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Development of some useful generators to obtain partially neighbor balanced designs

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ABSTRACT

Neighbor balanced designs are robust to neighbor effects, therefore, these designs are used to balance out the neighbor effects. If a large number of experimental material is required for combinatorial neighbor balance then partially neighbor balanced designs should be recommended. In this study, some useful generators are developed to obtain the partially neighbor balanced designs in linear blocks of sizes 3–7.

1. Introduction

If v treatments are applied in b blocks of size k ($k \leq v$) in such a manner that treatments occur next to each others once (i.e. $\lambda_1' = 1$) but at least one does not occur as neighbor (i.e. $\lambda_2 = 0$) then design is called partially neighbor balanced design (PNBD). PNBDs are always economical and useful for situation where some treatments have neighbor effects in opposite direction than that of others. Nutan (2007), Kedia and Misra (2008), Ahmed et al. (2009), Akhtar et al. (2010) and Zafaryab et al. (2010) generated some generalized neighbor designs in circular blocks. Partially neighbor balanced designs are constructed for some cases by Ahmed and Akhtar (2012) but this construction was in circular blocks. Some more references for the construction of neighbor designs are discussed by Ahmed et al. (2011). Hamad (2014) generated infinite series of one-dimensional partially neighbor balanced designs for v = n treatments in circular blocks. Hamad and Hanif (2016) constructed two new series of non-binary partially neighbor balanced designs for $v = 2n$ and $v = 2n + 1$ in non-binary circular blocks where no treatment is ever a neighbor to itself. They described that neighbor balanced designs become un-necessary when neighbor effect between any two particular treatments is not important or very costly or has known adverse effect. For example sometimes in agriculture experiment, such response variable is required when certain treatments are repeated side by side more than others. Under such conditions, we prefer partially neighbor balanced designs.

Applications of neighbor designs can be observed in biological, agricultural, agro-forestry and horticulture experiments. Particularly, (i) In plant breeding experiments, neighbor effects may be caused by differences in height, root vigor, or germinating date, especially on small plots; (ii) in crop variety trials where inter-plot neighbors usually affects only the outer plants of plots (units) (Kempton and Lockwood, 1984) (iii) Pearce (1957) described the fruit trees experiments where a treatment allotted to a branch (unit) affected the response of all other branches on the same tree (block) (iv) In serology, every antigen from a set of antigens has other two antigens as neighbors (v) Welham et al. (1996) explored the plants, such as potatoes, that obtain nutrients from neighboring plots on both sides.

If λ' takes only two values, λ_1' and $\lambda_1' - 1$ then partially-balanced designs are also called weakly balanced designs. Bailey et al. (2017) constructed universally optimal WBRMDs for $p = v$. In this paper, some useful generators to obtain PNBDs are developed in linear blocks for $p < v$. A block formed in a line where its first and last units

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are not considered as neighbors is called linear block. These are constructed under the model

$$\boldsymbol{y} = \mathbf{X_0}\mu + \mathbf{X_1}\boldsymbol{\tau} + \mathbf{X_2}\boldsymbol{\alpha} + \mathbf{X_3}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

where **y** and $\mathbf{X_0}$ are vectors of observations and 1's each of order (bk × 1) respectively, $\mathbf{X_1}$ and $\mathbf{X_2}$ are incidence matrices of order (bk × v) for treatment and neighbor effects respectively, and $\mathbf{X_3}$ is the (bk × b) incidence matrix for block effects, where k is equal block size. The vector error term ε is identical and independent and has normal distribution with mean vector 0 and variance-covariance matrix Σ. In this article, PNBDs are constructed in which treatment having opposite neighbor effect does not occur next to each other, therefore, no need for incorporating opposite neighbor effect in the model. Proposed designs are constructed in linear blocks using method of cyclic shifts which is explained briefly in Section 2. In Section 3, some useful generators are developed to obtain the PNBDs in linear blocks of sizes 3–7.

2. Method of cyclic shifts

Here we explain the method of cyclic shifts only for partially neighbor balanced designs in linear blocks. For further detail see Ahmed and Akhtar (2011).

Let S = $[q_1, q_2, \ldots, q_{k-1}]$ be a set of shifts, where $1 \le q_i \le v-1$. Then for linear blocks S* be a set which consists each element of S along with their complements. Here, complement of q_i is $v - q_i$.

(a) *Condition of partially neighbor balance:* S* consists each of 1, 2, …, $(v-1)$ either 0 or one time.
(b) *Condition of binary design:* Sum of any two, three, …, and (k − 1) consecutive elements of S is not zero (mod v).

Method is illustrated here with the help of an example.

Example 2.1: Following PNBD in which one pair (2, 6) does not appear as neighbor for v = 7 in linear blocks of size three is constructed through the set of shifts [1, 2] + [3, 3](3/7).

Here S_1 = [1, 2] generate v = 7 blocks. 0, 1, …, 6 be the 1st unit of each block respectively. Add q_1 = 1 to each of 1st unit modulo 7 to get the second unit of each block. Similarly get the 3rd unit by adding q_2 = 2. Then we may get seven more blocks through [3, 3] but we will take only first three of these seven due to (3/7) of send set of shift. Hence we obtain the desire design in 10 blocks of size 3.

B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9	B_{10}
0	1	2	3	4	5	6	0	1	2
1	2	3	4	5	6	0	3	4	5
3	4	5	6	0	1	2	6	0	1

3. Generators to obtain PNBDs

Generator 3.1. Following $(v+3)/4$ sets of shifts generate PNBD for $v = 4i+1$ in linear blocks of size 3. In this design, one pair $((v-3)/2,\ v-1)$ does not appear as neighbor while all other pairs of treatments appear once as neighbors.

$S_{l+1} = [2l+1, 2l+2]$; $l = 0, 1, \ldots i-1$.
$S_{(v+3)/4} = [(v\text{-}3)/2, (v\text{-}1)/2]\ ((v\text{-}1)/v)$

Example 3.1: PNBD is constructed for v = 19 in linear blocks of size three through the following sets of shifts. Here, one pair (8, 18) does not appear as neighbors.

[1, 2] + [3, 4] + [5, 6] + [7, 8] + [9, 9](9/19).

Generator 3.2. Following $(v+1)/4$ sets of shifts generate PNBD for $v = 4i+3$ in linear blocks of size 3. In this design, one treatment $(\nu-5)/2$ does not appear with $(v-1)$ and $(v-3)$ as neighbor while all other pairs of treatments appear once as neighbors.

$S_{l+1} = [2l+1, 2l+2]$; $l = 0, 1, \ldots, i-1$.
$S_{(v+3)/4} = [(\nu-1)/2, (\nu-1)/2]\ ((\nu-1)/2\nu)$

Example 3.2: PNBD is constructed for v = 17 in linear blocks of size three through the following sets of shifts. Here, two pairs (6, 16) and (6, 14) does not appear as neighbors.

[1, 2] + [3, 4] + [5, 6] + [7, 8] (16/17).

Generator 3.3. Following $(v-2)/4$ sets of shifts generate PNBD for $v = 4i+2$ in linear blocks of size 3. In this design, $v/2$ pairs $(0, v/2), (1, (v+2)/2), \ldots, ((v-2)/2,\ v-1)$ do not appear as neighbors while all other pairs of treatments appear once as neighbors.

$S_{l+1} = [2l+1, 2l+2]$; $l = 0, 1, \ldots, i-1$.

Example 3.3: PNBD is constructed for v = 18 in linear blocks of size three through the sets of shifts. Here, the pairs (0, 9), (1, 10), (2, 11), (3, 12), (4, 13), (5, 14), (6, 15), (7, 16), (8, 17) do not appear as neighbors.

[1, 2] + [3, 4] + [5, 6] + [7, 8].

Generator 3.4. Following $v/4$ sets of shifts generate PNBD for $v = 4i$ in linear blocks of size 3. In this design, $2i$ pairs $(v/2,\ v-1)$ and $(0+j,\ j+(v+2)/2)$; $j = 0,\ 1, \ldots, 2i-2$ do not appear as neighbors.

$S_{l+1} = [2l+1, 2l+2]$; $l = 0, 1, \ldots, i-1$. $S_{v/4} = [(v-2)/2, v/2](1/2)$

Example 3.4: PNBD is constructed for v = 16 in linear blocks of size three through the following sets of shifts. Here, the pairs (8, 15),(0, 9),(1, 10),(2, 11),(3, 12),(4, 13),(5, 14),(6, 15) do not appear as neighbors while all others appear once.

[1, 2] + [3, 4] + [5, 6] + [7, 8](1/2).

Generator 3.5. Following $(v+9)/6$ sets of shifts generate PNBD for $v = 6i+3$ in linear blocks of size 4. In this design, three pairs $((v-5)/2,\ v-3)$, $(v-5)/2,\ v-2)$ & $(v-3)/2,\ v-2)$ do not appear as neighbors.

$S_{l+1} = [3l+1, 3l+2, 3l+3]$; $l = 0, 1, \ldots, i-1$.
$S_{(v+3)/6} = [(v\text{-}1)/2, (v\text{-}1)/2, (v\text{-}1)/2](1/3)$ Every third (1st, 4th, 7th, …)
$S_{(v+9)/6} = [(v+1)/2, (v+1)/2, (v+1)/2](1/3)$ Every third (1st, 4th, 7th, …)

Example 3.5: PNBD is constructed for v = 21 in linear blocks of size four through the following sets of shifts. Here, The pairs (8, 18), (8, 19), (9, 19) do not appear as neighbor while all other pairs appear once.

[1, 2, 3] + [4, 5, 6] + [7, 8, 9] + [10, 10, 10](3/21) + [11, 11, 11](3/21).

Generator 3.6. Following $(v-2)/6$ sets of shifts generate PNBD for $v = 6i+2$ in linear blocks of size 4. In this design, $v/2$ pairs $(0,\ v/2), (1,\ (v+2)/2), \ldots, ((v-2)/2,\ v-1)$ do not appear as neighbors.

$S_{l+1} = [3l+1, 3l+2, 3l+3]$; $l = 0, 1, \ldots, i-1$.

Example 3.6: PNBD is constructed for v = 26 in linear blocks of size four through the following sets of shifts. Here, the pairs (0, 13), (1, 14), (2, 15), (3, 16), (4, 17), (5, 18), (6, 19), (7, 20), (8, 21), (9, 22), (10, 23), (11, 24), (12, 25) do not appear as neighbors while all other pairs appear once.

[1, 2, 3] + [4, 5, 6] + [7, 8, 9] + [10, 11, 12].

Generator 3.7. Following $(v-1)/6$ sets of shifts generate PNBD for $v = 6i+1$ in linear blocks of size 4. In this design, three pairs $(3i-3,\ 6i)$, $(3i-3,\ 6i-4)$ & $(3i-5,\ 6i-4)$ do not appear as neighbors.

$S_{l+1} = [3l+1, 3l+2, 3l+3]$; $l = 0, 1, \ldots, i-1$.
$S_{(v-1)/6} = [(v-5)/2, (v-3)/2, (v-1)/2]\ ((v-1)/v)$

Example 3.7: PNBD is constructed for v = 25 in linear blocks of size four through the following sets of shifts, Here, the pairs (9, 24), (9, 20), (7, 20) do not appear as neighbors.

[1, 2, 3] + [4, 5, 6] + [7, 8, 9] + [10, 11, 12](24/25).

Generator 3.8. Following $(v+2)/6$ sets of shifts generate PNBD for $v = 6i+4$ in linear blocks of size 4. In this design, three pairs $(3i-1,\ 6i+1)$, $(3i-1,\ 6i+2)$ & $(3i,\ 6i+3)$ do not appear as neighbors.

$S_{l+1} = [3l + 1, 3l + 2, 3l + 3]$; $l = 0, 1, \ldots, i - 1$.

$S_{(v+2)/6} = [(v\text{-}2)/2, (v\text{-}2)/2, v/2]$ $((v\text{-}2)/2v)$

Example 3.8: PNBD is constructed for $v = 28$ in linear blocks of size four through the following sets of shifts. Here, the pairs (11, 25), (12, 26), (12, 27) do not appear as neighbors.

[1, 2, 3] + [4, 5, 6] + [7, 8, 9] + [10, 11, 12] + [13, 13, 14](13/28).

Generator 3.9. Following $(v - 2)/8$ sets of shifts generate PNBD for $v = 8i + 2$ in linear blocks of size 5. In this design, $v/2$ pairs $(0, v/2)$, $(1, (v + 2)/2)$, …, $((v - 2)/2, v - 1)$ do not appear as neighbors.

$S_{l+1} = [4l + 1, v - (4l + 2), 4l + 3, 4l + 4]$; $l = 0, 1, \ldots, i - 1$.

Example 3.9: PNBD is constructed for $v = 34$ in linear blocks of size five through the following sets of shifts. Here, the pairs (0, 17), (1, 18), (2, 19), (3, 20), (4, 21), (5, 22), (6, 23), (7, 24), (8, 25), (9, 26), (10, 27), (11, 28), (12, 29), (13, 30), (14, 31), (15, 32), (16, 33) do not appear as neighbors.

[1, 32, 3, 4] + [5, 28, 7, 8] + [9, 24, 11, 12] + [13, 20, 15, 16].

Generator 3.10. Following $(v - 1)/8$ sets of shifts generate PNBD for $v = 8i + 1$ in linear blocks of size 5. In this design, four pairs $(4i - 4, v - 2)$, $(4i - 4, v - 1)$, $(4i - 3, v\text{-}2)$ and $(4i - 3, v - 4)$ do not appear as neighbors.

$S_{l+1} = [4l + 1, v - (4l + 2), 4l + 3, 4l + 4]$; $l = 0, 1, \ldots, i - 1$.

$S_{(v-1)/8} = [(v - 7)/2, (v + 5)/2, (v - 3)/2, (v - 1)/2]$ $((v - 1)/v)$

Example 3.10: PNBD is constructed for $v = 33$ in linear blocks of size five through the following sets of shifts. Here, the pairs (12, 31), (12, 32), (13, 31), (13, 29) do not appear as neighbors.

[1, 31, 3, 4] + [5, 27, 7, 8] + [9, 23, 11, 12] + [13, 19, 15, 16] (32/33)

Generator 3.11. Following $(v - 2)/10$ sets of shifts generate PNBD for $v = 10i + 2$ in linear blocks of size 6. In this design, $v/2$ pairs $(0, v/2)$, $(1, (v + 2)/2)$, …, $((v - 2)/2, v - 1)$ do not appear as neighbors.

$S_{l+1} = [5l + 1, v - (5l + 2), 5l + 3, v - (5l + 4), 5l + 5]$; $l = 0, 1, \ldots, i - 1$.

Example 3.11: PNBD is constructed for $v = 32$ in linear blocks of size six through the following sets of shifts. Here, the pairs (0, 16), (1, 17), (2, 18), (3, 19), (4, 20), (5, 21), (6, 22), (7, 23), (8, 24), (9, 25), (10, 26), (11, 27), (12, 28), (13, 29), (14, 30), (15, 31) do not appear as neighbors.

[1, 30, 3, 28, 5] + [6, 25, 8, 23, 10] + [11, 20, 13, 18, 15].

Generator 3.12. Following $(v - 1)/10$ sets of shifts generate PNBD for $v = 10i + 1$ in linear blocks of size 6. In this design, five pairs $(5i - 5, v - 2)$, $(5i - 5, v - 1)$, $(5i - 4, v - 3)$, $(5i - 4, v - 2)$ & $(5i - 3, v - 3)$ do not appear as neighbors.

$S_{l+1} = [5l + 1, v - (5l + 2), 5l + 3, v - (5l + 4), 5l + 5]$; $l = 0, 1, \ldots, i - 1$.

$S_{(v-1)/10} = [(v - 9)/2, (v + 7)/2, (v - 5)/2, (v + 3)/2, (v - 1)/2]$ $((v - 1)/v)$

Example 3.12: PNBD is constructed for $v = 31$ in linear blocks of size six through the following sets of shifts. Here, the pairs (10, 29), (10, 30), (11, 28), (11, 29), (12, 28) do not appear as neighbors.

[1, 29, 3, 27, 5] + [6, 24, 8, 22, 10] + [11, 19, 13, 17, 15](30/31).

Generator 3.13. Following $(v - 2)/12$ sets of shifts generate PNBD for $v = 12i + 1$ in linear blocks of size 7. In this design, $v/2$ pairs $(0, v/2)$, $(1, (v + 2)/2)$, …, $((v - 2)/2, v - 1)$ do not appear as neighbors.

$S_{l+1} = [6l + 1, v - (6l + 2), 6l + 3, v - (6l + 4), 6l + 5, 6l + 6]$; $l = 0, 1, \ldots, i - 1$.

Example 3.13: PNBD is constructed for $v = 38$ in linear blocks of size seven through the following sets of shifts. Here, the pairs (0, 19), (1, 20), (2, 21), (3, 22), (4, 23), (5, 24), (6, 25), (7, 26), (8, 27), (9, 28), (10, 29), (11, 30), (12, 31), (13, 32), (14, 33), (15, 34), (16, 35), (17, 36), (18, 37) do not appear as neighbors.

[1, 36, 3, 34, 5, 6] + [7, 30, 9, 28, 11, 12] + [13, 24, 15, 22, 17, 18].

Generator 3.14. Following $(v - 1)/12$ sets of shifts generate PNBD for $v = 12i + 2$ in linear blocks of size 7. In this design, six pairs $(6i - 6, v - 2)$, $(6i - 6, v - 1)$, $(6i - 5, v - 3)$, $(6i - 5, v - 2)$, $(6i - 4, v - 5)$ & $(6i - 4, v - 3)$ do not appear as neighbors.

$S_{l+1} = [6l + 1, v - (6l + 2), 6l + 3, v - (6l + 4), 6l + 5, 6l + 6]$; $l = 0, 1, \ldots, i - 1$.

$S_i = [(v - 11)/2, (v + 9)/2, (v - 7)/2, (v + 5)/2, (v - 3)/2, (v - 1)/2]$ $((v - 1)/v)$

Example 3.14: PNBD is constructed for $v = 37$ in linear blocks of size seven through the following sets of shifts. Here, **t**he pairs (12, 35), (12, 36), (13, 34), (13, 35), (14, 32), (14, 34) do not appear as neighbors.

[1, 35, 3, 33, 5, 6] + [7, 29, 9, 27, 11, 12] + [13, 23, 15, 21, 17, 18] (36/37).

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