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ORIGINAL ARTICLE

Explicit solutions of nonlinear (2 + 1)-dimensional dispersive long wave equation

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Abstract In this work,we construct the travelling wave solutions involving parameters of the (2 + 1)-dimensional dispersive long wave equation, by using a new approach, namely, the (G'/G)-expansion method, where $G = G(\xi)$ satisfies a second order linear ordinary differential equation. When the parameters are taken special values, the solitary waves are derived from the travelling waves.

1. Introduction

Searching for explicit solutions of nonlinear evolution equations by using various methods has become the main goal for many authors, and many powerful methods to construct exact solutions of nonlinear evolution equations have been established and developed such as the tanh-function expansion and its various extension, the Jacobi elliptic function expansion (Inc and Evans, 2004; Liu et al., 2001; Yan, 2003; Yan and Zhang, 1999; Zayed et al., 2005, 2007; Abdou, 2007; Fan, 2000; Bekir, 2008; Chow, 1995; Zhang et al., 2008a,b), very recently, Wang et al. introduced a new method called the $\frac{G'}{G}$-expansion method (Wang et al., 2008) to look for travelling wave solutions of nonlinear evolution equations. The $\frac{G'}{G}$-expansion method is based on the assumptions that the travelling wave solutions can be expressed by a polynomial in $\frac{G'}{G}$, and that $G = G(\xi)$ satisfies a second order linear ordinary differential equation (ODE). The objective of this paper is to use a new method which is called the (G'/G)-expansion method. The paper is arranged as follows. In Section 2, we describe briefly the $\frac{G'}{G}$-expansion method. In Sections 3, we apply the method to the (2 + 1)-dimensional dispersive long wave equation. In Section 4 some conclusions are given.

2. Description of the $\frac{G'}{G}$-expansion method

Suppose that a nonlinear equation, say in two independent variables x, y and t, is given by

$$P(u, u_t, u_x, u_y, u_{tt}, u_{xt}, u_{xx}, u_{yy}, \cdots) = 0, \tag{1}$$

where $u = u(x, y, t)$ is an unknown function, P is a polynomial in $u = u(x, y, t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. In the following we give the main steps of the $\frac{G'}{G}$-expansion method.

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step 1: Combining the independent variables x and t into one variable $\xi = x + y + wt$, we suppose that

$$u(x, y, t) = u(\xi), \quad \xi = x + y + wt. \tag{2}$$

The travelling wave variable (2) permits us to reduce Eq. (1) to an ODE for $u = u(\xi)$, namely

$$P(u, wu', u', u', v^2u'', wu'', u'', \cdots) = 0. \tag{3}$$

step 2: Suppose that the solution of ODE (3) can be expressed by a polynomial in $\frac{G'}{G}$ as follows

$$u(\xi) = \alpha_m \left(\frac{G'}{G}\right)^m + \cdots, \tag{4}$$

where $G = G(\xi)$ satisfies the second order LODE in the form

$$G'' + \lambda G' + \mu G = 0, \tag{5}$$

$\alpha_m, \ldots, \lambda$ and μ are constants to be determined later, $\alpha_m \neq 0$, the unwritten part in (4) is also a polynomial in $\frac{G'}{G}$, but the degree of which is generally equal to or less than $m - 1$, the positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in ODE (3).

step 3: By substituting (4) into Eq. (3) and using the second order linear ODE (5), collecting all terms with the same order of $\frac{G'}{G}$ together, the left-hand side of Eq. (3) is converted into another polynomial in $\frac{G'}{G}$. Equating each coefficient of this polynomial to zero yields a set of algebraic equations for $\alpha_m, \ldots, \lambda$ and μ.

step 4: Assuming that the constants $\alpha_m, \ldots, \lambda$ and μ can be obtained by solving the algebraic equations in step 3, since the general solutions of the second order LODE (5) have been well known for us, then substituting $\alpha_m, \ldots, w$ and the general solutions of Eq. (5) into (4) we have more travelling wave solutions of the nonlinear evolution Eq. (1).

3. (2 + 1)-Dimensional dispersive long wave equation

In this section, we study the following (2 + 1)-DDLW equation in the form

$$u_{yt} + v_{xx} + (uu_x)_y = 0, \tag{6}$$
$$v_t + u_x + (uv)_x + u_{xxy} = 0. \tag{7}$$

The travelling wave variable below

$$u(x, y, t) = u(\xi), \quad \xi = x + y + wt. \tag{8}$$

Permits us converting Eqs. (6) and (7) into an O.D.E for $u = u(\xi), \xi = x + y + wt$

$$u'' + v'' + \left(\frac{1}{2}(u^2)'\right)' = 0, \tag{9}$$
$$wv' + u' + (uv)' + u''' = 0. \tag{10}$$

Integrating twice of Eq. (9), we have

$$u + v + \frac{1}{2}u^2 = c, \tag{11}$$

where c is the integration constant, and the first integrating constant is taken to zero. And integrating it with respect to ξ of Eq. (10), once yields

$$wv + u + uv + u'' = 0. \tag{12}$$

Also first integrating constant of this equation is taken to zero. On substituting (11) into (12) we obtain

$$u'' - \frac{1}{2}u^3 - \frac{3}{2}wu^2 + (w^2 + c + 1)u + wc = 0. \tag{13}$$

Suppose that the solution of O.D.E (13) can be expressed by a polynomial in $\left(\frac{G'}{G}\right)$ as follows:

$$u(\xi) = \alpha_m \left(\frac{G'}{G}\right)^m + \cdots, \tag{14}$$

where $G = G(\xi)$ satisfies the second order LODE in the form

$$G'' + \lambda G'' + \mu G = 0, \tag{15}$$

$\alpha_1, \alpha_0, \lambda$ and μ are to be determined later.

By using (14) and (15) and considering the homogeneous balance between u'' and u^3 in Eq. (13) we required that $3m = m + 2$ then $m = 1$. So we can write (14) as

$$u(\xi) = \alpha_1 \left(\frac{G'}{G}\right) + \alpha_0. \tag{16}$$

And therefore

$$u^3 = \alpha_1^3 \left(\frac{G'}{G}\right)^3 + 3\alpha_1^2\alpha_0 \left(\frac{G'}{G}\right)^2 + 3\alpha_1\alpha_0^2 \left(\frac{G'}{G}\right) + \alpha_0^3, \tag{17}$$

$$u^2 = \alpha_1^2 \left(\frac{G'}{G}\right)^2 + 2\alpha_1\alpha_0 \left(\frac{G'}{G}\right) + \alpha_0^2. \tag{18}$$

By using (15) and (16) it is derived that

$$u'' = 2\alpha_1 \left(\frac{G'}{G}\right)^3 + 3\alpha_1\lambda \left(\frac{G'}{G}\right)^2 + (\alpha_1\lambda^2 + 2\alpha_1\mu) \left(\frac{G'}{G}\right) + \alpha_1\lambda\mu. \tag{19}$$

By substituting (16)–(19) into Eq. (13) and collecting all terms with the same power of $\left(\frac{G'}{G}\right)$ together, the left-hand side of Eq. (13) is converted into another polynomial in $\left(\frac{G'}{G}\right)$. Equating each coefficient of this polynomial to zero, yields a set of simultaneous algebraic equations for $\alpha_1, \alpha_0, w, \lambda, \mu$ and c as follows:

$$2\alpha_1 - \frac{1}{2}\alpha_1^3 = 0,$$
$$3\alpha_1\lambda - \frac{3}{2}\alpha_1^2\alpha_0 - \frac{3}{2}w\alpha_1^2 = 0,$$
$$\alpha_1\lambda^2 + \alpha_1\mu - \frac{3}{2}\alpha_1\alpha_0^2 - 3w\alpha_1\alpha_0 + (w^2 + c + 1)\alpha_1 = 0,$$
$$\alpha_1\lambda\mu - \frac{1}{2}\alpha_0^3 - \frac{3}{2}w\alpha_0^2 + (w^2 + c + 1)\alpha_0 + wc = 0. \tag{20}$$

On solving the above algebraic equations above by using the Maple package, we get

$$\alpha_1 = \pm 2. \tag{21}$$

If $\alpha_1 = 2$, then

$$\alpha_1 = 2, \quad \alpha_0 = -\lambda \mp \sqrt{3\lambda^2 + 4\mu + 2c + 2},$$
$$w = 2\lambda \pm \sqrt{3\lambda^2 + 4\mu + 2c + 2}. \tag{22}$$

By using (22), expression (16) can be written as

$$u(\xi) = 2\left(\frac{G'}{G}\right) - \lambda \mp \sqrt{3\lambda^2 + 4\mu + 2c + 2}, \tag{23}$$

where $\xi = x + y - (2\lambda \pm \sqrt{3\lambda^2 + 4\mu + 2c + 2})t$. Eq. (23) is the formula of a solution of Eq. (13), and by substituting Eq. (22) into Eq. (20) we obtain the integration constant c.

On solving the Eq. (15), we deduce after some reduction that

$$\frac{G'}{G}=\frac{1}{2}\sqrt{\lambda^2-4\mu}\times\left(\frac{C_1 sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2 cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1 cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2 sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}\right)-\frac{\lambda}{2},$$

where C_1 and C_2 are arbitrary constants. Substituting the general solutions of Eq. (15) into (23) we have three types of travelling wave solutions of the DDLW Eqs. (6) and (7) as follows:

When $\lambda^2-4\mu>0$

$$u(\xi)=\sqrt{\lambda^2-4\mu}\times\left(\frac{C_1 sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2 cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1 cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2 sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}\right)-\frac{3}{2}\lambda\pm\sqrt{3\lambda^2+4\mu+2c+2}, \tag{24}$$

where $\xi=x-(2\lambda\pm\sqrt{3\lambda^2+4\mu+2c+2})t$. C_1 and C_2, are arbitrary constants. And by substituting (24), into (11) we have solution of v.

In particular, if $C_1\neq 0, C_2=0, \lambda>0, \mu=0$, u become

$$u(\xi)=2\lambda tgh\frac{1}{2}\lambda\xi-\frac{3}{2}\lambda\pm\sqrt{3\lambda^2+4\mu+2c+2}.$$

When $\lambda^2-4\mu<0$

$$u(\xi)=\sqrt{4\mu-\lambda^2}\times\left(\frac{-C_1 sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2 cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1 cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2 sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}\right)-\frac{3}{2}\lambda\pm\sqrt{3\lambda^2+4\mu+2c+2}. \tag{25}$$

Also in this case we obtain v by substituting (25), into (11). When $\lambda^2-4\mu=0$

$$u(\xi)=\frac{2C_2}{C_1+C_2\xi},$$

where C_1 and C_2 are arbitrary constants.

And for $\alpha_1=-2$ we have

$$\alpha_0=\lambda\mp\sqrt{3\lambda^2+4\mu+2c+2}, w=-2\lambda\pm\sqrt{3\lambda^2+4\mu+2c+2}. \tag{26}$$

By using (26) we obtain three types of travelling wave solutions of the DDLW Eqs. (6) and (7) as follows:

When $\lambda^2-4\mu>0$

$$u(\xi)=-\sqrt{\lambda^2-4\mu}\times\left(\frac{C_1 sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2 cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1 cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2 sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}\right)+\frac{1}{2}\lambda\pm\sqrt{3\lambda^2+4\mu+2c+2}, \tag{27}$$

where $\xi=x-(-2\lambda\pm\sqrt{3\lambda^2+4\mu+2c+2})t$. C_1 and C_2, are arbitrary constants. And by substituting (24), into (11) we have solution of v.

In particular, if $C_1\neq 0, C_2=0, \lambda>0, \mu=0$, u become

$$u(\xi)=2\lambda tgh\frac{1}{2}\lambda\xi+\frac{1}{2}\lambda\pm\sqrt{3\lambda^2+4\mu+2c+2}.$$

When $\lambda^2-4\mu<0$

$$u(\xi)=-\sqrt{4\mu-\lambda^2}\times\left(\frac{-C_1 sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2 cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1 cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2 sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}\right)+\frac{1}{2}\lambda\pm\sqrt{3\lambda^2+4\mu+2c+2}. \tag{28}$$

Also in this case we obtain v by substituting (25), into (11). When $\lambda^2-4\mu=0$

$$u(\xi)=\frac{-2C_2}{C_1+C_2\xi},$$

where C_1 and C_2 are arbitrary constants.

4. Conclusion

In this article we have seen that three types of explicit travelling wave solutions of the (2 + 1)-dimensional dispersive long wave equation are successfully found out by using the $\frac{G'}{G}$-expansion method. The solutions of these nonlinear evolution equations have many potential applications in physics. These equations are very difficult to be solved by traditional methods. The performance of this method is reliable, simple and gives many new exact solutions.

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