analytic at $t = t_0$, then their Taylor series

$$x_{i,0}(t) = \sum_{n=0}^\infty a_n(t - t_0)^n, \quad f_i(t) = \sum_{n=0}^\infty a_n^*(t - t_0)^n, \tag{10}$$

can be used in Eq. (8), where $a_0^*, a_1^*, a_2^*, \dots$ are known coefficients and $a_0 a_1 a_2 \dots$ are unknown ones, which must be computed.

We would explain this method by considering several examples.

3. Examples

In this section we present two examples. These examples are considered to illustrate the NHPM for systems of integro-differential equations.

Example 1. Consider the following system of integro-differential equations with the exact solutions $x_1(t) = e^t$ and $x_2(t) = e^{-t}$,

$$\frac{dx_1(t)}{dt} = t^4 - t^3 - 2t^2 - 6 + (3t^2 - 6t + 7)x_1(t) + 2t^2(t + 1)x_2(t) + \int_0^t ((s^3 - t^3)x_1(s) + t^2(s^2 - t^2)x_2(s))ds, x_1(0) = 1,$$

$$\frac{dx_2(t)}{dt} = -t^4 - 3t^2 + 2 + 2(t-1)x_1(t) + (2t^4 + 2t^3 + 2t^2 - 1)x_2(t) + \int_0^t ((s^2 - t^2)x_1(s) + t^2(s^2 + t^2)x_2(s))ds, x_2(0) = 1. \tag{11}$$

For solving system (11), by NHPM, we construct the following homotopy:

$$\begin{aligned} \frac{dX_1(t)}{dt} &= x_{1,0}(t) - p(x_{1,0}(t) - t^4 + t^3 + 2t^2 + 6 \\ &\quad - (3t^2 - 6t + 7)X_1(t) - 2t^2(t+1)X_2(t) \\ &\quad - \int_0^t ((s^3 - t^3)X_1(s) + t^2(s^2 - t^2)X_2(s))ds), \\ \frac{dX_2(t)}{dt} &= x_{2,0}(t) - p(x_{2,0}(t) + t^4 + 3t^2 - 2 - 2(t-1)X_1(t) \\ &\quad - (2t^4 + 2t^3 + 2t^2 - 1)X_2(t) \\ &\quad - \int_0^t ((s^2 - t^2)X_1(s) + t^2(s^2 + t^2)X_2(s))ds). \end{aligned} \tag{12}$$

Assuming that, $x_{1,0}(t) = \sum_{n=0}^{\infty} \alpha_n P_n(t)$, $x_{2,0}(t) = \sum_{n=0}^{\infty} \beta_n P_n(t)$, $P_i(t) = t^i$, $X_1(0) = X_2(0) = 1$.

By integration of Eq. (12) we have

$$\begin{aligned} X_1(t) &= 1 + \sum_{n=0}^{\infty} \frac{\alpha_n}{n+1} t^{n+1} - p \left(\sum_{n=0}^{\infty} \frac{\alpha_n}{n+1} t^{n+1} - \frac{t^5}{5} + \frac{t^4}{4} \right. \\ &\quad \left. + \frac{2t^3}{3} + 6t - \int_0^t ((3s^2 - 6s + 7)X_1(s) + 2s^2(s+1)X_2(s))ds \right. \\ &\quad \left. - \int_0^t \int_0^\tau ((s^3 - \tau^3)X_1(\tau) + \tau^2(s^2 - \tau^2)X_2(\tau))dsd\tau \right), \end{aligned} \tag{13}$$

$$\begin{aligned} X_2(t) &= 1 + \sum_{n=0}^{\infty} \frac{\beta_n}{n+1} t^{n+1} - p \left(\sum_{n=0}^{\infty} \frac{\beta_n}{n+1} t^{n+1} + \frac{t^5}{5} + t^3 - 2t \right. \\ &\quad \left. - \int_0^t (2(s-1)X_1(s) + (2s^4 + 2s^3 + 2s^2 - 1)X_2(s))ds \right. \\ &\quad \left. - \int_0^t \int_0^\tau ((s^2 - \tau^2)X_1(\tau) + \tau^2(s^2 + \tau^2)X_2(\tau))dsd\tau \right). \end{aligned} \tag{14}$$

Suppose the solutions of system (13) have the following form:

$$X_i(t) = X_{i,0}(t) + pX_{i,1}(t) + p^2X_{i,2}(t) + \dots, \quad i = 1, 2, \tag{15}$$

where $X_{i,j}(t)$, $i = 1, 2$ and $j = 0, 1, 2, \dots$ are functions which should be determined.

Substituting (14) into (13) and equating the coefficients of p with the same powers leads to

$$p^0 : \begin{cases} X_{1,0}(t) = 1 + \sum_{n=0}^{\infty} \frac{\alpha_n}{n+1} t^{n+1}, \\ X_{2,0}(t) = 1 + \sum_{n=0}^{\infty} \frac{\beta_n}{n+1} t^{n+1}, \end{cases}$$

$$\begin{cases} p^1 : \begin{cases} X_{1,1}(t) = - \sum_{n=0}^{\infty} \frac{\alpha_n}{n+1} t^{n+1} + \frac{t^5}{5} - \frac{t^4}{4} - \frac{2t^3}{3} - 6t \\ \quad + \int_0^t ((3s^2 - 6s + 7)X_{1,0}(s) \\ \quad + 2s^2(s+1)X_{2,0}(s))ds \\ \quad + \int_0^t \int_0^\tau ((s^3 - \tau^3)X_{1,0}(\tau) \\ \quad + \tau^2(s^2 - \tau^2)X_{2,0}(\tau))dsd\tau, \\ X_{2,1}(t) = - \sum_{n=0}^{\infty} \frac{\beta_n}{n+1} t^{n+1} - \frac{t^5}{5} - t^3 \\ \quad + 2t + \int_0^t (2(s-1)X_{1,0}(s) \\ \quad + (2s^4 + 2s^3 + 2s^2 - 1)X_{2,0}(s))ds \\ \quad + \int_0^t \int_0^\tau ((s^2 - \tau^2)X_{1,0}(\tau) \\ \quad + \tau^2(s^2 + \tau^2)X_{2,0}(\tau))dsd\tau, \end{cases} \\ p^m : \begin{cases} X_{1,m}(t) = \int_0^t ((3s^2 - 6s + 7)X_{1,m-1}(s) \\ \quad + 2s^2(s+1)X_{2,m-1}(s))ds \\ \quad + \int_0^t \int_0^\tau ((s^3 - \tau^3)X_{1,m-1}(\tau) \\ \quad + \tau^2(s^2 - \tau^2)X_{2,m-1}(\tau))dsd\tau, \\ X_{2,m}(t) = \int_0^t (2(s-1)X_{1,m-1}(s) + (2s^4 \\ \quad + 2s^3 + 2s^2 - 1)X_{2,m-1}(s))ds \\ \quad + \int_0^t \int_0^\tau ((s^2 - \tau^2)X_{1,m-1}(\tau) \\ \quad + \tau^2(s^2 + \tau^2)X_{2,m-1}(\tau))dsd\tau, \\ m = 2, 3, \dots \end{cases} \end{cases}$$

Now if we set $X_{1,1}(t) = 0$, then

$$\begin{aligned} (1 - \alpha_0)t + \left(\frac{7\alpha_0}{2} - 3 - \frac{\alpha_1}{2} \right)t^2 + \left(\frac{7\alpha_1}{6} - 2\alpha_0 - \frac{\alpha_2}{3} + 1 \right)t^3 \\ + \left(\frac{7\alpha_2}{12} + \frac{3\alpha_0}{4} - \frac{3\alpha_1}{4} - \frac{\alpha_3}{4} + \frac{1}{4} + \frac{\beta_0}{2} \right)t^4 \\ + \left(\frac{7\alpha_3}{20} - \frac{2\alpha_2}{5} + \frac{3\alpha_1}{10} - \frac{\alpha_4}{5} + \frac{2\beta_0}{5} + \frac{\beta_1}{5} + \frac{1}{20} \right)t^5 + \dots = 0, \end{aligned}$$

and if we set $X_{2,1}(t) = 0$, then

$$\begin{aligned} -(1 + \beta_0)t + \left(1 - \frac{\beta_1}{2} - \frac{\beta_0}{2} - \alpha_0 \right)t^2 \\ + \left(\frac{2\alpha_0}{3} - \frac{\alpha_1}{3} - \frac{\beta_1}{6} - \frac{\beta_2}{3} - \frac{1}{3} \right)t^3 \\ + \left(\frac{\alpha_1}{4} - \frac{\alpha_2}{6} + \frac{\beta_0}{2} - \frac{\beta_3}{4} - \frac{\beta_2}{12} + \frac{1}{3} \right)t^4 \\ + \left(\frac{\beta_1}{5} + \frac{2\beta_0}{5} + \frac{2\alpha_2}{15} - \frac{\alpha_0}{20} - \frac{\beta_4}{5} - \frac{\alpha_3}{10} - \frac{\beta_3}{20} + \frac{1}{5} \right)t^5 + \dots = 0. \end{aligned}$$

It can be easily shown that

$$\begin{aligned} \alpha_0 = 1, \quad \alpha_1 = 1, \quad \alpha_2 = \frac{1}{2!}, \quad \alpha_3 = \frac{1}{3!}, \quad \alpha_4 = \frac{1}{4!}, \quad \alpha_5 = \frac{1}{5!}, \dots, \\ \beta_0 = -1, \quad \beta_1 = 1, \quad \beta_2 = -\frac{1}{2!}, \quad \beta_3 = \frac{1}{3!}, \quad \beta_4 = -\frac{1}{4!}, \quad \beta_5 = \frac{1}{5!}, \dots \end{aligned}$$

Therefore, the exact solutions of the system of integral-differential equation (11) can be expressed as

$$x_1(t) = 1 + \sum_{n=0}^{\infty} \frac{\alpha_n}{n+1} t^{n+1} = \sum_{n=0}^{\infty} \frac{t^n}{n!} = e^t,$$

$$x_2(t) = 1 + \sum_{n=0}^{\infty} \frac{\beta_n}{n+1} t^{n+1} = \sum_{n=0}^{\infty} (-1)^n \frac{t^n}{n!} = e^{-t}.$$

Example 2. Consider the following system of integro-differential equations with the exact solutions $x_1(t) = \cosh t$ and $x_2(t) = \sinh t$,

$$\begin{aligned} \frac{dx_1(t)}{dt} &= -t^3 - 6t - 1 + x_1(t) + (7 - 2t)x_2(t) \\ &\quad + \int_0^t ((s+t)x_1(s) + (s-t)^3 x_2(s)) ds, \quad x_1(0) = 1, \\ \frac{dx_2(t)}{dt} &= -3t^2 + t - 6 + (7 - 2t)x_1(t) + x_2(t) \\ &\quad + \int_0^t ((s-t)^3 x_1(s) + (s+t)x_2(s)) ds, \quad x_2(0) = 0. \end{aligned} \tag{16}$$

For solving system (15) by NHPM, we construct the following homotopy:

$$\begin{aligned} \frac{dX_1(t)}{dt} &= x_{1,0}(t) - p(x_{1,0}(t) + t^3 + 6t + 1 - X_1(t) - (7 - 2t)X_2(t) \\ &\quad - \int_0^t ((s+t)X_1(s) + (s-t)^3 X_2(s)) ds), \\ \frac{dX_2(t)}{dt} &= x_{2,0}(t) - p(x_{2,0}(t) + 3t^2 - t + 6 - (7 - 2t)X_1(t) - X_2(t) \\ &\quad - \int_0^t ((s-t)^3 X_1(s) + (s+t)X_2(s)) ds). \end{aligned} \tag{17}$$

Assuming that, $x_{1,0}(t) = \sum_{n=0}^{\infty} \alpha_n P_n(t)$, $x_{2,0}(t) = \sum_{n=0}^{\infty} \beta_n P_n(t)$, $P_i(t) = t^i$, $X_1(0) = 1$, $X_2(0) = 0$.

Applying the inverse operator, $L^{-1} = \int_{t_0}^t (\cdot) dt$ to both sides of Eq. (17), we obtain

$$\begin{aligned} X_1(t) &= 1 + \sum_{n=0}^{\infty} \frac{\alpha_n}{n+1} t^{n+1} - p \left(\sum_{n=0}^{\infty} \frac{\alpha_n}{n+1} t^{n+1} + \frac{t^4}{4} + 2t^3 + t \right. \\ &\quad \left. - \int_0^t (X_1(s) + (7 - 2s)X_2(s)) ds \right. \\ &\quad \left. - \int_0^t \int_0^\tau ((s+\tau)X_1(s) + (s-\tau)^3 X_2(s)) ds d\tau \right), \\ X_2(t) &= \sum_{n=0}^{\infty} \frac{\beta_n}{n+1} t^{n+1} - p \left(\sum_{n=0}^{\infty} \frac{\beta_n}{n+1} t^{n+1} + t^3 - \frac{t^2}{2} + 6t \right. \\ &\quad \left. - \int_0^t ((7 - 2s)X_1(s) + X_2(s)) ds \right. \\ &\quad \left. - \int_0^t \int_0^\tau ((s-\tau)^3 X_1(s) + (s+\tau)X_2(s)) ds d\tau \right). \end{aligned} \tag{18}$$

Suppose the solutions of system (17) have the form (14), substituting (14) into (17) and equating the coefficients of p with the same power leads to

$$p^0 : \begin{cases} X_{1,0}(t) = 1 + \sum_{n=0}^{\infty} \frac{\alpha_n}{n+1} t^{n+1}, \\ X_{2,0}(t) = \sum_{n=0}^{\infty} \frac{\beta_n}{n+1} t^{n+1}, \end{cases}$$

$$p^1 : \begin{cases} X_{1,1}(t) = - \sum_{n=0}^{\infty} \frac{\alpha_n}{n+1} t^{n+1} - \frac{t^4}{4} - 3t^2 - t \\ \quad + \int_0^t (X_{1,1}(s) + (7 - 2s)X_{2,1}(s)) ds \\ \quad + \int_0^t \int_0^\tau ((s+\tau)X_{1,1}(s) \\ \quad + (s-\tau)^3 X_{2,1}(s)) ds d\tau, \\ X_{2,1}(t) = - \sum_{n=0}^{\infty} \frac{\beta_n}{n+1} t^{n+1} - t^3 + \frac{t^2}{2} - 6t \\ \quad + \int_0^t ((7 - 2s)X_{1,1}(s) + X_{2,1}(s)) ds \\ \quad + \int_0^t \int_0^\tau ((s-\tau)^3 X_{1,1}(s) \\ \quad + (s+\tau)X_{2,1}(s)) ds d\tau, \end{cases}$$

$$p^m : \begin{cases} X_{1,m}(t) = \int_0^t (X_{1,m-1}(s) + (7 - 2s)X_{2,m-1}(s)) ds \\ \quad + \int_0^t \int_0^\tau ((s+\tau)X_{1,m-1}(s) \\ \quad + (s-\tau)^3 X_{2,m-1}(s)) ds d\tau, \\ X_{2,m}(t) = \int_0^t ((7 - 2s)X_{1,m-1}(s) + X_{2,m-1}(s)) ds \\ \quad + \int_0^t \int_0^\tau ((s-\tau)^3 X_{1,m-1}(s) \\ \quad + (s+\tau)X_{2,m-1}(s)) ds d\tau, \\ m = 2, 3, \dots \end{cases}$$

If we set $X_{1,1}(t) = 0$, then

$$\begin{aligned} -\alpha_0 t + \left(\frac{\alpha_0}{2} + \frac{7\beta_0}{2} - 3 - \alpha_1 \right) t^2 \\ + \left(\frac{1}{2} + \frac{\alpha_1}{6} + \frac{7\beta_1}{6} - \frac{2\beta_0}{3} - \frac{\alpha_2}{3} \right) t^3 \\ + \left(\frac{\alpha_2}{12} + \frac{5\alpha_0}{24} + \frac{7\beta_2}{12} - \frac{\beta_1}{4} - \frac{1}{4} - \frac{\alpha_3}{4} \right) t^4 \\ + \left(\frac{\alpha_3}{20} - \frac{\alpha_4}{5} + \frac{7\beta_3}{20} - \frac{2\beta_2}{15} + \frac{7\alpha_1}{120} \right) t^5 + \dots = 0. \end{aligned}$$

Further assume that $X_{2,1}(t) = 0$. Then we have

$$\begin{aligned} (1 - \beta_0)t + \left(\frac{7\alpha_0}{2} - \frac{1}{2} - \frac{\beta_0}{2} - \frac{\beta_1}{2} \right) t^2 \\ + \left(\frac{7\alpha_1}{6} - \frac{2\alpha_0}{3} + \frac{\beta_1}{6} - \frac{\beta_2}{3} - 1 \right) t^3 \\ + \left(\frac{7\alpha_2}{12} + \frac{5\beta_0}{24} - \frac{\beta_3}{4} - \frac{\alpha_1}{4} + \frac{\beta_2}{12} \right) t^4 \\ + \left(\frac{7\alpha_3}{20} + \frac{7\beta_1}{120} + \frac{\beta_3}{20} - \frac{2\alpha_2}{15} - \frac{\beta_4}{5} - \frac{1}{20} \right) t^5 + \dots = 0. \end{aligned}$$

It can be easily shown that

$$\begin{aligned} \alpha_0 = 0, \quad \alpha_1 = 1, \quad \alpha_2 = 0, \quad \alpha_3 = \frac{1}{3!}, \quad \alpha_4 = 0, \quad \alpha_5 = \frac{1}{5!}, \dots, \\ \beta_0 = 1, \quad \beta_1 = 0, \quad \beta_2 = \frac{1}{2!}, \quad \beta_3 = 0, \quad \beta_4 = \frac{1}{4!}, \quad \beta_5 = 0, \dots \end{aligned}$$

Thus

$$x_1(t) = 1 + \sum_{n=0}^{\infty} \frac{\alpha_n}{n+1} t^{n+1} = \sum_{n=0}^{\infty} \frac{t^{2n}}{(2n)!} = \cosh t,$$

$$x_2(t) = \sum_{n=0}^{\infty} \frac{\beta_n}{n+1} t^{n+1} = \sum_{n=0}^{\infty} \frac{t^{2n-1}}{(2n-1)!} = \sinh t.$$

which are exact solutions.

4. Conclusion

In this work, we considered a new homotopy perturbation method for solving systems of linear integro-differential equations. New method is a powerful straightforward method. Using this method we obtained new efficient recurrent relations to solve these systems. The new homotopy perturbation method is apt to be utilized as an alternative approach to current techniques being employed to a wide variety of mathematical problems. The computations associated with the examples in this paper were performed using maple 10.

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